Adaptive Companding as a PAPR Reduction Technique of an OFDM Signal

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Abstract—**In this paper, a new adaptive PAPR reduction scheme is proposed by which the PAPR of the OFDM signal can be reduced to keep the value lower than a predefined threshold level. A conventional companding is used as a technique to reduce the PAPR of the OFDM signal. If the PAPR of the OFDM signal is higher than any predefined level, one or more times companding operations are performed until the PAPR becomes lower than that level. On top of that, no companding is applied if the signal's PAPR is lower than the predefined level. However, there is a trade-off between the improvement of the PAPR and BER which may be optimized based on any specific application requirements**

Index Terms—**Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Bit Error Rate (BER), Complementary Cumulative Distribution Function (CCDF), Adaptive Companding**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a gracious Multi Carrier Modulation (MCM) technique that is able to provide robustness in frequency selective fading channels, high spectral efficiency, immunity to inter-symbol interference and can combat very strong multipath fading. The main feature of OFDM system which differentiates this system from other MCM techniques is its characteristic of orthogonality. This means, the split subcarriers of the total bandwidth maintain the rule of orthogonality by placing the subcarriers in such a way that the spacing between those is equal to the reciprocal of the useful symbol period [1]. OFDM can be implemented efficiently by using fast Fourier transforms (IFFTs and FFTs) at the transmitter and receiver ends, respectively. Because of its advantages and easy implementation OFDM has been applied into digital audio broadcasting (DAB), digital video broadcasting terrestrial (DVB-T), wireless local area network (WLAN), broadband wireless access (BWA) among others [2].

Because of the time domain addition of many narrowband orthogonal signals, sometimes the cumulative signal becomes very large in magnitude and sometimes become low. As a result of this fluctuation in magnitude the peak of the signal becomes very large than the average signal. This creates the high PAPR problem. When The OFDM signal passes through high power amplifier (HPA), it increases the dynamic range of digital-to-analog converter (DAC) and creates the problem of spectral broadening, which results in high cost and reduced efficiency [3]. Because of the spectral broadening, in-band distortion and out-of-band radiation occurs. Performance degradation in system and adjacent channel interference (ACI) that affects systems working in neighbor band are the eventual effects of these two problems. As a result of the whole problem bit error rate (BER) performance degrades.

To overcome this severe problem of OFDM system, many techniques have been proposed so far. Some techniques are designed based on employing redundancy; some are achieved by using extended signal constellation and some uses external coded sequence and interleaving. Coding [4], selective mapping [5, 6], tone reservation [1, 7], tone injection [8], multi-amplitude Continuous Phase Modulation (CPM) [9], interleaving [10] are some of the examples of the cited types. The associated drawbacks of these techniques are reduction in transmission rates [4], increased power [7], implementation complexity [9] etc. But the simple PAPR reduction methods can be achieved by clipping and filtering [11] or companding [12, 13]. However, use of the clipping technique causes both in-band and out-of-band distortion which results in an increased bit error rate (BER) in the system [14]. As an alternative approach, the companding technique shows better performance than the clipping technique, because the inverse companding transform (expanding) can be applied at the receiving end to reduce the distortion of signal.

A proposal of new companding based PAPR reduction scheme called adaptive companding is elaborated in this paper. Due to its ability of adjustment with the required number of companding to achieve a certain level of PAPR, this technique is named as adaptive companding. In each step of this technique, the calculated PAPR is compared to a predefined PAPR level and goes for companding if it is not lower than or equal to the mentioned level.

II. THEORITICAL MODELING

A. A Generic OFDM System & PAPR

In an OFDM system, N symbols are divided into a set of orthogonal subcarriers $\boldsymbol{X} = X_k \{k = 0, 1, 2 \ldots., N - 1\}$. Each of the subcarriers is modulated using M-PSK or

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M-QAM and transmitted independently over their assigned system bandwidth. To maintain orthogonality, the subcarrier frequencies are equally spaced with $fk = k\Delta f$, where $\Delta f \equiv 1/NTs$. Ts is the symbol duration.The resulting complex baseband OFDM signal can be expressed as [15, 16],

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t} \quad 0 \le t \le NT_s \tag{1}$$

The Ratio of the maximum instantaneous power and the average power of the signal $x(t)$ is the PAPR of that OFDM system. In dB it can be written as [15, 16],

$$\text{PAPR} = \frac{\max_{0 \le t \le NT_S}[|x(t)^2|]}{\frac{1}{NT_S}\int_0^{NT_S} |x(t)^2| dt} \tag{2}$$

Since, IFFT is a linear operation; the transmitted signal $x(t)$ follows a complex Gaussian distribution. Large number of N results to a high PAPR. Reducing $\max|x(t)|$ is the principle goal of any companding technique

OFDM signal needed to be oversampled by a factor L for better approximation. The discrete time OFDM signal after oversampling the signal of eq. 1 with a sampling frequency $f_s = L/T$ can be written as [15,16],

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{NL}kn} \quad 0 \le t \le NL - 1 \tag{3}$$

To implement eq.2 on this new expression of discrete OFDM signal, an IFFT operation of length NL is needed. The changed input vector is obtained by padding $(L-1)N$ number of zeros in the middle of vector, X. The new PAPR can be expressed as [15,16],

$$\text{PAPR}[x(n)]_{\text{dB}} = \frac{\max_{0 \le t \le NL-1}[|x(n)^2|]}{E[|x(n)^2|]} \tag{4}$$

Where, $E[.]$ is the expectation value of the signal and it is taken on a frame of an OFDM signal.

B. The CCDF of PAPR

The performance of PAPR can be evaluated with Cumulative Distribution Function (CDF) of OFDM system; the most frequently used criteria of measuring the efficiency of PAPR. The CDF can be expressed as [4],

$$F(\lambda) = 1 - e^{\lambda} \tag{5}$$

In practical use, the Complementary CDF (CCDF) is used instead of CDF which denotes the PAPR of a certain data block exceeds a given threshold. For a large number of subcarriers (N) the CCDF of the PAPR can be expressed as [4],

$$Pr(PAPR > \lambda) = 1 - \left(F(\lambda)\right)^N$$
$$= 1 - (1 - e^{-\lambda})^N \tag{6}$$

C. Conventional Companding System

Companding is an algorithm to reduce the dynamic range of a signal by increasing the amplitudes of the smaller signals. The average signal power is increased with this technique. PAPR is reduced to an extent if dynamic range is reduced.

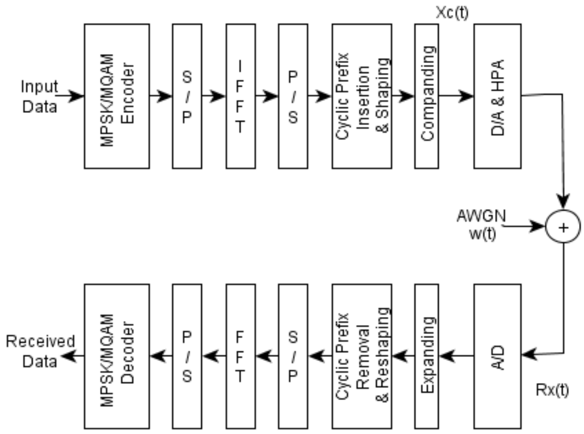

Figure 1. A baseband OFDM system with conventional companding scheme

Fig.1 depicts the block diagram of a baseband OFDM system which utilizes the conventional companding system [16]. Additive White Gaussian Noise (AWGN) channel is used as the noise channel. The μ-law companding is basically used as the non-linear companding technique. The value of μ is varied for different level of compression.

In the case of μ-law companding, for a selected μ the baseband signal can be compressed as follows [16,17],

$$x_c(n) = C\{x(n)\} = sgn[x(n)] \frac{ln\left[1+\mu\frac{|x(n)|}{x_{max}}\right]}{ln[1+\mu]}\ x_{max} \tag{7}$$

Where, sgn(.) is the signum function and x_{max} is the peak value of the signal.

In the receiving end, the receiver receives the companded signal with addition of noise. The received signal can be expressed as [18],

$$R_x(n) = X_c(n) + w(n) \tag{8}$$

Where, w (t) is the AWGN signal.

The received signal is then decompanded using companding transform. The formula which is used usually for decompanding is as follows [17, 18],

$$X_r = sgn[R_x(n)] \left[\frac{1}{\mu}\left\{(1+\mu)^{\frac{R_x(n)}{x_{max}}} - 1\right\}\right] x_{max} \tag{9}$$

Where, x_{max} is the peak value of the transmitted signal.

The expanded signal is then demodulated to reproduce the original information. The BER is calculated from the comparison of original signal and reconstructed signal.

Fig. 2 shows the time domain output of a baseband OFDM signal and compressed OFDM signal with μ-law companding where μ=255 [19].

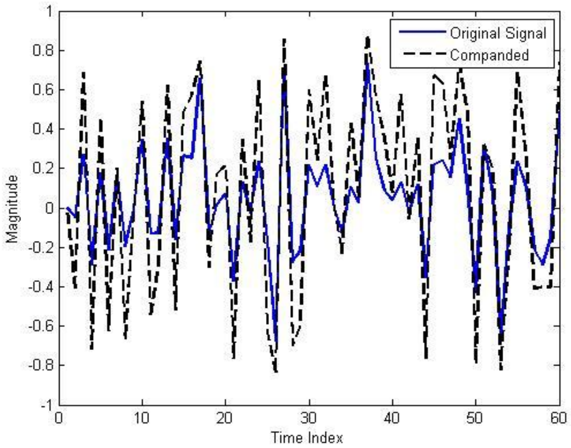

Figure 2. Time domain output of uncompanded and companded envelope of a baseband OFDM signal

III. Proposed Adaptive Companding Technique

To reduce the PAPR of an OFDM system a new technique based on conventional companding is proposed in this paper. This technique gives the opportunity to keep the PAPR to an adequate level by doing the task of multiple companding which is illustrated in fig. 3.

The key signal processing steps are described as below:

Step 1: An IFFT is applied on the vector X yielding:

$$x = [x(1), x(2), \ldots\ldots, x(N)]^{T_s}$$

Step 2: PAPR of the signal $x(n)$ which is $PAPR_1$, is calculated using eq.4.

Step 3: A threshold value of PAPR based on application requirement is set for comparison.

i.e. for level 2 the value of the threshold is set at $PAPR_{th} = 2dB$

Step 4: $PAPR_1$ is compared with $PAPR_{th}$.

Step 5: If $PAPR_1 \leq PAPR_{th}$, then the original signal is transmitted.

i.e. Transmitted signal, $t_r(n) = x(n)$

Step 6: A side information is transmitted which indicates how many times the companding are performed.

i.e. When $t_r(n) = x(n)$, side information, $s = 0$

Step 7: If $PAPR_1 > PAPR_{th}$, A companding transform is then applied to $x(n)$.

i.e. $x_1(n) = C\{x(n)\}$

Step 8: Step 2 & step 4 are repeated for $x_1(n)$. Here the calculated PAPR for $x_1(n)$ is $PAPR_2$.

Step 9: If $PAPR_2 \leq PAPR_{th}$ then $t_r(n) = x_1(n)$, $s = 1$

Step 10: If $PAPR_2 > PAPR_{th}$, Another companding transform is applied to $x_1(n)$.

i.e. $x_2(n) = C\{x_1(n)\}$

Step 11: This whole procedure is repeated unless$PAPR_m \leq PAPR_{th}$. Where m=1,2,3,….

Step 12: From the side information receiver understands whether it is original, companded, double companded or multi-companded which helps it to decompand the signal accurately. For decompression eq. 9 is applied.

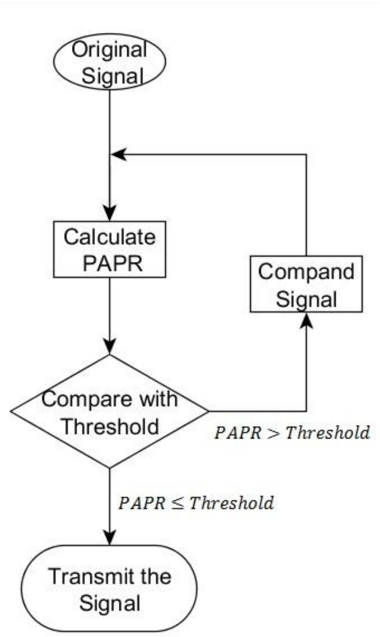

Figure 3. Logical diagram for adaptive companding technique

IV. Simulation Results and Discussion

To show the PAPR reduction capacity and BER performance of the adaptive companding technique, an OFDM system using 64 subcarriers and 16 QAM modulation scheme, based on randomly generated data is considered. CCDF (Pr [PAPR>PAPR0]), is used to present the range of PAPR in term of a probability of occurrence. Also BER performance is evaluated to analyze the system degradation.

By applying this technique, it is possible to keep PAPR as the demanded level. Fig. 4 shows CCDF plot of the different level of demanded PAPR. Level 2, 3 and 4 are representing the 2 dB, 3dB and 4 dB PAPR respectively.

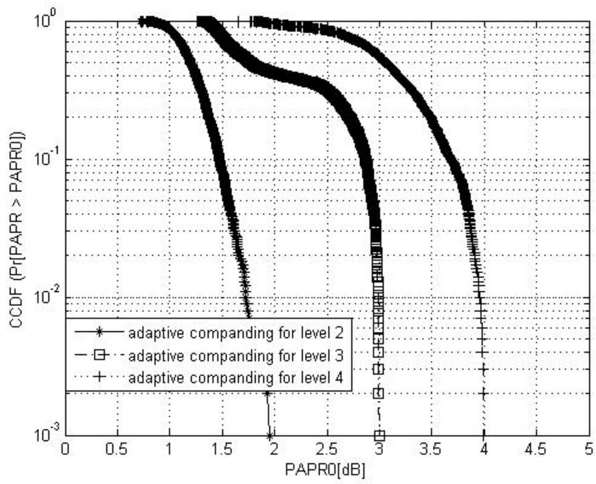

Figure 4. CCDF plot of adaptive companding for different levels

The corresponding BER performance is shown in fig. 5.It can be seen from this figure, to keep the BER at 10^{-4}, the required SNRs are approximately 26dB, 27dB and 30dB for adaptive companding level 4, 3 and 2 respectively.

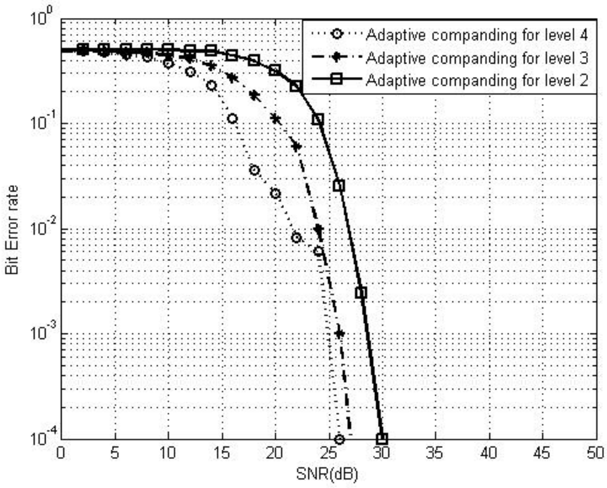

Figure 5. BER performance plot for different PAPR levels of adaptive companding.

The result in Fig. 6 is showing the performances of PAPR of an OFDM signal without companding, using conventional companding scheme, double companding scheme and adaptive companding scheme with level 3. At CCDF= 10^{-3}, the uncompanded envelope's PAPR is at near 11dB. Whereas, for single companding; PAPR can be kept near 5dB, for double companding it is near 2.2 dB and for adaptive companding it is 3dB. Hence, for adaptive companding method it is possible to vary the PAPR as demanded.

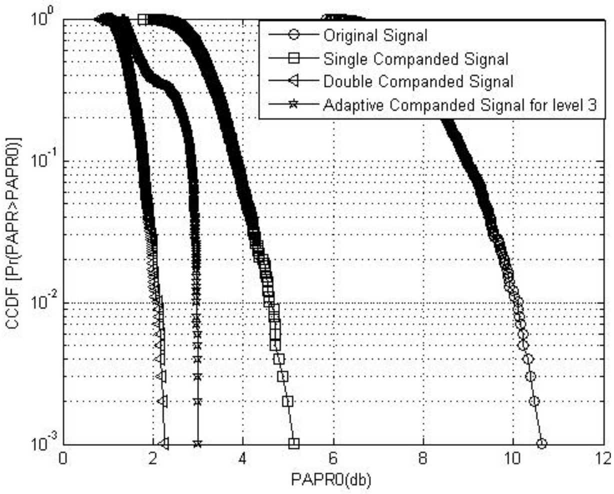

Figure 6. Comparison of the CCDF plots for different companding schemes

Fig.7 shows the Bit Error Rate (BER) Vs Sigal-to- Noise Ratio (SNR) under different companding schemes. Comparing to the single companding scheme, the double companding scheme offers much higher BER. Whereas, the adaptive companding offers the intermediate value. To keep the value of BER at $10^{-3}$the required SNRs are approximately 15dB, 20dB, 25dB and 29dB for original signal, single companded signal, double companded signal and adaptive companded for level 3 signal respectively. This indicates about the occurrence of a tradeoff between PAPR reduction and BER performance. More PAPR reduction gives high bit error rate. In that case the system should be optimized based on application requirements.

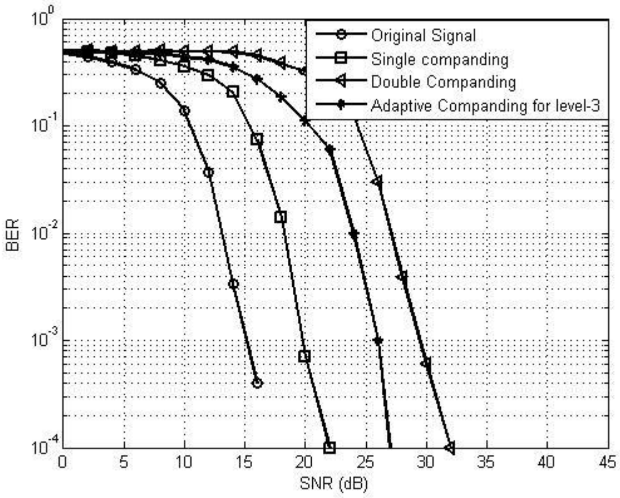

Figure 7. BER plot of different Companding schemes

V. CONCLUSION

In this paper, a promising technique for PAPR reduction has been proposed and numerically analyzed. The main attraction of this technique is to keep the PAPR at a level which is lower than or equal to the pre-specified PAPR level, by performing multiple companding. However, multiple companding may not be required if the signal is already within the level of PAPR. The proposed technique is based on conventional companding scheme and the implementation complexity is relatively simple than the other PAPR reduction techniques. The performance analysis of the scheme shows a trade-off relation between the CCDF and the BER performance. This indicates, improvement in PAPR reduction degrades the BER performance and vice versa.

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