MOTION PARAMETER ESTIMATION OF MULTIPLE TARGETS IN MULTISTATIC PASSIVE RADAR THROUGH SPARSE SIGNAL RECOVERY

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ABSTRACT

The problem of estimating motion parameters of multiple closely located ground moving targets in a multistatic passive radar system is considered, with a focus on weak signal conditions. The proposed method provides a means of combining signal energy from all available, spatially separated, illuminators of opportunity to achieve multistatic diversity and overall signal enhancement. The proposed technique is based on sparse signal recovery and exploits a two-step process that sequentially estimates the acceleration and velocity vectors in order to reduce the dimensionality of parameter search space.

I. INTRODUCTION

Motion parameter estimation of ground moving targets has been an area of interest in radar signal processing. While extensive literature is available for motion parameter estimation in conventional radar systems, limited work has been done in multistatic passive radar (MPR) systems (e.g., [1], [2]). Motion parameter estimation in MPR systems differs from conventional radar systems because they operate at low signal-to-noise ratio (SNR) and narrowband conditions. These create additional challenges for target detection, localization, and tracking in MPR systems [3].

Existing motion parameter estimation techniques (e.g., [3], [4], [5]) are based on the time-frequency analysis of the Doppler signatures of the received signals, which are commonly modeled as second-order polynomials for a moderately long coherent processing interval (CPI). In low SNR situations as encountered in typical MPR systems, it is desirable to combine the data from all available illuminators of opportunity and, thereby, yielding overall signal enhancement and multistatic diversity. When multiple moving targets fall in the same range cells, time-frequency based techniques may become inconvenient as they require association of the target Doppler signatures. Application of sparse signal reconstruction based methods has been explored in some recent works (e.g., [6], [7]) for effectively combining radar data from multiple bistatic links.

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Techniques used for estimating motion parameters of closely spaced multiple ground moving targets in a typical MPR systems suffer from the following:

1) Despite the availability of multiple transmitters in an MPR system, it is rather difficult to combine the Doppler signatures corresponding to different bistatic links directly in the time-frequency domain.
2) Although a longer CPI can be used to enhance the SNR corresponding to each bistatic link, target range migration due to higher-order motion parameters (e.g., acceleration and jerk) emerges as a critical issue, and is difficult to compensate for.
3) Any method based on an exhaustive search becomes computationally inefficient for multiple target scenarios as they require a multi-dimensional search, thus exponentially increasing the computational complexity.

In this paper, we develop a new motion parameter estimation technique based on sparse signal reconstruction which enables us to effectively fuse the data corresponding to all available illuminators. This enables the accumulation of sufficient signal energy without further extending the CPI. In order to reduce the computational load associated with motion parameter estimation of multiple closely located targets, we propose a sequential process which estimates the acceleration and velocity in tandem. Simulation results are provided to demonstrate the applicability of the proposed method in estimating motion parameters of multiple closely-located ground moving targets in low SNR conditions.

The following notations are used in this paper. A lower (upper) case bold letter denotes a vector (matrix). $(.)^*$ and $(.)^T$ respectively denote complex conjugation and transpose operations. $\|\cdot\|_1$ and $\|\cdot\|_2$ respectively denote the l_1 and l_2 norm of a vector.

II. SIGNAL MODEL

A. Geographical relationship

We consider the problem of estimating motion parameters of multiple closely located ground moving targets in a standard MPR system. We assume that the MPR system operates in a multiple-frequency network, such that N broadcast stations located at known stationary positions $\mathbf{b}^{(i)}$, $i = 1, ..., N$, transmit waveforms in non-overlapping frequency bands which are respectively centered at $f^{(i)}$, $i = 1, ..., N$.

An airborne receiver, initially located at $\mathbf{r}_0$, is assumed to be moving along its track direction with a uniform velocity $\mathbf{v}_r$, whereas there are K closely located ground moving targets. The kth target is assumed to be initially located at $\mathbf{p}_0^{(k)}$, moving with an initial velocity of $\mathbf{v}^{(k)}$ and an acceleration of $\mathbf{a}^{(k)}$. Because only ground targets are considered, the z-axis components of $\mathbf{p}_0^{(k)}$, $\mathbf{v}_0^{(k)}$, and $\mathbf{a}^{(k)}$ are assumed to be 0.

The direct range between the ith illuminator and the receiver is expressed as

$$r^{(i)}(t) = \|\mathbf{r}(t) - \mathbf{b}^{(i)}\|_2, \tag{1}$$

where $\mathbf{r}(t)$ represents the trajectory of the receiver at time t. Likewise, the bistatic range between the ith transmitter, the kth target, and the receiver is expressed as

$$\rho^{(i,k)}(t) = \|\mathbf{p}^{(k)}(t) - \mathbf{b}^{(i)}\|_2 + \|\mathbf{p}^{(k)}(t) - \mathbf{r}(t)\|_2, \tag{2}$$

where $\mathbf{p}^{(k)}(t) = \mathbf{p}_0^{(k)} + \mathbf{v}_0^{(k)}t + \frac{1}{2}\mathbf{a}^{(k)}t^2$ represents the trajectory of the kth target at time t.

The direct path signal received from the ith transmitter can be expressed as

$$s_{\mathrm{r}}^{(i)}(t) = u^{(i)}(t - r^{(i)}(t)/c) \exp\left(-j2\pi f^{(i)} r^{(i)}(t)/c\right), \tag{3}$$

where $u^{(i)}(t)$ is the baseband representation of the signal transmitted from the ith illuminator and c is the velocity of wave propagation. We assume that the transmitted signal is perfectly reconstructed at the receiver after demodulation and forward error correction.

The surveillance signal reflected from the kth target, on the other hand, is given for the ith illuminator by,

$$\begin{aligned} s_{\mathrm{s}}^{(i,k)}(t) = {} & \sigma^{(i,k)} u^{(i)}(t - \rho^{(i,k)}(t)/c) \exp\left(-j2\pi f^{(i)} \rho^{(i,k)}(t)/c\right) \\ & + n_{\mathrm{s}}^{(i)}(t), \end{aligned} \tag{4}$$

where $\sigma^{(i,k)}$ is the target reflection coefficient corresponding to the kth target, and $n_{\mathrm{s}}^{(i)}(t)$ is the additive noise.

Since the motion parameters of the receiver platform are precisely known, we can compensate for the range migration due to its movement about a ground reference position in close vicinity of the actual position of the targets, referred to as the scene origin [3]. For the kth target, considering a scene origin at $\mathbf{q}^{(k)}$, the range difference at the mth azimuthal sampling instant can be expressed as

$$\begin{aligned} \tilde{R}^{(i,k)}(t_m) \approx {} & \rho^{(i,k)}(t_m) - \zeta^{(i)}(t_m) \\ = {} & \|\mathbf{q}^{(k)} + \mathbf{v}^{(k)}t_m + \mathbf{a}^{(k)}t_m^2/2 - \mathbf{b}^{(i)}\|_2 \\ & + \|\mathbf{q}^{(k)} + \mathbf{v}^{(k)}t_m + \mathbf{a}^{(k)}t_m^2/2 - \mathbf{r}_0 \ - \mathbf{v}_r t_m\|_2 \\ & - \|\mathbf{q}^{(k)} - \mathbf{b}^{(i)}\|_2 - \|\mathbf{q}^{(k)} - \mathbf{r}_0 \ - \mathbf{v}_r t_m\|_2, \end{aligned} \tag{5}$$

where t_m are the azimuthal sampling instants, $m = 1, ..., M$ and $\zeta^{(i)}(t_m) = \|\mathbf{q}^{(k)} - \mathbf{b}^{(i)}\|_2 + \|\mathbf{q}^{(k)} - \mathbf{r}(t_m)\|_2$ is the bistatic range between the ith transmitter, the scene origin and the receiver at t_m.

B. Observed Doppler signature

The output of the receiver matched filter at azimuthal time t_m corresponding to the ith illuminator, after range compensation due to the motion of the receiver platform at the scene origin, can be expressed as a linear sum of K different Doppler signatures, as

$$s^{(i)}(t_m) = \sum_{k=1}^{K} \xi^{(i,k)} \exp\left(-j2\pi f^{(i)} \tilde{R}^{(i,k)}(t_m)/c\right) + n^{(i)}(t_m), \tag{6}$$

where $\xi^{(i,k)}$ is the magnitude of the matched filter output corresponding to the return from the kth target, and $n^{(i)}(t_m)$ is the additive Gaussian white noise. The phase term of the matched filter output, as discussed in the preceding section, is determined by the range difference, depicted in (5).

For a moderately long CPI, the phase term of $s^{(i,k)}(m)$, for the kth target, denoted as $\phi^{(i,k)}(m)$, follows the following quadratic relationship,

$$\phi^{(i,k)}(m) = \phi_0^{(i,k)} + 2\pi f_0^{(i,k)} m + \pi \beta^{(i,k)} m^2, \tag{7}$$

where $\phi_0^{(i,k)}$ is the initial phase, $f_0^{(i,k)}$ is the initial Doppler frequency, and $\beta^{(i,k)}$ is the chirp rate.

III. MOTION PARAMETER ESTIMATION THROUGH SPARSE SIGNAL RECOVERY

From (5), the target motion is determined by four unknown motion parameters, i.e., x- and y- axis components of target's acceleration and velocity. The Doppler signatures corresponding to different bistatic links share the same set of unknown motion parameters. As such, the problem can be modeled as a standard sparse signal reconstruction problem. The problem of sparse signal representation requires finding the sparsest signal $\mathbf{x}$ that satisfies $\mathbf{y} = \mathbf{A}\mathbf{x}$, where $\mathbf{y}$ represents an $P \times 1$ observation vector and $\mathbf{A} \in \mathcal{C}^{P \times Q}$ represents an overcomplete basis, i.e., $P < Q$.

In the underlying problem, the dictionary matrix represents a discretized four-dimensional (4-D) space of the unknown motion parameters, such that each point in the discretized space represents a hypothetical combination of target motion parameters (v_x, v_y, a_x, a_y). From a practical standpoint, such a 4-D search, for all possible values of target velocity and acceleration, is computationally infeasible. Therefore, we propose a two-step sequential estimation process. First, we obtain estimates of target acceleration by applying sparsity based signal recovery in the ambiguity domain. Then, the estimated values of target acceleration are used for estimating the velocities of the respective targets.

A. Estimation of acceleration of multiple targets

It is established in [6] that, for a radar return whose Doppler signature is characterized by a linear FM signal, the chirp rate depends largely on the target acceleration, whereas the initial velocity of the target has an insignificant effect on the chirp rate, especially when the target-receiver distance is large. The slope of a chirp signal signature viewed in the ambiguity domain is not affected by its initial frequency. In essence, the ambiguity function of a target's Doppler signature is a straight line passing through the origin, irrespective of the initial Doppler frequency, where the slope of the straight line is determined by the chirp rate. With multiple targets, the ambiguity function of the radar return, defined in (6), constitutes multiple lines. These lines all pass through the origin but with different slopes, depending on the respective target accelerations. The slope, however, can only be estimated when the SNR is sufficiently high. By applying sparse signal reconstruction methods in the ambiguity domain, nevertheless, it is possible to simultaneously utilize the signal energy in all available links for estimating the acceleration of the multiple targets, using the process detailed as follows.

The ambiguity function of the signal $s^{(i)}(m)$ is defined in the discrete-time representation as

$$\boldsymbol{\chi}^{(i)}(\theta,\tau)=\sum_{m=1}^{M} s^{(i)}(m+\tau)[s^{(i)}(m-\tau)]^{*}\exp(-j2\pi\theta m), \tag{8}$$

where θ represents the Doppler frequency and τ represents the delay. The discretized two-dimensional (2-D) ambiguity function corresponding to the ith broadcast station is, thus, a matrix $\boldsymbol{\chi}^{(i)} \in \mathcal{C}^{N_\theta \times N_\tau}$, where N_θ and N_τ, respectively, represent the number of Doppler bins and the number of delay bins considered in the analysis.

For the estimation of target acceleration, we are concerned only about the chirp rate. Therefore, in order to estimate the target acceleration by applying sparsity based signal reconstruction in the ambiguity domain, we define a $N_\theta N_\tau \times 1$ column vector $\tilde{\mathbf{x}}^{(i)} = \mathrm{vec}[|\boldsymbol{\chi}^{(i)}|]$ by vectorizing the magnitude of the discretized 2-D ambiguity function. To combine information from all bistatic links, a long observation vector $\tilde{\mathbf{x}} \in \mathcal{C}^{N_\theta N_\tau N\times 1}$ is defined for the N illuminators as

$$\tilde{\mathbf{x}}=\left[(\tilde{\mathbf{x}}^{(1)})^T,\cdots,(\tilde{\mathbf{x}}^{(N)})^T\right]^T. \tag{9}$$

The entire acceleration space is represented by a 2-D discrete space comprising N_{ax} and N_{ay} points along the x-axis and y-axis, respectively. Let an $N_{ax}N_{ay}\times 1$ vector $\mathbf{u}$ be the unknown sparse vector which vectorizes the discretized 2-D acceleration space. Following (5), for the pth hypothetical target acceleration vector $\mathbf{a}_{[p]} = [a_{[p]x}, a_{[p]y}]^T$, the bistatic range at the mth azimuthal sampling instant, after performing range compensation, can be expressed as,

$$\begin{aligned}\tilde{R}_{[p]}^{(i,k)}(t_m) &= \|\mathbf{q}^{(k)}+\mathbf{a}_{[p]}^{(k)}t_m^2/2-\mathbf{b}^{(i)}\|_2\\ &+\|\mathbf{q}^{(k)}+\mathbf{a}_{[p]}^{(k)}t_m^2/2-\mathbf{r}_0\ -\mathbf{v}_r t_m\|_2\\ &-\|\mathbf{q}^{(k)}-\mathbf{b}^{(i)}\|_2-\|\mathbf{q}^{(k)}-\mathbf{r}_0\ -\mathbf{v}_r t_m\|_2.\end{aligned} \tag{10}$$

Since the target velocity does not have a significant impact on the ambiguity function magnitude of the chirp Doppler signature, the velocity vector for all the targets is ignored in (10). As such, the output of the receiver matched filter at the mth azimuthal sample can be expressed as,

$$s_{[p]}^{(i)}(m)=\sum_{k=1}^{K}\xi^{(i,k)}\exp\left(-j2\pi f^{(i)}\tilde{R}_{[p]}^{(i,k)}(m)/c\right). \tag{11}$$

The corresponding ambiguity function in the discrete-time representation is defined as

$$\boldsymbol{\chi}_{[p]}^{(i)}(\theta,\tau)=\sum_{m=1}^{M}s_{[p]}^{(i)}(m+\tau)\left[s_{[p]}^{(i)}(m-\tau)\right]^{*}\exp(-j2\pi\theta m). \tag{12}$$

Vectorizing $|\boldsymbol{\chi}_{[p]}^{(i)}|$, we obtain a $N_\theta N_\tau \times 1$ column vector defined as $\tilde{\mathbf{x}}_{[p]}^{(i)} = \mathrm{vec}[|\boldsymbol{\chi}_{[p]}^{(i)}|]$. Combining the vectorized ambiguity functions corresponding to the N available broadcast stations, a long vector is defined such that

$$\tilde{\mathbf{x}}_{[p]}=\left[(\tilde{\mathbf{x}}_{[p]}^{(1)})^T,\cdots,(\tilde{\mathbf{x}}_{[p]}^{(N)})^T\right]^T. \tag{13}$$

An $N_\theta N_\tau N \times N_{ax}N_{ay}$ dictionary matrix $\boldsymbol{\Psi}$ is defined such that its pth column represents the vectorized ambiguity function corresponding to the pth hypothetical target acceleration vector $\mathbf{a}_{[p]}$, as defined in (13). Therefore, the problem of acceleration estimation can be formulated as the following l_1-norm minimization problem [8],

$$\min ||\mathbf{u}||_1 \quad \text{subject to} \quad \tilde{\mathbf{x}}=\boldsymbol{\Psi}\mathbf{u}, \tag{14}$$

which can be readily solved using a number of methods available for sparse signal reconstruction. As such, we obtain estimates of acceleration of the K ground moving targets, $\hat{\mathbf{a}}^{(k)} = [\hat{a}_x^{(k)}, \hat{a}_y^{(k)}, 0]^T$, where $k = 1, \cdots, K$. The estimated acceleration is used in the following to estimate the respective target velocities.

B. Estimation of velocity of multiple targets

For estimating the velocity vectors of K targets, we again exploit the sparsity based signal recovery method. Define an NM-element complex vector $\mathbf{y}$ as an observation vector which stacks the matched filter output vectors corresponding to the N broadcast stations. The output corresponding to each station contains M azimuthal samples as defined in (6). The 2-D space of the unknown velocity can be modeled as an $N_{vx}\times N_{vy}$ search space such that each point in the discretized space represents a hypothetical target velocity vector, where N_{vx} and N_{vy} denote the number of discrete points used to represent the entire target velocity space along the x-axis and y-axis respectively. As such, for a given estimate of target acceleration and the p'th hypothetical velocity vector $v_{[p']} = [v_{[p']x}, v_{[p']y}, 0]^T$, the bistatic range at the mth azimuthal sampling instant, after range compensation, can be expressed as,

$$\begin{aligned}\tilde{R}_{[p']}^{(i,k)}(m) &= \|\mathbf{q}^{(k)}+\mathbf{v}_{[p']}^{(k)}m+\hat{\mathbf{a}}^{(k)}m^2/2-\mathbf{b}^{(i)}\|_2\\ &+\|\mathbf{q}^{(k)}+\mathbf{v}_{[p']}^{(k)}m+\hat{\mathbf{a}}^{(k)}m^2/2-\mathbf{r}_0\ -\mathbf{v}_r m\|_2\\ &-\|\mathbf{q}^{(k)}-\mathbf{b}^{(i)}\|_2-\|\mathbf{q}^{(k)}-\mathbf{r}_0\ -\mathbf{v}_r m\|_2.\end{aligned} \tag{15}$$

Therefore, the output of the receiver matched filter, for the p'th hypothetical target velocity, can be expressed as

$$s_{[p']}^{(i)}(m)=\sum_{k=1}^{K}\xi^{(i,k)}\exp\left(-j2\pi f^{(i)}\tilde{R}_{[p']}^{(i,k)}(m)/c\right). \tag{16}$$

An unknown and sparse vector $\mathbf{u}'$, which vectorizes the discretized 2-D search space, is to be estimated. We define an $NM \times N_{vx}N_{vy}$ dictionary matrix $\boldsymbol{\Psi}'$ such that its p'th column represents the receiver matched filter output, as defined in (16), corresponding to the p'th hypothetical target velocity vector $\mathbf{v}_{[p']}$. As discussed in the previous section, such problem can be casted as an l_1-norm minimization problem, as follows

$$\min ||\mathbf{u}'||_1 \quad \text{subject to} \quad \mathbf{y} = \mathbf{\Psi}'\mathbf{u}', \tag{17}$$

The solutions to the l_1-norm minimization problems in (14) and (17) can be obtained using convex optimization algorithms. In this paper, the Compressive Sampling Matching Pursuit (CoSaMP) [9] is used.

IV. SIMULATION RESULTS

In the simulations, we consider a geolocation scenario as illustrated in Fig. 1, where 7 digital audio broadcast (DAB) stations [10] are respectively located at $[10, 10, 0.1]^T$ km, $[-10, 12, 0.1]^T$ km, $[10, -18, 0.1]^T$ km, $[0, 20, 0.1]^T$ km, $[-10, -10, 0.1]^T$ km, $[-5, 15, 0.1]^T$ km, and $[5, 5, 0.1]^T$ km. The respective carrier frequencies of these seven illuminators are 225, 227, 229, 231, 233, 235, and 237 MHz. The initial position of the air-borne receiver is $[0, 0, 5]^T$ km, and it moves with a constant velocity of $[150, 0, 0]^T$ m/s.

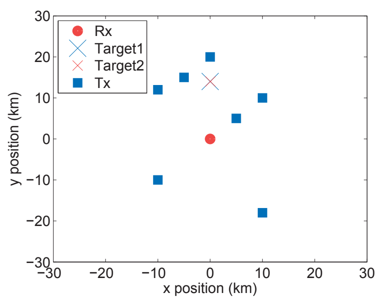


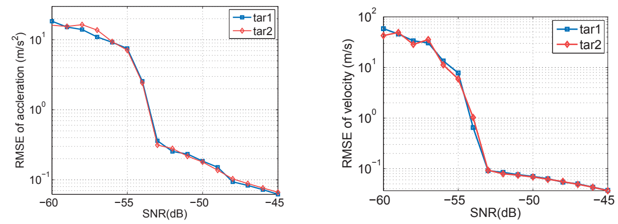
Fig. 1. Relative positions of transmitters, receiver and two closely spaced ground moving targets.

We consider two ground moving targets which are closely located, respectively, at $[0, 14, 0]^T$ km and $[0.05, 14, 0]^T$ km. The first target is assumed to be moving with an initial speed of $[-10, -10, 0]^T$ m/s and an acceleration of $[-4, -4, 0]^T$ m/s^2, whereas the second target is assumed to be moving with an initial speed of $[10, 10, 0]^T$ m/s and an acceleration of $[4, 4, 0]^T$ m/s^2. We use a search grid resolution of 0.01 m/s and 0.03 m/s^2, respectively, for target velocity and acceleration.

The receiver data is sampled at 2.048 MHz, and the matched filter output yields a 200 Hz azimuthal sampling frequency. The overall CPI is assumed to be 2 second, which generates 400 azimuthal samples per illuminator. For the given simulation scenario, through 100 independent trials, we obtain the root-mean-square error (RMSE) for the estimation of acceleration and velocity of the two targets presented in Fig. 2. Results show that the proposed method can achieve robust estimation even at low SNR conditions. It is also important to note that the performance can be further improved, when more illuminators are available.

V. CONCLUSIONS

In this paper, we have developed a method for the estimation of motion parameters of multiple, closely located ground moving target in a multistatic passive radar platform. We focus on weak signal conditions where the signal-to-noise ratio (SNR) of individual bistatic link is poor and, therefore, it is desirable to exploit the availability of multiple transmitters. In the proposed method, we exploit the sparsity of motion parameters and obtain a robust motion parameter estimates for multiple targets through the fusion of data from all bistatic links. Also, the proposed method obtains a sequential estimation of motion parameters of multiple targets to avoid the need for computationally demanding multidimensional exhaustive search. Simulation results were presented to demonstrate that the proposed method achieves robust estimations, even in low SNR conditions.



(a) Acceleration (b) Velocity

Fig. 2. RMSE of motion parameter estimates versus SNR.

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