Channel Estimation and Tracking for MC-CDMA Signals

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Abstract: This paper investigates the impact of channel estimation errors in the uplink and downlink of a multicarrier code-division multiple-access system. Pilot blocks are periodically inserted into the downlink data stream to perform a least-squares channel estimation. It is shown that a single-user minimum-mean-square-error (MMSE) detector endowed with the proposed channel estimator has relatively low complexity, good performance and is robust to estimation errors even under full-loaded conditions. In the uplink scenario channel estimates are computed through a least-mean-square algorithm and are passed to a linear MMSE multi-user detector or a parallel interference cancellation (PIC) receiver. Simulations indicate that in this case the system performance depends heavily on the quality of the channel estimates, meaning that they are particularly critical.

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I. INTRODUCTION

Multi-Carrier Code-Division Multiple-Access (MC-CDMA) is a multiplexing technique that combines orthogonal frequency division multiplexing (OFDM) with direct sequence CDMA [1]-[2]. Because of attractive features such as high spectral efficiency, robustness to frequency selective fading and flexibility to support integrated applications [3], MC-CDMA has been proposed as a viable candidate for future generation broadband communications.

In an MC-CDMA system the data of different users are spread in the frequency domain using orthogonal signature sequences. In the presence of multipath propagation, however, signals undergo frequency-selective fading and the spreading codes loose their orthogonality. This gives rise to multiple-access interference (MAI) at the receiver, which strongly limits the system performance. Some advanced signal-processing techniques are available to mitigate interference and multipath distortion. They are categorized into space-time processing with antenna array and multi-user detection [4]-[5]. Linear multiuser receivers in the form of decorrelating detectors [6] or minimum mean square error (MMSE) detectors are usually proposed to achieve a reasonable trade-off between performance and complexity. Alternatively, non-linear techniques are adopted in the form of parallel interference cancellation (PIC) receivers that are very promising for applications on the uplink channel.

All of the above techniques require explicit knowledge of the channel impulse response of each user. In the downlink all the signals arriving from the base station (BS) at a given terminal propagate through the same channel. Accordingly, channel estimation in the downlink can be accomplished with the same methods employed in OFDM applications. In particular, the schemes proposed in [7]-[8] make use of known symbols (pilots) inserted in both the frequency and time dimensions. Channel estimates are obtained interpolating between pilots through the cascade of two mono-dimensional (1D) Wiener filters. The main drawback of this approach is that it requires knowledge of the channel statistics. In practice this entails some performance loss since the system is designed for fixed values of the channel correlations. The scheme studied in [9] employs a channel-sounding approach in which a train of pulses are transmitted periodically within the OFDM frame. Finally, reference [10] investigates decision-directed (DD) channel estimators that operate in an iterative fashion through least-mean-square (LMS) or recursive-least-square (RLS) algorithms. These solutions require crucial information about the active users, including their spreading codes and power levels. Unfortunately, this information is not generally available at the mobile terminal.

The problem of channel estimation in the uplink of an MC-CDMA system has received little attention in the literature and only few results are available [11]-[12]. The main problem is that the channel responses of the active users are different from one another as the users transmit from different locations. Accordingly, the BS must estimate a large number of parameters and this is expected to degrade the quality of the estimates as compared to the downlink where only a single channel response is involved.

In this paper we address the channel estimation problem in both uplink and downlink and we investigate the impact of estimation errors on the system performance. The downlink channel is estimated through a least-squares (LS) approach assuming that pilot blocks are periodically inserted into the transmitted data stream. The estimate obtained from a given pilot block is used to detect the data symbols until the arrival of the next pilot block. The proposed scheme is reminiscent of the method discussed in [13] for OFDM applications.

In the uplink we consider a quasi-synchronous system in which each user is time-aligned to the BS reference in a way similar to that discussed in [14]. A least-squares method is employed for channel acquisition while channel tracking is pursued by means of the LMS algorithm. Both linear and non-linear multiuser detectors are considered in the uplink, while a single-user receiver is adopted in the downlink.

The rest of the paper is organized as follows. Next section describes the signal model and introduces basic notation. In section III we discuss downlink and uplink data detectors. Channel estimation in the downlink is addressed in section IV while channel acquisition and tracking schemes are proposed in section V for the uplink. Simulation results are discussed in section VI and some conclusions are offered in section VII.

II. SIGNAL MODEL

A. MC-CDMA system

We consider an MC-CDMA network in which the total number of subcarriers, N, is divided into smaller groups of Q elements [8]. Several users within a group are simultaneously active and are separated by their specific spreading codes. Without loss of generality we concentrate on a single group of K users ($K \le Q$) and assume that the Q subcarriers are uniformly spread over the signal bandwidth so as to better exploit the channel frequency diversity. We denote $\{i_n ; 1 \le n \le Q\}$ the subcarrier indexes in the considered group, with $i_n = 1 + (n-1)N/Q$. The channel is assumed static over an MC-CDMA block (slow fading) and a N_G-point cyclic prefix (longer than the channel impulse response) is used to eliminate inter-block interference.

At the receiver side the incoming waveform is first filtered and then sampled with period $T_s = T_B/(N+N_G)$, where T_B is the block duration. Next, the cyclic prefix is removed and the remaining samples are passed to an N-point discrete Fourier transform (DFT) unit. We concentrate on the m-th MC-CDMA block and denote $\boldsymbol{X}(m) = [X(m,i_1), X(m,i_2), \ldots, X(m,i_Q)]^T$ the DFT outputs corresponding to the Q subcarriers in the considered group (the superscript $(\cdot)^T$ means transpose operation). We have

$$\boldsymbol{X}(m) = \sum_{k=1}^{K} a_k(m)\boldsymbol{d}_k(m) + \boldsymbol{w}(m) \qquad (1)$$

where $a_k(m)$ is the symbol of the k-th user, $\boldsymbol{w}(m) = [w(m,i_1), w(m,i_2), \ldots, w(m,i_Q)]^T$ is thermal noise that is modeled as a white Gaussian process with zero mean and variance $\sigma_w^2 = \mathrm{E}\{|w(m,i_n)|^2\}$, and $\boldsymbol{d}_k(m)$ is a Q-dimensional vector with entries

$$d_k(m,n) = H_k(m,i_n)c_k(n) \qquad 1 \le n \le Q. \qquad (2)$$

In the above equation $c_k(n) \in \{\pm 1/\sqrt{Q}\}$ is the (unit-energy) spreading code of the k-th user and $H_k(m,i_n)$ is the channel frequency response over the i_n-th subcarrier of the m-th block. Note that in the downlink $H_k(m,i_n)$ does not depend on the index k since the signals from the BS arrive at the i-th user through the same channel, i.e., we set $H_k(m,i_n) = H_i(m,i_n)$ for $k = 1,2,\ldots,K$.

Inspection of (1) reveals that $\boldsymbol{X}(m)$ may also be written as

$$\boldsymbol{X}(m) = \boldsymbol{D}(m)\boldsymbol{a}(m) + \boldsymbol{w}(m) \qquad (3)$$

where $\boldsymbol{D}(m) = [\boldsymbol{d}_1(m) \;\; \boldsymbol{d}_2(m) \cdots \boldsymbol{d}_K(m)]^T$ and $\boldsymbol{a}(m) = [a_1(m), a_2(m), \ldots, a_K(m)]^T$.

B. Channel model

We consider a multipath channel with N_p distinct paths. Thus, the *k*-th baseband channel impulse response (CIR) during the *m*-th OFDM block takes the form

$$h_k(m,t) = \sum_{\ell=1}^{N_p} a_{\ell,k}(m)\, g(t-\tau_{\ell,k}(m)) \tag{4}$$

where $g(t)$ is the convolution between the impulse responses of the transmit and receive filters, $\tau_{\ell,k}(m)$ is the delay on the ℓ-path and $a_{\ell,k}(m)$ is the corresponding complex amplitude. The gain $a_{\ell,k}(m)$ is modeled as a narrow-band independent Gaussian random process with zero-mean and average power $\sigma_\ell^2 = \mathrm{E}\{|a_{\ell,k}(m)|^2\}$.

The channel frequency response $H_k(m,i_n)$ is computed as the DFT of $h_k(m,pT_s)$, i.e.,

$$H_k(m,i_n) = \sum_{p=1}^{L_k} h_k(m,pT_s)\, e^{-j2\pi\, p i_n/N} \tag{5}$$

where L_k is the duration of $h_k(m,t)$ in sampling periods. Note that $L_k = \mathrm{int}\{(\tau_k^{(\max)} + T_g)/T_s\}$, where T_g is the duration of $g(t)$, $\tau_k^{(\max)} = \max_\ell\{\tau_{\ell,k}\}$ is the maximum path delay of the *k*-th user and $\mathrm{int}(x)$ denotes the maximum integer not exceeding *x*. Since $\tau_k^{(\max)}$ is usually unknown, in practice L_k is estimated taking the maximum *expected* value of $\tau_k^{(\max)}$.

III. DATA DETECTION

Multi-user detectors are not suited for downlink applications due to their complexity. Accordingly, in the sequel we apply single-user detection in the downlink while we consider both linear and non-linear multiuser receivers in the uplink.

A. Downlink data detection

We adopt the single-user MMSE receiver as it combines low complexity with relatively good performance. The decision statistic for the *i*-th user at the m-th epoch is

$$y_i(m) = \sum_{n=1}^{Q} \xi_{i,n}(m) X(m,i_n) \tag{6}$$

where the coefficients $\xi_{i,n}(m)$ minimize the mean-square-error (MSE) between $y_i(m)$ and $a_i(m)$. For this to happen it must be

$$\xi_{i,n}(m)=\frac{H_i^*(m,i_n)c_i(n)}{|H_i(m,i_n)|^2+\sigma_w^2} \qquad n=1,2,\ldots,Q. \tag{7}$$

An estimate of $a_i(m)$ is then obtained feeding $y_i(m)$ to a threshold device. Note that the MMSE single-user detector restores the orthogonality between the spreading codes if the thermal noise is small relative to the received signal power while it acts as a channel-matched receiver when the thermal noise is dominant.

B. Uplink data detection

We first consider the MMSE multi-user detector. Its decision statistic at the *m*-th block is

$$\boldsymbol{Y}(m)=\boldsymbol{G}(m)\boldsymbol{X}(m) \tag{8}$$

where matrix $\boldsymbol{G}(m)$ minimizes the MSE between $\boldsymbol{Y}(m)$ and $\boldsymbol{a}(m)$. From (3) it is found

$$\boldsymbol{G}(m)=[\boldsymbol{D}^H(m)\boldsymbol{D}(m)+\sigma_w^2\boldsymbol{I}_K]^{-1}\boldsymbol{D}^H(m) \tag{9}$$

where $\boldsymbol{I}_K$ denotes the identity matrix of order K and $(\cdot)^H$ means Hermitian transposition. The entries of $\boldsymbol{Y}(m)$ are fed to a threshold device to produce an estimate $\hat{\boldsymbol{a}}(m)$ of the transmitted symbols.

Next we concentrate on the PIC detector. Here MAI is estimated using tentative data decisions and is subtracted from the DFT output. Without loss of generality we concentrate on the k-th user. At the $\ell-$th stage the PIC detector computes the vector

$$\boldsymbol{Z}_k^{(\ell)}(m)=\boldsymbol{X}(m)-\sum_{\substack{j=1\\ j\neq k}}^{K}\hat{a}_j^{(\ell-1)}(m)\boldsymbol{d}_j(m) \tag{10}$$

where $\{\hat{a}_j^{(\ell-1)}(m)\}$ are data decisions from the previous stage. A decision statistic is obtained from $\boldsymbol{Z}_k^{(\ell)}(m)$ in the form

$$Y_k^{(\ell)}(m)=\boldsymbol{d}_k^H(m)\boldsymbol{Z}_k^{(\ell)}(m) \tag{11}$$

where $\boldsymbol{d}_k(m)$ is defined in (2). Finally, passing $Y_k^{(\ell)}(m)$ to a threshold device produces the estimate $\hat{a}_k^{(\ell)}(m)$ at the ℓ-th stage. The performance of the PIC detector depends on the quality of the initial estimate $\hat{\boldsymbol{a}}^{(0)}(m)$. In our simulations we take $\hat{\boldsymbol{a}}^{(0)}(m)$ as the output of the MMSE multi-user detector.

IV. CHANNEL ESTIMATION IN THE DOWNLINK

From (6)-(7) we see that the data detection in the downlink requires knowledge of the channel response $\{H_i(m,i_n)\}$. Channel estimation in the downlink can be accomplished with any method adopted in OFDM applications. Here, we choose the pilot-aided scheme described in [13] since it is simple to implement and does not require a-priori information about the channel statistics. To this purpose we assume that the MC-CDMA blocks are organized in frames and the pilot blocks (carrying known symbols) are periodically inserted into the frame structure as shown in Fig. 1a. We denote M the distance between two consecutive pilot blocks and assume that the channel variations are negligible (slow fading). The channel estimate from a pilot block can be used in the detection of the subsequent $M-1$ data blocks.

Let us concentrate on the channel response estimation. For notational simplicity we drop the block index m and assume that the pilot blocks have an OFDM structure (i.e., the pilot symbols are not spread). Denoting $\boldsymbol{X}_i^{(p)} = [X_i^{(p)}(1), X_i^{(p)}(2), \ldots, X_i^{(p)}(N)]^T$ the DFT outputs corresponding to the pilot block (at the i-th terminal), we have

$$\boldsymbol{X}_i^{(p)} = \boldsymbol{P}\boldsymbol{H}_i + \boldsymbol{w}_i^{(p)} \tag{12}$$

where $\boldsymbol{w}_i^{(p)}$ is the noise contribution and $\boldsymbol{H}_i = [H_i(1), H_i(2), \ldots, H_i(N)]^T$ is the frequency response of the channel between the BS and the i-th user. Finally, $\boldsymbol{P}$ is a diagonal matrix

$$\boldsymbol{P} = \mathrm{diag}\{p_1, p_2, \ldots, p_N\} \tag{13}$$

where $\{p_n; n = 1, 2, \ldots, N\}$ are pilot symbols taken from a PSK constellation, i.e., $|p_n| = 1$. Letting $L = \max_k\{L_k\}$, from (5) we see that $\boldsymbol{H}_i$ can also be written as

$$\boldsymbol{H}_i = \boldsymbol{F}\boldsymbol{h}_i \tag{14}$$

where $\boldsymbol{h}_i = [h_i(1), h_i(2), \ldots, h_i(L)]^T$ is the CIR vector and $\boldsymbol{F}$ is an $N \times L$ matrix with entries

$$[\boldsymbol{F}]_{n,\ell} = e^{-j2\pi(n-1)(\ell-1)/N} \qquad 1 \le n \le N, \quad 1 \le \ell \le L. \tag{15}$$

Substituting (14) into (12) yields

$$\boldsymbol{X}_i^{(p)} = \boldsymbol{P}\boldsymbol{F}\boldsymbol{h}_i + \boldsymbol{w}_i^{(p)} \tag{16}$$

and the LS estimate of $\boldsymbol{h}_i$ is obtained as

$$\hat{\boldsymbol{h}}_i = (\boldsymbol{F}^H\boldsymbol{P}^H\boldsymbol{P}\boldsymbol{F})^{-1}\boldsymbol{F}^H\boldsymbol{P}^H\boldsymbol{X}_i^{(p)}. \tag{17}$$

An estimate of $\boldsymbol{H}_i$ is computed by pre-multiplying both sides of (17) by $\boldsymbol{F}$. This produces

$$\hat{\boldsymbol{H}}_i = \frac{1}{N}\boldsymbol{F}\boldsymbol{F}^H\boldsymbol{P}^H\boldsymbol{X}_i^{(p)} \tag{18}$$

where we have borne in mind that $\boldsymbol{P}^H\boldsymbol{P} = \boldsymbol{I}_N$ and $\boldsymbol{F}^H\boldsymbol{F} = N \times \boldsymbol{I}_L$.

In Appendix A it is shown that $\hat{\boldsymbol{H}}_i$ is unbiased and has the following mean square estimation error

$$\mathrm{E}\left\{\left|\hat{H}_i(n) - H_i(n)\right|^2\right\} = \frac{L}{N}\sigma_w^2 . \tag{19}$$

V. CHANNEL ESTIMATION IN THE UPLINK

Inspection of (8)-(11) reveals that data detection in the uplink requires knowledge of the K vectors $\{\boldsymbol{d}_k(m)\,; 1 \le k \le K\}$, which are related to the channel frequency responses $\boldsymbol{H}_k(m) = [H_k(m,i_1), H_k(m,i_2), \ldots, H_k(m,i_Q)]^T$ as indicated in (2). Compared to the downlink situation, channel estimation in the uplink is much more challenging since the channel responses of the active users are different from one another and the BS must estimate a large number of parameters. As explained later, joint estimation of the channel responses of all active users require several training blocks while a single pilot block is sufficient in the downlink.

In the following we aim at directly estimating $\boldsymbol{d}_k(m)$ rather than $\boldsymbol{H}_k(m)$. As shown in Fig. 1b, each frame in the uplink is preceded by some training blocks that provide initial estimates of $\boldsymbol{d}_k(m)$ (*acquisition*). These estimates are then updated during the data section of the frame (*tracking*). We consider the situation in which different groups of subcarriers are assigned to different groups of users. In these circumstances the subcarriers belonging to a given group can only be exploited to estimate the channel responses of the users within that group (this is in contrast to the downlink situation where all the subcarriers can be used to estimate the channel response between the BS and a given mobile terminal).

A. Acquisition

Denote N_T the number of training blocks and assume that the channel variations are negligible over the entire training sequence, i.e., $\boldsymbol{d}_k(m) = \boldsymbol{d}_k$ for $m = 0,1,\ldots,N_T - 1$. Then, collecting $\{\boldsymbol{d}_k; k = 1,2,\ldots,K\}$ into a single $KQ-$dimensional vector $\boldsymbol{d} = [\boldsymbol{d}_1^T\ \boldsymbol{d}_2^T \cdots \boldsymbol{d}_K^T]^T$, from (1) we have

$$\boldsymbol{X}(m) = \boldsymbol{B}(m)\boldsymbol{d} + \boldsymbol{w}(m) \qquad m = 0,1,\ldots,N_T - 1 \tag{20}$$

where $\boldsymbol{X}(m)$ is the DFT output corresponding to the Q subcarriers of the group and $\boldsymbol{w}(m)$ is thermal noise. Also, $\boldsymbol{B}(m)$ is a $Q \times QK$ matrix of K blocks

$$\boldsymbol{B}(m) = [a_1(m)\boldsymbol{I}_Q \;\; a_2(m)\boldsymbol{I}_Q \cdots a_K(m)\boldsymbol{I}_Q] \tag{21}$$

and $\{a_k(m); m = 0,1,\ldots,N_T - 1\}$ is the training sequence of the k-th user. The observations $\{\boldsymbol{X}(m); m = 0,1,\ldots,N_T - 1\}$ are now exploited to get an LS estimate of $\boldsymbol{d}$ in the form

$$\hat{\boldsymbol{d}} = \boldsymbol{R}^{-1} \sum_{m=0}^{N_T - 1} \boldsymbol{B}^H(m)\boldsymbol{X}(m) \tag{22}$$

with

$$\boldsymbol{R} = \sum_{m=0}^{N_T - 1} \boldsymbol{B}^H(m)\boldsymbol{B}(m). \tag{23}$$

Substituting (20) into (22) it is seen that $\hat{\boldsymbol{d}}$ is unbiased and the mean square estimation error is

$$\mathrm{E}\left\{\left\|\hat{\boldsymbol{d}} - \boldsymbol{d}\right\|^2\right\} = \sigma_w^2 \times \mathrm{tr}\{\boldsymbol{R}^{-1}\} \tag{24}$$

where $\mathrm{tr}\{\cdot\}$ is the trace of a matrix and $\|\cdot\|$ denotes Euclidean norm.

The following remarks are of interest:

i) Inspection of (21) reveals that $\mathrm{rank}[\boldsymbol{B}(m)] = Q$ so that $\mathrm{rank}[\boldsymbol{B}^H(m)\boldsymbol{B}(m)] = Q$. On the other hand, using the inequality $\mathrm{rank}[\boldsymbol{A} + \boldsymbol{B}] \le \mathrm{rank}[\boldsymbol{A}] + \mathrm{rank}[\boldsymbol{B}]$ [15, p.13], from (23) we see that $\mathrm{rank}[\boldsymbol{R}] \le QN_T$. Finally, bearing in mind that $\boldsymbol{R}$ is a matrix of order QK, it follows that a necessary condition for the existence of $\boldsymbol{R}^{-1}$ is $N_T \ge K$, i.e., the number of training blocks cannot be less than the number of active users within the group.

ii) The complexity of the estimator (22) can be greatly reduced if the training sequences are orthogonal, i.e., they satisfy the identity

$$\sum_{m=0}^{N_T - 1} a_k^*(m)a_\ell(m) = E_k \times \delta(k - \ell) \tag{25}$$

where E_k is the energy of the k-th sequence and $\delta(\ell)$ is the Kronecker delta function. In these circumstances $\boldsymbol{R}$ becomes diagonal

$$\boldsymbol{R}=\begin{bmatrix} E_1\boldsymbol{I}_Q & 0 & \cdots & 0 \\ 0 & E_2\boldsymbol{I}_Q & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_K\boldsymbol{I}_Q \end{bmatrix} \tag{26}$$

and (22) reduces to

$$\hat{\boldsymbol{d}}_k=\frac{1}{E_k}\sum_{m=0}^{N_T-1} a_k^*(m)\boldsymbol{X}(m) \qquad k=1,2,\ldots,K. \tag{27}$$

Finally, collecting (24) and (26) we have

$$\mathrm{E}\left\{\left|\hat{d}_k(n)-d_k(n)\right|^2\right\}=\frac{\sigma_w^2}{E_k} \tag{28}$$

where $\hat{d}_k(n)$ is the n-th entry of $\hat{\boldsymbol{d}}_k$.

B. Tracking

In principle the variations of $\boldsymbol{d}_k(m)$ during the data section of the frame can be tracked as discussed for the downlink, where a new channel estimate is periodically computed for each pilot block. Unfortunately this solution is not suited for the uplink since it would entail a too large overhead. The reason is that while a single block is sufficient in the downlink to get a channel estimate, at least K training blocks are needed in the uplink. In other words, groups of K training blocks should be periodically inserted into the transmitted data stream. A way out consists of tracking the channel variations adaptively in a decision-directed mode. We propose the following LMS algorithm

$$\hat{\boldsymbol{d}}_k(m+1)=\hat{\boldsymbol{d}}_k(m)+\mu\,\hat{a}_k^*(m)\left[\boldsymbol{X}(m)-\sum_{\ell=1}^{K}\hat{a}_\ell(m)\hat{\boldsymbol{d}}_\ell(m)\right] \qquad m\geq N_T \tag{29}$$

in which $\{\hat{a}_\ell(m)\}$ are data decisions from either the MMSE or PIC detectors and the initial estimate $\hat{\boldsymbol{d}}_k(N_T)$ is computed from (22) or (27). The step-size μ affects the convergence of the algorithm and is chosen as a trade-off between steady-state performance and tracking capabilities.

The performance of the tracker (29) over a static channel (i.e., $\boldsymbol{d}_k(m)=\boldsymbol{d}_k$) is investigated in Appendix B assuming statistically independent data symbols with zero-mean and variance A_2. It turns out that $\hat{\boldsymbol{d}}_k(m)$ is unbiased and has the following mean square estimation error

$$\mathrm{E}\left\{\left|\hat{d}_k(m,n)-d_k(n)\right|^2\right\}=\frac{2B_LT_B\sigma_w^2}{A_2} \tag{30}$$

where T_B is the time duration of an MC-CDMA block (including the cyclic prefix) and $B_LT_B=\mu A_2/[2(2-\mu A_2)]$ is the noise equivalent bandwidth of the recursion (29), normalized to $1/T_B$.

VI. SIMULATION RESULTS

Computer simulations have been run to assess the performance of an MC-CDMA receiver employing the proposed channel estimation schemes. The system parameters are as follows.

A. System parameters

The cellular system operates over a typical urban area with a cell radius of 1 km. The transmitted symbols belong to a QPSK constellation and are obtained from the information bits through a Gray map. The total number of subcarriers is $N=64$ and Walsh-Hadamard codes of length $Q=8$ are used for spreading purposes. The signal bandwidth is $B=8$ MHz, so that the useful part of each MC-CDMA block has length $T=N/B=8$ µs. The sampling period is $T_s=T/N=0.125$ µs and a cyclic prefix of $T_G=2$ µs is adopted to eliminate inter-block interference. This corresponds to an extended block (including the cyclic prefix) of 10 µs. The users are synchronous within the cyclic prefix and have the same power in the downlink and in the uplink. The channel impulse response of the k-th user is generated as indicated in (4) with eight paths ($N_p=8$). Pulse $g(t)$ is a raised-cosine function with roll-off 0.22 and duration $T_g=8T_s=1$ µs. The path delays are kept constant over a frame and are uniformly distributed within [0, 1 µs]. This corresponds to a maximum CIR length of 2 µs (i.e., $L=16$). The path gains have powers $\sigma_\ell^2=\exp(-\ell)$ $(0\le\ell\le 7)$ and vary independently of each other within a frame. They are generated by filtering statistically independent white Gaussian processes in a third-order low-pass Butterworth filter. The 3-dB bandwidth of the filter is taken as a measure of the Doppler rate $f_D=f_0v/c$, where $f_0=2$ GHz is the carrier frequency, v denotes the speed of the mobile terminal and $c=3\times10^8$ m/s is the speed of light.

An uplink simulation begins with the generation of the channel responses of each user. Channel acquisition is then performed exploiting Walsh-Hadamard training sequences of length

$N_T = 8$ while the LMS algorithm tracks the channel variations during the data section of the frame. Each frame has 64 blocks (corresponding to an overhead of 12.5%).

For the downlink a single (time-varying) channel response is generated for all the active users and pilot blocks are periodically inserted within the frame with period M. A channel estimate is computed based on the observation of a pilot block and is kept fixed over the next data blocks. Different values are given to M and to the mobile velocity so as to assess their impact on the system performance.

B. Uplink performance

We begin with the effect of the step-size μ on the performance of the LMS channel tracker. Figure 2 shows the bit-error-rate (BER) vs. μ with either 4 or 8 active users (half load or full load). The signal-to-noise ratio E_b / N_0 is 10 dB (E_b is the energy per bit and $N_0 / 2$ the two-sided noise spectral density). Three Doppler bandwidths are considered, corresponding to mobile speeds of 30, 60 and 120 km/h. The PIC receiver is used for data detection. As expected, an optimal μ exists for any combination of fading rate and number of users. For mobile speeds between 30 and 120 km/h the optimum is given by the rule-of-thumb formula $\mu = 0.15 / \sqrt{K}$.

Figure 3 shows the MSE of the channel estimates vs. $1/\sigma_w^2$ as obtained during acquisition and tracking. The channel is static over the frame and the step-size is either $\mu = 0.08$ or $\mu = 0.04$. Marks indicate simulations, solid lines represent analytical results as given by (28) and (30). Good agreement is observed between simulations and theory.

Figure 4 illustrates the BER performance of MMSE and PIC detectors. The fading rate is 110 Hz (corresponding to $v = 60$ km/h) and the number of users is $K = 4$ (half load). For comparison, the single user bound (SUB) and the performance with perfect channel knowledge (PCK) are also shown. We see that the PIC detector with PCK approaches the SUB while the MMSE looses 2 dB. The loss due to channel estimation errors is comparable with both detectors and is approximately 3 dB.

Figure 5 shows simulation results with a full-loaded system ($K = 8$). The loss with respect to the SUB is now significantly larger than in Fig. 4, even with perfect channel knowledge. Note that channel estimation errors lead to an irreducible error floor with both MMSE and PIC detectors.

Figure 6 illustrates the performance of the PIC detector with $K = 4$ and different mobile speeds. Note that the results with PCK do not depend on the fading rate since the channel is assumed constant on an OFDM block. As expected, the BER deteriorates as the mobile speed increases. For an error probability of 10^{-2}, the gap from PCK is approximately 2 dB with $v = 30$ km/h and becomes 5 dB with $v = 120$ km/h.

C. Downlink performance

Figure 7 shows the BER of the MMSE single-user receiver vs. E_b / N_0 for a half-loaded system ($K = 4$). The mobile speed is 60 km/h and the separation between pilot blocks is $M = 8$, 16 and 32. The SUB and the BER with PCK are also indicated as benchmarks. Comparing with Fig. 4 we see that, in the uplink and with PCK, both PIC and MMSE multi-user detectors perform much better than the single-user receiver in the downlink. This is not surprising since we know that single-user detectors have worse performance than their multi-user counterparts. It is interesting to note that the impact of channel estimation errors is much stronger in the uplink than in the downlink. For an error probability of 10^{-3}, the loss with respect to PCK in the uplink is 3 dB with either PIC or MMSE multi-user detector, while it reduces to less than 1 dB in the downlink with $M = 8$. Note that $M = 8$ corresponds to a pilot overhead of 12.5%, the same overhead incurred in the uplink due to the training sequence. As expected, the BER deteriorates as M increases. An irreducible floor is observed with $M = 32$.

Figure 8 shows BER results in the operating conditions of Fig. 7, except that the system is now full-loaded ($K = 8$). We see that the loss with respect to PCK is slightly larger with respect to the half-loaded situation of Fig. 7. Comparing with the corresponding results of Fig. 5 it is seen that channel estimation is more critical in the uplink than in the downlink. For an error probability of 10^{-3} and $M = 8$, the loss due to channel estimation is approximately 1.5 dB in the downlink while a floor shows up in the uplink.

VII. CONCLUSIONS

We have discussed channel estimation in MC-CDMA systems. Orthogonal training sequences are exploited in the uplink to perform LS channel acquisition while the LMS algorithm is employed to track the channel variations. An LS approach is also adopted in the downlink where pilot blocks are periodically inserted in the data stream for channel estimation purposes. Either MMSE or PIC receivers are used in the uplink while a single-user MMSE detector is employed in the downlink.

Computer simulations have been run to evaluate the impact of channel estimation errors on the system performance. It is shown that the loss due to imperfect channel knowledge is stronger in the uplink and increases with the number of active users and the fading rate. For a full-loaded system and a mobile speed of 60 Km/h, a loss of 1.5 dB is incurred in the downlink with respect to a system with ideal channel information while an irreducible error floor shows up in the uplink. We conclude that while MC-CDMA is well suited for downlink applications, its use in the uplink is critical due to the high sensitivity to channel estimation errors.

APPENDIX A

In this Appendix we compute the MSE of the channel estimator in the downlink. Substituting (16) into (18) and using the identities $\boldsymbol{P}^H\boldsymbol{P} = \boldsymbol{I}_N$ and $\boldsymbol{F}^H\boldsymbol{F} = N \times \boldsymbol{I}_L$ produces

$$\hat{\boldsymbol{H}}_i = \boldsymbol{F}\boldsymbol{h}_i + \frac{1}{N}\boldsymbol{F}\boldsymbol{F}^H\boldsymbol{P}^H\boldsymbol{w}_i^{(p)}. \tag{A1}$$

Then, bearing in mind that $\boldsymbol{w}_i^{(p)}$ has zero mean, from (A1) and (14) it is seen that $\hat{\boldsymbol{H}}_i$ is unbiased.

Using (A1), the covariance matrix of $\hat{\boldsymbol{H}}_i$ is found to be

$$\boldsymbol{C}_{\hat{\boldsymbol{H}}} = \frac{\sigma_w^2}{N}\boldsymbol{F}\boldsymbol{F}^H \tag{A2}$$

where we have taken into account that $\mathrm{E}\{\boldsymbol{w}_i^{(p)}[\boldsymbol{w}_i^{(p)}]^H\} = \sigma_w^2\boldsymbol{I}_N$.

Finally, collecting (15) and (A2) yields

$$\mathrm{E}\left\{\left|\hat{H}_i(n) - H_i(n)\right|^2\right\} = \frac{L}{N}\sigma_w^2. \tag{A3}$$

APPENDIX B

In this Appendix we highlight the major steps leading to (30) in the text. For simplicity we assume that the channel is static and the data decisions are correct, i.e., we set $\boldsymbol{d}_k(m) = \boldsymbol{d}_k$ and $\hat{a}_k(m) = a_k(m)$. Also, we rewrite (29) in the equivalent form

$$\hat{\boldsymbol{d}}(m+1) = \hat{\boldsymbol{d}}(m) + \mu \boldsymbol{e}(m) \tag{B1}$$

where $\hat{\boldsymbol{d}}(m) = [\hat{\boldsymbol{d}}_1^T(m) \;\; \hat{\boldsymbol{d}}_2^T(m) \cdots \hat{\boldsymbol{d}}_K^T(m)]^T$ is an estimate of $\boldsymbol{d} = [\boldsymbol{d}_1^T \;\; \boldsymbol{d}_2^T \cdots \boldsymbol{d}_K^T]^T$ at the m-th block, $\boldsymbol{e}(m)$ is given by

$$\boldsymbol{e}(m) = \boldsymbol{B}^H(m)[\boldsymbol{X}(m) - \boldsymbol{B}(m)\hat{\boldsymbol{d}}(m)] \tag{B2}$$

and $\boldsymbol{B}(m)$ is the matrix defined in (21).

We first concentrate on the conditional expectation $\mathrm{E}\{\boldsymbol{e}(m) | \hat{\boldsymbol{d}}(m)\}$. To this purpose substituting (20) into (B2) gives

$$\boldsymbol{e}(m) = \boldsymbol{B}^H(m)\boldsymbol{B}(m)\Delta\hat{\boldsymbol{d}}(m) + \boldsymbol{B}^H(m)\boldsymbol{w}(m) \tag{B3}$$

where $\Delta\hat{\boldsymbol{d}}(m) = \boldsymbol{d} - \hat{\boldsymbol{d}}(m)$ is the estimation error at the m-th step and $\{\boldsymbol{w}(m)\}$ are statistically independent Gaussian vectors with zero mean and covariance matrix $\sigma_w^2 \boldsymbol{I}_Q$. Then, using the identity $\mathrm{E}\{\boldsymbol{B}^H(m)\boldsymbol{B}(m)\} = A_2 \times \boldsymbol{I}_{QK}$ (which is valid for independent data symbols with zero mean and variance A_2), produces

$$\mathrm{E}\left\{\boldsymbol{e}(m) \middle| \hat{\boldsymbol{d}}(m)\right\} = A_2 \times \Delta\hat{\boldsymbol{d}}(m). \tag{B4}$$

The above indicates that $\boldsymbol{e}(m)$ may be thought of as the sum of $A_2 \times \Delta\hat{\boldsymbol{d}}(m)$ plus some zero-mean disturbance term $\boldsymbol{\eta}(m)$. Accordingly, recursion (B1) may be rewritten as

$$\Delta\hat{\boldsymbol{d}}(m+1) = (1 - \mu A_2) \times \Delta\hat{\boldsymbol{d}}(m) - \mu\boldsymbol{\eta}(m) \tag{B5}$$

with

$$\boldsymbol{\eta}(m) = [\boldsymbol{B}^H(m)\boldsymbol{B}(m) - A_2 \times \boldsymbol{I}_{QK})\Delta\hat{\boldsymbol{d}}(m) + \boldsymbol{B}^H(m)\boldsymbol{w}(m)\,. \tag{B6}$$

Since $\hat{\boldsymbol{d}}(m) \approx \boldsymbol{d}$ (i.e., $\Delta\hat{\boldsymbol{d}}(m) \approx \boldsymbol{0}$) in the steady-state, it is reasonable to approximate $\boldsymbol{\eta}(m)$ as

$$\boldsymbol{\eta}(m) \approx \boldsymbol{B}^H(m)\boldsymbol{w}(m). \tag{B7}$$

Inspection of (B5) reveals that $\Delta\hat{\boldsymbol{d}}(m)$ may be viewed as the response to $\boldsymbol{\eta}(m)$ of a digital filter with impulse response

$$\rho_k = \begin{cases} -\mu(1-\mu A_2)^{k-1} & k \geq 1 \\ 0 & \text{otherwise.} \end{cases} \tag{B8}$$

Thus, (B5) becomes

$$\Delta\hat{\boldsymbol{d}}(m) = \sum_i \rho_i \boldsymbol{\eta}(m-i)\,. \tag{B9}$$

Recalling that $\boldsymbol{\eta}(m)$ has zero-mean, from (B9) we see that $\mathrm{E}\{\Delta\hat{\boldsymbol{d}}(m)\} = \mathbf{0}$, which means that $\hat{\boldsymbol{d}}(m)$ is unbiased.

Returning to (B7) we observe that vectors $\{\boldsymbol{\eta}(m)\}$ are statistically independent and have the covariance matrix $\boldsymbol{C}_\eta = \sigma_w^2 A_2 \times \boldsymbol{I}_{QK}$. Putting these facts together, from (B9) we have

$$\mathrm{E}\left\{\Delta\hat{\boldsymbol{d}}(m)\Delta\hat{\boldsymbol{d}}^H(m)\right\} = \left[\sigma_w^2 A_2 \sum_k \rho_k^2\right] \times \boldsymbol{I}_{QK}\,. \tag{B10}$$

Next, substituting (B8) into (B10) produces

$$\mathrm{E}\left\{\Delta\boldsymbol{d}(m)\Delta\boldsymbol{d}^H(m)\right\} = \frac{\mu\sigma_w^2}{2-\mu A_2} \times \boldsymbol{I}_{QK} \tag{B11}$$

or, equivalently,

$$\mathrm{E}\left\{\left|\hat{d}_k(m,n) - d_k(n)\right|^2\right\} = \frac{\mu\sigma_w^2}{2-\mu A_2}\,. \tag{B12}$$

At this stage we introduce the noise equivalent bandwidth of the filter in (B8) [16, p.126]

$$B_L = \frac{\mu A_2}{2(2-\mu A_2)T_B} \tag{B13}$$

where T_B is the updating rate of the recursion (B5). Then, collecting (B12) and (B13) yields (30) in the text.

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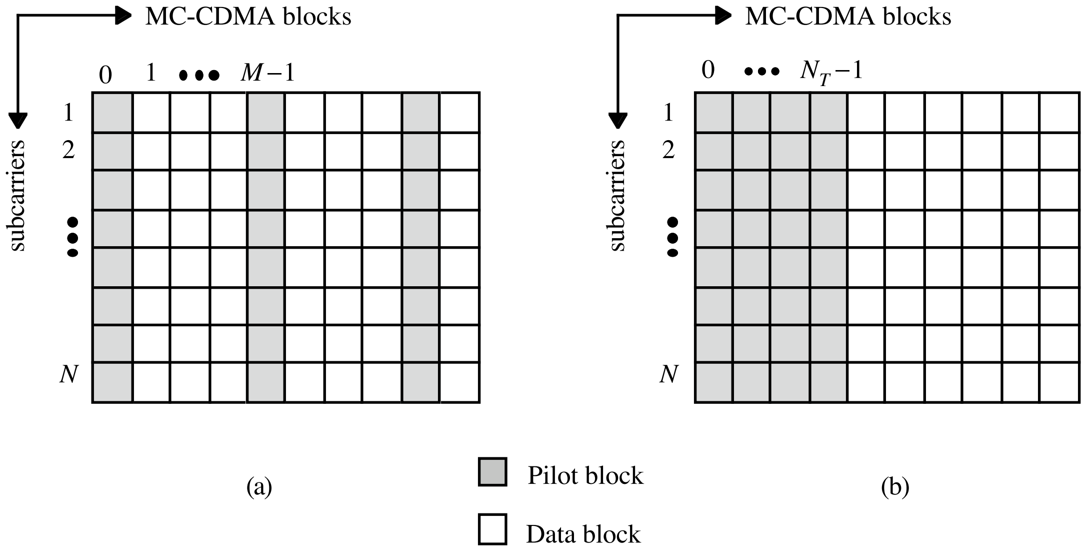

Fig. 1 – (a) Frame structure in the downlink . (b) Frame structure in the uplink

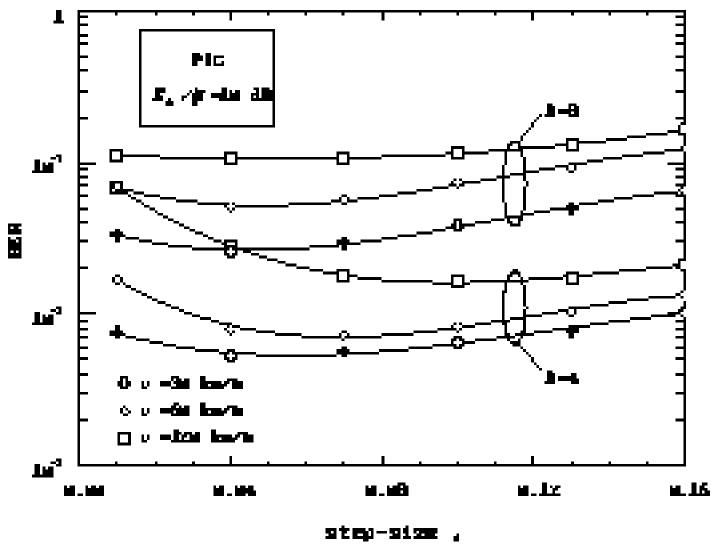

Fig. 2 – Performance of the PIC detector vs. μ for $E_b / N_0 = 10$ dB

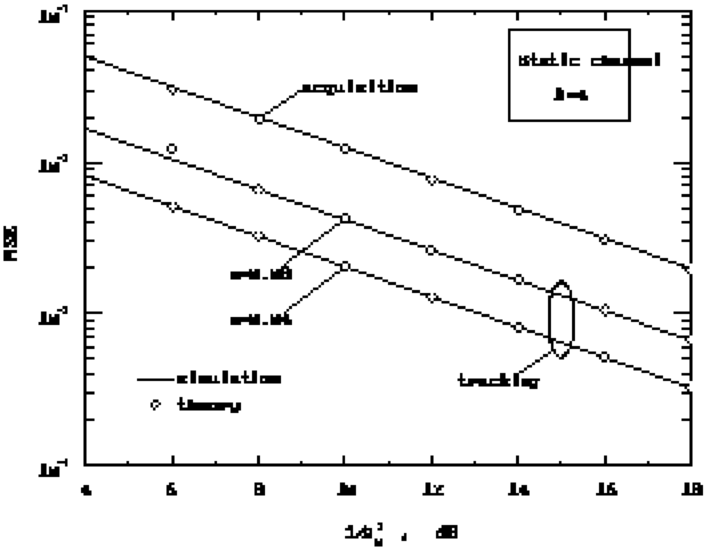

Fig. 3 – MSE of the channel estimates vs. $1/\sigma_w^2$ for $K = 4$

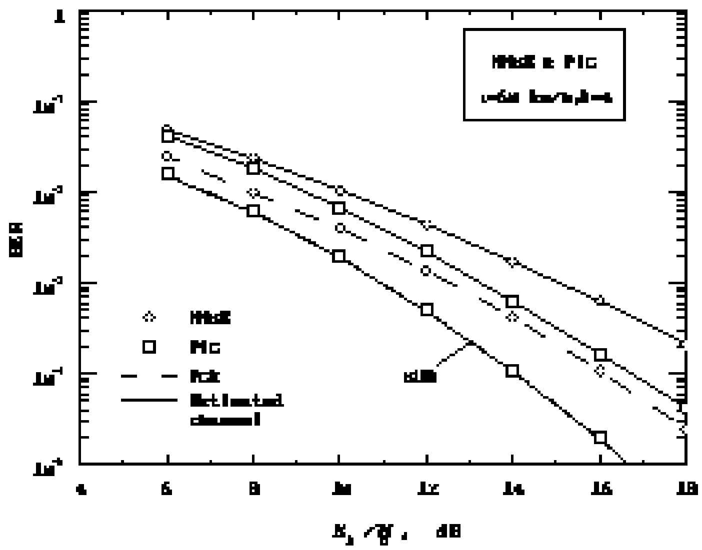

Fig. 4 – Performance of the MMSE and PIC detectors with $v = 60$ km/h and $K = 4$

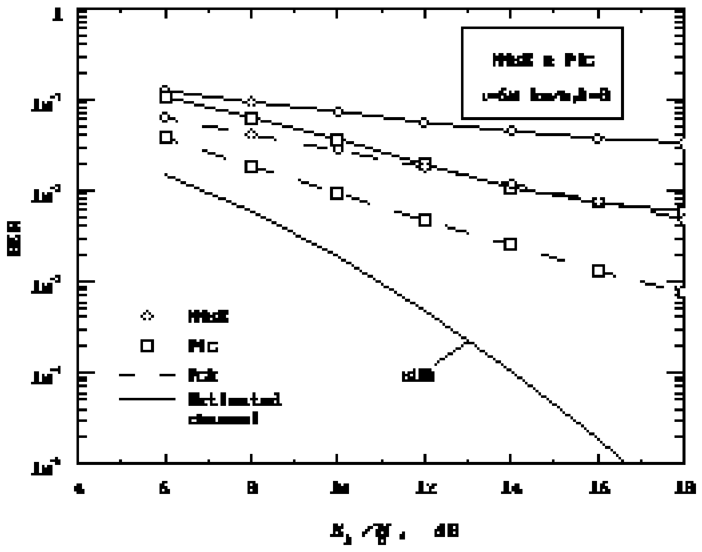

Fig. 5 – Performance of the MMSE and PIC detectors with $v = 60$ km/h and $K = 8$

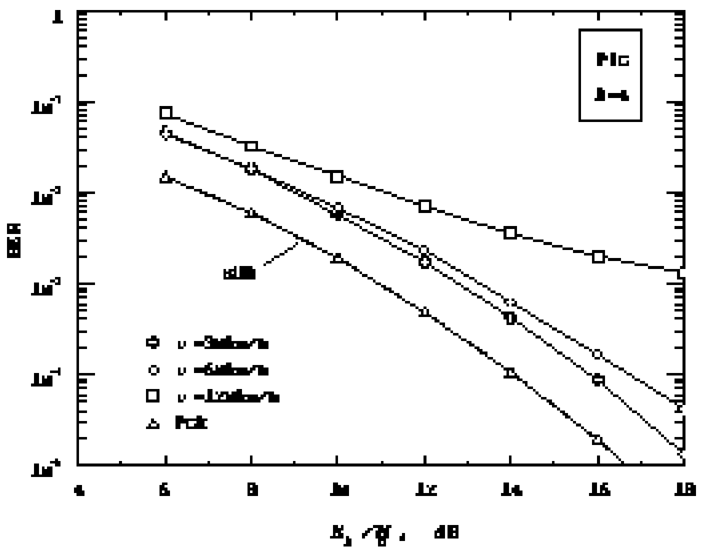

Fig. 6 – Performance of the PIC detector with $K = 4$ and various mobile speeds

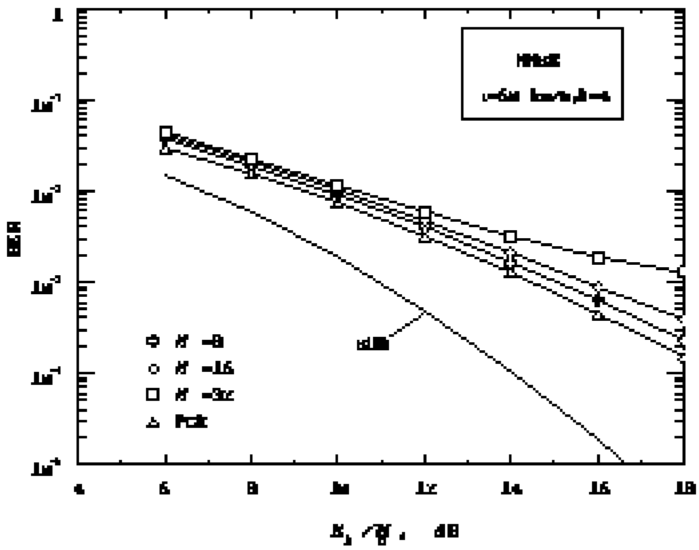

Fig. 7 – Downlink performance with $v = 60$ km/h, $K = 4$ and various M

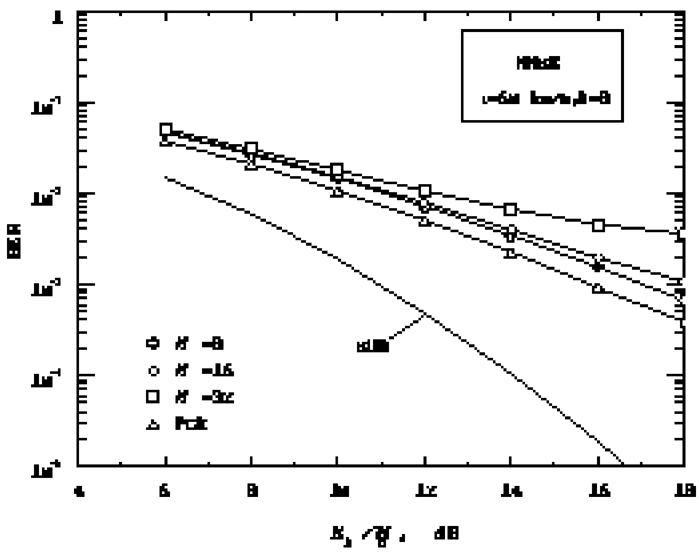

Fig. 8 – Downlink performance with $v = 60$ km/h, $K = 8$ and various M