

Incentive Mechanisms for Economic and Emergency Demand Responses of Colocation Datacenters

Nguyen H. Tran*, Cuong T. Do*, Shaolei Ren[‡], Zhu Han[§] and Choong Seon Hong*

*Department of Computer Engineering, Kyung Hee University, Korea; email: {nguyenth, dtuong, cshong}@khu.ac.kr

[‡]School of Computing and Information Sciences, Florida International University, U.S.A; email: sren@cis.fiu.edu

[§]Electrical and Computer Engineering Department, University of Houston, U.S.A; email: zhan2@uh.edu

Abstract—Demand response programs have been considered critical for power grid reliability and efficiency. Especially, the demand response of datacenters has recently received encouraging efforts due to huge demands and flexible power control knobs of datacenters. However, most current efforts focus on owner-operated datacenters, omitting another critical segment of datacenter business: multi-tenant colocation. In colocation datacenters, while there exist multiple tenants who manage their own servers, the colocation operator only provides facilities such as cooling, reliable power, and network connectivity. Therefore, colocation has a unique feature that challenges any attempts to design a demand response program: uncoordinated power management among tenants. To tackle this challenge, two incentive mechanisms are proposed to coordinate tenant power consumption for demand response under two different scenarios. First, in the case of economic demand response where the operator can adjust an elastic energy reduction target, we show that there is an interaction between the operator and tenant strategies, where each side maximizes its own benefit. Hence, we apply a two-stage Stackelberg game to analyze this scenario and derive this game’s equilibria. Second, in the case of emergency demand response where a fixed energy reduction target must be fulfilled, we devise two incentive schemes with distributed algorithms that can achieve the same optimal social cost, considering two types of tenants: price-taking and price-anticipating tenants. Finally, trace-based simulations are also provided to illustrate the efficacy of our proposed incentive schemes.

I. INTRODUCTION

Demand response programs have been adopted in many countries in order to improve the reliability and efficiency of power grids and to incorporate renewable energy into the power grid (see the survey [1] and references therein). Emergency, standby and economic demand response make up the majority of current demand response programs according to megawatt usage, representing 87% of demand reduction capabilities across all reliability regions [2]. In these programs, participating customers reduce their load/energy consumption upon requests from a load-serving entity (LSE) in order to receive financial reimbursements. Among potential customers, large-scale datacenters are considered vital participants due to two essential properties: (i) their power demands are extremely large, e.g., 91 billion kWh in 2013 in the U.S. [3], and (ii) their energy usage is flexible with many IT computing knobs (e.g., workload shedding/migration) as well as non-IT knobs (e.g., cooling) [4]. The huge yet flexible energy demands of datacenters are considered by grid operators as

a valuable energy buffer to help balance the grid’s power supply and demand [1]. From a practical viewpoint, a field study by Lawrence Berkeley National Laboratory showed that datacenters can reduce their power consumption by 10-25%, without affecting operations [5]. Recently, the U.S. EPA has identified datacenters as a crucial component of demand response [6], evidenced by an event on July 22, 2011 in which hundreds of datacenters worked to prevent an emergency blackout by cutting their electricity usage [7].

However, most of the research efforts have focused mainly on owner-operated datacenters (e.g., Google) [1], [5], [8], [9], while paying less attention to colocation datacenters (e.g., Equinix), simply called *colos*, which represent a crucial segment in datacenter industry. There are many reasons to advocate more research efforts on colos. First, with their critical role in datacenter business, colos provide a universal solution to all types of companies, especially for those who neither want to build their own datacenters nor completely outsource their entire computing demands to any public cloud providers. For example, colos’ customers diversely include many popular Internet websites such as Twitter and Wikipedia [10], [11] and various cloud-computing services such as Salesforce and Box [12]. Second, colos will play a critical role in network traffic infrastructure, since they are increasingly becoming the major physical homes for content delivery network providers that are predicted to support half of the Internet traffic by 2018 [13]. Third, the growth of colos continues to increase sharply: currently there are more than 1200 colos in the U.S. alone [14], and the colos market is expected to grow from \$25 billion to \$43 billion in the next five years [13]. Finally, colos are ideal contributors, at least on par with owner-operated datacenters, to the demand response programs: (i) Colos also have extreme power demands, e.g., colos’ demands in New York exceed 400MW, which is comparable to Google’s global datacenters demand [14], [15]. Moreover, while colos have been shown to consume up to 40% of the datacenter energy in the U.S, owner-operated datacenters like Google only consume 8% [3]; (ii) Colos are often located in urban areas, e.g., Los Angeles [14], where demand responses are required more often than in rural areas where owner-operated datacenters are typically situated, e.g., Google’s datacenters [14], and (iii) colos often have heterogeneous workloads (i.e. different delay sensitivities, peak load, etc.) due to the diversity of tenant’s business models, which helps colos conduct smooth demand

response.

With those great potentials of demand response participation, colos, however, have their unique challenges that invalidate existing demand response methods proposed for owner-operated datacenters [4], [5], [8], [16]–[18]. Instead of fully controlling all IT and non-IT facilities like owner-operated datacenters, a colo is a shared multi-tenant datacenter where multiple tenants house and fully control their servers, while the colo’s operator is mainly responsible for facility support such as power, cooling, and network access. Thus, there exists a split-incentive hindrance for colos’ demand response: the operator may need to reduce energy usage upon the request of an LSE in order to receive financial reimbursement, while tenants have little intention to cut down their power demand because their billings are based mainly on peak-power subscription with fixed rates, which is independent of actual usage [19]. Even if tenants have incentives to reduce demand (e.g., by the operator directly passing down the LSE’s incentives to tenants), they lack coordination to systematically achieve this. Therefore, operator incentives for tenants to coordinate in demand response poses a significant challenge.

Incentive mechanisms have been widely employed for demand-side management in smart grids [20], [21]. However, datacenter demand response is different from that of smart grids due to various control knobs such as cooling, IT load, renewable and/or backup power, etc., requiring a holistic optimization approach. Furthermore, very little effort is focused on colos, which significantly limits the applicability of datacenter demand response because of the colos’ importance and suitability for demand response. Therefore, in this study, we attempt to break the uncoordinated tenants for colos’ demand response based on incentive mechanism design. In the proposed mechanisms, the operator actively and wisely chooses its monetary reward rate and/or demand allocation rules to incentivize tenants to coordinately reduce their energy consumption. Based on reward information, tenants will decide to participate by bidding/announcing their reduced energy to maximize their benefit-minus-cost problems. Specifically, we propose two incentive mechanisms for different demand response scenarios as follows.

- We first examine colos’ economic demand response, where the operator has full control over an adjustable (elastic) demand response target for its own benefit. In this case, the operator will reward tenants with monetary incentives to perform demand response up to a level that can maximize the operator benefit, which can be financial compensation from the LSE or receipt of green certificates. Consequently, upon receiving the announced reward from the operator, self-optimized tenants will individually maximize their net utility. We model this mechanism as a Stackelberg game and analyze its equilibrium. We also propose an algorithm to obtain the optimal solutions of the operator’s mixed-boolean nonlinear problem.
- We next study colos’ emergency demand response. In this scenario, there is a fixed (inelastic) demand target requested by the LSE, and the operator has to solicit the tenants’ demand response to exactly match that target. We

first present a dual-based distributed algorithm for price-taking tenants. Then, we propose an incentive mechanism to deal with price-anticipating (strategic) tenants. Both proposals are designed to achieve colo-wide social cost minimization.

- In the above scenarios, our key contributions are not only reflected in the efficient performance guarantee, but also validated by trace-based simulations. In the former case, a wide range of numerical case studies demonstrate that our linear-complexity scheme can achieve the same performance as the exhaustive search method for the mixed-boolean programming problem. In the latter case, we show that our mechanisms designed for price-taking and strategic tenants can achieve the optimal social cost, which outperforms a random incentive scheme in a 12-hour emergency demand response case study.

The rest of this paper is organized as follows. In Section II, we review the related work. Section III presents the system model. We provide the proposed mechanisms for economic and emergency demand response in Section IV and Section V, respectively. Section VII demonstrates the trace-based simulation results, and Section VIII concludes our work.

II. RELATED WORK

In this section, we first concentrate on the demand response of datacenters. We then discuss how our work contributes to the recent trends in colo demand response.

Demand response is identified as a high-prioritized area, with its potential to reduce up to 20% of the total peak electricity demand of the U.S. [22]. Most initial demand response proposals targeted residential customers [20], [23]. However, demand response of datacenters has recently received significant attention, with various approaches for different types of demand response being considered, such as price response of datacenters to grid operator [24] for economic demand response, or controlling the IT (e.g., turning servers on/off) and non-IT (e.g., cooling) knobs for ancillary and/or emergency demand responses [1], [4], [5], [25]–[27].

While most of the mentioned results focus on owner-operated datacenters, studies on colo demand response are very limited in number. The first study of colos’ economic demand response is [12], though its mechanism is simple and relies on the tenants’ best-effort, which cannot assure the truthfulness of strategic tenants. In terms of emergency demand response, the work in [28] proposes a randomized auction mechanism that can guarantee a 2-approximation of social welfare cost and is approximately truthful. While both are based on a *reverse auction* where tenants must *voluntarily* submit bids first, and the operator will decide winning bids as well as reward amount later. Tenants at first are not concerned with power reduction, so treating their bids as voluntary tasks can lead to pessimistic results on the number of participating tenants. Hence, it is expected that an upfront incentive by the operator will effectively increase tenant participation. Furthermore, in the reverse auction, tenants need to first calculate and disclose complex bids (e.g., cost functions), which might leak their private information. In contrast, we take a forward-mechanism

approach, where the energy reduction and reward allocation rules are announced in advance in order to align tenants' interests to the socially optimal performance. A recent work [29] also studies emergency demand response using supply function bidding. However, while the supply function bidding approach is restricted to a particular "parameterized" function that inherently suffers from social welfare loss, our mechanism aims to achieve the optimal social welfare.

III. SYSTEM MODEL

We consider a colo-datacenter in which a set of $\mathcal{I} = \{1, \dots, I\}$ tenants house their servers. Tenant i has M_i homogeneous servers. A tenant with heterogeneous servers can be viewed as multiple virtual tenants, each having homogeneous servers. We consider a one-period demand response, as in [8], [12], [17], [28], where its duration T is controlled by an LSE, e.g., 15 minutes or 1 hour. During a period, the workload arrival rate to tenant i is denoted by λ_i .

Even though tenants may use various control knobs (e.g., scaling down CPU frequencies, migrating loads to other places) for energy saving, the simple yet widely-studied approach that our study adopts as an example is turning off idle servers [12], [28], [30]. If tenant i has no intention to participate in demand response, all of its servers are active, and the workload will be evenly distributed to all servers to optimize performance [30]; hence, the energy consumption of this case is $e_i = M_i(p_{i,s} + p_{i,d} \frac{\lambda_i}{M_i \mu_i})T$ [12], where $p_{i,s}$ and $p_{i,a}$ are the static and active powers of each server, respectively, μ_i is a server's service rate measured in terms of the amount of workload processed per unit time, and $\frac{\lambda_i}{M_i \mu_i}$ is the server utilization with M_i active servers. In contrast, when performing demand response by turning off m_i servers, the energy consumption of tenant i is $e'_i = (M_i - m_i)(p_{i,s} + p_{i,d} \frac{\lambda_i}{(M_i - m_i) \mu_i})T$. Therefore, IT-only (e.g., not including cooling) energy reduction by tenant i is

$$\Delta e_i = e_i - e'_i = m_i \frac{p_{i,s} \cdot T}{PUE}, \quad (1)$$

where PUE is the power usage effectiveness measuring the energy efficiency of the colo. In the sequel, we assume $\frac{p_{i,s} \cdot T}{PUE} = 1$ without loss of generality (w.l.o.g.); hence, we will use Δe_i and m_i interchangeably.

Turning servers off can have negative effects on tenant performance, inducing tenant costs. We rely on two typical costs that are widely used for tenants: the wear-and-tear cost and Service Level Agreement (SLA) cost [12], [30].

Wear-and-tear cost: This cost, which occurs when tenants switch/toggle servers between active and idle states in every period, is linear with the number of turned-off servers and so can be modeled as $\omega_{i,1} \cdot m_i$, where $\omega_{i,1}$ is a monetary weight (i.e., \$/server.)

SLA cost: Since many Internet services hosted in datacenters are sensitive to response/delay time, the SLA cost can be viewed proportionally to tenant average response time. Using the M/M/1 queue, the average response time of each tenant i 's workload is $\frac{1}{\mu_i - \frac{\lambda_i}{M_i - m_i}}$. We note that the queueing model has been widely used as a reasonable approximation for the actual service process [31], [32]. The total SLA cost of a tenant can

be modeled as $\omega_{i,2} \cdot d_i(m_i)$, where $d_i(m_i) = \frac{\lambda_i}{\mu_i - \frac{\lambda_i}{M_i - m_i}}$, and $\omega_{i,2}$ is a monetary weight (i.e., \$/delay.)

Therefore, tenant i 's total cost when turning off m_i servers is

$$C_i(m_i) = \omega_{i,1} \cdot m_i + \omega_{i,2} \cdot d_i(m_i). \quad (2)$$

IV. INCENTIVE MECHANISM FOR COLOS' ECONOMIC DEMAND RESPONSES

In this section, we first present the motivation for studying the economic demand response of colos using a two-stage Stackelberg game between the operator (leaders) and tenants (followers). We then analyze this game's equilibria using backward induction method, and propose an algorithm to achieve equilibria. Finally, we discuss about the practical implementations of the proposed algorithm.

A. Economic Demand Response: A Two-stage Stackelberg Game Approach

Economic demand response programs generally indicate how customers can actively respond to price signals [33]. For example, during peak times with high wholesale prices, the customers (i.e., colos), who receive signals from the LSE, can reduce their consumption to receive some economic benefits corresponding to the amount of energy reduction. Since the reduction volume is not necessarily fixed, many customers find this program appealing due to its flexibility.

In this scenario, even though a colo can freely determine a desired reduction volume, its operator cannot directly control the tenants' servers to proceed the demand response. Therefore, the operator's purpose is to incentivize tenants to reduce their energy to a level that can maximize the operator's benefit. Consequently, upon receiving the announced reward from the operator, rational tenants will individually maximize their own profits. Observing this hierarchical structure between the operator and tenants, we study this economic demand response for colos by using a Stackelberg game approach. The strategies of players in each stage of this game are presented sequentially.

Tenants (Stage II). Since the operator is the leader with a first-move advantage, it will first announce a reward rate r (e.g., \$/kWh) that it is willing to pay tenants for turning off their servers. Given r , at Stage II, each rational tenant i 's strategy is to choose a number of turned-off servers m_i that will maximize its net utility as follows

$$\underset{m_i}{\text{maximize}} \quad u_i(m_i, r) = r m_i - C_i(m_i) \quad (3)$$

$$\text{s.t.} \quad m_i \geq 0. \quad (4)$$

Since the number of servers can be very large, e.g., thousands, we can relax m_i as a continuous variable [8]. We have

$$C_i''(m_i) = \frac{2\lambda_i^2 \mu_i \omega_{i,2}}{((M_i - m_i) \mu_i - \lambda_i)^3}, \quad (5)$$

which means $C_i(m_i)$ is a strictly convex function when tenant i 's server workload is less than its service rate, i.e., $C_i''(m_i) > 0$ when $\frac{\lambda_i}{M_i - m_i} < \mu_i$. We further relax the feasible constraint

$0 \leq m_i \leq M_i$ to (4), which has no effect on problem (3) since its feasible solutions are always strictly less than M_i (i.e., $C_i(m'_i) = \infty$, $m'_i \geq M_i$). Then, since $u_i(m_i)$ is strictly concave, there exists a unique solution $m_i^*(r)$, $\forall i$, for a given r in Stage II.

Operator (Stage I). Knowing that each tenant i 's strategy will be $m_i^*(r)$, the operator's strategy is to choose an optimal r^* of the following profit maximization problem

$$\max_{r \geq 0} \mathcal{U}(r, \{m_i^*\}) = U\left(\sum_{i \in \mathcal{I}} m_i^*(r)\right) - r \sum_{i \in \mathcal{I}} m_i^*(r), \quad (6)$$

where $U(\cdot)$ is the colo utility, which represents a financial compensation from the LSE or a green certificate achieved with respect to energy reduction, balanced with the cost spent for incentivizing tenants $r \sum_{i \in \mathcal{I}} m_i^*(r)$. Even though we have no assumption on a specific utility function, some typical candidates are provided for case studies in Section VII.

Stackelberg Equilibrium. Denoting a solution to the operator's profit maximization by r^* , we have the following definition.

Definition 1. $(r^*, \{m_i^*\})$ is a Stackelberg equilibrium if it satisfies the following conditions for any values of r and $\{m_i\}$

$$\mathcal{U}(r^*, \{m_i^*\}) \geq \mathcal{U}(r, \{m_i^*\}), \quad (7)$$

$$u_i(m_i^*, r^*) \geq u_i(m_i, r^*), \forall i. \quad (8)$$

Next, we use the backward-induction method to analyze the Stackelberg equilibria: the Stage-II problem is first solved to obtain $\{m_i^*\}$, which is then used to solve the Stage-I problem to obtain r^* .

B. Stackelberg Equilibrium: Analysis and Algorithm

By the first-order condition $\frac{\partial u_i}{\partial m_i} = r - C'_i(m_i) = 0$, we have the unique solution m_i^* of tenant i for a given r as follows

$$m_i^*(r) = [f_i(r)]^+ := \left[M_i - \rho_i \left(1 + \sqrt{\frac{\omega_{i,2}}{r - \omega_{i,1}}} \right) \right]^+, \forall i, \quad (9)$$

where $[x]^+ = \max\{x, 0\}$, and $\rho_i := \frac{\lambda_i}{\mu_i}$.

Then, by substituting (9) into (6), the operator's problem is formulated as follows

$$\begin{aligned} & \maximize_r \quad U\left(\sum_{i \in \mathcal{I}} [f_i(r)]^+\right) - r \sum_{i \in \mathcal{I}} [f_i(r)]^+ \quad (10) \\ & \text{s.t.} \quad r \geq 0. \end{aligned}$$

Due to the operator $[\cdot]^+$, problem (10) is non-convex. Specifically, if we define a new variable

$$z_i = \begin{cases} 1, & r > \kappa_i; \\ 0, & \text{otherwise,} \end{cases} \quad (11)$$

where

$$\kappa_i := \omega_{i,1} + \frac{\omega_{i,2}\rho_i^2}{(M_i - \rho_i)^2}, \quad (12)$$

Algorithm 1 Operator's Revenue Maximizer

- 1: Sort tenants according to $\kappa_1 < \kappa_2 < \dots < \kappa_I$.
- 2: $\mathcal{A} = \{\}$, $\mathcal{B} = \mathcal{I}$, $j = I$;
- 3: **while** $j > 0$ **do**
- 4: Find the solutions r_j to the following problem

$$\max_{r \geq \kappa_1} U\left(\sum_{i \in \mathcal{B}} f_i(r)\right) - r \sum_{i \in \mathcal{B}} f_i(r) \quad (14)$$

- 5: **if** $r_j > \kappa_j$, **then** $\mathcal{A} = \mathcal{A} \cup \{r_j\}$;
 - 6: **end if**
 - 7: $\mathcal{B} = \mathcal{B} \setminus j$;
 - 8: $j = j - 1$;
 - 9: **end while**
 - 10: Return $r_j \in \mathcal{A}$ with highest optimal values of (14).
-

then $m_i^*(r) > 0$ when $z_i = 1$, and $m_i^*(r) = 0$ when $z_i = 0$. Therefore, problem (10) is equivalent to

$$\begin{aligned} & \maximize_{r, \{z_i\}_{i \in \mathcal{I}}} \quad U\left(\sum_{i \in \mathcal{I}} z_i \cdot f_i(r)\right) - r \sum_{i \in \mathcal{I}} z_i \cdot f_i(r) \quad (13) \\ & \text{s.t.} \quad r \geq 0, \\ & \quad \quad z_i \in \{0, 1\}, \forall i. \end{aligned}$$

We see that problem (13) is a mixed-boolean programming, for which we may acquire an exponential-complexity effort (i.e., 2^I configurations of $\{z_i\}_{i \in \mathcal{I}}$) to solve by the exhaustive search. However, by unveiling its special structure, we propose an algorithm, namely Algorithm 1, that can find the solutions of problem (13) with linear complexity as follows.

Proposition 1. *Algorithm 1 can solve the Stage-I equivalent problem (13) with linear complexity.*

Proof: Please see Appendix A. ■

Denoting the Algorithm 1' outputs as r^* (which can be multiple values) and $m_i^* := m_i^*(r^*)$, we have the following result.

Theorem 1. *The Stackelberg equilibria of colos' economic demand response are the set of pairs $(r^*, \{m_i^*\})$.*

Proof: Please see Appendix B. ■

Based on this equilibria analysis, we next examine the implementation of the Stackelberg game-based incentive mechanism.

C. Implementation Operations

The main operations of colos' economic demand response can be implemented in the following order:

Step 1: Each self-optimized tenant submits its best response (9) to the operator.

Step 2: After collecting all of these best responses, the operator determines its profit maximization (6) using Algorithm 1 to achieve r^* and broadcasts this r^* to all tenants.

Step 3: Based on this r^* , each tenant will correspondingly turn off m_i^* servers.

We have further remarks for the scheme's operation as follows

- The incentive mechanism with Algorithm 1 is one-round and is centralized: the operator needs to know the values $\omega_{i,1}$, $\omega_{i,2}$, M_i , and ρ_i of all tenants. In practice, the operator may have no such information, which inspires the distributed approaches in the following sections.
- A uniform reward rate r is applied to all tenants, which is meaningful in terms of fairness.

V. INCENTIVE MECHANISM FOR COLOS' EMERGENCY DEMAND RESPONSE

In this section, we first present the motivations for colos' emergency demand response with social welfare maximization. We then study this scenario for price-taking and price-anticipating tenants.

A. Emergency Demand Response: A Social Welfare Optimization

Emergency (or reliability) demand response indicates that the response is mandatory (with penalty for non-compliance) for the participants, who are not only compensated for their reduction during emergency events, but are also paid for their availability (i.e., even when no emergent signal is triggered) [33]. Such programs are currently employed by many Independent System Operators (ISO) such as New England or PJM, where the customers' contracts can be established three years in advanced [34]. In detail, if there are some reliability issues (e.g., forecast capacity shortages), the LSE will trigger a signal to customers from at least 10 minutes to one day in advance, and customers must comply with the notified reduction volume. In current practice, colos often participate in emergency demand response using onsite backup diesel generators. However, relying totally on diesel generators is not cost effective. Furthermore, frequently using diesel can be environmentally dirty, while datacenters are well motivated to reduce dirty energy for green certificate pursuit (e.g., LEED program [35]). Therefore, it is critical for colos to extract energy reduction from tenants.

In this scenario, the main concern of the operator is how to solicit the tenants to reduce their energy usage in order to satisfy at least a fixed demand target requested by the LSE [28]. Consequently, we consider a social welfare optimization problem (SWO) in a colo system such that the sum of tenant reductions is at least an amount D requested by the LSE as follows

$$\text{SWO : } \underset{\Delta e_i \geq 0, \forall i}{\text{minimize}} \quad \sum_{i \in \mathcal{I}} C_i(\Delta e_i) \quad (15)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{I}} \Delta e_i \geq D. \quad (16)$$

In this problem, we implicitly assume that tenant's power reduction is sufficient to satisfy the target D . If not, we use diesel generation to make up the shortfall in Section VI.

We see that the operator's benefit (i.e., LSE payment for colos) is not included in problem SWO since this benefit (as well as penalty for non-compliance) is often pre-determined via contracts and has no impact on how the operator achieves reduction D . In other words, the operator benefit from the LSE is independent of the reward that the operator grants to

tenants for emergency response. Clearly, it is different from the economic demand response where the operator's benefit, encoded by a utility function, flexibly depends on the LSE conditions (e.g., wholesale prices). Furthermore, the objective of SWO is only to minimize the total tenant costs since the internal reward transfer between the operator and tenants cancels and has no effect on the social cost. We note that several works also study the SWO of emergency demand response for non-colos contexts [36] or for colos with different approaches [28].

Since SWO is a convex problem, its optimal primal and dual variables $(\Delta e_i^*, \nu^*)$ can be characterized by the KKT condition as follows

$$\begin{cases} \nu^* = C'_i(\Delta e_i^*), & \text{if } \Delta e_i^* > 0; \\ \nu^* \leq C'_i(0), & \text{if } \Delta e_i^* = 0, \forall i; \\ \sum_{i \in \mathcal{I}} \Delta e_i^* = D. \end{cases} \quad (17)$$

Because the objective of SWO is strictly convex, if $(\Delta e_i^*, \nu^*)$ exists, then it is unique. We note that, if D is too large, there are no feasible solutions of SWO; therefore, we additionally consider this case in Section VI.

The main purpose of the operator is to set a reward rate that aligns the self-optimized tenants' interests to the solution of SWO characterized in (17). However, the operator's incentive mechanism should take into account whether tenants are price-taking users who just accept the reward rate, or they are price-anticipating on how their actions impact the rate. While the price-taking assumption justifies the large number of users where no one has a market power to alter the price, price-anticipating is more likely in wholesale colo-datacenters, where there are typically only a few large tenants, each having a large power demand. In the next section, we will design two different incentive schemes that can solve SWO for these two types of tenants.

B. SWO for Price-taking Tenants

We propose an incentive scheme for emergency demand response that can align the price-taking tenants' strategies to the solutions of SWO in Algorithm 2, which is an iterative and distributed algorithm based on dual-decomposition methods [37], [38].

The operations of Algorithm 2 can be explained as follows. According to dual methods, the Lagrangian variable ν plays the role of the reward rate (e.g., \$/kWh) that the operator is willing to pay tenants to reduce the energy usage. Therefore, given this reward rate announced by the operator at each iteration k , each tenant will submit to the operator a reduction volume that maximizes the following tenant's net utility

$$\max_{\Delta e_i \geq 0} \nu^{(k)} \Delta e_i - C_i(\Delta e_i), \forall i, \quad (20)$$

and the solution to this problem is given in line 3 of Algorithm 2, where $C_i'^{-1}(\cdot)$ is the inverse of the derivative of cost function $C_i(\cdot)$. Then, after collecting all tenants' submitted reduction levels, the operator will adjust the reward rate as in line 4 of Algorithm 2 with an appropriate step size rule $\gamma^{(k)}$ to balance the total reduced energy with target D : decrease the reward rate if over-provision ($\sum_{i \in \mathcal{I}} \Delta e_i > D$) and vice versa.

Algorithm 2 Distributed Algorithm for Price-taking Tenants

- 1: Operator initializes and broadcasts a random reward rate $\nu^{(0)} > 0$ to all tenants, $k = 0$;
- 2: **repeat**
- 3: Tenant i submits its reduced energy level

$$\Delta e_i^{(k)} = \left[C_i'^{-1}(\nu^{(k)}) \right]^+; \quad (18)$$

- 4: Operator updates its reward rate as follows

$$\nu^{(k+1)} = \left[\nu^{(k)} - \gamma^{(k)} \left(\sum_{i \in \mathcal{I}} \Delta e_i^{(k)} - D \right) \right]^+, \quad (19)$$

then broadcasts $\nu^{(k+1)}$ to all tenants;

- 5: $k = k + 1$;
 - 6: **until** $|\nu^{(k+1)} - \nu^{(k)}| < \epsilon$.
-

When the algorithm converges with a number of iterations, i.e., $\nu^{(k+1)} \approx \nu^{(k)}$, with a sufficiently small ϵ at line 6, we see that (18) and (19) will satisfy the KKT condition (17) of SWO, inducing the optimal solutions. We next provide the optimal performance of Algorithm 2.

Proposition 2. *Algorithm 2 converges to the unique solution of SWO with an appropriate step-size rule.*

The proof of Proposition 2 follows the lines of a similar technique in [37] so that it is omitted due to limited space.

We have some further remarks for Algorithm 2 as follows:

- In contrast to Algorithm 1 of economic demand response, tenants are not required to reveal their private information to the operator.
- However, similar to Algorithm 1, the operator can use a uniform reward rate ν for fairness.

C. SWO for Strategic Tenants

In this subsection, based on a formulated bidding game, we will design an incentive mechanism for this game to handle the operator's concern of how to align the strategic tenants' incentives to the social optimum point for emergency demand response.

Bidding game. We consider I strategic tenants bidding for a finite amount of D energy reduction to receive compensation rewards from the operator. Each tenant i is encouraged to bid θ_i , representing its aggressiveness of energy reduction. We denote the bid vector of all tenants by $\theta = (\theta_1, \dots, \theta_I)$. We also denote $\theta_{-i} = (\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_I)$ the bid vector of all tenants excluding i . We further denote $\Theta = \sum_{i \in \mathcal{I}} \theta_i$, $\Theta_{-i} = \sum_{j \neq i} \theta_j$. Based on the bids of tenants, the provider will reward tenant i an amount $R_i(\theta_i, \theta_{-i})$ for reducing a quantity $\Delta e_i(\theta_i, \theta_{-i})$. Hence, the payoff function of tenant i with bid θ_i is given as the following

$$u_i(\theta_i, \theta_{-i}) = R_i(\theta_i, \theta_{-i}) - C_i(\Delta e_i(\theta_i, \theta_{-i})). \quad (21)$$

Since the tenants unilaterally maximize their own payoff by adjusting their bids, we have a bidding game:

- *Players:* tenants in the set \mathcal{I} ;
- *Strategy:* $\theta_i \geq 0, \forall i \in \mathcal{I}$;

- *Payoff function:* $u_i(\theta_i, \theta_{-i}), \forall i \in \mathcal{I}$.

For this game, a bidding profile θ^{ne} is called a Nash Equilibrium (NE) if and only if

$$\theta_i^{ne} = \arg \max_{\theta_i \geq 0} u_i(\theta_i, \theta_{-i}^{ne}), \quad \forall i. \quad (22)$$

Efficient Mechanism Design. The existence of a NE of the bidding game is not obvious, and if it exists, it may not be unique. Therefore, the challenge boils down to how the operator designs its reward and energy reduction rules such that the result of the tenants' bidding game is the existence and uniqueness of an efficient NE (i.e., the same as SWO solutions). To do that, we design an Efficient and Proportional Mechanism (EPM) as follows.

EPM: Operator

Energy reduction rule:

$$\Delta e_i(\theta_i, \Theta_{-i}) = \begin{cases} \frac{\theta_i}{\theta_i + \Theta_{-i}} D, & \theta_i \neq 0; \\ 0, & \theta_i = 0. \end{cases} \quad (23)$$

Reward rule:

$$R_i(\theta_i, \Theta_{-i}) = \frac{\Theta_{-i} D}{\alpha + 1} (\Theta_{-i}^{-\alpha-1} - (\theta_i + \Theta_{-i})^{-\alpha-1}). \quad (24)$$

Tenants

Bidding Strategy:

$$\theta_i = \arg \max_{x \geq 0} [R_i(x, \Theta_{-i}) - C_i(\Delta e_i(x, \Theta_{-i}))], \forall i. \quad (25)$$

The basic operation of EPM in a demand response period can be described as follows.

Step 1: First, after receiving an emergency signal from LSE with a specific reduction amount D , the operator will announce the parameters, i.e., D , α , and Θ_{-i} , of energy reduction and reward rules as in (23) and (24), respectively, to tenant i , $\forall i$.

Step 2: Then, each rational tenant will strategically choose a bid to maximize its net utility according to (25).

Step 3: When an equilibrium θ^{ne} is reached, the emergency demand response proceeds: tenant i reduces energy usage by an amount $\Delta e_i(\theta_i^{ne}, \theta_{-i}^{ne})$ and receives its reward $R_i(\theta_i^{ne}, \theta_{-i}^{ne})$.

We notice that EPM is different from recent related incentive mechanisms proposed for colos [12], [28] based on the combinatorial reverse auction method where tenants first submit their bids and costs, and the operator then decides which bids to accept and how much money to reward tenants. The design of EPM is based on the principle of g -mechanism [39]. In these rules, while tenants are allocated their reduction proportional to their bids, the reward rule is designed to align tenant's interests with the social welfare. This mechanism can be briefly described with a proportional allocation rule similar to (23), where the payment of buyers to sellers is as follows

$$R_i(\theta_i, \Theta_{-i}) = \begin{cases} \Theta_{-i} \int_0^{\theta_i} \frac{g(t + \Theta_{-i})}{(t + \Theta_{-i})^2} dt, & \theta_i > 0; \\ 0, & \theta_i = 0. \end{cases} \quad (26)$$

The simple g -mechanism is proposed in the context of allocating a divisible network resource to finite users. This mechanism is flexible in that we can choose a function $g(\cdot)$

Algorithm 3 Distributed Algorithm for EPM

1: $k = 0$, the operator set a random $\tilde{v}^{(0)} > 0$ and broadcasts $\Theta_{-i}^{(0)} = g^{-1}(\tilde{v}^{(0)})$, D , and α to all tenants;

2: **repeat**

3: Tenant i submits $\theta_i^{(k)} \geq 0$ that satisfies

$$\frac{\theta_i^{(k)} D}{\theta_i^{(k)} + \Theta_{-i}^{(k)}} = C_i^{-1} \left((\theta_i^{(k)} + \Theta_{-i}^{(k)})^{-\alpha} \right), \forall i; \quad (27)$$

4: Operator updates its virtual reward rate as follows

$$\tilde{v}^{(k+1)} = \left[\tilde{v}^{(k)} - \gamma^{(k)} \left(\sum_{i \in \mathcal{I}} \frac{\theta_i^{(k)} D}{\theta_i^{(k)} + \Theta_{-i}^{(k)}} - D \right) \right]^+ \quad (28)$$

and sends $\Theta_{-i}^{(k+1)} = g^{-1}(\tilde{v}^{(k+1)}) - \theta_i^{(k)}$ to tenant i , $\forall i$;

5: $k = k + 1$;

6: **until** $|\tilde{v}^{(k+1)} - \tilde{v}^{(k)}| < \epsilon$.

that is suitable to a specific context. The authors in [39] also provide some typical functions $g(\cdot)$ intended to the users' costs; however, they cannot be applied in our context where we design a reward mechanism. In EPM, we choose $g(\Theta) = \Theta^{-\alpha}$, characterized by the parameter $\alpha > 0$, for the reward rule (24) to align the NE of the bidding game with the solution of SWO. We have the following result of the proposed EPM.

Theorem 2. *With EPM, the bidding game either has a trivial NE $\theta^{ne} = 0$ or a unique non-trivial NE θ^{ne} (with at least two tenants have positive bids) such that $\Delta e_i(\theta_i^{ne}, \theta_{-i}^{ne})$, $i \in \mathcal{I}$, is the unique solution of the SWO.*

Proof: Please see Appendix C. ■

From this proposition, we see that the existence of a non-trivial unique efficient NE is what the operator aims to. However, there is no guideline on how to achieve this desired equilibrium at step 3 of the EPM operation (i.e., the operator clearly wants to avoid the trivial equilibria).

If the operator can calculate the non-trivial NE in advance, then all steps of EPM can proceed in only one round. But this capability only exists when the operator can solve a set of fixed-point equations (22), which requires accessing each tenant i 's private cost information $C_i(\cdot)$, $\forall i$. In this case, the well-known VCG mechanism is favored. In contrast, we prefer a distributed algorithm that can help tenants protect their privacy and use their bids as the only means to communicate with the operator.

Distributed algorithm for EPM. Inspired by Algorithm 2, we propose a distributed implementation for EPM, which is presented in Algorithm 3. Intuitively, it is designed based on two principles: (i) the EPM rules to guarantee an efficient NE according to Proposition 2, and (ii) the dual-based gradient methods to enable the distributed fashion similar to Algorithm 2. We explain the operation of Algorithm 3 as follows.

At the beginning of each demand response period (line 1), the operator will broadcast D , α , and random initial values of $\Theta_{-i} > 0$ to all tenants i , $\forall i$, according to the

energy reduction and reward rules of EPM. The initial Θ_{-i} is randomly set to a positive value to avoid the trivial NE, as in Proposition 2. Then, the next loop (lines 2-5) is simply iterating the interaction between the operator and tenants in steps 1 and 2 of EPM until the equilibrium is reached. Specifically, at line 4, in each iteration k , the operator collects all bids and calculates a new value $\Theta_{-i}^{(k+1)}$ for tenant i based on an updated *virtual rate* $\tilde{v}^{(k+1)}$ that tracks the values of $g(\cdot)$ (EPM step 1). After receiving its value, at line 3, tenant i updates its bid (27), which is the solution to its net utility maximization problem (25) (EPM step 2). The algorithm will stop if the convergence condition is satisfied at line 6, where $\tilde{v}^{(k+1)} \approx \tilde{v}^{(k)}$ with a sufficiently small ϵ .

Proposition 3. *Algorithm 3 can converge to the unique efficient NE θ^{ne} with an appropriate step-size rule.*

Proof: Please see Appendix D. ■

We have some remarks for Algorithm 3 as follows:

- Similar to Algorithm 2, tenants need not reveal their private information (e.g., $\omega_{i,1}$, $\omega_{i,2}$, M_i , and ρ_i) to the operator. However, unlike Algorithm 2 where each tenant submits its Δe_i and the operator broadcasts the reward rate, in Algorithm 3, based on the bids of all tenants $\{\theta_i\}$, the operator announces the aggregate of other tenant bids Θ_{-i} for tenant i to update its strategy. Therefore, it is not necessary for tenant i to know each individual bid of other tenants.
- We can see that tenants have discriminate reward rates: $R_i(\theta_i^{ne}, \theta_{-i}^{ne}) / \Delta e_i(\theta_i^{ne}, \theta_{-i}^{ne})$. We observe through simulations (Section VII) that this rate is higher than the optimal rate ν^* of Algorithm 2, inducing that the operator has to give more incentives to strategic tenants than to price-taking ones in order to achieve the optimal social cost.

VI. PRACTICAL EXTENSION DISCUSSION

In this section, we discuss other practical scenarios for colos' demand responses. We first consider the operator with a fixed reward budget constraint in the case of economic demand response. We next examine how colos use their backup generator to fulfill an LSE's emergency demand request which cannot be achieved through tenant reduction.

A. Economic Demand Response: Colos with a Reward Budget

In Section IV, the Stage-I operator model has no restriction on budget. We include the budget, denoted by B , into this model, where the original operator's problem (6) can be modified as follows

$$\begin{aligned} & \underset{r \geq 0}{\text{maximize}} && U \left(\sum_{i \in \mathcal{I}} m_i^*(r) \right), && (29) \\ & \text{s.t.} && r \sum_{i \in \mathcal{I}} m_i^*(r) \leq B. \end{aligned}$$

Analogously, by introducing variable z_i according to (11), problem (29) is equivalent to

$$\begin{aligned} & \underset{r, \{z_i\}_{i \in \mathcal{I}}}{\text{maximize}} && U \left(\sum_{i \in \mathcal{I}} z_i f_i(r) \right), & (30) \\ & \text{s.t.} && r \sum_{i \in \mathcal{I}} z_i f_i(r) \leq B, \\ & && r \geq 0, \\ & && z_i \in \{0, 1\}, \forall i. \end{aligned}$$

By assuming a fixed configuration of $\{z_i = 1\}_{i \in \mathcal{I}}$, problem (30) is reduced to

$$\begin{aligned} & \underset{r \geq 0}{\text{maximize}} && U \left(\sum_{i \in \mathcal{I}} f_i(r) \right), & (31) \\ & \text{s.t.} && \sum_{i \in \mathcal{I}} r f_i(r) \leq B. \end{aligned}$$

Then we can solve (30) in a similar way as with Algorithm 1, replacing unconstrained problem (14) at line 4 by its constrained version (31). We note that the second derivative of $r f_i(r)$ is $\frac{(r-4\omega_{i,1})\sqrt{\omega_{i,2}\rho}}{4(r-\omega_{i,1})^{3/2}}$, which shows that $r f_i(r)$ is convex when $r > 4\omega_{i,1}$ and concave otherwise. Therefore, problem (31) is convex when $r > 4 \max_i \omega_{i,1}$ and non-convex otherwise, which complicates the analysis. Further simplifications are first solving the non-convex part in the range $0 \geq r \geq 4\omega_{i,1}$ using the branch-and-bound method and the convex part in the range $r > 4\omega_{i,1}$ using the interior-point method, then comparing and choosing the better solutions.

B. Emergency Demand Response: The Use of Backup Generators

In Section V, the social cost minimization problem (15) is infeasible if the request target D is higher than the tenant cost (e.g., $D > \sum_i M_i$). In this case, the operator has to rely on backup energy storage, e.g., diesel generator, pre-charged batteries [16], etc., to fulfill the mismatch. A similar model was studied in [28] with a different approach. Let y denote the backup energy used by the operator and β denote the cost of backup usage per kWh; a new social cost problem is then formulated as follows

$$\begin{aligned} \text{SWO}' : & \underset{y \geq 0, \{\Delta e_i\}_{i \in \mathcal{I}}}{\text{minimize}} && \beta y + \sum_{i \in \mathcal{I}} C_i(\Delta e_i) & (32) \\ & \text{s.t.} && y + \sum_{i \in \mathcal{I}} \Delta e_i = D. \end{aligned}$$

The Lagrangian of problem (32) is as follows

$$\begin{aligned} L(\{\Delta e_i\}, y, \{\mu_i\}, \nu, \zeta) &= \beta y + \sum_{i \in \mathcal{I}} C_i(\Delta e_i) \\ &- \nu(y + \sum_i \Delta e_i - D) - \sum_i \mu_i \Delta e_i - \zeta y, \end{aligned}$$

where $\{\mu_i\}, \nu$, and ζ are dual variables. Since SWO' is a convex problem, its optimal primal and dual variables are characterized by the KKT conditions provided as follows

$$\begin{cases} C'_i(\Delta e_i^*) - \nu^* - \mu_i^* &= 0, \quad \forall i; \\ \beta - \nu^* - \zeta^* &= 0; \\ y^* + \sum_{i \in \mathcal{I}} \Delta e_i^* &= D; \\ y^* \geq 0, \zeta^* \geq 0, \Delta e_i^* \geq 0, \mu_i^* \geq 0, \forall i; & (33) \\ \mu_i^* \Delta e_i^* &= 0, \quad \forall i; \\ \zeta^* y^* &= 0. \end{cases}$$

Algorithm 4 Distributed Algorithm for SWO'

1: Operator initializes and broadcasts a random reward rate $\nu^{(0)} > 0$ to all tenants, $k = 0$;

2: **repeat**

3: Tenant i submits its reduced energy level

$$\Delta e_i^{(k)} = \arg \max_{\Delta e_i \geq 0} \left[\nu^{(k)} \Delta e_i - C_i(\Delta e_i) \right]; \quad (35)$$

4: Operator updates its reward rate

$$\nu^{(k+1)} = \left[\nu^{(k)} - \gamma^{(k)} \left(y^{(k)} + \sum_{i \in \mathcal{I}} \Delta e_i^{(k)} - D \right) \right]^+, \quad (36)$$

then broadcasts $\nu^{(k+1)}$ to all tenants;

5: Operator updates its backup usage

$$y^{(k+1)} = \begin{cases} \left[D - \sum_{i \in \mathcal{I}} \Delta e_i^{(k)} \right]^+, & \text{if } \nu^{(k+1)} \geq \beta, \\ 0, & \text{otherwise.} \end{cases} \quad (37)$$

6: $k = k + 1$;

7: **until** $|\nu^{(k+1)} - \nu^{(k)}| < \epsilon$.

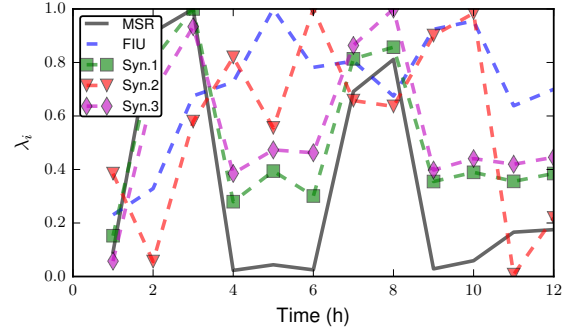


Fig. 1: Traced and synthesized workloads.

From the complementary slackness, i.e., the last two equations of (33), we have: i) if $\Delta e_i^* > 0$, then $\mu_i^* = 0$, $C'_i(\Delta e_i^*) = \nu^*$; and if $\mu_i^* > 0$, then $\Delta e_i^* = 0$, $C'_i(0) = \nu^* + \mu_i^* \geq \nu^*$; ii) if $\zeta^* > 0$, then $y^* = 0$, $\nu^* = \beta - \zeta^* \leq \beta$; and if $y^* > 0$, then $\zeta^* = 0$, $\nu^* = \beta$.

In summary, the KKT condition (33) can be simplified as follows

$$\begin{cases} \nu^* = \beta = C'_i(\Delta e_i^*), & \text{if } \Delta e_i^* > 0 \text{ and } y^* > 0; \\ \nu^* < \beta, & \text{if } y^* = 0; \\ \nu^* \leq C'_i(0), & \text{if } \Delta e_i^* = 0, \forall i; \\ y^* + \sum_{i \in \mathcal{I}} \Delta e_i^* = D. & \end{cases} \quad (34)$$

There are many interesting observations from condition (34). First, we always have $\nu^* \leq \beta$: the optimal incentive price is no greater than the backup energy cost. Intuitively, if incentivizing tenants is more costly than using backup, the operator is better off performing the demand response using its backup energy. Second, if D is too high such that all tenants cannot fulfill it, then the operator will turn on the backup to complement the mismatch: $\nu^* = \beta$, and $y^* > 0$ such

that $y^* + \sum_{i \in \mathcal{I}} \Delta e_i^* = D$. Finally, if D is small such that tenant reduction is sufficient to fulfill, then backup energy is not necessary (i.e., a feasible solution of the problem SWO in Section V): $\nu^* < \beta$, $y^* = 0$, and $\sum_{i \in \mathcal{I}} \Delta e_i^* = D$.

Based on these observations from KKT condition (34), the incentive mechanism for price-taking tenants in Algorithm 2 can be modified as in Algorithm 4 to include the backup energy constraints, which are reflected in lines 4 and 5 of Algorithm 4. Therefore, the convergence of Algorithm 4 to the solution of SWO' can be stated similarly to that of Algorithm 2.

However, due to the coupling among y , Δe_i and D , EPM cannot be extended to problem SWO' to deal with strategic tenants, which is an interesting problem for future work.

VII. SIMULATION RESULTS

In this section, we present the simulation settings, then provide the results to validate our proposal's efficacy.

A. Settings

We consider a colo with varying number of tenants for performance evaluation, where each tenant i has a number of maximum servers M_i that varies uniformly from 3,000 to 10,000, representing heterogeneous tenant business. The wear-and-tear and delay cost weights, $\omega_{i,1}$ and $\omega_{i,2}$, respectively, also are uniformly distributed on $[0.1, 3]$, which captures a wide range of tenant cost sensitivity. The total energy reduction requested by the operator is scaled to a ratio such that $D = 20$ kWh for every considered one-hour period (i.e., $T = 1$). Unless otherwise stated, we set $\alpha = 1$ in all scenarios.

In terms of workload of each tenant λ_i , we uses two basic traces "MSR" and "FIU", which were also used in [18], to generate synthetic workloads for all tenants. Each tenant's workload is normalized with respect to its service rate μ_i , which is set to 1000 jobs/s [12]. All workload samples of five tenants in 12 hours are illustrated in Fig. 1.

B. Results

Since we have two different scenarios, we will evaluate them separately and compare each individual with its corresponding baselines.

Economic demand response. We compare the performance of Algorithm 1 (Alg. 1) with two baselines. The first baseline, named OPT, is the optimal solutions of problem (10) using the exhaustive search. The second baseline, called RAND, is a random price ν^{rand} uniformly distributed in $[\min_i \{C'_i(0)\}, \max_i \{C'_i(0)\}]$ to enable feasible solutions, which represents a simple but inefficient scheme.

When the operator's utility is chosen to be $U = \omega_3 \log(1 + \sum_{i \in \mathcal{I}} m_i^*(r))$, where ω_3 is set to be uniformly distributed on $[0.2, 50]$ and log term reflects the diminishing return on the amount of reduced load, we show the values of the reward rates of different schemes and the corresponding operator's profit in Figs. 2a and 2b, respectively. When the operator's utility is affine $U = \omega_4 (\sum_{i \in \mathcal{I}} m_i^*(r)) + \omega_5$, where ω_4 and ω_5 are uniformly distributed on $[1, 2]$ and $[5, 10]$,

respectively, we show the operator's reward rate and profit of three schemes in Figs. 3a and 3b, respectively. Since the operator can have a wide range of possible utility values depending on many factors such as LSE's reimbursement, peak or non-peak demand response period, and colo characteristics, we have the freedom to choose the weight parameters in order to achieve feasible solutions. We also compare the operator profit and reward rates of the three schemes with a budget constraint of problem (29) in Figs. 5a and 5b, respectively. In all scenarios, while Alg. 1 and OPT achieve the same performance, the scheme RAND is not as efficient as the others.

We also examine the effect of ω_3 in the case of log utility function in Figs. 4a and 4b. We see that ω_3 has an impact on the operator profit. Specifically, the optimal operator profit increases linearly when ω_3 increases, while the optimal reward rates are unchanged. We observe a similar behavior in the case of linear utility function with varying parameters ω_4 and ω_5 .

Emergency demand response. We first illustrate the convergence of the proposed schemes with fixed number of five tenants (their workload traces are in Fig. 1). Considering the first period and setting $\epsilon = 10^{-3}$, $\gamma^{(k)} = 1/k$, we show in Fig. 6 that tenant bids and reduced energy and operator reward of EPM converge within an acceptable number of iterations (i.e., less than 90 iterations).

We further consider the impact of α in Fig. 7. There are two important observations to highlight: a) varying α has no effect on the convergence of the virtual reward rate $g(\Theta^{ne})$, which is the same value as the optimal price ν^* of the problem SWO (c.f., left plot of Fig. 7), and b) the total reward that the operator gives tenants increases with α (c.f., right plot of Fig. 7). Therefore, we expect that the operator will choose an α as small as possible. However, we observe that when α is small, more iterations are required for convergence, and when α is sufficiently small (e.g, less than 1), there is no convergence. We conclude numerically that there is a trade-off between convergence speed and operator payment via adjusting α .

To evaluate the efficacy of the proposed mechanisms, we compare EPM with two baselines. The first baseline, named SWO, is an efficient scheme that uses the optimal price ν^* satisfying (17) of the SWO problem. The second baseline is the random scheme RAND used in economic demand response comparison.

We first compare all schemes without backup energy. Fig. 8 shows the sum cost of all tenants of the three schemes. Different from RAND, SWO and EPM have the same performance in all periods, which illustrates that EPM can achieve the objective of social welfare maximization problem (15). Fig. 9 shows how different schemes respond to the energy reduction request D in 12 periods. While EPM and SWO have the same energy reduction levels for all tenants and can achieve the energy reduction target, RAND has off-target responses from tenants due to its random nature, which is not efficient. In terms of rewards, Fig. 10 compares how much the operator pays to tenants with different schemes. It is interesting to see that the operator has to pay approximately 25% more with EPM than with SWO. This observation indicates that in order

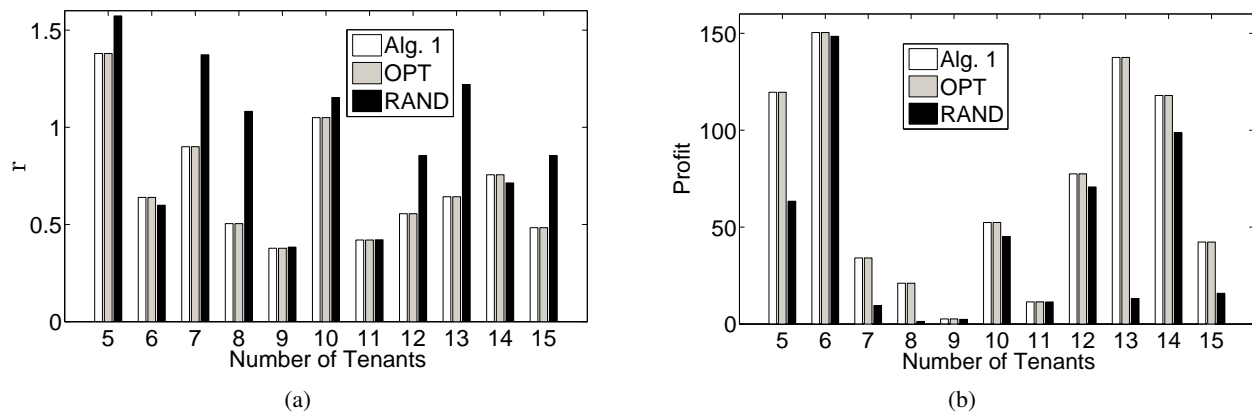


Fig. 2: Comparison among three schemes in economic demand response with utility U as a log function: a) Reward rate, b) Operator profit.

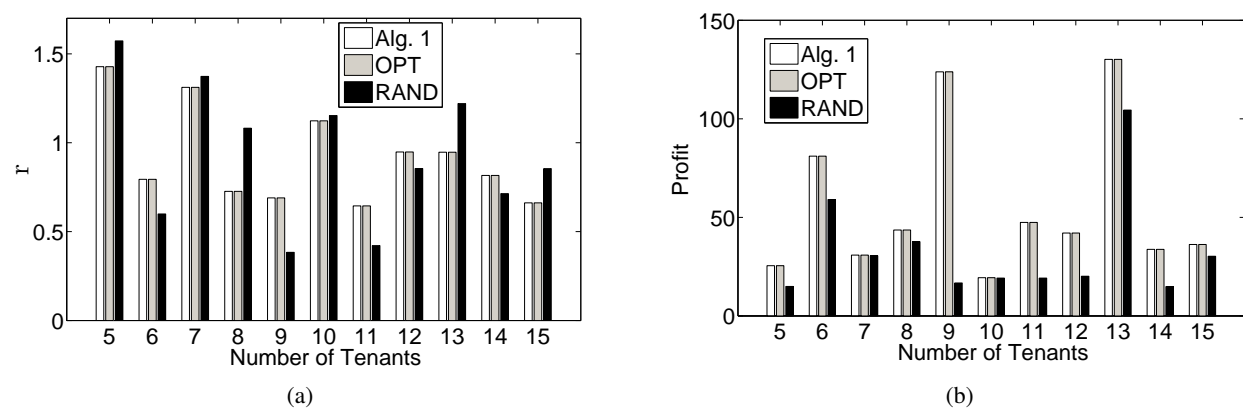


Fig. 3: Comparison among three schemes in economic demand response with utility U as a linear function: a) Reward rate, b) Operator profit.

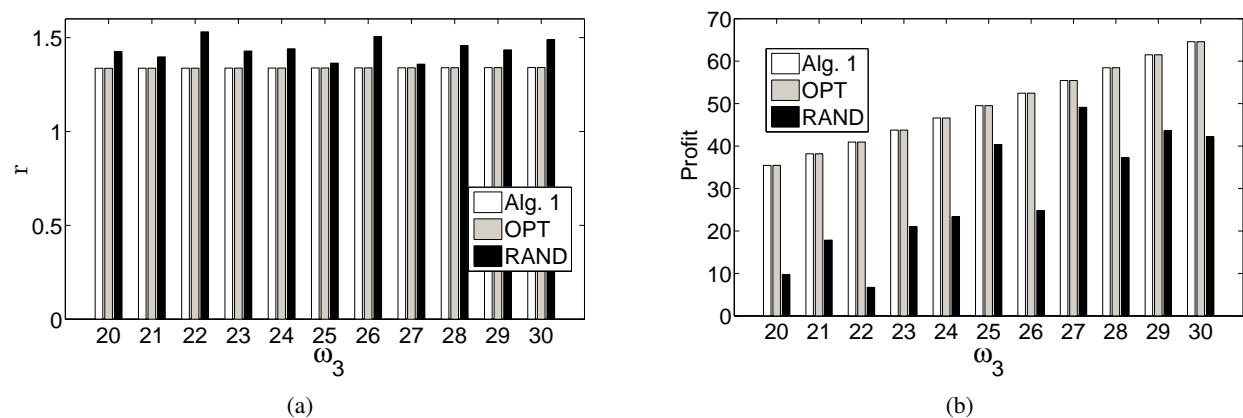


Fig. 4: Comparison among three schemes in economic demand response with varying ω_3 : a) Reward rate, b) Operator profit.

to achieve efficiency while dealing with the strategic behaviors of tenants, the operator must provide more incentives with EPM than those of SWO scheme with presumed price-taking tenants. Furthermore, Fig. 9 and Fig. 10 show that tenants receive their rewards proportionally to their reduced energy levels.

We also illustrate the demand response with backup energy in Algorithm 4. The value of β is set to 0.3 \$/kWh, corre-

sponding to a typical diesel cost [40]. Fig. 11 shows that all reduced and backup energy of Algorithm 4 converge to the optimal point of SWO'. We next demonstrate the effect of backup energy in Fig. 12 by increasing D linearly from 30 to 50 kWh in 12 periods. In this figure, we see that the backup energy also increases in order to fulfill the high demand target D since the total reduced energy of all tenants is not sufficient to match the target.

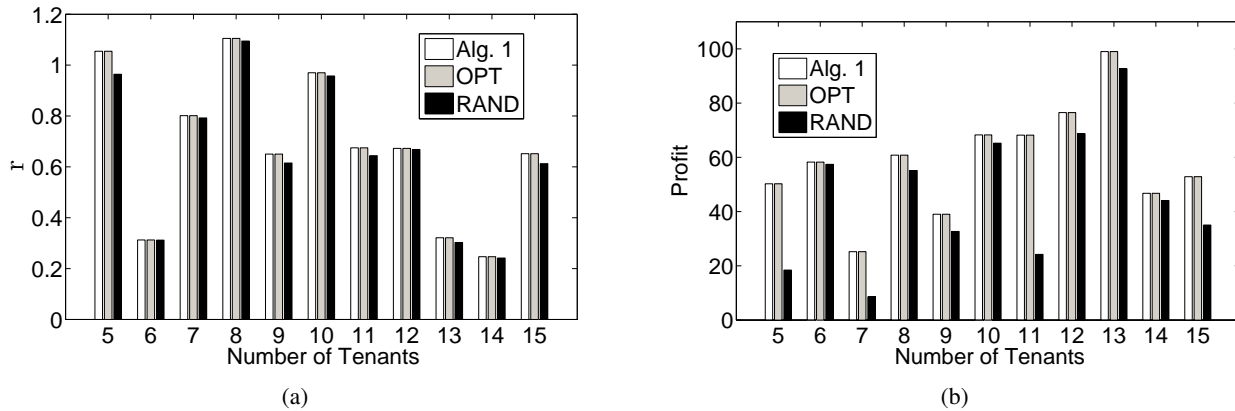


Fig. 5: Comparison among three schemes in economic demand response with a budget constraint: a) Reward rate, b) Operator profit.

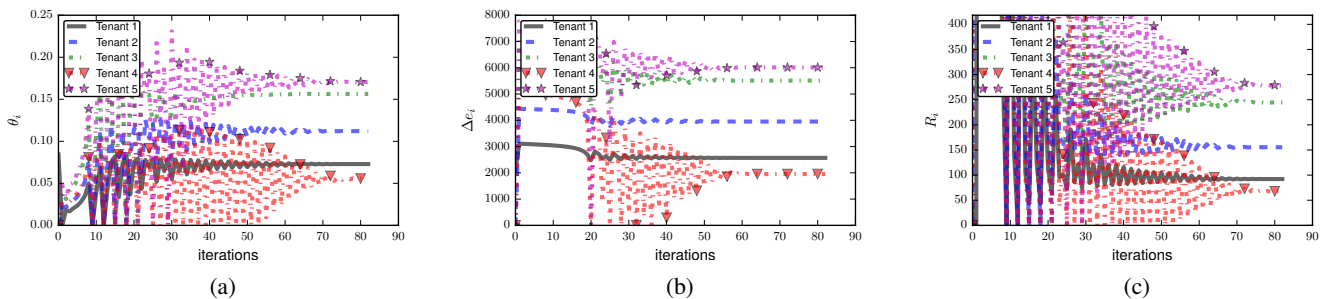


Fig. 6: Convergence of EPM: a) Tenant bids, b) Tenant energy reduction, and c) Operator payment.

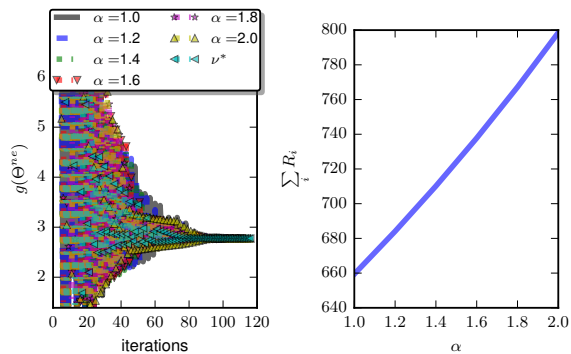


Fig. 7: The impact of α on EPM. The left plot shows that α has no effect on virtual reward rate's convergent value. The right plot shows that total payment of the operator increases with α .

VIII. CONCLUSIONS

In this paper, we addressed the demand response of a crucial but less-studied segment of datacenter market: colocation datacenters (colos). We tackled the split-incentive hindrance between colo tenants and operator, a unique feature of colos, by proposing two incentive schemes. The first scheme, which is appropriate for a controllable demand target of the operator, is based on the two-stage Stackelberg game, where the operator is the leader who sets its incentive reward rate, and the tenants are the followers who decide how much energy to reduce given

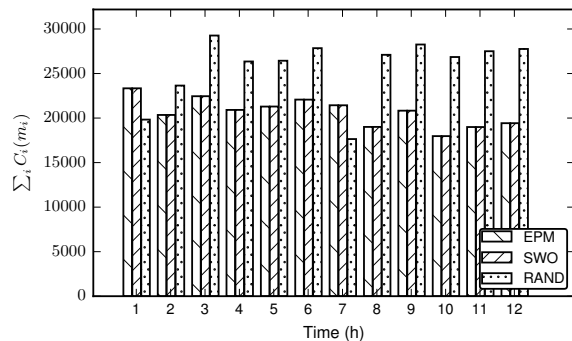


Fig. 8: Total tenant cost of three schemes.

the operator's reward. We first analyze this hierarchical game structure using the backward induction method and propose a linear time complexity to find its equilibrium. The second scheme, which is designed for fixed demand response target in many grid emergency incidents, is considered with two types of tenants: if tenants are price-takers, for which we propose a dual-based distributed algorithm that can achieve the optimal social cost; if otherwise, tenants are price-predictors, we propose a proportional mechanism with a distributed algorithm that can incentivize the tenants to reduce their energy in strategies that produce the same optimal social performance as in the previous price-taking case. Finally, the trace-based simulation results validate the efficacy of our proposals.

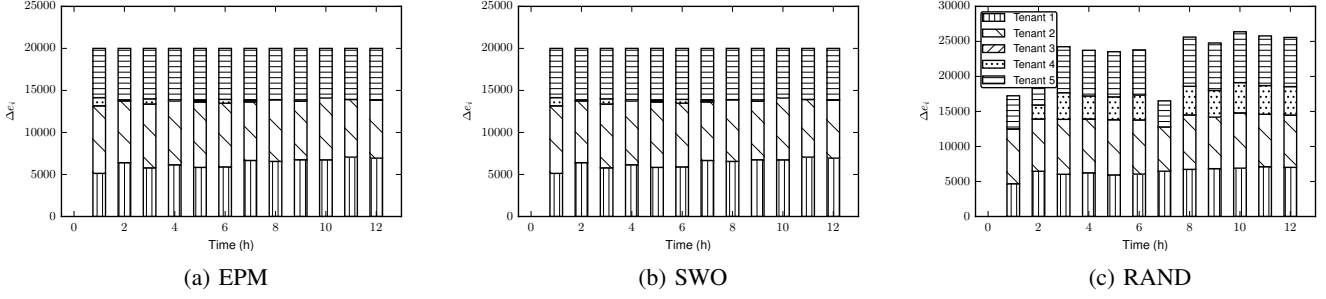


Fig. 9: Tenant energy reduction of three schemes.

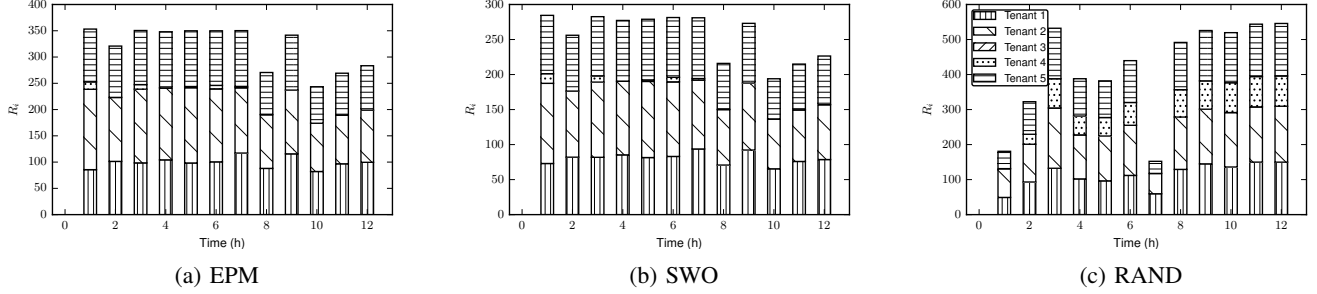


Fig. 10: Operator rewards to tenants of three schemes.

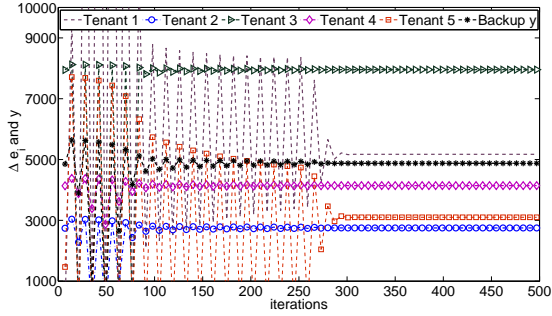
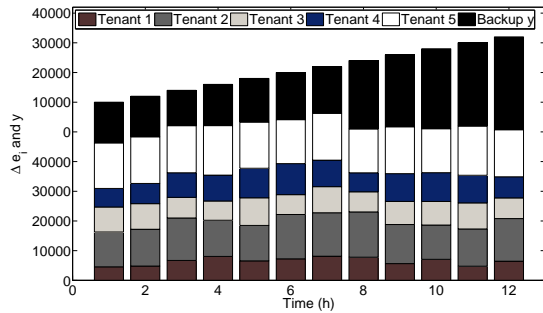


Fig. 11: Convergence of Algorithm 4 with backup energy.

Fig. 12: Backup usage y and tenant reduction energy.

APPENDIX A PROOF OF PROPOSITION 1

Since the tenants are sorted according to increasing κ_i (line 1), when the sufficient condition $r > \kappa_i$ is satisfied, we have $z_j = 1, \forall j \leq i$. In this case, the operator's problem (13) becomes (14), which is a single-variable and continuous

problem and can be solved efficiently using any numerical methods (e.g., bisection, Newton, etc.) (lines 1-4).

Therefore, we assume that (14) is available, then find its solutions and keep those satisfying the sufficient condition (line 5). By successively solving (14) and checking the sufficient condition (lines 5-8), we cover all possible cases of equivalence between problems (13) and (14). Finally, we compare and choose the solutions that result in the highest operator profit (line 10).

Clearly, with a single loop, Algorithm 1 has the complexity $O(cI)$, where c is complexity to solve problem (14).

APPENDIX B PROOF OF THEOREM 1

It is obvious that $\mathcal{U}(r^*, \{m_i\}) \geq \mathcal{U}(r, \{m_i\}), \forall r$, for any given $\{m_i\}$ since r^* is the solution to the Stage-I problem; hence, we have $\mathcal{U}(r^*, \{m_i^*\}) \geq \mathcal{U}(r, \{m_i^*\})$. Similarly, for any given values r and $\forall i$, we have $u_i(m_i^*, r) \geq u_i(m_i, r), \forall m_i$; hence, $u_i(m_i^*, r^*) \geq u_i(m_i, r^*), \forall m_i$. Combining these facts, we conclude the proof based on the definitions of (7) and (8).

APPENDIX C PROOF OF THEOREM 2

It is straightforward to see that $\theta = 0$ is an NE because, when $\theta_{-i} = 0$, tenant $i, \forall i$, receives reward $R_i(\theta_i, \theta_{-i}) = 0$ according to (24) so that it has no incentive to submit a positive bid.

We next show that if θ has only one positive element, then it is not an NE. Suppose, w.l.o.g., tenant 1 has $\theta_1 > 0$ and $\theta_{-1} = 0$, then the reward to tenant 1 is zero. Therefore, tenant 1 will decrease its bid to 0.

Finally, we show the existence and uniqueness of an efficient NE θ^{ne} (with at least two positive elements) via two steps. In step 1, we provide a necessary and sufficient condition for a bidding profile to be an NE. Based on this condition, in step 2, we show that the solution of SWO can lead to an NE of the bidding game and vice versa, which finishes the proof due to the existence of SWO's unique solution.

Step 1: For a profile $\hat{\theta}$ to be an NE according to (22), using the first-order condition, we have

$$\begin{aligned} \frac{\partial u_i}{\partial \hat{\theta}_i}(\hat{\theta}_i, \hat{\theta}_{-i}) &= 0 \\ &= \frac{\hat{\Theta}_{-i} D}{(\hat{\theta}_i + \hat{\Theta}_{-i})^2} \left((\hat{\theta}_i + \hat{\Theta}_{-i})^{-\alpha} - C'_i(\Delta e_i(\hat{\theta}_i, \hat{\theta}_{-i})) \right), \end{aligned} \quad (38)$$

if $\hat{\theta}_i > 0$, and

$$\frac{\partial u_i}{\partial \hat{\theta}_i}(0, \hat{\theta}_{-i}) = \frac{D}{\hat{\Theta}_{-i}} \left(\hat{\Theta}_{-i}^{-\alpha} - C'_i(0) \right) \leq 0, \quad (39)$$

if $\hat{\theta}_i = 0$, which implies

$$\begin{cases} g(\hat{\theta}_i + \hat{\Theta}_{-i}) = C'_i(\Delta e_i(\hat{\theta}_i, \hat{\theta}_{-i})), & \hat{\theta}_i > 0; \\ g(\hat{\theta}_i + \hat{\Theta}_{-i}) \leq C'_i(0), & \hat{\theta}_i = 0, \forall i. \end{cases} \quad (40)$$

Since $g(\hat{\theta}_i + \hat{\Theta}_{-i})$ is strictly decreasing (we can check that $\frac{\partial g}{\partial \hat{\theta}_i}(\hat{\theta}_i + \hat{\Theta}_{-i}) < 0, \forall i$) and $C'_i(\Delta e_i(\hat{\theta}_i, \hat{\theta}_{-i}))$ is strictly increasing with respect to $\hat{\theta}_i$, we see that, for a fixed $\hat{\theta}_{-i}$, there exists a unique solution $\hat{\theta}_i$ to (40), which is the solution to $\max_{\theta_i} u_i(\theta_i, \hat{\theta}_{-i}), \forall i$. Therefore, (40) is a necessary and sufficient condition for $\hat{\theta}$ to be an NE.

Step 2: Comparing the KKT conditions (17) with (40), we show the existence and uniqueness of an efficient NE θ^{ne} . That is, there exists a unique NE θ^{ne} such that $\Delta e^* = \Delta e(\theta^{ne})$.

First, if θ^{ne} with a corresponding Θ^{ne} is an NE satisfying (40), then by choosing $\nu' = g(\Theta^{ne})$, we see that $(\Delta e(\theta^{ne}), \nu')$ satisfies (17), which implies that $(\Delta e(\theta^{ne}), \nu')$ coincides with the unique primal-dual solution $(\Delta e^*, \nu^*)$ of the SWO.

Second, with the unique solution $(\Delta e^*, \nu^*)$ of SWO, we can construct a profile θ^{ne} as follows

$$g(\Theta^{ne}) = \nu^*, \quad (41)$$

$$\theta_i^{ne} = \frac{\Theta^{ne}}{D} \Delta e_i^*, \forall i. \quad (42)$$

We see that there exists a unique Θ^{ne} satisfying (41) since $g(\cdot)$ is strictly decreasing. Hence, with $(\Delta e^*, \nu^*)$, the constructed profile θ^{ne} is unique and satisfies (40), which implies an NE.

APPENDIX D PROOF OF PROPOSITION 3

We show that all updates of Algorithms 2 and 3 have the same functionalities. Therefore, with a chosen step-side rule, if the former converges to the unique solution of SWO, then the latter also converges to the same point due to a one-to-one relationship between the solution of SWO and the NE of EPM (c.f. Proposition 2).

First, according to EPM's rules, we see that the tenants' bid update (27) of Algorithms 3 can be rewritten as

$$\Delta e_i(\theta_i, \Theta_{-i}) = [C'_i{}^{-1}(g(\theta_i + \Theta_{-i}))]^+. \quad (43)$$

However, in line 4 of Algorithm 3, we know that the virtual reward rate $\tilde{\nu}$ tracks the values of $g(\cdot)$; therefore, (43) is equivalent to

$$\Delta e_i(\theta_i, \Theta_{-i}) = [C'_i{}^{-1}(\tilde{\nu})]^+, \forall i. \quad (44)$$

Second, according to EPM's rules, we see that the operator' virtual rate updates (27) of Algorithm 3 can be rewritten as

$$\tilde{\nu}^{(k+1)} = \left[\tilde{\nu}^{(k)} - \gamma^{(k)} \left(\sum_{i \in \mathcal{I}} \Delta e_i(\theta_i, \Theta_{-i}) - D \right) \right]^+, \quad (45)$$

It is obvious that updates (44) and (45) of Algorithm 3 are equivalent to the energy reduction level and reward rate updates (18) and (19) of Algorithm 2, respectively.

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