Iterative Design of MIMO Radar Transmit Waveforms and Receive Filter Bank

Y.-C. Wang*, *Member, IEEE,* Hongwei Liu†, *Member, IEEE,* and Z.-Q. Luo‡, *Fellow, IEEE*

***Abstract*—In this paper, we propose an iterative design approach to jointly optimize probing signal waveforms and a receive filter bank for a multiple-input multiple-output (MIMO) radar under a constant modulus constraint. The design goals are to approximate a desired beampattern and to minimize the auto-/cross- correlation levels of the probing signal waveforms for different time lags and between different spatial angles. Since the overall design problem is nonconvex, we propose to optimize the transmit probing signals and receive filter bank separately and alternately. The optimization of receive filter bank is a standard least squares problem, while the optimization of the constant modulus transmit signal waveforms is a norm-constrained least squares problem which can be approximately solved using a low-rank semidefinite relaxation procedure. We demonstrate the effectiveness of our proposed approach through a simulation example.**

***Index Terms*—Multiple-input multiple-output (MIMO) radar, Beampattern, Mismatched filter bank, Constant modulus probing signal, Auto correlation peak sidelobes, Cross correlation Level.**

I. INTRODUCTION

MIMO transmission and reception is a promising paradigm for the next generation radar systems. Unlike the phased-array radar, MIMO radar allows independent probing signals to be transmitted at different antennas. Through this additional waveform diversity, MIMO radar can deliver higher spatial resolution and better detection performance [1].

A central signal processing problem in MIMO radar research is waveform design. The basic issue is how to generate a pre-specified beampattern using independent constant modulus waveforms while minimizing the so called auto-/cross- correlation peak sidelobe levels. The constant modulus property is important since all radar systems typically require their power amplifiers to operate at saturation region where nonlinearity effect is significant, while low auto-/cross-correlation levels are necessary so that the echo signals from different targets or at different range cells do not interfere with each other. Existing approaches to MIMO waveform design consist of two phases: the first is to optimize the transmitted signal correlation matrix, and the second phase is to synthesize the derived signal correlation matrix using constant modulus signals. For example, in reference [2], Stoica et.al. formulated the signal correlation matrix design problem as convex semidefinite program. Their objective was to match a desired beampattern while minimizing peak sidelobe level of the generated beampattern. In [3] and [4], Li et.al. proposed a cyclic algorithm to synthesize a given signal correlation matrix using constant modulus probing signals. In reference [5], [6], Fuhrmannn showed how to create spatial beampatterns ranging from high directionality to omni-directionality through binary phase-shifted keyed signaling.

In this paper, we consider the MIMO waveform design problem by directly optimizing the constant modulus probing signals and a receive filter bank to achieve a given beampattern while maximally suppressing the auto-/cross- correlation peak sidelobe levels at different time delay and between different spatial angles. Unlike the existing work [2], [3], [5], [6], we directly impose the constant modulus constraint in the optimization process, and include the minimization of time-delayed auto-/cross- correlation levels in our formulation so that clutters' effect can be as small as possible. To facilitate efficient computation, we introduce a receive filter bank and separate the optimization of transmit waveforms and receive filter bank (not necessarily matched to the transmit waveform as in the work of [2], [3], [5], [6]). It turns out the optimization of the receive filter bank is a convex least squares problem, while the transmit waveform optimization is a norm-constrained least squares problem. Although the latter is a nonconvex (NP-hard in general) problem, we introduce an efficient low rank SDP relaxation method for this purpose. By alternating between the optimization of the transmit waveforms and the receive filter bank, we are able to achieve a high degree of suppression of auto-/cross- correlation levels at different time delays and between different spatial angles, while closely approximating a desired beampattern. A simulation example shows the effectiveness of the proposed approach.

II. BASIC CONCEPTS AND SYSTEM PARAMETERS

Consider a uniform linear array with M antennas. As shown in Fig.1, we denote the probing signal waveforms and and steering vector as a L-by-M matrix and a M-by-1 vector, respectively, where L denotes the length of the transmitted waveforms.

$$X = [\mathbf{x_1} \ \cdots \ \mathbf{x_M}] = \begin{bmatrix} x_{1,1} & \cdots & x_{1,M} \\ \vdots & \ddots & \vdots \\ x_{L,1} & \cdots & x_{L,M} \end{bmatrix} \tag{1}$$

$$a(\theta) = [e^{j0} \ e^{j\pi \sin(\theta)} \ \cdots \ e^{j\pi(M-1)\sin(\theta)}]^T, . \tag{2}$$

The work of the first author is supported by 111 Project (B08038). The research of the last author is supported in part by the National Science Foundation, Grant No. DMS-0610037.

*State Key Laboratory on Integrated Service Networks, Xidian University, Xi'an, 710071, China. Currently Y.-C Wang is a post doctorial fellow at the University of Minnesota. (e-mail: wyong@mail.xidian.edu.cn)

†National Lab of Radar Signal Processing, Xidian University, Xi'an, 710071, China.

‡ Department of Electrical Computer Science Engineering, University of Minnesota, Minneapolis, 55455, USA. (e-mail: luozq@umn.edu)

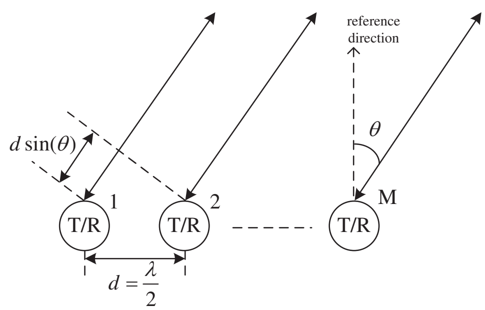


Fig. 1. MIMO radar system equipped with M antennas (uniform linear array - ULA and half wavelength inter-element spacing, $d = \frac{\lambda}{2}$). θ is the angle between the beampattern direction and reference direction.

The transmit beampattern is defined as the power of the probing signal at a given direction θ

$$P_t(\theta) = a(\theta)^\dagger X^\dagger X a(\theta). \tag{3}$$

Here, "$\dagger$" is conjugate transpose operator and θ belongs to a set Θ which covers the directions of interest. The concept of beampattern can be extended to the receiver, named herein the receive beampattern,

$$P_r(\theta) = a(\theta)^\dagger H^\dagger X a(\theta), \tag{4}$$

where $H \in \mathbb{C}^{L\times M}$ is a filter bank of the receiver. In particular, if the receiver uses a matched filter bank $H = X$, then the transmitter beampattern $P_t(\theta)$ and the receiver beampattern $P_r(\theta)$ coincide.

Similar with the traditional radar, the transmitted waveforms probing to different directions are required to hold good autocorrelation property, in order to improve the detection performance of the small target close to a large target, and avoiding false alarm introduced by clutter. This should be concerned when designing transmit waveforms in a MIMO radar system. MIMO radar has the ability to form multiple spatial beams simultaneously. The target echoed signal or clutter from different beams will be interference to each other. Therefore, it is ideal that the transmitted waveforms for different spatial directions of MIMO radar are orthogonal to each other or have small enough correlation coefficients. This can be handled by a careful design of transmit waveforms and mismatch filter bank. Notice that clutters have different time delays and spatial directions with respect to useful echoes. We can describe time delay characteristics of clutters by

$$\mathfrak{x}_m(\ell) = \begin{cases} [\underbrace{0 \ \cdots \ 0}_{\ell \ zeros} \ x_{1,m} \cdots x_{L-\ell,m}]^T, & \ell > 0 \\ [x_{-\ell+1,m} \cdots x_{L,m} \ \underbrace{0 \ \cdots \ 0}_{-\ell \ zeros}]^T, & \ell \le 0 \end{cases}, \tag{5a}$$

$$\mathfrak{X}(\ell) = [\mathfrak{x}_1(\ell) \ \cdots \ \mathfrak{x}_M(\ell)] = S(\ell)X, \tag{5b}$$

where $-L < \ell < L$ and $S(\ell)$ is a L-by-L matrix. The elements in the ℓ-th diagonal off-line of $S(\ell)$ are 1 and others are 0.

Assuming the use of a receive (possibly mismatched) filter bank H, the correlation characteristics can be described by

$$P_{\rm c}(\ell, \hat\theta_i, \hat\theta_k) = a(\hat\theta_i)^\dagger H^\dagger S(\ell) X a(\hat\theta_k), \tag{6}$$

where $\hat\theta_i, \hat\theta_k \in \hat\Theta = \{\hat\theta_1, ..., \hat\theta_{\hat K}\}$, $\hat\Theta$ is a set of angles representing the directions of interested targets, $1 \le i, k \le \hat K$, and $\hat K$ is the number of interested targets. Naturally, $\hat\Theta \subset \Theta$. From (6), two important parameters, named auto-correlation function and cross-correlation functions can be derived. The former is related to clutters and the latter can take effects on some adaptive techniques of radar system [2].

If $\hat\theta_i = \hat\theta_k = \hat\theta$,

$$P_{\rm c}(\ell, \hat\theta, \hat\theta) = P_{\rm ac}(\ell, \hat\theta) = a(\hat\theta)^\dagger H^\dagger S(\ell) X a(\hat\theta) \tag{7}$$

is referred to as the auto-correlation function. So $P_{\rm ac}(\ell, \hat\theta)$ ($\ell \neq 0$) is the auto-correlation sidelobes. If $\hat\theta_i \neq \hat\theta_k$,

$$P_{\rm c}(\ell, \hat\theta_i, \hat\theta_k) = P_{\rm cc}(\ell, \hat\theta_i, \hat\theta_k) = a(\hat\theta_i)^\dagger H^\dagger S(\ell) X a(\hat\theta_k) \tag{8}$$

is referred to as the cross correlation function. Let $P_d(\theta)$ denote the desired beampattern. The normalized auto-correlation function and cross-correlation function can also be defined accordingly

$$P'_{\rm ac}(\ell, \hat\theta) = \frac{P_{\rm ac}(\ell, \hat\theta)}{P_{\rm d}(\hat\theta)} \tag{9a}$$

$$P'_{\rm cc}(\ell, \hat\theta_i, \hat\theta_k) = \frac{P_{\rm cc}(\ell, \hat\theta_i, \hat\theta_k)}{\sqrt{P_{\rm d}(\hat\theta_i) P_{\rm d}(\hat\theta_k)}}. \tag{9b}$$

III. Problem Formulation

Let $\mathcal{C}^{L\times M}$ denote the set of matrices whose elements have unit modulus. Define

$$\begin{aligned} f(X, H) = & \sum_{\theta \in \Theta} |w_1 [P_{\rm d}(\theta) - a(\theta)^\dagger H^\dagger X a(\theta)]|^2 \\ & + \sum_{\substack{\ell=-L+1 \\ \ell \neq 0}}^{L-1} \sum_{\hat\theta \in \hat\Theta} |w_2 a(\hat\theta)^\dagger H^\dagger S(\ell) X a(\hat\theta)|^2 \\ & + \sum_{\ell=-L+1}^{L-1} \sum_{\substack{\hat\theta_i \neq \hat\theta_k \\ \hat\theta_i, \hat\theta_k \in \hat\Theta}} |w_3 a(\hat\theta_i)^\dagger H^\dagger S(\ell) X a(\hat\theta_k)|^2, \end{aligned} \tag{10}$$

where $w_i > 0$, $i = 1, 2, 3$, are some positive weights (chosen by the user). The cost function $f(X, H)$ captures the beampattern matching (the first term), auto-correlation level suppression (the second term), and the cross-correlation level suppression (the third term). Then the joint optimization of receive filter bank H and transmit waveforms X can be described as

$$\min_{X \in \mathcal{C}^{L\times M}, H \in \mathbb{C}^{L\times M}} f(X, H), \tag{11}$$

where the constant modulus condition is represented by $X \in \mathcal{C}^{L\times M}$. The above MIMO waveform design problem involves minimizing a nonconvex fourth-order polynomial which is computationally difficult. Below we propose a coordinate descent method for the joint design of X and H.

A. Optimizing the Mismatched filter Bank H

Fix a value of $X \in \mathcal{C}^{L\times M}$. We optimize H by

$$H_{\rm opt} = \underset{H\in\mathbb{C}^{L\times M}}{\rm argmin}\, f(X,H). \tag{12}$$

This problem can be formulated as a linear least squares problem. Specifically, if we define $b(\cdot,\ell) = S(\ell)Xa(\cdot)$ and $Y(\cdot,\cdot,\ell) = b(\cdot,\ell)a(\cdot)^\dagger$, the beampattern term in (10) can be formulated as

$$\begin{aligned}&\sum_{\theta\in\Theta}|w_1[P_{\rm d}(\theta) - a(\theta)^\dagger H^\dagger Xa(\theta)]|^2\\ &=\begin{bmatrix}1\\ {\rm vec}(H^\dagger)\end{bmatrix}^\dagger \begin{bmatrix}A_1 & A_2\\ A_3 & A_4\end{bmatrix}\begin{bmatrix}1\\ {\rm vec}(H^\dagger)\end{bmatrix},\end{aligned} \tag{13}$$

where

$$A_1 = \sum_{\theta\in\Theta} w_1^2 P_{\rm d}(\theta)^2, \tag{14a}$$

$$A_2 = -\sum_{\theta\in\Theta} w_1^2 P_{\rm d}(\theta){\rm vec}(Y(\theta,\theta,0)^T)^T, \tag{14b}$$

$$A_3 = -\sum_{\theta\in\Theta} w_1^2 P_{\rm d}(\theta){\rm vec}(Y(\theta,\theta,0)^T)^*, \tag{14c}$$

$$A_4 = \sum_{\theta\in\Theta} w_1^2 {\rm vec}(Y(\theta,0)^T)^*{\rm vec}(Y(\theta,\theta,0)^T)^T, \tag{14d}$$

and "*" is the conjugate operator and ${\rm vec}(\cdot)$ vectorizes a matrix by stacking its columns in sequence on top of one another. We further define

$$A_5 = \sum_{\substack{\ell=-L+1\\ \ell\neq 0}}^{L-1}\sum_{\hat\theta\in\hat\Theta} w_2^2{\rm vec}(Y(\hat\theta,\hat\theta,\ell)^T)^*{\rm vec}(Y(\hat\theta,\hat\theta,\ell)^T)^T, \tag{15a}$$

$$A_6 = \sum_{\ell=-L+1}^{L-1}\sum_{\substack{\hat\theta_i\neq\hat\theta_k\\ \hat\theta_i,\hat\theta_k\in\hat\Theta}} w_3^2{\rm vec}(Y(\hat\theta_i,\hat\theta_k,\ell)^T)^*{\rm vec}(Y(\hat\theta_i,\hat\theta_k,\ell)^T)^T. \tag{15b}$$

Then the auto-correlation term and the cross-correlation term in (10) can also be written as

$$\begin{aligned}&\sum_{\substack{\ell=-L+1\\ \ell\neq 0}}^{L-1}\sum_{\theta\in\hat\Theta}|w_2 a(\hat\theta)^\dagger H^\dagger S(\ell)Xa(\hat\theta)|^2\\ &={\rm vec}(H^\dagger)^\dagger A_5{\rm vec}(H^\dagger),\\ &\sum_{\ell=-L+1}^{L-1}\sum_{\hat\theta_i\neq\hat\theta_k}|w_3 a(\hat\theta_i)^\dagger H^\dagger S(\ell)Xa(\hat\theta_k)|^2\\ &={\rm vec}(H^\dagger)^\dagger A_6{\rm vec}(H^\dagger).\end{aligned} \tag{16}$$

Then (12) is equivalent to

$$H_{\rm opt} = \underset{H\in\mathbb{C}^{L\times M}}{\rm argmin}\begin{bmatrix}1\\ {\rm vec}(H^\dagger)\end{bmatrix}^\dagger A\begin{bmatrix}1\\ {\rm vec}(H^\dagger)\end{bmatrix}, \tag{17}$$

where

$$A = \begin{bmatrix}A_1 & A_2\\ A_3 & A_4+A_5+A_6\end{bmatrix}. \tag{18}$$

Because (12) guarantees A is a positive semi-definite matrix, its eigenvalue decomposition can be expressed as $A = UDU^\dagger = V^\dagger V$. Then (12) can be equivalent to a least square problem

$$\min \left\|V\begin{bmatrix}1\\ {\rm vec}(H^\dagger)\end{bmatrix}\right\|_2,\quad \text{s.t.}\quad H\in\mathbb{C}^{L\times M}, \tag{19}$$

where $\|\cdot\|_2$ is 2-norm. This is a standard convex least square problem which can be solved efficiently by a standard optimization software using YALMIP [7].

B. Optimization of Transmit Waveforms

For a fixed receive filter bank H, we can optimize the transmit waveforms X by solving a norm-constrained least squares problem

$$X_{\rm opt} = \underset{X\in\mathcal{C}^{L\times M}}{\rm argmin}\, f(X,H). \tag{20}$$

Similar to the optimization of H, we define

$$c(\cdot,\ell) = S(\ell)^\dagger Ha(\cdot), \tag{21a}$$

$$Z(\cdot,\cdot,\ell) = a(\cdot)c(\cdot,\ell)^\dagger, \tag{21b}$$

$$B_1 = \sum_{\theta\in\Theta} w_1^2 P_{\rm d}(\theta)^2, \tag{21c}$$

$$B_2 = -\sum_{\theta\in\Theta} w_1^2 P_{\rm d}(\theta){\rm vec}(Z(\theta,\theta,0)^T)^T, \tag{21d}$$

$$B_3 = -\sum_{\theta\in\Theta} w_1^2 P_{\rm d}(\theta){\rm vec}(Z(\theta,\theta,0)^T)^*, \tag{21e}$$

$$B_4 = \sum_{\theta\in\Theta} w_1^2 {\rm vec}(Z(\theta,\theta,0)^T)^*{\rm vec}(Z(\theta,\theta,0)^T)^T, \tag{21f}$$

$$B_5 = \sum_{\substack{\ell=-L+1\\ \ell\neq 0}}^{L-1}\sum_{\hat\theta\in\hat\Theta} w_2^2{\rm vec}(Z(\hat\theta,\hat\theta,\ell)^T)^*{\rm vec}(Z(\hat\theta,\hat\theta,\ell)^T)^T, \tag{21g}$$

$$B_6 = \sum_{\ell=-L+1}^{L-1}\sum_{\substack{\hat\theta_i\neq\hat\theta_k\\ \hat\theta_i,\hat\theta_k\in\hat\Theta}} w_3^2{\rm vec}(Z(\hat\theta_i,\hat\theta_k,\ell)^T)^*{\rm vec}(Z(\hat\theta_i,\hat\theta_k,\ell)^T)^T, \tag{21h}$$

$$B = \begin{bmatrix}B_1 & B_2\\ B_3 & B_4+B_5+B_6\end{bmatrix} = W^\dagger W. \tag{21i}$$

Then $X_{\rm opt}$ in (20) can be solved through

$$\min \left\|W\begin{bmatrix}1\\ {\rm vec}(X)\end{bmatrix}\right\|_2,\quad \text{s.t.}\quad X\in\mathcal{C}^{L\times M}. \tag{22}$$

In theory, optimization model (22) is a NP-hard problem. However, this problem can be approximately solved using an efficient low rank semidefinite relaxation procedure [8]. This procedure has been coded in a public software (available from the last author's homepage).

C. An Alternating Direction Algorithm

Combining the above two steps we obtain an alternating direction algorithm for the joint optimization of X and H.

Step 1. Initialization. Set maximum iteration number `max` and randomly generate a L-by-M random matrix X_0.

Step 2. Fix the probing signal matrix X. Determine H through optimization model (19). If the maximum iteration number $\texttt{max}$ is reached, output the current H and X.

Step 3. Fix H from Step 1 and solve optimization model (22) to determine probing signal X. Increase the iteration counter by 1 and go to Step 1.

IV. Simulation Results

We present a simulation example to illustrate the effectiveness of the proposed design method. Consider a MIMO radar system with a total of $M = 8$ ULA antennas. Each probing pulse consists of $L = 256$ samples. The angle set Θ covers $[-90^o,\ 90^o]$ with spacing 1^o. Consider three targets located in $\hat{\Theta} = \{\hat{\theta}_k = -40^o, 0^0, 40^0\}$, and the desired beampattern is given by

$$P_{\rm d}(\theta) = \begin{cases} 1, & \theta \in [\hat{\theta}_k - 10^o, \hat{\theta}_k + 10^o], k = 1,2,3 \\ 0, & \text{otherwise} \end{cases}. \quad (23)$$

Optimization models (19) and (22) are solved using YALMIP [7] and PSK Detector [8]. The maximum iteration number is 4.

The constant modulus probing signal matrix X generates a transmit beampattern $P_t(\theta)$, and a receive beampattern $P_r(\theta)$. They are plotted in Fig.2 together with the desired beampattern $P_{\rm d}(\theta)$. The beampattern weight w_1 and auto-correlation peak sidelobe weight w_2 are set to 10 and 1, respectively. The cross-correlation weight w_3 is set to 2 for $\ell = 0$ and 1 for other nonzero ℓ's. Fig.2 shows that both transmit beampattern $P_t(\theta)$ and receive beampattern $P_r(\theta)$ are close to the desired beampattern $P_{\rm d}(\theta)$. Compared to the transmit beampattern, some values of receive beampattern are negative. This is because that $H^\dagger X$ does not satisfy positive semidefinite condition. In theory, $P_r(\theta)$ can be complex. However in this simulation, its imagery part is so small that it can be neglected.

Fig.3(a) and Fig.3(b) show the normalized auto- and cross-correlation functions respectively. Compared to the desired beampattern values ($P_{\rm d}(\theta)$, $\theta = -40^o, 0^o$, and 40^o), both auto-correlation and cross- correlation levels have been suppressed by as much as -10dB.

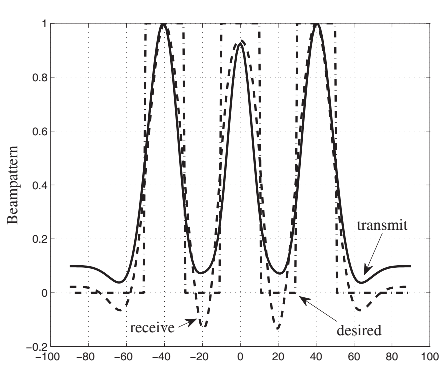
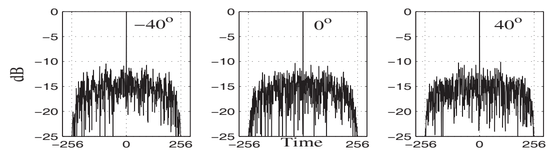
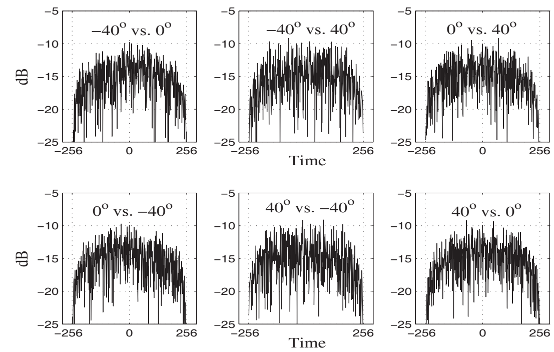


Fig. 2. Comparison of transmit beampattern $P_t(\theta)$, receive beampatten $P_r(\theta)$, and desired beampattern $P_{\rm d}(\theta)$.



(a) Normalized auto correlation function $P'_{\rm ac}$



(b) Normalized cross correlation function $P'_{\rm cc}$

Fig. 3. Suppressed auto- and cross- correlations.

V. Conclusion

By separating the optimization of transmit waveforms and a receive filter bank, we propose an efficient iterative approach for the joint optimization probing signals and a (possibly mismatched) receive filter bank. The designed probing signals satisfy the constant modulus constraint and can closely approximate the desired beampattern, while the corresponding normalized auto- and cross- correlation peak sidelobes are suppressed to as low as -10dB. Because of space reason, some important parameters, such as SNRLoss, will be discussed in our later works.

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