15th Triennial World Congress, Barcelona, Spain

ADAPTIVE FEEDBACK PASSIVITY OF A CLASS OF NONLINEAR SYSTEMS

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Abstract: Passivity is the property stating that any storage energy in a system is not larger than the energy supplied to it from external sources. This paper considers adaptive feedback passivation for a class of nonlinear systems. A nonlinear system with unknown constant parameters is transformed via feedback into a new system. An appropriate update law is designed so that the new transformed system is passive. In fact the system is passive via feedback if the unknown parameters are replaced with their suitable estimates.

Keywords: Passivity, nonlinear systems, sliding mode control, stability

1. INTRODUCTION

The link between Lyapunov stability and passivity increases the importance of passivity in the control area. In fact passivity is not only important because of this link, but also because there is a relation between passivity and optimality (Sepulchre *et al.*, 1997). Passivity has wide applications including electrical, mechanical and chemical process systems (Ortega *et al.*, 1998; Sepulchre et al., 1997).

The passivity concept is a particular case of dissipativity, which has been addressed by Willems (1971). Passivity has been considered in recent years in many different areas, the stability of feedback interconnected systems (Hill *et al.* 1977), applications to robotics and electro-mechanical systems (Ortega, 1991) and the geometric approach to feedback equivalence (Byrnes *et al.*, 1991). Global asymptotic stabilization of nonlinear passive systems with stable free dynamics using some techniques of feedback equivalence and bounded control, has been studied by Lin (1996). The full details of passivity of nonlinear systems including concepts, stability and applications can be found in Sepulchre *et al.* (1997), and for Euler-Lagrange systems with applications in Ortega *et al.* (1998).

The passivity of a general canonical form of nonlinear systems has been considered in a paper by Sira-Ramírez (1998), based upon the properties of projection operators. The system is converted into a generalized Hamiltonian system, and is passive whenever the appropriate symmetric matrix is negative definite. This work has been generalized to multivariable nonlinear systems (Sira-Ramírez and Ríos-Bolívar, 1999). An adaptive passivation procedure of SISO nonlinear system based control has been studied by Ríos-Bolívar *et al.* (2000).

In this paper the passivity of nonlinear systems is considered. A nonlinear affine system is transformed into a passive system via a transformed control. The resulting system is (strictly) passive regarding different choices of the rate function. The only assumption is the *transversality condition*, i.e. the directional derivative of the storage function in the input matrix direction is not zero.

A particularly important aspect in regulation and tracking tasks for systems is robustness in the presence of disturbances and unmodelled dynamics. We design sliding mode control (SMC) for the transformed passive systems to achieve robustness. When a nonlinear system contains an unknown parameter, the method allows one to design adaptive sliding mode (tracking) controllers. The resulting control law achieves robust asymptotic stability with considerably reduced *chattering*.

In Section 2 the passivation of nonlinear systems is considered. The sliding mode of nonlinear passive systems is studied in Section 3. In Section 4 feedback passivation is designed for parameterized nonlinear systems. In Section 5 an exothermic chemical reactor model illustrates the results. Conclusions are presented in Section 6.

2. PASSIVATION OF NONLINEAR SYSTEMS

Consider the nonlinear system

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned} \tag{1}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^n$ is the state; $u \in \mathcal{U} \subseteq \mathbb{R}^m$ the control input and $y \in \mathcal{Y} \in \mathbb{R}^m$ the output. Functions f and g are smooth on $\mathcal{X}$. Assume that $\mathcal{X}$ is a pathwise connected open subset of $\mathbb{R}^n$. The equilibrium point $x_e \in \mathcal{X}$ satisfies $f(x_e) + g(x_e)u_e = 0$ with $u_e \in \mathcal{U}$ constant. h is a smooth function defined on $\mathcal{X}$. Let us consider the well-known definitions (Ortega *et al.*,1998; Sepulchre *et al.*, 1997).

Definition 1. (*i*) The system (1) is dissipative with respect to the supply rate $w(u,y) : \mathcal{U} \times \mathcal{Y} \to \mathbb{R}$ if there exists a storage function $V : \mathcal{X} \to \mathbb{R}_+$ such that

$$V(x(T)) \leq V(x(t_0)) + \int_{t_0}^{T} w(u(t), y(t))dt \tag{2}$$

for all $u \in \mathcal{U}$, all T and $x(t_0)$ such that $x(t) \in \mathcal{X}$ for all $t_0 \leq t \leq T$.

(*ii*) The system (1) is said to be passive if it is dissipative with supply rate $w(u,y) = u^T y$.

(*iii*) The system (1) is said to be strictly passive if it is passive with supply rate $w(u,y) = u^T y - \Psi(t,x,u,y)$ with $\Psi(t,x,u,y)$ a positive function. If there exists a positive function γ such that $\Psi(t,x,u,y) = \gamma\|y\|^2$, the system is strictly output passive.

Definition 2. Let $V : \mathcal{X} \to \mathbb{R}_+$ with $V(0) = 0$, be a smooth positive definite storage function. Define

$$L_g V(x) = \frac{\partial V}{\partial x^T} g(x) \tag{3}$$

Assumption 3. It is assumed that $L_g V(x) \neq 0$ for all $x \in \mathcal{X}$, and this condition is known as the *transversality condition*.

It is assumed that Assumption 3 is satisfied throughout the paper. In general the transversality condition is a differential topology concept when the properties (such as dimension) of the intersection of two hyperplanes are considered. For instance, see Guckenheimer and Holmes (1983). This assumption is not restrictive. If the transversality condition is not satisfied, the storage function may be modified so that the new storage function satisfies the condition (Sira-Ramírez, 1998). We now follow our approach.

Theorem 4. Consider the system (1). Let

$$\Psi(t,x,u,y) : \mathbb{R}_+ \times \mathcal{X} \times \mathcal{U} \times \mathcal{Y} \to \mathbb{R}_+$$

be a positive real function. Assume that $f(x) = f_1(x) + f_2(x)$ where $\frac{\partial V}{\partial x^T} f_1 \leq 0$. The following input coordinate transformation with a new external independent control input v,

$$u = \frac{L_g^T V(x)}{\|L_g V(x)\|^2}\left(-\frac{\partial V}{\partial x^T} f_2(x) + h^T(x)v - \Psi(t,x,u,y)\right) \tag{4}$$

transforms the system (1) into

$$\begin{aligned}\dot{x} = &\left(I - g(x)\frac{L_g^T V(x)}{\|L_g V(x)\|^2}\frac{\partial V}{\partial x^T}\right) f(x) + g(x) \times \\ &\frac{L_g^T V(x)}{\|L_g V(x)\|^2}\left(h^T(x)v - \Psi + \frac{\partial V}{\partial x^T} f_1(x)\right)\end{aligned} \tag{5}$$

which is strictly passive with dissipation rate Ψ. If $\Psi = 0$ the system is passive. With $\Psi = \gamma\|y\|^2$, $\gamma > 0$, the system is strictly output passive.

Proof: Considering the time derivative V and substituting (4) yields

$$\begin{aligned}\dot{V} &= \frac{\partial V}{\partial x^T} f(x) + \frac{\partial V}{\partial x^T} g(x)u \\ &\leq \frac{\partial V}{\partial x^T} f_2(x) + \frac{\partial V}{\partial x^T} g(x)\frac{L_g^T V(x)}{\|L_g V(x)\|^2} \times \\ &\quad \left(-\frac{\partial V}{\partial x^T} f_2(x) + h^T(x)v - \Psi(t,x,u,y)\right) \\ &= h^T(x)v - \Psi(t,x,u,y)\end{aligned} \tag{6}$$

For simplicity and without loss of generality, for the rest of the paper, we assume that $f_1(x) = 0$, i.e. $f(x) = f_2(x)$. When $V(x)$ is a positive definite function with $V(0) = 0$, then from Theorem 4 the system (1) is (asymptotically) stable if $\big(h^T(x)v < \Psi(t,x,u,y)\big)$ $h^T(x)v \leq \Psi(t,x,u,y)$.

3. SLIDING MODE OF PASSIVE SYSTEMS

Now a sliding mode control v is designed so that the system (5) is converted to a reduced order system. This system could be the zero dynamics of the nonlinear system. Here we consider output sliding mode control but the approach can be easily extended to the general case by replacing $h(x)$ with $s(x)$, where $s(x) = 0$ is an arbitrary sliding surface. The rest of the theory remains intact. We assume that rank($g(0)$)=rank($dh(0)$) $= m$ and the rank $L_g h(x)$ is constant in a neighbourhood of 0, i.e. 0 is a regular point for the system. Then $L_g h(0)$ is nonsingular and the system has relative degree $[1, 1, \ldots, 1]$ at $x = 0$ (Byrnes *et al.*, 1991). In this method the system (1) is first converted into a new system (5) via the feedback transformation (4). Then a sliding mode control is designed for system (5). Consider $h(x) = 0$ as a sliding hyperplane. Ideal sliding motion occurs when $h(x) = 0$. The system equation of the ideal sliding mode is

$$\dot{x} = \left(I - g(x) \frac{L_g^T V(x)}{\|L_g V(x)\|^2} \frac{\partial V}{\partial x^T} \right) f(x) - g(x) \times \frac{L_g^T V(x)}{\|L_g V(x)\|^2} \Psi(t, x, u, y) \tag{7}$$

and the control (4) is now

$$u = \frac{L_g^T V(x)}{\|L_g V(x)\|^2} \left(-\frac{\partial V}{\partial x^T} f(x) - \Psi(t, x, u, y) \right) \tag{8}$$

When the ideal sliding mode occurs, it is required that on the manifold $h(x) = 0$, $\dfrac{\partial h}{\partial t} = \dfrac{\partial h}{\partial x} f_{eq} = 0$ with

$$f_{eq} = \left(I - g(x) \frac{L_g^T V(x)}{\|L_g V(x)\|^2} \frac{\partial V}{\partial x^T} \right) f(x) - g(x) \frac{L_g^T V(x)}{\|L_g V(x)\|^2} \Psi(t, x, u, y)$$

In a system with a boundary layer, $h(x)$ and/or $\dfrac{\partial h}{\partial t}$ may not be zero outside of the boundary layer. Assume that $h(x) \neq 0$ and $h^T(x)h(x)$ is invertible. It can be shown from (5) that the control

$$\tilde{v}_s = -h(x)(h(x)^T h(x))^{-1} L_g V(x) \left(\frac{\partial h}{\partial x} g(x) \right)^{-1} \frac{\partial h}{\partial x} f_{eq}$$

yields $\dfrac{\partial h}{\partial t} = 0$. v_s is not the equivalent control (when $h(x) = 0$). The equivalent control is $v_{eq} = 0$. However, v_s ensures that the trajectories remain inside a neighbourhood of the sliding surface for future time. For the occurrence of a proper sliding mode, the trajectories may cross the sliding manifold repeatedly and remain in the boundary of the sliding manifold. Along this manifold they tend to an equilibrium point whenever the sliding (zero) dynamics of the system is stable. A control is designed next so that the trajectories tend to a neighbourhood of $h(x) = 0$, an attractive sliding region, and remain inside for future time.

Consider the following control for the passive system (5)

$$v = -h(x)(h(x)^T h(x))^{-1} L_g V(x) \left(\frac{\partial h}{\partial x} g(x) \right)^{-1} \times \frac{\partial h}{\partial x} \left[f_{eq} + K \operatorname{sgn} \left((\frac{\partial h}{\partial x})^T h(x) \right) \right] \tag{9}$$

where $K > 0$ is a constant sliding mode gain. The control (9) enforces the system trajectories to the sliding manifold $s = h(x) = 0$, since

$$\begin{aligned} s^T \dot{s} &= h^T \frac{\partial h}{\partial x} \dot{x} \\ &= h^T \frac{\partial h}{\partial x} \left\{ f_{eq} - g \left(\frac{\partial h}{\partial x} g \right)^{-1} \frac{\partial h}{\partial x} \left[f_{eq} + K \operatorname{sgn} \left((\frac{\partial h}{\partial x})^T h \right) \right] \right\} \\ &= -K h^T \frac{\partial h}{\partial x} \operatorname{sgn} \left((\frac{\partial h}{\partial x})^T h \right) \leq 0 \end{aligned} \tag{10}$$

and

$$\dot{x} = \left(I - g \left(\frac{\partial h}{\partial x} g \right)^{-1} \frac{\partial h}{\partial x} \right) f_{eq} - g \left(\frac{\partial h}{\partial x} g \right)^{-1} \frac{\partial h}{\partial x} \operatorname{sgn} \left((\frac{\partial h}{\partial x})^T h \right)$$

4. ADAPTIVE PASSIVATION OF NONLINEAR SYSTEMS

Consider the parameterized multivariable nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + \phi(x)\theta + g(x)u \\ y &= h(x) \end{aligned} \tag{11}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^n$ is the state; $u \in \mathcal{U} \subseteq \mathbb{R}^m$ the control input and $y \in \mathcal{Y} \in \mathbb{R}^m$ the output. Functions f and g are smooth on $\mathcal{X}$. Assume that $\mathcal{X}$ is a pathwise connected open subset of $\mathbb{R}^n$. The equilibrium point $x_e \in \mathcal{X}$ satisfies $f(x_e) + g(x_e)u_e = 0$ with $u_e \in \mathcal{U}$ constant. h is a smooth function defined on $\mathcal{X}$. $\phi \in \mathbb{R}^{n \times p}$ are smooth and $\theta = [\theta_1 \quad \theta_2 \quad \ldots \quad \theta_p]^T$ is a vector of unknown constant parameters.

The system (11) can be written as

$$\begin{aligned} \dot{x} &= f(x) + \phi(x)\hat{\theta} + g(x)u + \phi(x)\tilde{\theta} \\ y &= h(x) \end{aligned} \tag{12}$$

where $\hat{\theta}$ is an estimate of θ and $\tilde{\theta} = \theta - \hat{\theta}$. According to Theorem 4 the nominal system of (12)

$$\begin{aligned} \dot{x} &= \hat{f}(x, \hat{\theta}) + g(x)u \\ y &= h(x) \end{aligned} \tag{13}$$

with $\hat{f}(x, \hat{\theta}) = f(x) + \phi(x)\hat{\theta}$, is a passive system via feedback passivation (4) by replacing f with

$\hat{f}$. Consider the Lyapunov function V and assume the transversality condition $L_gV \neq 0$ holds. The nominal passive system with new control input v is given by (5) by replacing $\hat{f}$ with f.

Now consider the extended Lyapunov function

$$W = V + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{14}$$

where Γ is a positive definite matrix. Then

$$\begin{aligned}\dot{W} &= \frac{\partial V}{\partial x}\dot{x} + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}(-\dot{\hat{\theta}}) \\ &= h^Tv - \Psi + \frac{\partial V}{\partial x}\phi\tilde{\theta} + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}(-\dot{\hat{\theta}}) \\ &= h^Tv - \Psi + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\left(\Gamma\left[\frac{\partial V}{\partial x}\phi\right]^T - \dot{\hat{\theta}}\right)\end{aligned} \tag{15}$$

The update estimate function is selected to be

$$\dot{\hat{\theta}} = \Gamma\phi^T\frac{\partial V}{\partial x^T} \tag{16}$$

which eliminates the last term of equation (15). Then

$$\dot{W} = h^Tv - \Psi \tag{17}$$

$$u = \frac{L_g^TV(x)}{\|L_gV(x)\|^2}\left(-\frac{\partial V}{\partial x^T}\hat{f}(x,\hat{\theta}) + h^T(x)v - \Psi\right) \tag{18}$$

and the transformed system is

$$\begin{aligned}\dot{x} = &\left(I - g(x)\frac{L_g^TV(x)}{\|L_gV(x)\|^2}\frac{\partial V}{\partial x^T}\right)\hat{f}(x,\hat{\theta}) + g(x) \times \\ &\frac{L_g^TV(x)}{\|L_gV(x)\|^2}\left(h^T(x)v - \Psi\right) + \phi(x)\tilde{\theta}\end{aligned} \tag{19}$$

which is a passive system.

5. EXAMPLES

An exothermic chemical reactor model

It is desired to globally stabilize the temperature of a chemical reactor at an arbitrary set point by designing an appropriate control. The stabilization of chemical reactors has been studied during the second half of the last century, e.g. Aris and Admundson (1958). A feedback control has been designed by Viel *et al.* (1997) to stabilize chemical reactors globally. Their method is based upon saturation control with some assumptions. One of the assumptions is that the zero dynamics of the isothermal system is globally asymptotically stable when the temperature tends to the desired temperature. In this case the global stabilization of both temperature and concentrations is achieved. We design a control via two steps; first the system is converted into a passive system via a feedback transformation and then sliding control is designed to stabilize the system. The sliding control ensures that the reactor temperature regulates to a desired value. Global stabilization is achieved without any assumptions except for the minimum phase condition on the system, which is necessary for stabilizing the concentrations.

Consider the multivariable reactor model (Sira-Ramírez and Ríos-Bolívar, 1999; Viel *et al.*, 1997) in which a first order and exothermic reaction $A \to B$ occurs:

$$\begin{aligned}\dot{x}_1 &= -k(x_3)x_1 - \beta x_1 + \beta w \\ \dot{x}_2 &= k(x_3)x_1 - \beta x_2 \\ \dot{x}_3 &= \alpha k(x_3)x_1 - qx_3 + u \\ y &= x_3\end{aligned} \tag{20}$$

where x_1 and x_2 are the concentrations in the reactor of reactant A and the product B, respectively. x_3 denotes the reactor temperature. The positive constant α is the exothermicity of the reaction. $\beta > 0$ is a constant associated with the dilution rate. The control input w represents the concentration of the reactant A in the feed flow, denoted by x_1^{in} when it is not considered as a positive constant input (Viel *et al.*, 1997), while the control input u corresponds to a suitable and well-known combination of the feed temperature T_{in} and the coolant temperature T_w, i.e. $u = \beta T_{in} + eT_w$ where $e > 0$ is the heat transfer rate constant. Let $q = \beta + e > 0$. $k(x_3)$ is a nonnegative bounded function of temperature. The operating region of the system is $\mathbb{R}^3_+$. The constant equilibrium point of the system may be parameterized in terms of the equilibria for the variables x_1 and x_2 as follows:

$$\begin{aligned}\bar{x}_3 &= \frac{k_1}{\ln\left(\frac{k_0\bar{x}_1}{\beta\bar{x}_2}\right)} \\ w &= \bar{x}_1 + \bar{x}_2 \\ u &= q\bar{x}_3 - \alpha\bar{x}_1k(\bar{x}_3) = q\bar{x}_3 - \alpha\beta\bar{x}_2\end{aligned}$$

Although w is a positive constant, we can consider it as control input

$$w = \begin{cases} x_1^{in} & x_3 \leq X_3^\star \\ 0 & \text{otherwise}\end{cases} \tag{21}$$

where x_1^{in} is positive constant value and $X_3^\star$ is the positive constant temperature. w affects the system by reducing the temperature and therefore reduces the velocity of exothermic action. Whenever the system temperature is larger than $X_3^\star$, the control input w cuts off at the value x_1^{in}. $X_3^\star$ may be selected to be the desired value X_3, but they also could be two different values. Consider the storage function

$$V(x) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$$

Then

$$
\begin{aligned}
\dot{V} &= \dot{x}_1 x_1 + \dot{x}_2 x_2 + \dot{x}_3 x_3 \\
&= -x_1^2 k(x_3) - \beta x_1^2 + \beta x_1 w + x_1 x_2 k(x_3) \\
&\qquad -\beta x_1 x_2 - \alpha x_1 x_3 k(x_3) - q x_3^2 + x_3 u \\
&\le \beta x_1 w + x_1 x_2 k(x_3) + \alpha x_1 x_3 k(x_3) + x_3 u \\
&= x_3 v - \Psi
\end{aligned}
$$

where v is a new control and Ψ is a nonnegative function. Therefore,

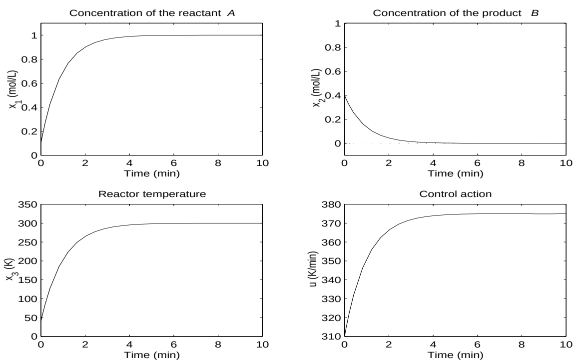

Fig. 1. An exothermic chemical reactor model with $K = 0.1$ and $W = 1$

$$
u = -\alpha x_1 k(x_3) - \frac{1}{x_3}(\beta x_1 w + x_1 x_2 k(x_3) + \Psi) + v \tag{22}
$$

and the transformed system is

$$
\begin{aligned}
\dot{x}_1 &= -k(x_3)x_1 - \beta x_1 + \beta w \\
\dot{x}_2 &= k(x_3)x_1 - \beta x_2 \\
\dot{x}_3 &= -q x_3 - \frac{1}{x_3}\left(\beta x_1 w + x_1 x_2 k(x_3) + \Psi\right) + v \\
y &= x_3
\end{aligned} \tag{23}
$$

Since the transformed system (23) is passive, it is

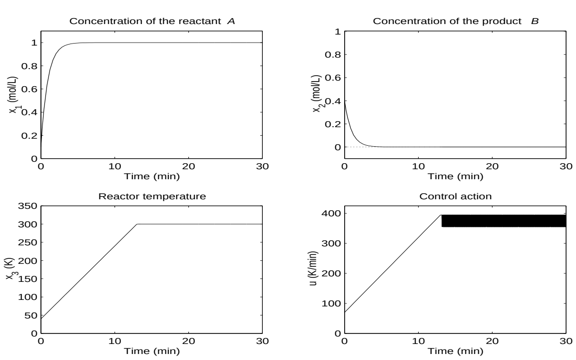

Fig. 2. An exothermic chemical reactor model with $K = 20$ and $W = 0$

stable. Because $x_3 \dot{x}_3 < 0$, the temperature tends to an equilibrium point asymptotically without any restriction on the selection of v even if $\Psi = 0$. However, it is desired that the temperature tends to a given set point. Now the control input v is designed so that the state x_3 tracks the desired value X_3. Consider the sliding tracking control

$$
\begin{aligned}
v = q x_3 + \frac{1}{x_3}\left(\beta x_1 w + x_1 x_2 k(x_3) + \Psi\right) \\
-W(x_3 - X_3) - K \operatorname{sgn}(x_3 - X_3)
\end{aligned} \tag{24}
$$

where K and W are nonnegative numbers such that $W + K \neq 0$. Substituting (24) in (22)

$$
u = -\alpha x_1 k(x_3) + q x_3 - W(x_3 - X_3) - K \operatorname{sgn}(x_3 - X_3)
$$

Since $\alpha x_1 k(x_3)$ is very small and $q x_3$ is large, the control takes nonnegative values. The nonnegativity of the control is a physical restriction of the problem. However, one can select the control as

$$
\begin{aligned}
u = \max \big\{ 0, -\alpha x_1 k(x_3) + q x_3 - W(x_3 - X_3) \\
-K \operatorname{sgn}(x_3 - X_3) \big\}
\end{aligned} \tag{25}
$$

to ensure the control is nonnegative. Note that the system may be stabilized via many other choices of v. Sliding control (24) has been considered because it is a simple and suitable control. The isothermal dynamics, the sliding zero dynamics of the system, is

$$
\begin{aligned}
\dot{x}_1 &= -k(X_3)x_1 - \beta x_1 + \beta w \\
\dot{x}_2 &= k(X_3)x_1 - \beta x_2
\end{aligned}
$$

which is asymptotically stable. Successful simulation results are shown in Fig. 1 for $X_3 = 300$, $X_3^\star = 340$, $\psi = 0$, $k_0 = 7.2e + 10\text{min}^{-1}$, $k_1 = 8700\text{K}$, $\alpha = 209\text{KL/mol}$, $\beta = 1.1\text{min}^{-1}$, $e = 0.15\text{min}^{-1}$, $K = 0.1$, $W = 1$, $X_3^\star = 340\text{K}$, $X_3 = 300\text{K}$, $x_1^{in} = 1\text{mol/L}$. We consider $k(x_3) = k_0 e^{-\frac{k_1}{x_3}}$, i.e. Arrhenius Law. For simulation we consider $k_0 = 7.2e + 10\text{min}^{-1}$ and $k_1 = 8700K$. The control input w is constant, $w = 1$ for all time, because the temperature does not exceed $X_3^\star$. The sliding gains W and K affect the reaching time of the desired temperature X_3. If the value W is larger than K the chattering is very small, while for a constant value W, for increasing K chattering appears. Note that $\lim_{t\to\infty} x_2 = 0$ because $\lim_{t\to\infty} k(x_3) = 0$, however $x_2 > 0$ for all t. The gain values W and K should be selected so that the value u takes a reasonable value from a practical viewpoint. Table 1 shows that suitable gains W are $0 \le W \le 2$. A large value of K causes undesired chattering, and the control switches between at least two different values. However, the chattering phenomenon is negligible if we select K sufficiently small (see Table 1 and Fig. 2). Note that for all values of K and W, the desired temperature is obtained.

Now assume that w is not a constant input and consider the output of the system as $y = [x_1 \;\; x_3]^T$. Then $L_g V(x) = [x_1 \;\; x_3] = y^T$. The control is now

$$
\begin{aligned}
\begin{bmatrix} w \\ u \end{bmatrix} = \frac{-1}{x_1^2 + x_3^2} \Big\{ \Big((k(x_3) + \beta) x_1^2 + \beta x_2^2 + q x_3^2 + \\
-x_1 x_2 k(x_3) - \alpha x_1 x_3 k(x_3) - \Psi \Big) \begin{bmatrix} \frac{1}{\beta} x_1 \\ x_3 \end{bmatrix} - v \Big\}
\end{aligned}
$$

K	W	initial value of u	$u_{\min}$	$u_{\max}$	final value u
2	0	52	52	376.97	373 and 376.9
4	0	54	54	378.98	371 and 378.9
6	0	56	56	380.9	380.89 and 369
20	0	70	70	394.99	355 and 394.99
50	0	100	100	424.98	325.1 and 424.98
0.1	0.1	76.1	76.1	375	375
0.1	0.5	180.1	180.1	375.33	375
0.1	1	310.1	310.1	375.1	375
0.1	2	570.1	374.68	570.1	375
0.1	4	10901	370	10901	376
0.1	10	26051	345	26051	376
0.1	100	26050	177	26050	292 − 492
0.5	1	310.5	310.5	375.5	375(Avg.) (374.5 − 375.5)
1	1	311	311	375.49	75(Avg.) (374 − 376)
1	2	571	372.9	571	374(Avg.) (373.97 − 376.1)
2	2	572	372.6	372.6	375(Avg.) (372 − 377)

Table 1. Initial, maximum, minimum and equilibrium (final) values of the control u with respect to different gain values K and W.

It is desired to regulate x_1 and x_3 to given values X_1 and X_3, respectively. The sliding surface is the intersection of the surfaces $x_1 = X_1$ and $x_3 = X_3$. The sliding control $v = [v_1 \;\; v_2]^T$ is given by

$$\begin{aligned}
v_1 =& \frac{1}{\beta}\Big[x_1^3 x_3^2(\beta + k(x_3)) + x_1^3 x_2(x_1 k(x_3) - \beta x_2) \\
&+ x_1^3 x_3(x_1 k(x_3) - q x_3) - x_1^2 x_3^2(x_1 k(x_3) + \beta x_1) \\
&+ x_1 x_2 x_3^2(x_1 k(x_3) - \beta x_2) + x_1^3 x_3(\alpha x_1 k(x_3) - q x_3) \\
&+ K x_1^2 \operatorname{sgn}(x_1 - X_1) + K x_1 x_3 \operatorname{sgn}(x_3 - X_3)\Big] \\
v_2 =& x_1^2 x_3^3(\beta + k(x_3)) + x_1^2 x_2 x_3(x_1 k(x_3) - \beta x_2) \\
&+ x_1^2 x_3^2(x_1 k(x_3) - q x_3) - x_1^2 x_3^3(k(x_3) + \beta) + \\
&x_2 x_3^3(x_1 k(x_3) - \beta x_2) + x_1^2 x_2(\alpha x_1 k(x_3) - q x_3) \\
&+ K x_1 x_3 \operatorname{sgn}(x_1 - X_1) + K x_3^2 \operatorname{sgn}(x_3 - X_3)
\end{aligned} \tag{26}$$

The sliding condition is

$$s^T \dot{s} = -K(|x_1 - X_1| + |x_3 - X_3| < 0)$$

The sliding zero dynamics system is obtained when $v_1 = v_2 = 0$.

6. CONCLUSIONS

Passivity based control has been studied for affine nonlinear systems. In this method, the system is transformed into a new system, the so-called passive system via control feedback (passivation). The sliding mode of the passive system has been considered. An exothermic chemical reactor model has been presented to show the results.

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