

GENERALIZED AMBIGUITY FUNCTION FOR THE MIMO RADAR WITH CORRELATED WAVEFORMS

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ABSTRACT

An ambiguity function (AF) for the multiple-input multiple-output (MIMO) radar with correlated waveforms is derived. It serves as a generalized AF for which the phased-array and the traditional MIMO radar AFs are important special cases. A simplified expression for the AF for the case of far-field targets and narrow-band waveforms is also derived. We establish relationships between the generalized MIMO radar AF metric and the previous works on AF including the Woodward's AF and the AF defined for the traditional colocated MIMO radar. Moreover, we compare the AF of the MIMO radar with correlated waveforms with the squared-summation-form AF definition. Simulation results show that the generalized MIMO radar AF achieves lower relative sidelobe level with proper design of the waveform correlation matrix or, equivalently, the transmit beamspace matrix.

Index Terms— Ambiguity function, correlated waveforms, generalized, MIMO radar, transmit beamspace.

1. INTRODUCTION

Recently, the multiple-input multiple-output (MIMO) radar has become the focus of intensive research [1]–[3]. Despite the benefits due to the use of waveform diversity [3], the traditional MIMO radar with colocated transmit antenna elements suffers from the loss of coherent processing gain that can be achieved in the phased-array (PA) radar system [4]. The transmit beampattern can be formed by designing a proper correlation matrix for the waveforms at the transmitter [5]–[7]. Such waveform correlation matrix design can be simplified to transmit beamspace (TB) matrix design (see for example [8]). It allows to achieve the coherent processing gain by focusing the energy of multiple transmitted orthogonal waveforms within a certain angular sector where a target is likely to be

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located using beamforming techniques [4], [8]. For the MIMO radar systems, accurate estimation and detection capabilities are critically important. For example, one important task is to estimate the direction-of-arrivals of potential targets utilizing the extra degrees-of-freedom offered by the MIMO radar. To efficiently characterize the resolution performance, ambiguity function (AF) [9]–[13] can be employed. There are some works on the traditional MIMO radar AF [11]–[13], and their starting point is the well-known Woodward's AF [9], [10]. The research questions of great significance are to see how the AF of the MIMO radar with correlated waveforms behaves, and what the relationships with the previous works on AF are.

In this paper, we define the AF for the MIMO radar with correlated waveforms, in which the phase shift information conveyed by the target echo is contained. This phase shift results from the array geometry and the relative position between the target and the transmit/receive colocated array. Equivalent phase centers are used for its calculation. Moreover, we derive a simplified AF expression for the case of far-field targets and narrow-band waveforms. Based on this expression relationships between the AF of the MIMO radar with correlated waveforms and the previous works on AF are established. It can be utilized as a generalized AF for the PA and traditional MIMO radars, as well as the MIMO radar with correlated waveforms. We propose a method to reduce the relative sidelobe level of the AF and compare it with the traditional MIMO radar AF defined in [13] by simulations.

2. SIGNAL MODEL

Consider a colocated MIMO radar system with a transmit array of M antenna elements and a receive array of N antenna elements. The complex envelope of the transmitted waveforms in the case of the traditional MIMO radar can be modeled as $s_m(t) = \sqrt{E/M}\phi_m(t)$, $m = 1, 2, \dots, M$ where E is the total transmit energy, t is the continuous time index, i.e., time within the pulse, and $\phi_m(t)$ is the m th orthogonal baseband waveform. Without loss of generality, we assume that the transmitted waveforms are normalized to have unit-energy, i.e., $\int_T |\phi_m(t)|^2 dt = 1$, $m = 1, 2, \dots, M$ where T is the time

duration of the pulse.

In the MIMO radar with correlated waveforms formulated using the TB matrix design, K (in general, $K \leq M$) initially orthogonal waveforms are transmitted. For each waveform, a transmit beam is formed by illuminating a particular area within an angular sector Ω . The signal radiated towards the target that is located at the spatial direction θ via the k th transmit beam can be modeled as [8]

$$s_k(t) = \sqrt{\frac{E}{K}} \mathbf{c}_k^T \mathbf{a}(\theta) \phi_k(t), \quad k = 1, 2, \dots, K \quad (1)$$

where $\mathbf{a}(\theta)$ is the transmit array steering vector, \mathbf{c}_k denotes the k th column of the $M \times K$ TB matrix \mathbf{C} which is defined as $\mathbf{C} \triangleq [\mathbf{c}_1, \dots, \mathbf{c}_K]$, and $(\cdot)^T$ stands for the transpose operation. Each column of \mathbf{C} with M elements is designed to form a certain transmit beam within the sector-of-interest Ω , and it corresponds to one of the K transmitted waveforms. Let c_{mk} be the m th element of \mathbf{c}_k , then the signal $\tilde{s}_m(t)$ radiated by the m th antenna element can be expressed as

$$\tilde{s}_m(t) = \sqrt{\frac{E}{K}} \sum_{k=1}^K c_{mk} \phi_k(t), \quad m = 1, \dots, M. \quad (2)$$

The transmitted energy focused within the sector-of-interest Ω can be maximized by properly designing the TB matrix \mathbf{C} .

3. GENERALIZED MIMO RADAR AF WITH CORRELATED WAVEFORMS

3.1. AF Definition

Let a point target be described by the parameter Θ which contains the information of the target position vector \mathbf{p} and the velocity vector \mathbf{v} . Based on (2), the received signal at the j th receive antenna element after demodulation to the base band can be written as (3), shown at the bottom of this

page. Here α_{mj} denotes the complex reflection coefficient for the (m, j) th transmit-receive channel, $\tau_{mj}(\mathbf{p})$ is the two-way time delay due to a target located at \mathbf{p} , f_c is the operating frequency, $f_{mj}(\Theta)$ is the Doppler shift associated with the (m, j) th transmit-receive channel for the target characterized by the parameter Θ and $z_j(t)$ is the white Gaussian noise observed at the j th receive antenna element with power σ_z^2 .

At the receiving end, $\hat{r}_j(t, \Theta)$ is matched to each of the waveforms $\phi_k(t)$, $k = 1, \dots, K$ with a specific target parameter Θ' due to the fact that it is composed of the cumulative echoes of the known transmitted waveforms. Thus, the received signal component $\bar{r}_{ji}(\Theta, \Theta')$ that is associated with the i th transmitted waveform can be obtained as (4), shown at the bottom of this page. Here $(\cdot)^*$ is the conjugate operator, $q(i)$ is the equivalent phase center for the i th transmitted waveform and $\bar{z}_{ji}(t)$ is the noise after matched filtering.

We define the AF as the square of the coherent summation of all noise-free matched filtering outputs $\bar{r}'_{ji}(\Theta, \Theta')$, $j = 1, \dots, N$ and $i = 1, \dots, K$. Then the AF of the MIMO radar with correlated waveforms can be mathematically expressed as (5), shown at the bottom of this page. Let us introduce an $M \times K$ matrix \mathbf{R} whose (m, i) th element is defined as (6), shown at the bottom of the next page. Using (6), the AF (5) can be expressed in a simplified form as (7), shown at the bottom of the next page.

The AF (7) is composed of summation terms, and each term contains two more components in addition to the complex reflection coefficient. One is the matched-filtered component denoted by (6), which stands for the effect of waveform correlation. The other is composed of the last two exponential terms in (7), which stands for the phase shift information caused by the relative position and velocity of the target with respect to the array geometry. It can also be understood as follows. The m th transmit antenna element emits a signal that is composed of all the K initially orthogonal waveforms windowed by the elements of the m th row in the TB matrix \mathbf{C} . Thus, the TB

$$\hat{r}_j(t, \Theta) = \sqrt{\frac{E}{K}} \sum_{m=1}^M \sum_{k=1}^K \alpha_{mj} c_{mk} \phi_k(t - \tau_{mj}(\mathbf{p})) \exp\{-j2\pi\tau_{mj}(\mathbf{p})(f_c + f_{mj}(\Theta))\} \exp\{j2\pi f_{mj}(\Theta)t\} + z_j(t) \quad (3)$$

$$\begin{aligned} \bar{r}_{ji}(\Theta, \Theta') &= \int \hat{r}_j(t, \Theta) \phi_i^*(t, \Theta') dt = \sqrt{\frac{E}{K}} \sum_{m=1}^M \sum_{k=1}^K \alpha_{mj} \int c_{mk} \phi_k(t - \tau_{mj}(\mathbf{p})) \phi_i^*(t - \tau_{q(i)j}(\mathbf{p}')) \\ &\times \exp\{-j2\pi\tau_{mj}(\mathbf{p})(f_c + f_{mj}(\Theta))\} \exp\{j2\pi\tau_{q(i)j}(\mathbf{p}')(f_c + f_{q(i)j}(\Theta'))\} \exp\{j2\pi(f_{mj}(\Theta) - f_{q(i)j}(\Theta'))t\} dt \\ &+ \bar{z}_{ji}(t) \triangleq \bar{r}'_{ji}(\Theta, \Theta') + \bar{z}_{ji}(t) \end{aligned} \quad (4)$$

$$\begin{aligned} \chi(\Theta, \Theta') &\triangleq \left| \sum_{j=1}^N \sum_{i=1}^K \bar{r}'_{ji}(\Theta, \Theta') \right|^2 = \left| \sqrt{\frac{E}{K}} \sum_{j=1}^N \sum_{i=1}^K \sum_{m=1}^M \sum_{k=1}^K \alpha_{mj} \int c_{mk} \phi_k(t - \tau_{mj}(\mathbf{p})) \phi_i^*(t - \tau_{q(i)j}(\mathbf{p}')) \right. \\ &\times \exp\{-j2\pi\tau_{mj}(\mathbf{p})(f_c + f_{mj}(\Theta))\} \exp\{j2\pi\tau_{q(i)j}(\mathbf{p}')(f_c + f_{q(i)j}(\Theta'))\} \exp\{j2\pi(f_{mj}(\Theta) - f_{q(i)j}(\Theta'))t\} dt \left. \right|^2 \end{aligned} \quad (5)$$

matrix \mathbf{C} is playing a role of transforming the original $K \times K$ waveform covariance matrix to the $M \times K$ matrix \mathbf{R} . The main purpose of adding the phase shift information in (7) is to incorporate the property of coherent processing introduced by the array configuration. By properly designing the matrix \mathbf{C} and selecting the equivalent phase centers, the AF (7) can serve also as the AF for the PA and traditional MIMO radars.

3.2. Far-Field Targets and Narrow-Band Waveforms

One most common scenario that radar systems deal with is the case of far-field targets and narrow-band waveforms. The effect of the complex reflection coefficients in different transmit-receive channels can be neglected in this case because the contributions of different transmit-receive channels to the generalized MIMO radar AF are constant at any given time t . They remain constant even when considering multiple pulses and inter-pulse varying reflection coefficients, if long pulse width is employed and no range folding occurs [12].

Assume that the antenna elements of the transmit and receive arrays have locations $\{\mathbf{q}_{T,1}, \dots, \mathbf{q}_{T,M}\}$ and $\{\mathbf{q}_{R,1}, \dots, \mathbf{q}_{R,N}\}$, respectively, and the equivalent phase centers have locations $\{\mathbf{q}_{TE,1}, \dots, \mathbf{q}_{TE,K}\}$ whose elements are vectors in three-dimensional Cartesian coordinates. Let $\mathbf{u}(\Theta)$ be a unit-norm steering vector pointing from the transmit/receive array to the target with parameter Θ . Then the AF (7) can be simplified as

$$\chi(\Theta, \Theta') = |\mathbf{a}_R^H(\Theta) \mathbf{a}_R(\Theta')|^2 |\mathbf{a}_T^H(\Theta) \bar{\mathbf{R}} \mathbf{a}_{TE}(\Theta')|^2 \quad (8)$$

where the (m, i) th element of the $M \times K$ matrix $\bar{\mathbf{R}}$ is expressed as

$$\begin{aligned} [\bar{\mathbf{R}}]_{mi}(\Delta\tau, \Delta f_d, \mathbf{C}) & \quad (9) \\ &= \sqrt{\frac{E}{K}} \sum_{k=1}^K c_{mk} \int \phi_k(t) \phi_k^*(t - \Delta\tau) \exp\{j2\pi \Delta f_d t\} dt \end{aligned}$$

and $(\cdot)^H$ denotes the conjugate transpose. Here also $\Delta\tau \triangleq \tau(\mathbf{p}) - \tau(\mathbf{p}')$, $\Delta f_d \triangleq f(\Theta) - f(\Theta')$, and $\mathbf{a}_T(\Theta) \triangleq [\exp\{\tilde{\mathbf{u}}^T(\Theta) \mathbf{q}_{T,1}\}, \dots, \exp\{\tilde{\mathbf{u}}^T(\Theta) \mathbf{q}_{T,M}\}]^T$, $\mathbf{a}_R(\Theta) \triangleq [\exp\{\tilde{\mathbf{u}}^T(\Theta) \mathbf{q}_{R,1}\}, \dots, \exp\{\tilde{\mathbf{u}}^T(\Theta) \mathbf{q}_{R,N}\}]^T$, $\mathbf{a}_{TE}(\Theta) \triangleq [\exp\{\tilde{\mathbf{u}}^T(\Theta) \mathbf{q}_{TE,1}\}, \dots, \exp\{\tilde{\mathbf{u}}^T(\Theta) \mathbf{q}_{TE,K}\}]^T$ are the $M \times 1$ transmit array steering vector, the $N \times 1$ receive array

steering vector, and the $K \times 1$ equivalent transmit array steering vector, respectively, with $\tilde{\mathbf{u}}(\Theta) \triangleq j2\pi f'(\Theta) \cdot \mathbf{u}(\Theta)/c$ and $f'(\Theta) \triangleq f_c + f(\Theta)$. The dependence of $\bar{\mathbf{R}}$ from $\Delta\tau$, Δf_d , and \mathbf{C} is not shown in (8) for brevity, and the subscript indices for τ and f are omitted for the far-field narrow-band case.

3.3. Relationships With Other AFs

Let the $K \times K$ matrix $\bar{\chi}(\tau, f_d)$ be the AF matrix of the K orthogonal waveforms, whose (j, k) th element is given by

$$[\bar{\chi}]_{jk}(\tau, f_d) = \int \phi_j(t) \phi_k^*(t - \tau) \exp\{j2\pi f_d t\} dt. \quad (10)$$

Using (9) and (10), the simplified AF (8) can be expressed as

$$\begin{aligned} \chi(\Theta, \Theta') &= \frac{E}{K} |\mathbf{a}_R^H(\Theta) \mathbf{a}_R(\Theta')|^2 \quad (11) \\ &\quad \times |\mathbf{a}_T^H(\Theta) \mathbf{C} \bar{\chi}(\Delta\tau, \Delta f_d) \mathbf{a}_{TE}(\Theta')|^2 \end{aligned}$$

where $\bar{\chi}(\Delta\tau, \Delta f_d)$ is the $K \times K$ matrix whose elements are given by (10). Considering that $\Delta\tau$ and Δf_d depend on Θ and Θ' , we employ these two parameters to denote the AF of the MIMO radar with correlated waveforms.

Equation (11) establishes the connection between the generalized MIMO radar AF and the well known Woodward's AF. If the number of transmitted waveforms K is increased to M , \mathbf{C} is simply the identity matrix \mathbf{I}_M , and the equivalent phase centers are selected to be the positions of the M transmit antenna elements, then the AF (11) becomes

$$\begin{aligned} \chi_{\text{MIMO}}(\Theta, \Theta') &= \frac{E}{M} |\mathbf{a}_R^H(\Theta) \mathbf{a}_R(\Theta')|^2 \quad (12) \\ &\quad \times |\mathbf{a}_T^H(\Theta) \bar{\chi}(\Delta\tau, \Delta f_d) \mathbf{a}_T(\Theta')|^2. \end{aligned}$$

Expression (12) denotes the traditional MIMO radar AF and has the same form as the definition in [11] except for the magnitude term. This term represents the general expression of the transmit power allocation for the traditional MIMO radar. Therefore, if E is selected to be equal to M , then the expression (12) and the AF definition [11] have identical expressions. Furthermore, the generalized MIMO radar AF (11) is also related to the traditional MIMO radar AF with K uniform subarrays [14]. In this case, \mathbf{C} is designed as a block diagonal matrix whose block elements are associated with the

$$[\mathbf{R}]_{mi}(\Theta, \Theta', \mathbf{C}, j) \triangleq \sqrt{\frac{E}{K}} \sum_{k=1}^K c_{mk} \int \phi_k(t - \tau_{mj}(\mathbf{p})) \phi_k^*(t - \tau_{q(i)j}(\mathbf{p}')) \exp\{j2\pi (f_{mj}(\Theta) - f_{q(i)j}(\Theta')) t\} dt \quad (6)$$

$$\begin{aligned} \chi(\Theta, \Theta') & \quad (7) \\ &= \left| \sum_{j=1}^N \sum_{i=1}^K \sum_{m=1}^M \alpha_{mj} [\mathbf{R}]_{mi}(\Theta, \Theta', \mathbf{C}, j) \exp\{-j2\pi \tau_{mj}(\mathbf{p})(f_c + f_{mj}(\Theta))\} \exp\{j2\pi \tau_{q(i)j}(\mathbf{p}')(f_c + f_{q(i)j}(\Theta'))\} \right|^2 \end{aligned}$$

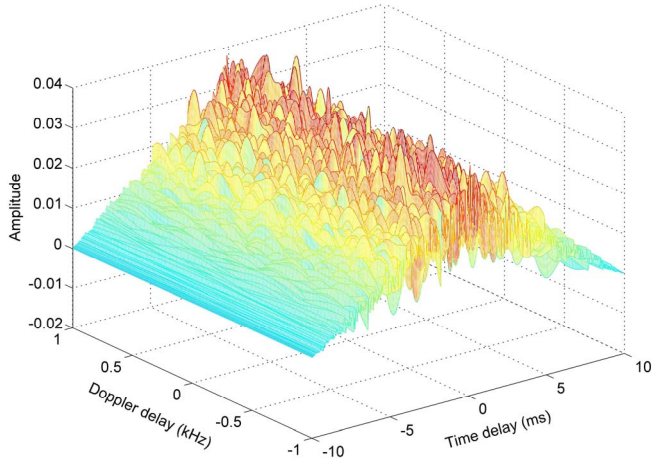


Fig. 1. Difference between the AF (8) and that defined in [13].

subarrays. The corresponding equivalent phase centers are selected as the centers of the subarrays. When K is selected as 1, the generalized MIMO radar AF (11) also boils down to the PA radar AF. Consequently, the AF defined in this paper serves as a unified definition of AF, and it links to the Woodward's AF by using the expression of the Woodward's AF matrix.

For the MIMO radar with correlated waveforms, if each coherent processing gain $\Upsilon_j \triangleq \mathbf{a}_T^H(\Theta)\mathbf{c}_j$, $j \in \{1, \dots, K\}$ is designed to have constant magnitude, then the rotational invariance property [15], [16] holds. Hence, we can design the TB matrix \mathbf{C} to guarantee that the j th coherent processing gain and the j th element of the equivalent transmit array steering vector have opposite phases, i.e., $\angle \Upsilon_j = -\angle \mathbf{a}_{TE,j}(\Theta)$, $j \in \{1, \dots, K\}$, in order to reduce the effect of the relative sidelobes of the generalized MIMO radar AF.

4. SIMULATION RESULTS

Throughout our simulations, we assume uniform linear arrays of $M = 8$ omni-directional transmit antennas and $N = 8$ receive antennas spaced half a wavelength apart. Both the transmit and receive arrays are located at the same position on the x -axis. The total transmit energy is fixed to $E = M$. Polyphase-coded sequences [17] are employed as the transmitted waveforms. The code length of each waveform is 256. We employ a single pulse whose pulse width is selected to be $T = 10$ ms to simulate the AF. The time-bandwidth product is set to be $BT = 128$, and the sampling rate is set to be $f_s = 2B$. Two targets are assumed to be located on the y -axis, sharing the same spatial angle $\theta = 0^\circ$. The simulated AFs for the case of far-field targets and narrow-band waveforms are normalized to their maximal value.

In the first example, we investigate the difference between the generalized MIMO radar AF metric defined in this paper and the AF metric defined in [13]. 8 waveforms for the traditional MIMO radar case are employed, and the TB matrix \mathbf{C} is given as the identity matrix \mathbf{I}_M . It can be seen that the differ-

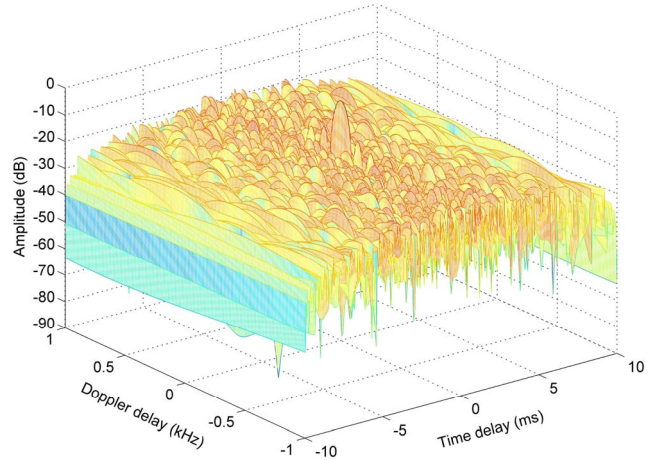


Fig. 2. Generalized MIMO radar AF with 4 waveforms.

ences are nearly all above zero, which means that the relative sidelobe level of the AF in [13] is higher than that obtained using the AF (8). The largest difference of the relative sidelobe level reaches 4% of the normalized AF metric peak (i.e., 1), demonstrating that the AF (8) gives a better relative sidelobe level than that in [13]. This means that the maximum possible region which is free of sidelobes achieved in the generalized AF for the MIMO radar with correlated waveforms is always larger than that achieved in the AF in [13].

In the second example, we present the generalized MIMO radar AF with $K = 4$ waveforms. The first 4 waveforms used in the first example are exploited. The TB matrix \mathbf{C} of size 8×4 is designed to meet the aforementioned condition that the rotational invariance property at the receive array holds. It can be seen that the mainlobe peak of the generalized MIMO radar AF is obtained at the point (0, 0), i.e., no time and Doppler delays for the two targets. The relative sidelobe level of the generalized MIMO radar AF in this case ranges from -50 dB to -20 dB. The thumbtack shape of the AF clearly demonstrates how the AF of the generalized MIMO radar with 4 orthogonal transmitted waveforms behaves.

5. CONCLUSIONS

We have derived the AF for the MIMO radar with correlated waveforms that facilitates obtaining waveform diversity and coherent processing gain simultaneously. Our definition generalizes the AFs for the PA and traditional MIMO radars, as well as the AF of the MIMO radar with correlated waveforms. A simplified AF expression for the case of far-field targets and narrow-band waveforms is obtained. We have established the relationships between the generalized AF defined in this paper and the previous works on AF including the Woodward's AF and the AF for the traditional MIMO radar. It is shown that the proposed generalized MIMO radar AF can achieve lower relative sidelobe level by properly designing the TB matrix or, equivalently, the waveform correlation matrix.

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