On the Distribution of the Weighted Sum of L Independent Rician and Nakagami Envelopes in the Presence of AWGN

George K. Karagiannidis and Stavros A. Kotsopoulos

Abstract: **An alternative, unified, semi-analytical approach for the evaluation of the cumulative distribution function (cdf) of the weighted sum of L independent Rician (or Rayleigh as a special case) and m-Nakagami envelopes with or without the presence of Additive White Gaussian Noise (AWGN) is presented. The cdf is evaluated directly in a nested mode via the Hermite numerical integration technique. The proposed formulation avoids the calculation of complex functions and can be efficiently applied to practical wireless applications when $L \leq 3$, using arbitrary statistical characteristics for the modeling parameters. Moreover, it can be also used to control the accuracy of other techniques when $L > 3$. Comments, comparison with other existing techniques and useful curves for several practical wireless applications such as the calculation of the error bounds for coding on fading channels in mobile satellite applications and the Equal Gain Combining (EGC), are also presented. Finally, the relation between the distribution of the sum of m-Nakagami and Rice envelopes is investigated and discussed.**

Index Terms: **Rician fading, Nakagami fading, diversity, equal gain combining, mobile satellite communications.**

I. INTRODUCTION

In several practical wireless applications which involve Rician (or Rayleigh as a special case) and Nakagami fading, there is a need for the calculation of the cdf or the complementary cdf (ccdf) of the sum (or generally the weighted sum) of L statistically independent random variables (RVs) with or without the presence of AWGN. Such weighted sums occur in the calculation of the error bounds for coding on Rician fading channels in mobile satellite applications in which the cdf of the weighted sum of L statistically independent Rician RVs needs to be calculated [1]. Other important applications involve the evaluation of the error performance in equal gain and maximal ratio combining systems, signal detection, linear equalizers, outage probability, intersymbol interference, and phase jitter.

One solution to this problem is related to the extraction of a simple expression for the characteristic function (chf) of the sum (or the weighted sum) of L independent Rician or m-Nakagami fading amplitudes, which has not been solved adequately yet. It must be noted here that the derivation of the chf in Rician fading channels is more complicated compared to the Nakagami fading case, because the Rician probability density function (pdf) contains an explicit term of the modified Bessel function of the first kind [2].

In the previous years, many researchers tried to meet this need using several techniques. Scanning the literature we can find several attempts for the extraction of the cdf or the pdf of the sum of RVs (related to wireless applications), but they are limited to sine waves and Rayleigh RVs [3]–[6] and they do not investigate the case of the weighted sum.

The most well known approach for the Nakagami fading was made by Beaulieu and Abu-Dayya [7], who used the infinite series representation presented in [8] to obtain the ccdf. This is an important result since the proposed series approximation is general and simpler compared to previous published techniques. Other authors as Alouini and Simon [9] proposed another approximate approach for Nakagami fading channels using Hermite numerical integration. Recently, the same authors obtained closed-form expressions for the average signal-to-noise ratio (SNR) over diversity paths with exponentially decaying power delay profile [10]. The authors of the present paper proposed [11] an alternative approach for the evaluation of the error probability (ERRP) in Nakagami fading EGC systems, which is efficient for low order diversity. Other researchers who tried to cope with the problems arising in wireless applications in which the sum of L m-Nakagami RVs is involved are listed in [12], [13]. A comprehensive summary for the performance analysis of digital communications techniques over generalized fading channels can be found in [14].

As far as the Rician fading is concerned, Abu-Dayya and Beaulieu [15] have proposed a method similar to [7] for the evaluation of the ERRP for EGC diversity in Rician slow fading environment for coherent BPSK and non-coherent BFSK. Later, the same authors examined the performance of MPSK in the presence of co-channel interference for EGC in Nakagami and Rician fading environments [16]. Recently, Annamalai *et al.* in [2] presented a direct technique—expressed in terms of single or double finite integrals—for the evaluation of the ERRP of EGC systems in Rayleigh, Rician, and Nakagami fading channels. Other researchers such as Zhang in [5], [6], presented a simpler approach for the evaluation of the ERRP for coherent and non-coherent modulation schemes in slow Rayleigh fading

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channels using the Gil-Palaez lemma. Finally, recently the authors of this paper presented a method for the calculation of the ERRP in Rayleigh fading EGC wireless applications, which is very efficient compared to other techniques, especially when the number of diversity branches is lower than four [17]. It must be noted here that to our knowledge the solution to the problem of extracting the cdf of a weighted sum of fading amplitudes is limited only to the Rayleigh case in [8].

In this paper, we attempt to extract the cdf of the weighted sum of L m-Nakagami and Rician RVs in the presence or not of additive uncorrelated white noise avoiding the calculation of complex functions and using arbitrary values for the modeling parameter. The proposed semi-analytical approach assumes independent fading envelopes, which are not necessarily identically distributed. The cdf of the weighted sum of two RVs is evaluated directly using the definition and the properties of the chf of a RV and a formula which can be evaluated via the Hermite numerical integration method is derived. Then, this formula is used in a nested mode for the derivation of the cdf of the sum of L RVs. Although the proposed approach needs the evaluation of well known tabulated functions, its nested form makes it computationally cumbersome for. In such cases, it is useful to be used as a reference point in order to control the accuracy of other techniques. On the other hand, for the proposed in this paper method is very efficient and simpler compared to other techniques. Hence, it can be easily used for applications involving a low number of fading RVs. But, in any case, to our knowledge, results for the distribution of the weighted sum of Rice and m-Nakagami fading RVs, which are not identically distributed (different powers), have not been published previously.

In Section II, the problem of the Rician fading case is formulated with the necessary mathematical analysis and the final expressions for the cdf with or without the presence of AWGN are presented. In Section III, the problem for the m-Nakagami fading case is solved. In Section IV, comments are made and computer results for practical wireless applications illustrate the proposed formulation. In the same section, the relation between the distribution of the sum of m-Nakagami and Rice envelopes is investigated and discussed. Finally, Section V presents the paper's concluding remarks.

II. THE DISTRIBUTION OF THE WEIGHTED SUM OF L RICIAN RVs

Let $x_1, x_2, \cdots, x_L$ be the amplitudes of L statistical independent envelopes, which follow the well-known Rice distribution

$$f_{RICE}\left(x_k\right) = \frac{x_k}{\sigma_k^2}\exp\left(-\frac{x_k^2+u_k^2}{2\sigma_k^2}\right) I_0\left(\frac{x_k u_k}{\sigma_k^2}\right), x_k \geq 0, \tag{1}$$

where x_k is the signal amplitude, I_0 is the zero-order modified Bessel function of the first kind, $2\sigma_k^2$ is the average fading-'scatter' component and u_k^2 the line-of-sight (LOS) power component. The Local Mean Power (LMP) Ω_k is defined as $\Omega_k = 2\sigma_k^2 + u_k^2$ and the Rice factor K_k of the k-th envelope is defined as the ratio of the signal power in dominant component over the scattered power, i.e., $K_k = \frac{u_k^2}{2\sigma_k^2}$. When K_k goes to zero, the channel statistic becomes Rayleigh, whereas, if K_k goes to infinity, the channel becomes a nonfading channel. Values of Rice factor in outdoor and indoor systems usually range from 0 to 25 [18], [19].

Let $X_{1L} = c_1x_1 + c_2x_2 + \cdots + c_Lx_L$ be the variable which represents the weighted sum of the L Rician RVs with $c_1, c_2, \cdots, c_L$ being constants. First, the cdf of the sum of the variables c_1x_1 and c_2x_2—denoted as X_{12}—will be formulated and then, this sum will be used in a nested mode for the derivation of the cdf of X_{1L}.

Let $\Phi_{12}(s), \Phi_{c_1x_1}(s)$, and $\Phi_{c_2x_2}(s)$ be the chfs of the variable X_{12}, c_1x_1, and c_2x_2 respectively. Then, due to the independence between x_1 and $x_2, \Phi_{12}(s)$ can be written as [20]

$$\Phi_{12}\left(s\right) = \Phi_{C_1X_1}\left(s\right)\Phi_{C_2X_2}\left(s\right) = \Phi_{X_1}\left(c_1s\right)\Phi_{X_2}\left(c_2s\right) \tag{2}$$

or from the definition of the chf

$$\Phi_{12}\left(s\right) = \Phi_{X_1}\left(c_1s\right)\int_0^{\infty}\exp\left(jc_2st_2\right)f_2\left(t_2\right)dt_2, \tag{3}$$

with $f_2(t_2)$ being the Rician pdf of the RV x_2. Using (1) and (3) and after the transformation $t_2 = \sigma_2\sqrt{2}r_2, \Phi_{12}(s)$ can be written as

$$\begin{aligned}\Phi_{12}\left(s\right) = {} & \Phi_{X_1}\left(c_1s\right)2e^{-K_2} \\ & \cdot\int_0^{\infty}e^{-r_2^2}r_2I_0\left(2r_2\sqrt{K_2}\right) \\ & \cdot\exp\left(jsc_2\sqrt{2}r_2\right)dr_2.\end{aligned} \tag{4}$$

The cdf of x_{12} is defined as

$$F_{12}\left(v\right) = \mathrm{Prob}\left[\mathrm{X_{12}} \leq \mathrm{v}\right] \tag{5}$$

or

$$\begin{aligned}F_{12}\left(v\right) &= \int_0^{v}f_{X_{12}}\left(\tau\right)d\tau \\ &= \frac{1}{2\pi}\int_0^{v}\int_0^{\infty}\Phi_{12}\left(s\right)\exp\left(-js\tau\right)dsd\tau,\end{aligned} \tag{6}$$

where τ is another auxiliary variable. Now, using (4) and (6) and taking into account the fact that by definition

$$\begin{aligned}&\int_0^{\infty}\Phi_{X_1}\left(c_1s\right)\exp\left[-jsc_1\left(\frac{\tau-\sigma_2c_2\sqrt{2}r_2}{c_1}\right)\right]ds \\ &\quad = 2\pi f_1\left(\frac{\tau}{c_1}-\frac{\sqrt{2}\sigma_2r_2c_2}{c_1}\right),\end{aligned} \tag{7}$$

the cdf of X_{12} can be written following a straightforward procedure as

$$\begin{aligned}F_{12}\left(v\right) = {} & 2e^{-K_2}\int_0^{\infty}\left[F_1\left(v-\frac{r_2\sqrt{2}c_2\sigma_2}{c_1}\right)\right]r_2 \\ & \cdot I_0\left(2r_2\sqrt{K_2}\right)\exp\left(-r_2^2\right)dr_2,\end{aligned} \tag{8}$$

with $F_1(v)$ being the cdf of the c_1x_1. A formulation for the cdf of the cx, when x follows Rician pdf and c is constant, is easily found to be related to the Marcum Q_1-function (or simply Q-function) as

$$F_1(v) = 1 - Q\left(\sqrt{2K_1}, \frac{v}{c_1\sigma_1}\right). \tag{9}$$

An efficient formulation for the Marcum Q-function has been given in [14]. This formulation is used in this paper for the evaluation of $F_1(v)$ in (9).

The second part of (8) can be calculated numerically with desired accuracy using the Hermite numerical integration method [21, p. 875]. Hence, the final semi analytical closed form for the calculation of F_{12} can be written as

$$\begin{aligned} F_{12}(v) = 2e^{-K_2}\sum_{i=1}^{n} a_i \Bigg[& F_1\left(v - z_i\sqrt{2}\sigma_2\frac{c_2}{c_1}\right) z_i \\ & \cdot I_0\left(2z_i\sqrt{K_2}\right)\Bigg], \end{aligned} \tag{10}$$

where a_i, z_i, and $2n$ are the weighting factors, the abscissas and the order of the Hermite numerical integration method, respectively [21]. It is mentioned here that only the positive values of abscissas are used, because the integrals are defined over the positive half axe. Moreover, only the values which satisfy the condition, $v - z_i\sqrt{2}\sigma_2\frac{c_2}{c_1} > 0$ are taken into account in the summation of (10).

Following the same mathematical analysis in a nested mode, the formulae for the cdf of the weighted sum of L independent Rician RVs are shown below

$$\begin{aligned} F_{12}(v) =\ & 2e^{-K_2}\sum_{i=1}^{n} a_i z_i I_0\left(2z_i\sqrt{K_2}\right) \\ & \cdot\left[F_1\left(v - z_i\sqrt{2}\sigma_2\frac{c_2}{c_1}\right)\right], \\ F_{13}(v) =\ & 2e^{-K_3}\sum_{i=1}^{n} a_i z_i I_0\left(2z_i\sqrt{K_3}\right) \\ & \cdot\left[F_{12}\left(v - z_i\sqrt{2}\sigma_3\frac{c_3}{c_1}\right)\right], \\ & \vdots \\ F_{1L}(v) =\ & 2e^{-K_L}\sum_{i=1}^{n} a_i z_i I_0\left(2z_i\sqrt{K_L}\right) \\ & \cdot\left[F_{1L-1}\left(v - z_i\sqrt{2}\sigma_L\frac{c_L}{c_1}\right)\right]. \end{aligned} \tag{11}$$

The approach proposed in (11) can be used for Rician envelopes with arbitrary values for K and σ.

A. *The Weighted Sum of L Rician Envelopes in the Presence of AWGN*

If the L independent Rician envelopes are transmitted over an AWGN channel, the total sum of the desired signals plus the noise can be written as

$$XPN_{1L} = \sum_{i=1}^{L} c_i x_i + \sum_{i=1}^{L} w_i, \tag{12}$$

where x_k is the output signal amplitude and w_k represents the complex Gaussian noise, which affects at the k-th envelope with zero mean and variance $N_k/2$. The chf of the Gaussian variate w_k is well known and can be expressed as

$$\Phi_{w_k}(s) = \exp\left(-\frac{N_k}{4}s^2\right), \tag{13}$$

which is the chf of a Gaussian pdf with zero mean and variance $N_k/2$.

Due to the independence between desired signals and noise, the chf $\Phi_{XPN_{1L}}(S)$ of the envelope XPN_{1L} can be written as

$$\Phi_{XPN_{1L}}(s) = \prod_{k=1}^{L}\Phi_{w_k}(s)\prod_{k=1}^{L}\Phi_{x_k}(s) \tag{14}$$

or from (13)

$$\Phi_{XPN_{1L}}(s) = \Phi_{NORM(0,\eta_L/2)}(s)\prod_{k=1}^{L}\Phi_{X_k}(s), \tag{15}$$

with η_L being the total power of the Gaussian noise.

Following the same mathematical analysis as in the case without noise, the cdfs of the weighted sum of L independent Rician envelopes in the presence of AWGN are shown below

$$\begin{aligned} FN_{11}(v) =\ & 2e^{-K_1}\sum_{i=1}^{n} a_i z_i I_0\left(2z_i\sqrt{K_1}\right) \\ & \cdot\left[F_{NORM(0,\eta_L/2)}\left(v - z_i\sqrt{2}\sigma_1 c_1\right)\right], \\ FN_{12}(v) =\ & 2e^{-K_2}\sum_{i=1}^{n} a_i z_i I_0\left(2z_i\sqrt{K_2}\right) \\ & \cdot\left[F_{11}\left(v - z_i\sqrt{2}\sigma_2 c_2\right)\right], \\ & \vdots \\ FN_{1L}(v) =\ & 2e^{-K_L}\sum_{i=1}^{n} a_i z_i I_0\left(2z_i\sqrt{K_L}\right) \\ & \cdot\left[F_{1L-1}\left(v - z_i\sqrt{2}\sigma_L c_L\right)\right], \end{aligned} \tag{16}$$

with $F_{NORM(0,\eta_L/2)}(x)$ being the well known standard normal cdf with zero mean and variance $\eta_L/2$.

III. THE DISTRIBUTION OF THE WEIGHTED SUM OF L m-NAKAGAMI RVs

Nakagami fading (m-distribution [22]) describes multipath scattering with relatively large delay-time spreads with different clusters of reflected waves. Within any one cluster the phases of individual reflected waves are random, but the delay times are approximately equal for all waves. As a result, the envelope of each cumulated cluster is Rayleigh distributed. The m-Nakagami model is also often used to describe the interference cumulated from the multiple independent Rayleigh fading sources, particularly if these are identically distributed (same LMPs) [22].

Let $x_1, x_2, \cdots, x_L$ be L statistical independent RVs, which follow the well-known Nakagami m-distribution with pdf given by [22]

$$f(x_k) = 2(\frac{m_k}{\Omega_k})^{m_k} \frac{x_k^{2m_k-1}}{\Gamma(m_k)} \exp(-\frac{m_k}{\Omega_k} x_k^2), \quad x_k \geq 0, \quad k = 1, 2, \cdots, L, \tag{17}$$

where $\Gamma(x)$ is the Gamma function, Ω_k controls the spread of the distribution (for signal applications represents the LMP) and m represents the inverse normalized variance.

Following the same mathematical analysis as in the Rice case described above in Section II, the formulae for the cdf of the weighted sum of L independent m-Nakagami RVs are shown below

$$\begin{aligned} F_{12}(v) = & \frac{2}{\Gamma(m_2)} \sum_{i=1}^{n} a_i z_i^{2m_2-1} \\ & \cdot \left[F_1 \left(v - z_i c_2 c_1^{-1} \sqrt{\frac{\Omega_2}{m_2}} \right) \right], \\ F_{13}(v) = & \frac{2}{\Gamma(m_3)} \sum_{i=1}^{n} a_i z_i^{2m_3-1} \\ & \cdot \left[F_{12} \left(v - z_i c_3 c_1^{-1} \sqrt{\frac{\Omega_3}{m_3}} \right) \right], \\ & \vdots \\ F_{1L}(v) = & \frac{2}{\Gamma(m_L)} \sum_{i=1}^{n} a_i z_i^{2m_L-1} \\ & \cdot \left[F_{1L-1} \left(v - z_i c_L c_1^{-1} \sqrt{\frac{\Omega_L}{m_L}} \right) \right], \end{aligned} \tag{18}$$

with $F_1(v)$ being the cdf of the cx, easily found to be

$$F_1(v) = P\left(m_1, \frac{m_1}{\Omega_1 c_1^2} v^2\right), \tag{19}$$

and P being the incomplete Gamma function. This function can be evaluated using a routine from a mathematical software package (e.g., MATHEMATICA). The approach proposed in (18) can be used for random m-Nakagami variates with arbitrary values for Ω and m parameters.

A. *The Weighted Sum of L m-Nakagami Envelopes in the Presence of AWGN*

In the presence of AWGN, following similar mathematical analysis as in Section II-A, the formulae for the cdf of the weighted sum of L independent m-Nakagami RVs are shown below

$$\begin{aligned} FN_{11}(v) = & \frac{2}{\Gamma(m_1)} \sum_{i=1}^{n} a_i z_i^{2m_1-1} \\ & \cdot \left[F_{NORM(0,\eta_L/2)} \left(v - z_i c_1 \sqrt{\frac{\Omega_1}{m_1}} \right) \right], \\ FN_{12}(v) = & \frac{2}{\Gamma(m_2)} \sum_{i=1}^{n} a_i z_i^{2m_2-1} \\ & \cdot \left[FN_{11} \left(v - z_i c_2 \sqrt{\frac{\Omega_2}{m_2}} \right) \right], \\ & \vdots \\ FN_{1L}(v) = & \frac{2}{\Gamma(m_L)} \sum_{i=1}^{n} a_i z_i^{2m_L-1} \\ & \cdot \left[FN_{1L-1} \left(v - z_i c_L \sqrt{\frac{\Omega_L}{m_L}} \right) \right]. \end{aligned} \tag{20}$$

IV. PRACTICAL APPLICATIONS, COMMENTS, AND NUMERICAL RESULTS

Some comments on the Beaulieu and Abu-Dayya's technique presented in [7], [8], and [15] are as follows: a) The need for the evaluation of special complex functions as the confluent hypergeometric presents serious time overflow problems under some circumstances [23, Appendix B]. Moreover, using this series approximation two errors occur [24]. The first arises due to the assumption of bounded random variables and the second due to termination of the infinite series. These errors are also discussed in [24]. b) The Nakagami m-parameter is constrained to take integer values. This is not true for real mobile radio environments. c) The accuracy of computation for a specific value depends strongly on the appropriate selection of the parameter T and the number of non-zero terms in infinite series in [7], which are different for different values of L and m.

In order to show the general applicability of the proposed technique, we assume that the envelopes have arbitrary values for the statistical parameters m and K. We also assume that the LMPs Ω_k follow uniform $(\Omega_1 = \Omega_2 = \cdots = \Omega_L)$ or exponential power decay profile, given by $\Omega_k = \Omega_1 e^{d(k-1)}, k = 1, 2, \cdots, L$ with d being the power decay factor [14].

A. *The Rician Fading Case*

In Fig. 1, the cdf of the weighted sum of L Rician envelopes is plotted versus $x/L^{0.5}$ (for normalization purposes as in [4]) for several numbers of L (2, 3, and 4) and arbitrary values of Rice factor K (including the Rayleigh case of $K = 0$) and for the weighting coefficients. In the same figure, it is assumed that $\sigma = 1$ for all envelopes.

In Fig. 2, the cdf is plotted versus $x/L^{0.5}$ for $L = 2, 3$, and 4 and arbitrary values for the weighting coefficients. In this figure, it is assumed that a) the K factor is the same for all envelopes and b) the LMP of each envelope follows uniform $(d = 0)$ or

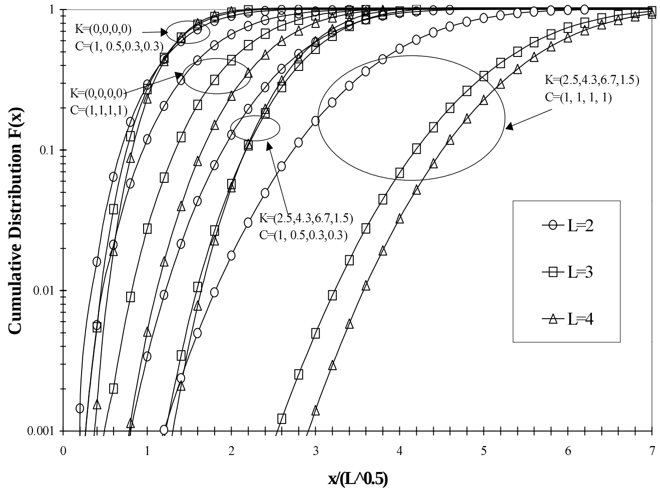

Fig. 1. The cdf of the weighted sum of L Rician envelopes for arbitrary values for the Rice K factor and the weighting coefficients. $\sigma = 1$ for all envelopes.

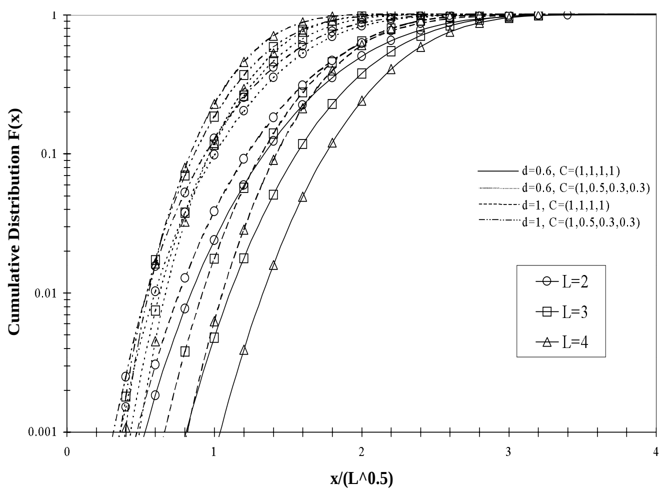

Fig. 2. The cdf of the weighted sum of L Rician envelopes for arbitrary values for the weighting coefficients and the power decay factor d. $K = 2.8$ for all envelopes.

exponential decay profile $(d \neq 0)$. To the best of the authors' knowledge, such results have not been previously presented.

The curves of Figs. 1 and 2 can be efficiently used for the calculation of the error bounds for coding on Rician fading channels in mobile satellite applications [1].

As referred above, a practical application of (16) is in the error analysis of EGC systems. In an EGC system with coherent detection, the signals received in each branch are co-phased, summed and coherently demodulated. The decision variable γ (which is assumed to be constant within symbol duration) for a coherent BPSK can be formulated as

$$\gamma\left(L\right) = \pm \sum_{k=1}^{L} x_k + \sum_{k=1}^{L} w_k, \tag{21}$$

where x_k is the output signal amplitude at the k-th branch and w_k represents the complex Gaussian noise at the k-th branch with zero mean and variance $N_k/2$. The sign in (21) is positive if the transmitted symbol equals one and is negative if zero. It is assumed that x_k remains constant within symbol duration, but changes from symbol to symbol following the Rician pdf. The average SNR at the k-th branch is defined as

$$\rho_k = \frac{\Omega_k}{\eta_L / L}, \tag{22}$$

with η_L being the total (in all branches) power of the Gaussian noise and Ω_k the total signal power-sum of the LOS and scattered at the k-th branch. Hence, assuming for simplicity that the Rice factor is the same for all branches $(K_1 = K_2 = \cdots = K_L = K)$, ρ_k can be written as

$$\rho_k = \frac{2\sigma_k^2 \left(K+1\right)}{\eta_L / L}. \tag{23}$$

The Error Probability (ERRP) for coherent BPSK detection is defined as

$$P_e\left(L\right) = \mathrm{Prob}\left[\gamma\left(\mathrm{L}\right) < 0\right], \tag{24}$$

which is equal to $FN_{1L}(0)$, with $FN_{1L}(v)$ defined in (16). Hence

$$\begin{aligned} P_e\left(L\right) = {} & 2^L e^{-LK} \sum_{i=1}^{\nu} a_i \sum_{j=1}^{\nu} a_j \cdots \sum_{n=1}^{\nu} a_n \\ & \cdot Q\left(\sum_{k=i,j,\cdots,n} \sqrt{\frac{2\rho_k}{L\left(K+1\right)}} z_k\right) \\ & \cdot \prod_{k=i,j,\cdots,n} t_k I_0\left(2 z_k \sqrt{K}\right) \end{aligned} \tag{25}$$

which is independent to η_L and $Q(x)$ is the well-known Gaussian Q-function [14, p. 70]. At this point, it must be mentioned here again that only the positive values of abscissas are used, because the integrals are defined over the positive half axes. Also, it must be noted that for the case of perfect coherent detection and no co-channel interference, the ERRP performance of BPSK is identical to that of QPSK. In the case of coherent BFSK, the noise variance is doubled compared to the case of coherent BPSK. Hence, the proposed formulation is also valid with ρ_k replaced by $\rho_k/2$ in (25).

In Fig. 3, the ERRP for coherent BPSK is depicted as a function of the SNR at the first branch $\rho_1 = \rho$ for several orders of diversity $(L = 2, 3)$, assuming that the signals arriving at each branch have the same Rice factor $K = 0, 5, 10$ dB. We also assume that the LMPs Ω_k and consequently the SNRs ρ_k at each branch follow exponentially power decay profile. In this case $\rho_k = \rho e^{-d(k-1)}, k = 2, \cdots, L$, with d being the power decay factor.

Some comments and comparison between the method proposed in [15] and the technique proposed in this paper are as follows: a) Eqs. (11) and (16) are in a nested form with $L-1$ summation loops for L envelopes (without the presence of AWGN) or L loops for L envelopes (in the presence of AWGN). In this case, which is comparable to [15], each summation loop consists of ν non-zero terms. Hence, the non-zero terms that need to be summed here are $M_1 = \nu^L$. The infinite series in [15, Eq. (23a)] is also in a nested form [15, Eqs. (23a), (23b), (23c),

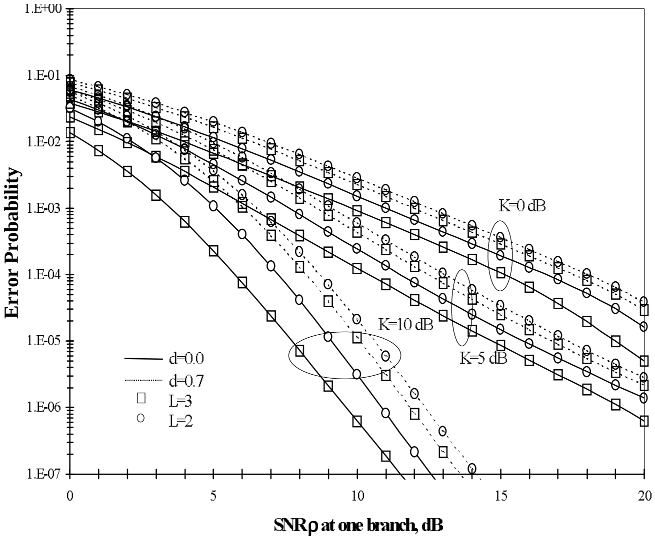

Fig. 3. ERRP for coherent BPSK, versus SNR in one branch ρ for several diversity orders L, decay factor d and Rice factor K.

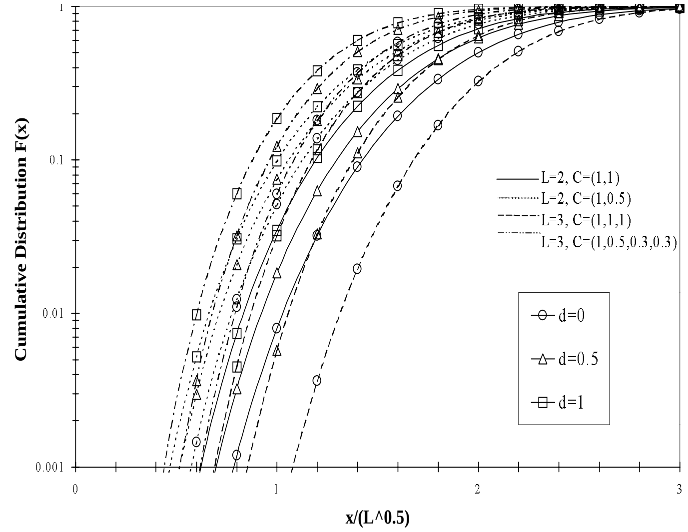

Fig. 4. The cdf of the weighted sum of L Nakagami envelopes for arbitrary values for the weighting coefficients and the power decay factor $(m = (2.5, 3, 3.2), \Omega_1 = 4)$.

(14), (15)] with three summation loops for every L, two of them being infinite series and the third consists of L non-zero terms. If n_1 is the selected number of the non-zero terms that needs to be summed in infinite series [15, Eq. (23a)] (for a specific accuracy) and n_2 is the corresponding number for the infinite series of [15, Eqs. (14), (15)], the total number of the terms needs to be summed is $M_2 = n_1 L n_2$. The number n_1 decreases as the L increases as referred in [8]. For $L = 2$ the values for M_1 and M_2 are 256 and 6720 and for $L = 3$ the corresponding values are 2197 and 8400. For $L = 4$ these values are 50625 and 7800, respectively. The desired accuracy is chosen to be up to sixth digit for both methods. It is obvious that the technique proposed in this paper is an efficient alternative to the infinite series concept of [15] and [8] for $L \leq 3$, as far as the speed of computation is concerned. On the other hand, for $L > 3$ and for higher values of L it becomes computationally cumbersome and Dayya and Beaulieu's technique seems to be a more attractive tool for the calculation of the ERRP with accuracy and speed, which increases as the L increases. In this case $(L > 3)$ and due to its non-complicated form, the proposed in this paper technique can be used for accuracy comparing purposes with other methods. b) As far as the complexity of the calculations is concerned, the functions that needed to be evaluated in every non-zero term using Eqs. (12) and (16) are the well-known Gaussian Q-function (in the presence of AWGN), the Marcum Q-function (without AWGN) and the zero-order modified Bessel of the first kind. However, the term $\prod_{k=i,j,\cdots,n} t_k I_0\left(2t_k\sqrt{K}\right)$ seems error prone, since $I_0(x)$ gets large quickly with the increase of its argument, and the multiplication of several large numbers could lead to inaccurate results. In order to solve this problem, the authors propose the calculation of $e^{-x} I_0(x)$ using an expansion in series of Chebyshev polynomials [21, 9.37] instead of the direct calculation of $I_0(x)$. In this case, the term e^{-LK} must be included in the summations. On the other hand, the technique presented in [15] needs the evaluation of two kinds of the confluent hypergeometric function in every non-zero term [15, Eqs. (14), (15)]. The calculation of such functions—as it is also referred in [23, Appendix B]—presents serious time overflow problems under some circumstances and the development of appropriate methods is necessary in order to avoid them. c) The error that is introduced using Eqs. (11) and (16) can be calculated using the formula for the remainder quantity of the Hermite numerical integration method [21, 25.4.46]. The total remainder for $L = 2$ is the ν-order summation of the partial remainders and for $L = 3$ is the ν^2-order corresponding summation. The error that occurs due to the termination of the infinite series of [15, p. 2261] is discussed in [24].

B. The m-Nakagami Fading Case

Eqs. (18) and (20) are used for the evaluation of the distribution of the weighted sum of L m-Nakagami RVs with or without the presence of AWGN. The accuracy of computations is under control ($n = 20$ is needed for an accuracy of six digits) and the disadvantage of lengthy computation time arises when the number of RVs is greater than three.

In Fig. 4, the cdf is plotted versus $x/L^{0.5}$ for $L = 2$ and 3, arbitrary values for the Nakagami m parameter, several values for the weighted coefficients and for the power decay factor d. To the best of the authors' knowledge, such results have not been previously presented. The technique proposed in this paper is very efficient for small values of v since, for example, the calculation of $F_{12}(2)$ needs the summation of 9 non-zero terms and the corresponding value for $F_{12}(7)$ is 20. The number of non-zero terms that need to be summed for the evaluation of the infinite series of [7] (under the same accuracy demands) is about 450, without taking into account the calculation of the involved two types of confluent hypergeometrics function. The number of non-zero terms that need to be summed for $F_{13}(2)$ are 81, and 400 for $F_{13}(7)$. The corresponding number of non-zero terms for the method presented in [7] is about 420.

Following the same analysis as in Section II-A for the Rician case, the ERRP for BPSK EGC systems in Nakagami fading

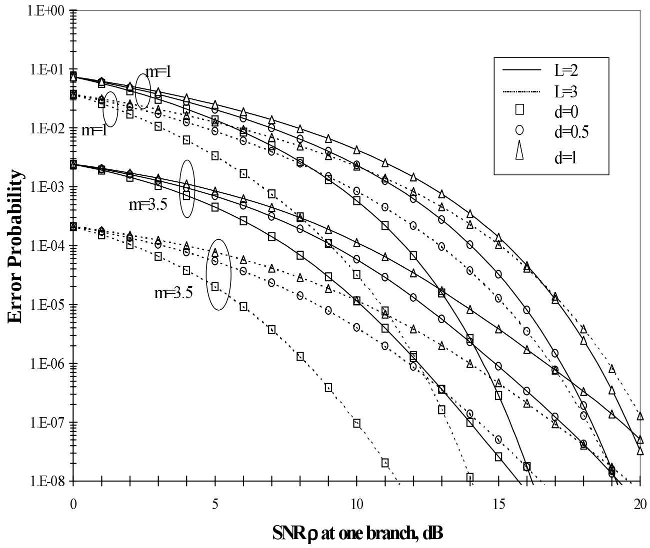

Fig. 5. ERRP for coherent BPSK versus SNR at a reference branch for several diversity orders L, decay factors and Nakagami m parameter.

channels is given by

$$P_e(L) = \frac{1}{[\Gamma(m)]^L} \sum_{i=1}^{\nu} w_i \sum_{j=1}^{\nu} w_j \cdots \sum_{n=1}^{\nu} w_n \cdot Q\left(\sum_{k=i,j,\cdots,n} \sqrt{\frac{2\rho_k z_k}{Lm}} \right) \prod_{k=i,j,\cdots,n} z_k^{m-1}, \tag{26}$$

where $Q(x)$ is the Gaussian Q-function. In (26), for simplicity purposes, it is assumed that all of the envelopes have the same value for the Nakagami m parameter. In Fig. 5, the ERRP for coherent BPSK is depicted as a function of the SNR at the first branch $\rho_1 = \rho$ for several orders of diversity $(L = 2, 3)$, assuming that the signals arriving at each branch have the same m parameter $(m = 1, 3.5)$. We also assume that the LMPs Ω_k and consequently the SNRs ρ_k at each branch follow exponential power decay profile with $d = 0, 0.5$, and 1.

C. Approximation of the Rician cdf by the m-Nakagami cdf

Sometimes the m-Nakagami model is used to approximate a Rician distribution [25] because of its form simplicity compared to the Rician case. Nakagami in [22, pp. 17–18] has given some formulae for the relation between m-distribution and n-distribution (Rice). In Fig. 6, the Rice pdf and cdf with $K = 4.45$ and $\Omega = 3$ are depicted together with an approximation of this distribution by a m-Nakagami pdf [22, pp. 17–18].

Although, this approximation seems to be accurate for the main body of the pdf (and cdf), it becomes highly inaccurate for the tails. As BER mainly occurs during deep fades, the tail of the pdf mainly determines these performance measures the corresponding cdf.

In Fig. 7, the cdf of the weighted sum of $L = 2$ and 3 Rician RVs with arbitrary values for the K factor is approximated by the corresponding m-Nakagami one, using the formulae of [22, pp. 17–18].

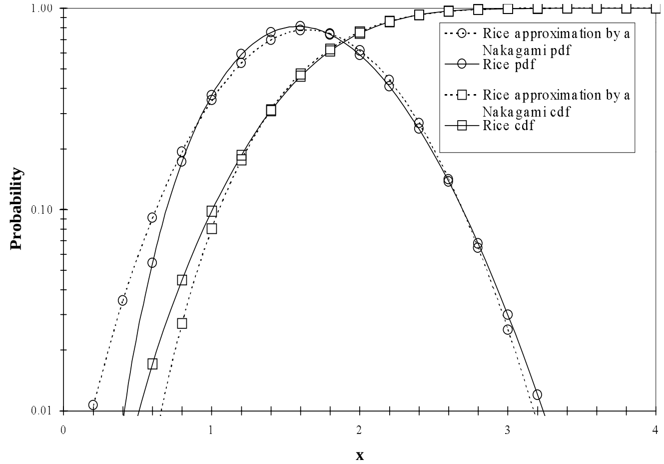

Fig. 6. Rice pdf and cdf for $K = 4.45, \Omega = 3$ and the corresponding Nakagami approximation using the formulae presented in [22].

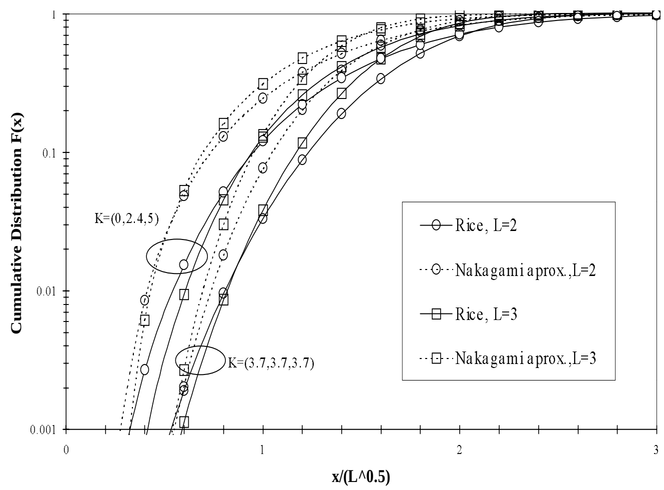

Fig. 7. Rice cdfs of the weighted sum of L RVs and the corresponding Nakagami approximated cdfs for arbitrary values of K. $C(1, 0.8, 0.4), \Omega_1 = 3,\ d = 0.8$.

As it is apparent here, the Nakagami approximation overestimates the performance of the "equivalent" Rician model. These results are in accordance to the comments, which are included in [7, pp. 2265–2266]. Moreover, in [7] this approximation is discussed in depth for EGC systems. But, it has to be mentioned that this overestimation is greater for weighted sums and not uniform distributed envelopes. Therefore, in our opinion it is not correct to approximate the cdf of the sum of L Rician envelopes signal by the corresponding m-Nakagami one if the application is a BER analysis.

V. CONCLUSIONS

A novel, simple and flexible approach for the evaluation of the cdf of the weighted sum of L Rician and m-Nakagami statistically independent envelopes in the presence (or not) of AWGN has been analyzed and presented. The obtained formulation provides accuracy and speed for $L \leq 3$ and can be easily used in a wide range of wireless applications, which involve Nakagami and Rice fading channels (Equal Gain Combining and calculation of the error bounds for coding on fading channels in mobile satellite applications). Moreover, when $L > 3$ it can also be used to control the accuracy of other techniques. Following the same analysis, the proposed method can be adapted to extract the cdf of the sum of the powers of L m-Nakagami or Rice distributed RVs. Such a result can be used in outage probability analysis in cellular networks.

REFERENCES

[1] D. Divsalar and M. Simon, "Trellis coded modulation for 4800-9600 bps transmission over a fading mobile satellite channel," *IEEE J. Select. Areas Commun.*, vol. SAC-5, no. 2, pp. 162–175, Feb. 1987.

[2] A. Annamalai, C. Tellambura, and V. Bhargava, "Unified analysis of equal-gain diversity on Rician and Nakagami fading channels," in *Proc. IEEE WCNC'99*, Sept. 1999, pp. 10–14.

[3] S. Rice, "Probability distributions for noise plus several sine waves—The problem of computation," *IEEE Trans. Commun.*, pp. 851–853, June 1974.

[4] C. Helstrom, "Computing the distribution of sums of random sine waves and of Rayleigh-distributed random variables by saddle-point integration," *IEEE Trans. Commun.*, vol. 45, no. 11, pp. 1487–1494, Nov. 1997.

[5] Q. Zhang, "Probability of error for equal gain combiners over Rayleigh channels," *IEEE Trans. Commun.*, vol. 45, no. 3, pp. 270–273, Mar. 1997.

[6] Q. Zhang, "A simple approach to probability of error for equal gain combiners over Rayleigh channels," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1151–1154, July 1999.

[7] N. Beaulieu and A. Abu-Dayya, "Analysis of equal gain diversity on Nakagami fading channels," *IEEE Trans. Commun.*, vol. 39, no. 2, pp. 225–234, Feb. 1991.

[8] N. Beaulieu, "An infinite series for the computation of the complementary probability distribution function of a sum of independent random variables and its application to the sum of Rayleigh random variables," *IEEE Trans. Commun.*, vol. 38, no. 9, pp. 1463–1474, Sept. 1990.

[9] M. Simon and M.-S. Alouini, "A unified approach to performance analysis of digital communication over generalized fading channels," in *Proc. IEEE*, vol. 86, Sept. 1998. pp. 1860–1877.

[10] M.-S. Alouini and M. Simon, "Performance analysis of coherent equal gain combining over Nakagami-m fading channels," Submitted for publication in *IEEE Trans. Veh. Technol.*

[11] G. Karagiannidis and S. Kotsopoulos, "Exact evaluation of egual-gain diversity in the presence of Nakagami fading," *IEE Electron. Lett.*, vol. 36, no. 14, July 2000.

[12] Y.-C. Ko, M.-S. Alouini, and M. Simon, "Outage probability of diversity systems over generalized fading channels," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT'00)*, Sorrento, Italy, June 2000.

[13] M.-S. Alouini and A. Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1324–1334, Sept. 1999.

[14] M. Simon and M.-S. Alouini, *Digital Communications over Fading Channels: A Unified Approach to Performance Analysis*, 1st ed., Wiley & Sons, July 2000.

[15] A. Abu-Dayya and N. Beaulieu, "Microdiversity on Rician fading channels," *IEEE Trans. Commun.*, vol. 42, no. 6, pp. 2258–2267, June 1994.

[16] A. Abu-Dayya and N. Beaulieu, "Diversity MPSK receivers in cochannel interference," *IEEE Trans. Veh. Technol.*, vol. 48, no. 6, pp. 1959–1965, Nov. 1999.

[17] S. Kotsopoulos and G. Karagiannidis, "Error performance for equal-gain combiners over Rayleigh fading channels," *IEE Electron. Lett.*, vol. 36, no. 10, May 2000.

[18] G. Gibson, *The Mobile Communications Handbook*, CRC and IEEE Press, 1996.

[19] S. Kotsopoulos and G. Karagiannidis, *Mobile Communications*, Greek ed., Papasotiriou S.A Publication, 1997.

[20] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd ed., McGraw-Hill, 1991.

[21] M. Abramovitz and I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th ed., New York, NY: Dover Publications, 1970.

[22] M. Nakagami, "The m-distribution—A general formula of intensity distribution of rapid fading," in Statistical Methods in Radio Wave Propagation, pp. 3–36, Pergamon Press, Oxford, U.K, 1960.

[23] A. Annamalai, C. Tellambura, and V. Bhargava, "Exact evaluation of maximal-ratio and equal-gain diversity receivers for M-ary QAM on Nakagami fading channels," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1335–1344, Sept. 1999.

[24] C. Tellambura, "Cochannel interference computation for arbitrary Nakagami fading," *IEEE Trans. Veh. Technol.*, vol. 48, no. 2, pp. 487–489, Mar. 1999.

[25] L. Wang and C. Lea, "Co-channel interference analysis of shadowed Rician channels," *IEEE Commun. Lett.*, vol. 2, no. 3, pp. 67–69, Mar. 1999.

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