A Unified Approach to the Probability of Error for Noncoherent and Differentially Coherent Modulations Over Generalized Fading Channels

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***Abstract*— We present a unified approach to determine the exact bit error rate (BER) of noncoherent and differentially coherent modulations with single and multichannel reception over additive white Gaussian noise and generalized fading channels. The multichannel reception results assume independent fading in the channels and are applicable to systems that employ postdetection equal gain combining. Our approach relies on an alternate form of the Marcum Q-function and leads to expressions of the BER involving a single finite-range integral which can be readily evaluated numerically. Aside from unifying the past results, the new approach also allows for a more general solution to the problem in that it includes many situations that in the past defied a simple solution. The best example of this occurs for multichannel reception where the fading on each channel need not be identically distributed nor even distributed according to the same family of distributions.**

***Index Terms*—Multichannel reception, Nakagami fading, noncoherent and differentially coherent communications, postdetection diversity.**

I. Introduction

IN MANY applications, the phase of the received signal cannot be tracked accurately, and it is therefore not possible to perform coherent detection. In such scenarios, communications systems must rely on i) noncoherent detection techniques such as envelope or square law detection of frequency-shift-keying (FSK) signals [1, Ch. 5] or on ii) differentially coherent detection techniques such as differential phase-shift keying (DPSK) [1, Ch. 7].

There is a large number of papers dealing with the performance of noncoherent and differentially coherent communication and detection systems over additive white Gaussian noise (AWGN) as well as fading channels. For example, Proakis [2] developed a generic expression for evaluating the bit-error rate (BER) for multichannel noncoherent and differentially coherent reception of binary signals over L independent AWGN channels. Further, in [3, Sec. 7.4, p. 725], Proakis provides closed-form expressions for the average BER of binary orthogonal square-law detected FSK and binary DPSK with multichannel reception over L independent identically distributed (i.i.d.) Rayleigh fading channels. In [4], Lindsey derived a general expression for the average BER of binary correlated FSK with multichannel communication over L independent Rician fading channels in which the strength of the scattered component is assumed to be constant for all the channels. In [5], Charash analyzed the average BER performance of binary orthogonal FSK with multichannel reception over L i.i.d. Nakagami-m fading channels. More recently, Weng and Leung [6] derived a closed form expression for the average BER of binary DPSK with multichannel reception over L i.i.d. Nakagami-m fading channels. Patenaude *et al.* [7] extended the results of Charash [5] and Weng and Leung [6] by providing a closed form expression for the average BER performance of binary orthogonal square-law detected FSK and binary DPSK with multichannel reception over L independent but not necessarily identically distributed Nakagami-m fading channels. Their derivation is based on the characteristic function method and the resulting expression contains $L-1$ order derivatives, which can be found for small L but which become more complicated to find as L increases. In addition, because of its adoption in the most recent North American and Japanese digital cellular systems standards, differential quadrature PSK (DQPSK) has also received a lot of attention in the literature. For instance, Tjhung *et al.* [8] and Tanda [9] analyzed the average BER performance of this particular scheme over Rician and Nakagami-m fading channels, respectively. Further, Tellambura and Bhargava [10] presented an alternate unified BER analysis of DQPSK over Rician and Nakagami-m fading channels.

In this paper, we unify and add to the above results by providing new generalized expressions for the average BER performance of noncoherent and differentially coherent communication systems with single- and multichannel reception over AWGN and fading channels. The multichannel reception results are applicable to independent channels which are not necessarily identically distributed nor even distributed according to the same family of distributions, and to systems that employ postdetection equal gain combining

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(EGC) [3, Section 4.4, p. 298], [11, Section 5.5.6, p. 253]. This unified framework relies on a new approach[1] which does not attempt to compute or approximate the probability density function (PDF) of the signal-to-noise ratio (SNR) at the output of the combiner and then average the conditional BER over that PDF. It rather exploits an alternate form of the Marcum Q-function [13, Appendix C], [14] and the resulting alternate integral representation of the conditional BER as well as some known Laplace transforms and/or Gauss–Hermite quadrature integration to independently average over the PDF of each channel that fades. In all cases, this approach leads to expressions of the average BER that involve a single finite-range integral whose integrand contains only elementary functions and which can therefore be easily computed numerically. Furthermore, this unified approach can easily be extended to include the performance of noncoherent and differentially coherent direct sequence code division multiple access (DS–CDMA) systems, thereby generalizing the results obtained, for example, by Eng and Milstein [15], Prasad *et al.* [16], and Efthymoglou *et al.* [17]. This particular interesting subject is reported by the authors in a forthcoming paper [18], as a part of a general analytical framework for the performance evaluation of coherent, noncoherent, and differentially coherent single carrier and multicarrier DS-CDMA systems. In addition, since the discussed modulation and fading combinations are far too numerous, numerical results for the error rates as well as a study of their dependence on the various fading parameters are omitted here. These numerical results will be presented in the more comprehensive treatment in preparation [19] which, in addition, will cover communication systems with error correction coding and multiple symbol observation.

The remainder of this paper is organized as follows. In the next section, the multichannel and the various fading models under consideration are described. Section III provides the performance of noncoherent and differentially coherent modulations with single channel reception. Section IV presents a new product form representation of the BER for multichannel reception with noncoherent or differentially coherent detection, and this representation is used to derive the average BER with multichannel reception. Finally, a summary of all the results and some concluding remarks are offered in Section V.

II. System and Channel Models

A. Transmitted Signals

Let

$$s(t)=\sum_{i=-\infty}^{\infty} A e^{j(2\pi f_i t+\phi_i)} \tag{1}$$

denote the generic complex baseband transmitted signal, where A is a constant amplitude related to the average signal power. For differentially coherent modulations, f_m is set equal to 0 and the information is conveyed via the phase $\phi_m = \pm(2m-1)\pi/M$ $(m=1, 2, \cdots, M/2)$, where M is the size of the transmitted symbol set. The modulator *differentially phase encodes* the transmitted symbols. Hence, if $\Delta\phi_i$ was the information to be communicated in the ith transmission interval, then the transmitter would first form $\phi_i = \phi_{i-1} + \Delta\phi_i$ modulo 2π and then modulate ϕ_i on the carrier. For noncoherent modulation, the information is transmitted via the frequency f_m. For instance, for binary FSK $\phi_m = 0$ and $f_m = \pm\Delta f$.

B. Multilink and Fading Channel Models

The transmitted signal is received over L independent channels, each of them being a slowly varying flat fading channel, as shown in Fig. 1. In Fig. 1, $\{r_l(t)\}_{l=1}^L$ is the set of received replicas of the signal, where l is the channel index, and $\{\alpha_l\}_{l=1}^L$, $\{\theta_l\}_{l=1}^L$, and $\{\tau_l\}_{l=1}^L$ are the random channel amplitudes, phases, and delays, respectively. The first channel is assumed to be the reference channel whose delay $\tau_1 = 0$ and, without loss of generality, we assume that $\tau_1 < \tau_2 < \cdots < \tau_L$. Because of the slow-fading assumption, we assume that the $\{\alpha_l\}_{l=1}^L$, $\{\theta_l\}_{l=1}^L$, and $\{\tau_l\}_{l=1}^L$ are all constant over a symbol interval.

The fading amplitude α_l on the lth channel, where l denotes the channel index $(l = 1, 2, \cdots, L)$, is assumed to be a random variable (RV) whose mean square value $\overline{\alpha_l^2}$ is denoted by Ω_l, and whose PDF is any of the family of distributions described in detail next. Furthermore, the fading amplitudes $\{\alpha_l\}_{l=1}^L$ are assumed to be statistically independent RV's. We call the multilink channel model under consideration a *generalized fading channel* in the sense that it is sufficiently general to include the case where the different channels are not necessarily identically distributed nor even distributed according to the same family of distributions. With such a general multilink channel model in hand, we are able to handle a large variety of diversity types such as antenna, frequency, site, or multipath diversity [11, p. 238].

After passing through the fading channel, each replica of the signal is perturbed by complex AWGN with a one-sided power spectral density which is denoted by $2N_l$ (W/Hz). The AWGN is assumed to be statistically independent from channel to channel, and independent of the fading amplitudes $\{\alpha_l\}_{l=1}^L$. Hence, the instantaneous SNR per bit of the lth channel is given by $\gamma_l = (\alpha_l^2 E_b)/N_e$, where $E_b(J)$ is the energy per bit, and $2N_l$ (W/Hz) is the complex AWGN power spectral density.

We now present the different fading PDF's considered in our analyzes and their relation to physical channels. Note that a more detailed treatment of this particular topic will be presented in [19, Ch. 2].

1) Multipath Fading: Multipath fading is due to the constructive and destructive combination of randomly delayed reflected, scattered, and diffracted signal components. Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of the multipath fading envelope.

[1] The approach used in this paper to unify the average BER performance of noncoherent and differentially coherent communication systems is introduced and discussed in more generic terms in [12] that also includes coherent communication systems as well as correlated fading and other forms of diversity, e.g., maximal ratio combining (MRC). It should be further noted that [12] is written in the style of a tutorial/survey paper, and as such does not contain the level of detail presented in this paper.

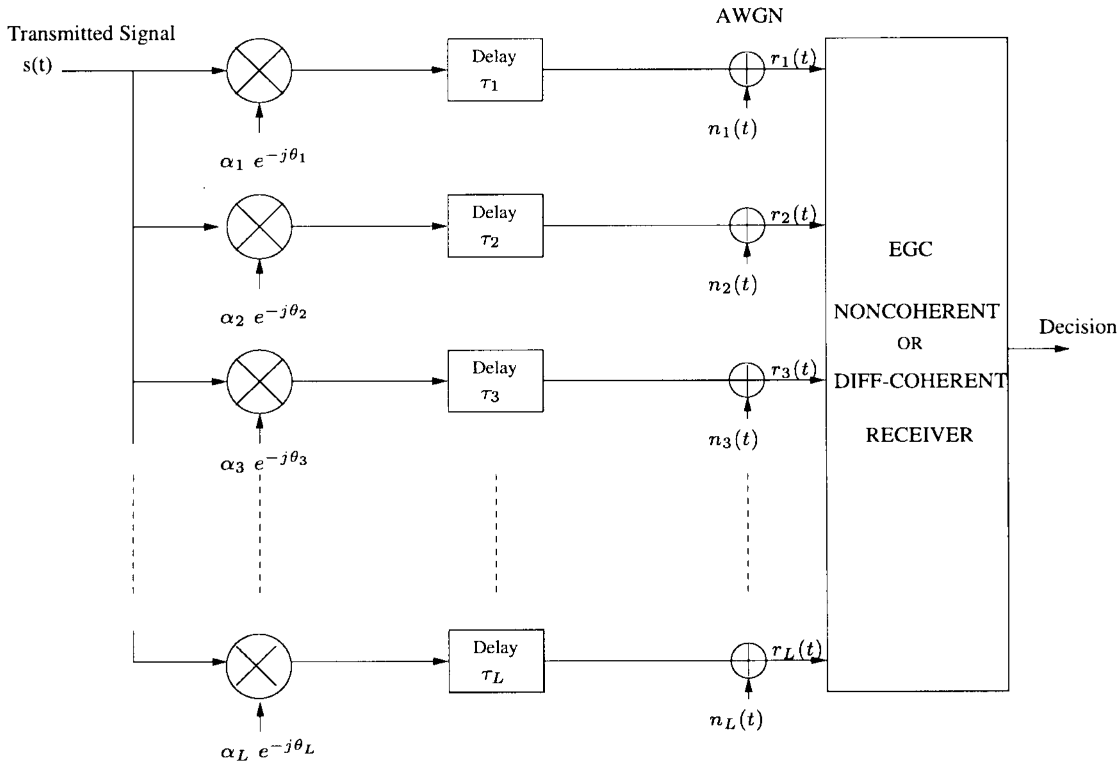

Fig. 1. Multilink channel model.

a) Rayleigh: The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. In this case, the lth channel fading amplitude α_l is distributed according to

$$p_{\alpha_l}(\alpha_l;\, \Omega_l) = \frac{2\alpha_l}{\Omega_l}\exp\left(-\frac{\alpha_l^2}{\Omega_l}\right); \qquad \alpha_l \geq 0 \tag{2}$$

and hence the instantaneous SNR per bit of the lth channel, γ_l, is distributed according to an exponential distribution given by

$$p_{\gamma_l}(\gamma_l;\, \overline{\gamma}_l) = \frac{1}{\overline{\gamma}_l}\exp\left(-\frac{\gamma_l}{\overline{\gamma}_l}\right); \qquad \gamma_l \geq 0 \tag{3}$$

where $\overline{\gamma}_l = \Omega_l E_s/N_0$ denotes the average SNR per bit of the lth channel. The Rayleigh distribution typically agrees very well with experimental data for mobile systems where no LOS path exists between the transmitter and receiver antennas. It also applies to the propagation of reflected and refracted paths through the troposphere [20] and ionosphere [21], [22], and ship-to-ship [23] radio links.

b) Nakagami-q (Hoyt): The Nakagami-q distribution, also referred to as the Hoyt distribution [24], is given in [25, eq. (52)] by

$$p_{\alpha_l}(\alpha_l;\, \Omega_l,\, q_l) = \frac{(1+q_l^2)\alpha_l}{q_l\Omega_l}\exp\left(-\frac{(1+q_l^2)^2\alpha_l^2}{4q_l^2\Omega_l}\right) \times I_0\left(\frac{(1-q_l^4)\alpha_l^2}{4q_l^2\Omega_l}\right); \qquad \alpha_l \geq 0 \tag{4}$$

where $I_0(.)$ is the zeroth-order modified Bessel function of the first kind, and q_l is the Nakagami-q fading parameter which ranges from 0 to 1. Using a change of variables, it can be shown that the SNR per bit of the lth channel, γ_l, is distributed according to

$$p_{q_l}(\gamma_l;\, \overline{\gamma}_l,\, q_l) = \frac{(1+q_l^2)}{2q_l\overline{\gamma}_l}\exp\left(-\frac{(1+q_l^2)^2\gamma_l}{4q_l^2\overline{\gamma}_l}\right) \times I_0\left(\frac{(1-q_l^4)\gamma_l}{4q_l^2\overline{\gamma}_l}\right); \qquad \gamma_l \geq 0. \tag{5}$$

The Nakagami-q distribution spans the range from one-sided Gaussian fading ($q_l = 0$) to Rayleigh fading ($q_l = 1$). It is typically observed on satellite links subject to strong ionospheric scintillation [26], [27]. Note that one-sided Gaussian fading corresponds to the worst case fading for all multipath distributions considered in this paper.

c) Nakagami-n (Rice): The Nakagami-n distribution is also known as the Rice distribution [28]. It is often used to model propagation paths consisting of one strong direct LOS component and many random weaker components. Here the lth channel fading amplitude follows the distribution [25, eq. (50)]

$$p_{\alpha_l}(\alpha_l;\, \Omega_l,\, n_l) = \frac{2(1+n_l^2)e^{-n_l^2}\alpha_l}{\Omega_l}\exp\left(-\frac{(1+n_l^2)\alpha_l^2}{\Omega_l}\right) \times I_0\left(2n_l\alpha_l\sqrt{\frac{1+n_l^2}{\Omega_l}}\right); \qquad \alpha_l \geq 0 \tag{6}$$

where n_l is the Nakagami-n fading parameter which ranges from 0 to ∞ and which is related to the Rician K_l factor by $K_l = n_l^2$. Here the SNR per bit of the lth channel, γ_l, is distributed according to a noncentral chi-square distribution given by

$$p_{n_l}(\gamma_l;\, \overline{\gamma}_l,\, n_l) = \frac{(1+n_l^2)e^{-n_l^2}}{\overline{\gamma}_l}\, \exp\left(-\frac{(1+n_l^2)\gamma_l}{\overline{\gamma}_l}\right) \times I_0\left(2n_l\sqrt{\frac{(1+n_l^2)\gamma_l}{\overline{\gamma}_l}}\right); \qquad \gamma_l \geq 0. \tag{7}$$

The Nakagami-n distribution spans the range from Rayleigh fading ($n_l = 0$) to no fading (constant amplitude) ($n_l = \infty$). This type of fading is typically observed in the first resolvable LOS paths of microcellular urban and suburban land mobile [29], picocellular indoor [30], and factory [31] environments. It also applies to the dominant LOS path of satellite [32], [33] and ship-to-ship [23] radio links.

d) Nakagami-m: The Nakagami-m PDF is in essence a central chi-square distribution given by [25, eq. (11)].

$$p_{\alpha_l}(\alpha_l;\, \Omega_l,\, m_l) = \frac{2m_l^{m_l}\alpha_l^{2m_l-1}}{\Omega_l^{m_l}\Gamma(m_l)}\, \exp\left(-\frac{m_l\alpha_l^2}{\Omega_l}\right); \quad \alpha_l \geq 0 \tag{8}$$

where $\Gamma(.)$ is the gamma function, and m_l is the Nakagami-m fading parameter which ranges from 1/2 to ∞. In this case, the SNR per bit, γ_l, of the lth channel is distributed according to a gamma distribution given by

$$p_{m_l}(\gamma_l;\, \overline{\gamma}_l,\, m_l) = \frac{m_l^{m_l}\gamma_l^{m_l-1}}{\overline{\gamma}_l^{m_l}\Gamma(m_l)}\, \exp\left(-\frac{m_l\gamma_l}{\overline{\gamma}_l}\right); \qquad \gamma_l \geq 0. \tag{9}$$

The Nakagami-m distribution spans via the m parameter the widest range of fading among all the multipath distributions considered in this paper. For instance, it includes the one-sided Gaussian distribution ($m_l = 1/2$) and the Rayleigh distribution ($m_l = 1$) as special cases. In the limit as $m_l \longrightarrow +\infty$, the Nakagami-$m$ fading channel converges to a nonfading channel. Furthermore, when $m_l < 1$, a one-to-one mapping between the m parameter and the q parameter allows the Nakagami-m distribution to closely approximate the Nakagami-q (Hoyt) distribution [25, eq. (59)]. Similarly, when $m_l > 1$, a one-to-one mapping between the m parameter and the n parameter (or, equivalently, the Rician factor) allows the Nakagami-m distribution to closely approximate the Nakagami-n (Rice) distribution [25, eq. (56)]. The Nakagami-m distribution often gives the best fit to land-mobile [34]–[36], indoor-mobile [37] multipath propagation, as well as scintillating ionospheric radio links [22], [38]–[41].

B. Log-Normal Shadowing

In terrestrial and satellite land-mobile systems, the link quality is also affected by slow variation of the mean signal level because of the shadowing from terrain, buildings, and trees. Communication system performance will depend only on shadowing if the radio receiver is able to average out the fast multipath fading, or if an efficient "micro"-diversity system is used to eliminate the effects of multipath. Based on empirical measurements, there is a general consensus that shadowing can generally be modeled by a log-normal distribution for various outdoor and indoor environments [34], [42]–[46], and the lth path SNR per bit γ_l has a PDF given by the standard log-normal expression

$$p_{\sigma_l}(\gamma_l;\, \mu_l,\, \sigma_l) = \frac{10}{\ln 10\sqrt{2\pi}\sigma_l\gamma_l}\, \exp\left[-\frac{(10\,\log_{10}\,\gamma_l - \mu_l)^2}{2\sigma_l^2}\right] \tag{10}$$

where μ_l (dB) and σ_l (dB) are the mean and the standard deviation of $10\,\log_{10}\,\gamma_l$, respectively.

C. Composite Multipath/Shadowing

In a multipath/shadowed fading environment, consisting of multipath fading superimposed on log-normal shadowing, the receiver does not average out the envelope fading due to multipath but rather reacts to the instantaneous composite multipath/shadowed signal [11, Sec. 2.4.2]. This is typically the scenario in congested downtown areas with slow moving pedestrians and vehicles [34], [47], [48]. This type of composite fading is also observed in land-mobile satellite systems subject to vegetative and/or urban [49]–[53] shadowing. There are two approaches and various combinations suggested in the literature for obtaining the composite distribution. Here, as an example, we present the composite gamma/log-normal PDF introduced by Ho and Stüber [48]. This PDF arises in Nakagami-m shadowed environments and is obtained by averaging the gamma distributed signal power (or, equivalently, SNR per bit) (9) over the conditional density of the log-normally distributed mean signal power (or, equivalently, average SNR per bit) (10) giving the following PDF for the lth channel:

$$p_{m_l\sigma_l}(\gamma_l;\, m_l,\, \mu_l,\, \sigma_l) = \int_0^\infty \frac{m_l^{m_l}\gamma_l^{m_l-1}}{w^{m_l}\Gamma(m_l)}\, \exp\left[-\frac{m_l\gamma_l}{w}\right] \frac{10}{\ln 10\sqrt{2\pi}\sigma_l w} \times \exp\left[-\frac{(10\,\log_{10}\,w - \mu_l)^2}{2\sigma_l^2}\right] dw. \tag{11}$$

For the special case where the multipath is Rayleigh distributed ($m_l = 1$), (11) reduces to a composite exponential/log-normal PDF which was initially proposed by Hansen and Meno [47].

D. Combined (Time-Shared) Shadowed/Unshadowed

From their land-mobile satellite channel characterization experiments, Lutz *et al.* [52] and Barts and Stutzman [54] found that the overall fading process for land-mobile satellite systems is a convex combination of unshadowed multipath fading and a composite multipath/shadowed fading. Here, as an example, we present in more detail the Lutz *et al.* model [52]. When no shadowing is present, the fading follows a Rice (Nakagami-n) PDF. On the other hand, when shadowing is present, it is assumed that no direct LOS path exists and the received signal power (or equivalently SNR per bit) is assumed to be an exponential/log-normal (Hansen–Meno) PDF. The combination is characterized by the shadowing time-share

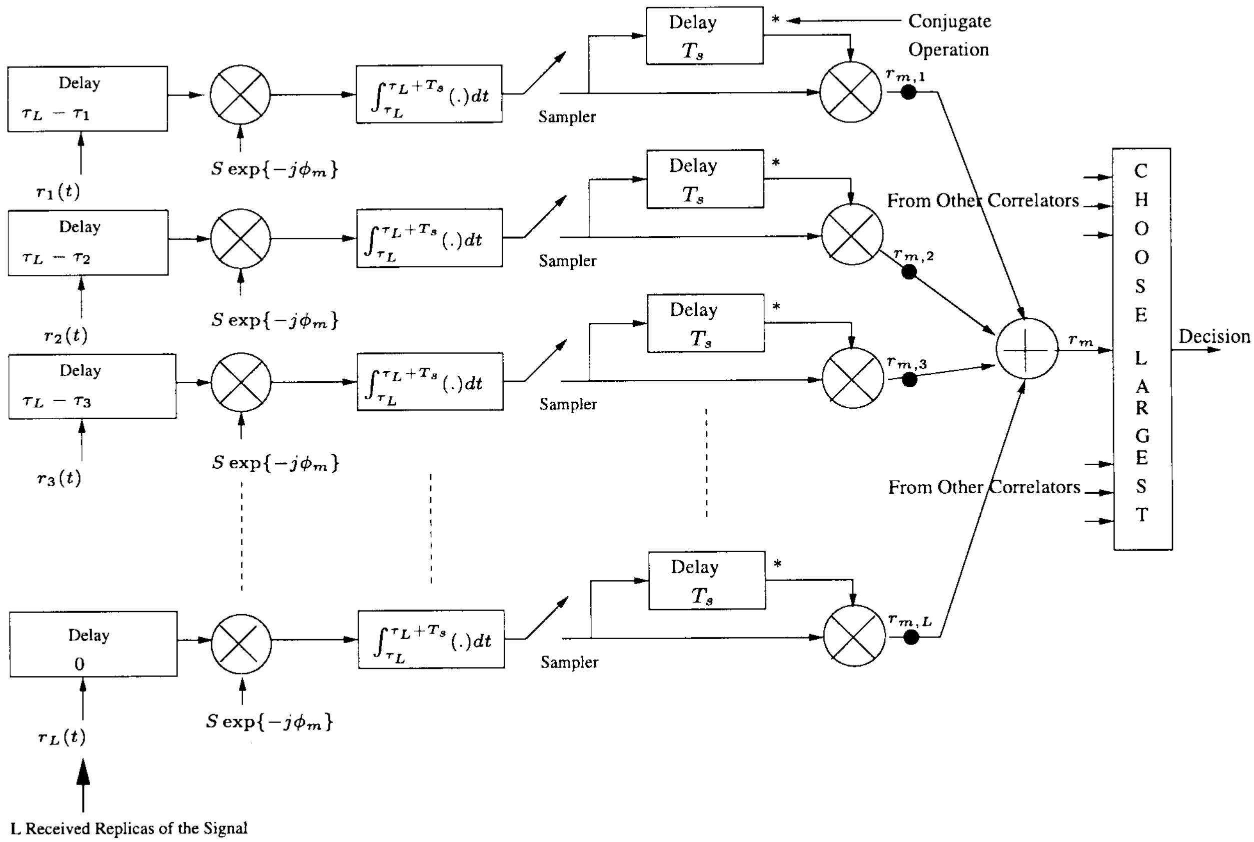

Fig. 2. Differentially coherent postdetection equal-gain combining receiver structure.

factor which is denoted by A, $0 \leq A \leq 1$, and hence the resulting combined PDF is given by

$$p_{A_l K_l \sigma_l}(\gamma_l;\, A_l,\, \overline{\gamma}_l^u,\, K_l;\, \mu_l^s,\, \sigma_l) = (1-A_l)p_{n_l}(\gamma_l;\, \overline{\gamma}_l^u,\, \sqrt{K_l}) + A_l p_{m_l \sigma_l}(\gamma_l;\, 1,\, \mu_l^s,\, \sigma_l) \quad (12)$$

where $\overline{\gamma}_l^u$ is the average SNR per bit during the unshadowed fraction of time, and μ_l^s is the average of $10\log_{10}\gamma_l$ during the shadowed fraction of time. The overall average SNR per bit, $\overline{\gamma}_l$, is then given by

$$\overline{\gamma}_l = (1-A_l)\overline{\gamma}_l^u + A_l 10^{(\mu_l^s/10)+(\ln 10\sigma_l^2/200)}. \quad (13)$$

C. Receiver Model

We consider L branch (finger) postdetection EGC receivers, as shown in Figs. 2 and 3, for differentially coherent and noncoherent detection, respectively. Both receivers utilize M correlators to detect the maximum *a priori* transmitted symbol. Without loss of generality, let us consider the mth symbol correlator. Each of the L received signals $r_l(t)$ is first delayed by $\tau_L - \tau_l$, then appropriately demodulated (symbol correlation followed by integration and dump then baud-rate sampling). These operations assume that the receiver is correctly time synchronized at every branch (i.e., perfect time delay $\{\tau_l\}_{l=1}^L$ estimates).

For differentially coherent detection (see Fig. 2), the receiver takes, at every branch l, the difference of two adjacent transmitted phases to arrive at the decision $r_{m,l}$. For noncoherent detection (see Fig. 3), no attempt is made to estimate the phase and the receiver yields the decision $r_{m,l}$ based on the squared envelope (i.e., square-law detection). Using EGC, the L decision outputs $\{r_{m,l}\}_{l=1}^L$ are summed to form the final decision variable r_m

$$r_m = \sum_{l=1}^{L} r_{m,l}; \qquad m = 1, 2, \cdots, M. \quad (14)$$

Last of all, the receiver selects the symbol corresponding to the maximum decision variable, as shown in Figs. 2 and 3.

For equally likely transmitted symbols, the total conditional SNR per bit, γ_t, at the output of the postdetection EGC combiner, is given by [3, p. 300, eq. (4.4.11)] and [3, p. 300, eq. (4.4.17)] as

$$\gamma_t = \sum_{l=1}^{L} \gamma_l. \quad (15)$$

III. BER WITH SINGLE CHANNEL RECEPTION ($L = 1$)

A. Desired Representation of the Conditional BER

A generic expression for the BER of differentially coherent and noncoherent modulations, $P_b(\gamma;\, a,\, b,\, \eta)$, over AWGN is given by [3, eq. (4B.21)]

$$P_b(\gamma;\, a,\, b,\, \eta) = Q_1(a\sqrt{\gamma},\, b\sqrt{\gamma}) - \frac{\eta}{1+\eta}\exp\left[-\frac{(a^2+b^2)\gamma}{2}\right] I_0(ab\gamma) \quad (16)$$

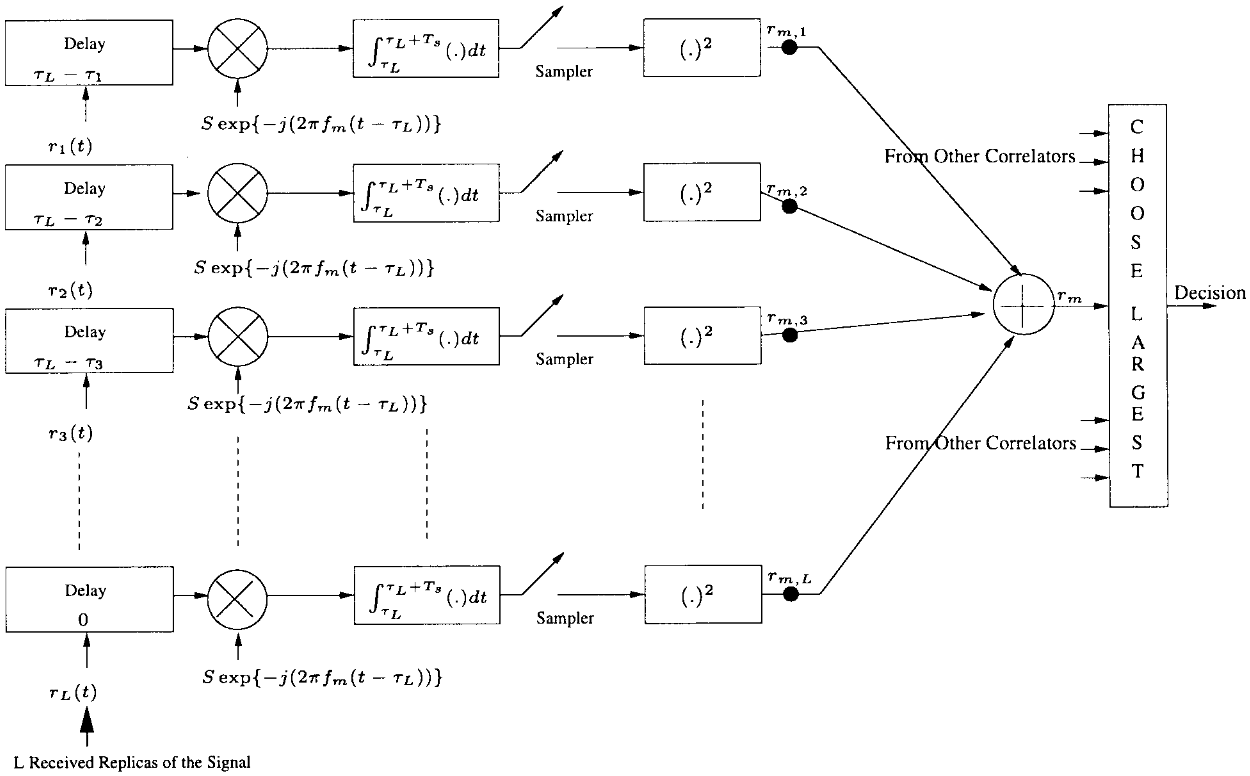

Fig. 3. Noncoherent postdetection equal-gain combining receiver structure.

where γ is the SNR per bit, and $Q_1(\cdot, \cdot)$ is the first-order Marcum Q-function traditionally defined by

$$Q_1(u, w) = \int_w^\infty x \exp\left[-\left(\frac{x^2+u^2}{2}\right)\right] I_0(ux)\,dx. \quad (17)$$

In (16) the parameters a and b are modulation-dependent and are defined in [3, eq. (4B.22)], and $\eta = v_2/v_1$, with the parameters v_1, v_2 defined in [3, eq. (4B.6)]. A number of special cases are of particular importance. For noncoherent detection of equal energy, equiprobable, correlated binary signals, $\eta = 1$ and

$$a = \left(\frac{1-\sqrt{1-|\rho|^2}}{2}\right)^{1/2}$$
$$b = \left(\frac{1+\sqrt{1-|\rho|^2}}{2}\right)^{1/2} \quad (18)$$

where $0 \le |\rho| \le 1$ is the magnitude of the cross-correlation coefficient between the two signals, and in this case (16) reduces to [3, eq. (4.3.15)]. The special case $\rho = 0$ corresponds to orthogonal binary FSK for which $a = 0$ and $b = 1$. Note that using the relation [55, eq. (9)]

$$Q_1(0, w) = e^{-w^2/2} \quad (19)$$

along with $I_0(0) = 1$, we see that (16) reduces in this particular case to the well-known expression reported by Proakis in [3, eq. (4.3.19)], namely,

$$P_b(\gamma; 0, 1, 1) = \tfrac{1}{2}e^{-\gamma/2}. \quad (20)$$

Furthermore, in the case of binary DPSK, $a = 0$, $b = \sqrt{2}$, and $\eta = 1$, whereupon using again (19) along with $I_0(0) = 1$, we see that (16) reduces to [3, eq. (4.2.117)]

$$P_b\left(\gamma; 0, \sqrt{2}, 1\right) = \tfrac{1}{2}e^{-\gamma}. \quad (21)$$

Finally, $a = \sqrt{2-\sqrt{2}}$, $b = \sqrt{2+\sqrt{2}}$, and $\eta = 1$ correspond to DQPSK with Gray coding and in this case (16) reduces to [3, eq. (4.2.118)].

To evaluate the average BER one must average the BER expression (16) (considered to be the conditional BER) over the statistics of the fading. Since the second argument of the function (which is proportional to the square-root of the SNR) appears in the lower limit of the integral in the traditional definition of the Marcum Q-function as given in (17), it is analytically difficult to perform such averages. We now introduce an alternate form of the Marcum Q-function, which leads to a desirable representation of the conditional BER. We then show in the next section how this representation circumvents this difficulty.

In virtually all applications of (16) to communication system performance analysis, the parameters a and b are typically independent of SNR, and furthermore $b > a$. Let us introduce the parameter $\beta = a/b$ which depends on the particular application, e.g., noncoherent detection of nonorthogonal signals, differential detection of PSK signals, etc., but is independent of SNR. With this in mind, an alternate integral form of the Marcum Q-function was presented in [14] which focused on having finite integration limits and an integrand with an exponential behavior in the argument w or u. In particular, it was shown in [13, eqs. (C-26) and (C-27)] or equivalently in

[14, eqs. (8) and (11)] that the first-order Marcum Q-function is given by

$$Q_1(u, w) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1+\beta \sin\phi}{1+2\beta\sin\phi+\beta^2} \times \exp\left[-\frac{w^2}{2}(1+2\beta\sin\phi+\beta^2)\right] d\phi; \quad 0 \le \beta = \frac{u}{w} < 1$$

$$Q_1(u, w) = 1 + \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\beta^2+\beta \sin\phi}{1+2\beta\sin\phi+\beta^2} \times \exp\left[-\frac{u^2}{2}(1+2\beta\sin\phi+\beta^2)\right] d\phi, \quad 0 \le \beta = \frac{w}{u} < 1. \tag{22}$$

Furthermore, using the integral representation of the zeroth-order modified Bessel function of the first kind [56], namely,

$$I_0(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-x\sin\phi}\, d\phi \tag{23}$$

it is straightforward to show that

$$\exp\left(-\frac{u^2+w^2}{2}\right) I_0(uw) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left[-\frac{w^2}{2}(1+2\beta\sin\phi+\beta^2)\right] d\phi, \quad \beta = \frac{u}{w}. \tag{24}$$

Substituting (22) and (24) in (16), the conditional BER (16) can be put in the desired representation given by

$$P_b(\gamma; a, b, \eta) = \frac{1}{2\pi(1+\eta)}\int_{-\pi}^{\pi} \frac{1-\eta\beta^2+\beta(1-\eta)\sin\phi}{1+2\beta\sin\phi+\beta^2} \times \exp\left[-\frac{b^2\gamma}{2}(1+2\beta\sin\phi+\beta^2)\right] d\phi, \quad 0 \le \beta = \frac{a}{b} < 1. \tag{25}$$

This form of the conditional BER (25) is more desirable since we can first integrate over a statistical distribution for γ and then perform the integral over ϕ, as described in more detail below. It should be noted that the specific form of the result in (25) with $\eta = 1$ can be obtained via the work of Pawula [57] who used certain relations between the Marcum Q-function and the Rice I_e-function which is defined by

$$I_e(k, x) = \int_0^x I_0(kt) e^{-t}\, dt. \tag{26}$$

In particular, combining (2a) and (2d) of Pawula [57] and making the substitutions $U = (b+a)/2$, $W = (b-a)/2$, and $V^2 = U^2 - W^2 = ab$ in these same equations, one arrives at the result

$$P_b(\gamma; a, b, 1) = \frac{1}{2\pi}\int_0^{\pi} \frac{1-\beta^2}{1-2\beta\cos\phi+\beta^2} \times \exp\left[-\frac{b^2\gamma}{2}(1+2\beta\cos\phi+\beta^2)\right] d\phi; \quad 0 \le \beta = \frac{a}{b} < 1 \tag{27}$$

which in view of the symmetry properties of the trigonometric functions over the interval $[-\pi, 0]$ and $[0, \pi]$ can be shown to be identically equivalent to (25) with $\eta = 1$. Also as a check, again setting $\eta = 1$, $a = 0$, and $b = 1$ ($b = \sqrt{2}$) in (25), it is straightforward to see that (25) reduces to (20) and (21) for orthogonal binary FSK (binary DPSK). Another special case of interest is DQPSK where $a = \sqrt{2-\sqrt{2}}$ and $b = \sqrt{2+\sqrt{2}}$. In this particular case, making the change of variable $\phi' = \phi + \pi/2$ in (25) and using the symmetry properties of the cosine function over the interval $[-\pi, 0]$ and $[0, \pi]$, it can be shown that (25) reduces to the expression reported in [10, eq. (3)], namely,

$$P_b\left(\gamma; \sqrt{2-\sqrt{2}}, \sqrt{2+\sqrt{2}}, 1\right) = \frac{1}{2\pi}\int_0^{\pi} \frac{\exp\left(-(2-\sqrt{2}\cos\phi)\gamma\right)}{\sqrt{2}-\cos\phi}\, d\phi. \tag{28}$$

B. Average BER

Recall that the conditional BER, conditioned on the SNR per bit γ, is given by (25). Since the fading is assumed to be independent of the AWGN, the unconditional BER, $\overline{P}_b(\overline{\gamma}, i; a, b, \eta)$, is obtained by averaging (25) over the underlying fading RV giving

$$\overline{P}_b(\overline{\gamma}, i; a, b, \eta) = \int_0^{\infty} P_b(\gamma; a, b, \eta) p_\gamma(\gamma; \overline{\gamma}, i)\, d\gamma \tag{29}$$

where i is the fading parameter associated with the distribution $p_\gamma(\gamma; \overline{\gamma}, i)$, and is hence denoted by $r, q, n, m, \sigma, m\sigma$, and $AK\sigma$ for the Rayleigh, Nakagami-q (Hoyt), Nakagami-n (Rice), Nakagami-m, log-normal shadowing, composite multipath/shadowing, and combined (time-shared) shadowed/unshadowed PDF's, respectively. Substituting (25) into (29) then interchanging the order of integration yields

$$\overline{P}_b(\overline{\gamma}, i; a, b, \eta) = \frac{1}{2\pi(1+\eta)}\int_{-\pi}^{\pi} \frac{1-\eta\beta^2+\beta(1-\eta)\sin\phi}{1+2\beta\sin\phi+\beta^2} \times \mathcal{J}_i(\overline{\gamma}, i; a, b; \phi)\, d\phi, \qquad 0 \le \beta = \frac{a}{b} < 1 \tag{30}$$

where

$$\mathcal{J}_i(\overline{\gamma}, i; a, b; \phi) \triangleq \int_0^{\infty} \exp\left[-\frac{b^2\gamma}{2}(1+2\beta\sin\phi+\beta^2)\right] p_\gamma(\gamma; \overline{\gamma}, i)\, d\gamma \tag{31}$$

is in the form of a Laplace transform. The form of the average BER in (30) is interesting in that the integrals $\mathcal{J}_i(\overline{\gamma}, i; a, b; \phi)$ can either be obtained in closed-form with the help of classical Laplace transform,s or can alternatively be efficiently computed by using Gauss–Hermite quadrature integration [58, p. 890, eq. (25.4.46)] for all previously mentioned fading channel models. We now evaluate these integrals for each of the fading models described in Section II-B. These integrals will also be useful to obtain the average BER with multichannel reception as described in Section IV-C.

1) Multipath Fading:

a) Rayleigh Fading: Substituting (3) into (31) then using the Laplace transform [56, (1), p. 1178]

$$\int_0^\infty e^{-sx}\,dx = \frac{1}{s}; \qquad s > 0 \tag{32}$$

yields[2]

$$\mathcal{J}_r(\overline{\gamma};\, a,\, b;\, \phi) = \left(1 + \frac{b^2\overline{\gamma}}{2}\,(1 + 2\beta\,\sin\,\phi + \beta^2)\right)^{-1}. \tag{33}$$

Inserting (33) in (30), we obtain the average BER performance over Rayleigh fading. For the special case of $\eta = 1$, one can proceed further to obtain a closed form expression for the average BER. Performing a partial fraction expansion on (30) with (33), then using the standard integral identity [56, eq.(3.661.4), p. 425]

$$\int_{-\pi}^{\pi} \frac{1}{u - w\,\sin\,\phi}\,d\phi = 2\int_0^{\pi} \frac{1}{u + w\,\cos\,\phi}\,d\phi = \frac{2\pi}{\sqrt{u^2 - w^2}}; \qquad u \geq |w| \tag{34}$$

it can be shown that the average BER in Rayleigh fading is given by

$$\overline{P}_b(\overline{\gamma},\, r;\, a,\, b,\, 1) = \frac{1}{2}\left[1 - \frac{(1-\beta^2)b^2\overline{\gamma}}{2\sqrt{1 + b^2\overline{\gamma}(1+\beta^2) + \left(\frac{b^2\overline{\gamma}}{2}\right)^2(1-\beta^2)^2}}\right], \quad 0 \leq \beta = \frac{a}{b} < 1. \tag{35}$$

Letting $a = 0$ and $b = 1$ in (35), it easy to see that $\overline{P}_b$ checks, as expected, with the expression reported by Proakis in [3, (7.3.12), p. 718], namely,

$$\overline{P}_b(\overline{\gamma},\, r;\, 0,\, 1,\, 1) = \frac{1}{2 + \overline{\gamma}} \tag{36}$$

for orthogonal binary FSK. Similarly, letting $a = 0$ and $b = \sqrt{2}$ in (35), it is easy to see that $\overline{P}_b$ checks, as expected, with the expression reported by Proakis in [3, eq. (7.3.10), p. 717], namely,

$$\overline{P}_b\left(\overline{\gamma},\, r;\, 0,\, \sqrt{2},\, 1\right) = \frac{1}{2(1 + \overline{\gamma})} \tag{37}$$

for binary DPSK. Another special case of interest is that for $a = \sqrt{2 - \sqrt{2}}$ and $b = \sqrt{2 + \sqrt{2}}$. In this case, (35) reduces to

$$\overline{P}_b\left(\overline{\gamma},\, r;\, \sqrt{2-\sqrt{2}},\, \sqrt{2+\sqrt{2}},\, 1\right) = \frac{1}{2}\left[1 - \frac{1}{\sqrt{\frac{(1+2\overline{\gamma})^2}{2\overline{\gamma}^2} - 1}}\right] \tag{38}$$

which is equivalent to the expression of $\overline{P}_b$ for DQPSK with Gray coding reported in [8, eq. (18)], [9, eq. (13)], namely,

$$\overline{P}_b\left(\overline{\gamma},\, r;\, \sqrt{2-\sqrt{2}},\, \sqrt{2+\sqrt{2}},\, 1\right) = \frac{1}{2\sqrt{1 + 4\overline{\gamma} + 2\overline{\gamma}^2}} \times \frac{\sqrt{2}\overline{\gamma} + (\sqrt{2}-1)\left(1 + 2\overline{\gamma} - \sqrt{1 + 4\overline{\gamma} + 2\overline{\gamma}^2}\right)}{\sqrt{2}\overline{\gamma} - (\sqrt{2}-1)\left(1 + 2\overline{\gamma} - \sqrt{1 + 4\overline{\gamma} + 2\overline{\gamma}^2}\right)} \tag{39}$$

or the one reported in [10, eq. (8)], namely,

$$\overline{P}_b\left(\overline{\gamma},\, r;\, \sqrt{2-\sqrt{2}},\, \sqrt{2+\sqrt{2}},\, 1\right) = \frac{1}{2}\left(1 - \frac{\sqrt{2}\overline{\gamma}}{\sqrt{1 + 4\overline{\gamma} + 2\overline{\gamma}^2}}\right). \tag{40}$$

b) Nakagami-q (Hoyt) Fading: Substituting (5) into (31), then using the Laplace transform [56, eq. (109), p. 1182]

$$\int_0^\infty I_0(ux)e^{-sx}\,dx = (s^2 - u^2)^{-1/2}, \qquad s > |u| \geq 0 \tag{41}$$

yields

$$\mathcal{J}_q(\overline{\gamma},\, q;\, a,\, b;\, \phi) = \left(1 + b^2\overline{\gamma}(1 + 2\beta\,\sin\,\phi + \beta^2) + \frac{q^2b^4\overline{\gamma}^2}{(1+q^2)^2}\,(1 + 2\beta\,\sin\,\phi + \beta^2)^2\right)^{-1/2}. \tag{42}$$

For $q = 0$, (42) becomes

$$\mathcal{J}_q(\overline{\gamma},\, 0;\, a,\, b;\, \phi) = (1 + b^2\overline{\gamma}(1 + 2\beta\,\sin\,\phi + \beta^2))^{-1/2} \tag{43}$$

which when inserted in (30) gives the BER performance over one-sided Gaussian fading. On the other hand, letting $q = 1$ in (42), it is easy to show that $\mathcal{J}_q(\overline{\gamma},\, 1;\, a,\, b;\, \phi)$ reduces to $\mathcal{J}_r(\overline{\gamma};\, a,\, b;\, \phi)$ as given by (33) and which corresponds to the Rayleigh fading case, as expected.

Letting $a = 0$ and $b = 1$ ($b = \sqrt{2}$) in (42) yields the average BER performance of orthogonal binary FSK (DPSK) over a Nakagami-q (Hoyt) fading channel as

$$\overline{P}_b(\overline{\gamma},\, q;\, 0,\, \sqrt{2g},\, 1) = \frac{1}{2}\left(1 + 2g\overline{\gamma} + \frac{4q^2g^2\overline{\gamma}^2}{(1+q^2)^2}\right)^{-1/2} \tag{44}$$

where $g = 1/2$ for orthogonal binary FSK and $g = 1$ for DPSK.

c) Nakagami-n (Rice) Fading: Substituting (7) into (31) then using the Laplace transform [58, eq. (29.3.81), p. 1026]

$$\int_0^\infty I_0(u\sqrt{x})e^{-sx}\,dx = \frac{e^{u^2/(4s)}}{s}; \qquad s > 0 \tag{45}$$

yields

$$\mathcal{J}_n(\overline{\gamma},\, n;\, a,\, b;\, \phi) = \frac{2(1+n^2)}{2(1+n^2) + b^2\overline{\gamma}(1 + 2\beta\,\sin\,\phi + \beta^2)} \times \exp\left[-\frac{n^2b^2\overline{\gamma}(1 + 2\beta\,\sin\,\phi + \beta^2)}{2(1+n^2) + b^2\overline{\gamma}(1 + 2\beta\,\sin\,\phi + \beta^2)}\right]. \tag{46}$$

[2]Note that for the Rayleigh fading case, the PDF has no dependency on the fading parameter, r. Hence, for simplicity of notations, we omit it in the argument sequence of the function $\mathcal{J}_r(\cdot;\, \cdot,\, \cdot;\, \cdot)$

For $n = 0$, (46) reduces, as expected, to $\mathcal{J}_r(\overline{\gamma};\, a,\, b;\, \phi)$ as given by (33), corresponding to the Rayleigh fading case. Furthermore, as $n \longrightarrow +\infty$,

$$\mathcal{J}_n(\overline{\gamma},\, n;\, a,\, b;\, \phi) \longrightarrow \exp\left[-\frac{b^2\overline{\gamma}}{2}\,(1+2\beta\,\sin\,\phi+\beta^2)\right]$$

which when substituted in (30) yields, as expected, the BER performance over the nonfading (i.e., AWGN) channel as given by (25).

Letting $a = 0$ and $b = 1$ ($b = \sqrt{2}$) in (46) yields the average BER performance of orthogonal binary FSK (DPSK) over a Nakagami-n (Rice) fading channel as

$$\overline{P}_b\left(\overline{\gamma},\, n;\, 0,\, \sqrt{2g},\, 1\right) = \frac{1+n^2}{2(1+n^2+g\overline{\gamma})}\,\exp\left(-\frac{n^2 g\overline{\gamma}}{1+n^2+g\overline{\gamma}}\right) \tag{47}$$

where $g = 1/2$ for orthogonal binary FSK and $g = 1$ for DPSK. Further, setting $a = \sqrt{2-\sqrt{2}}$ and $b = \sqrt{2+\sqrt{2}}$, it can be easily shown that $\overline{P}_b(\overline{\gamma},\, n;\, \sqrt{2-\sqrt{2}},\, \sqrt{2+\sqrt{2}},\, 1)$ is equivalent to the expression for DQPSK with Gray coding given in [10, eq. (6)], namely,

$$\overline{P}_b\left(\overline{\gamma},\, n;\, \sqrt{2-\sqrt{2}},\, \sqrt{2+\sqrt{2}},\, 1\right) = \frac{(1+n^2)e^{-n^2}}{2\pi} \times \int_0^{\pi} \frac{\exp\left(\dfrac{n^2(1+n^2)}{1+n^2+2\overline{\gamma}-\sqrt{2}\overline{\gamma}\,\cos\,\phi}\right)}{\left(\sqrt{2}-\cos\,\phi\right)\left(1+n^2+2\overline{\gamma}-\sqrt{2}\overline{\gamma}\,\cos\,\phi\right)}\, d\phi. \tag{48}$$

d) Nakagami-m Fading: Substituting (9) into (31) then using the Laplace transform [56,eq. (3), p. 1178]

$$\int_0^{\infty} x^{\nu}e^{-sx}\, dx = \frac{\Gamma(\nu+1)}{s^{\nu+1}}; \qquad s > 0,\, \nu > -1 \tag{49}$$

yields

$$\mathcal{J}_m(\overline{\gamma},\, m;\, a,\, b;\, \phi) = \left[1+\frac{b^2\overline{\gamma}}{2m}\,(1+2\beta\,\sin\,\phi+\beta^2)\right]^{-m}. \tag{50}$$

Note that for $m = 1/2$, $\mathcal{J}_m(\overline{\gamma},\, 1/2;\, a,\, b;\, \phi) = \mathcal{J}_q(\overline{\gamma},\, 0;\, a,\, b;\, \phi)$ which corresponds to the case of one-sided Gaussian fading. Further, for $m = 1$, $\mathcal{J}_m(\overline{\gamma},\, 1/2;\, a,\, b;\, \phi) = \mathcal{J}_r(\overline{\gamma};\, a,\, b;\, \phi)$ which corresponds to the Rayleigh fading case. Finally, as $m \rightarrow +\infty$, accounting for the identity

$$\lim_{m\rightarrow+\infty}\left(1+\frac{h}{m}\right)^{-m} = e^{-h}$$

we see that

$$\mathcal{J}_m(\overline{\gamma},\, m;\, a,\, b;\, \phi) \longrightarrow \exp\left[-\frac{b^2\overline{\gamma}}{2}\,(1+2\beta\,\sin\,\phi+\beta^2)\right]$$

which when substituted in (30) yields, as expected, the BER performance over the nonfading (i.e., AWGN) channel as given by (25).

For the special case of $\eta = 1$ and m restricted to positive integers values, one can proceed further to obtain a closed from expression for the average BER. Performing a partial fraction expansion on (30) with (50), then using the standard integral identity [56, eq. (3.661.4), p. 425]

$$\int_{-\pi}^{\pi} \frac{1}{(u-w\,\sin\,\phi)^{k+1}}\, d\phi = 2\int_0^{\pi} \frac{1}{(u+w\,\cos\,\phi)^{k+1}}\, d\phi = \frac{2\pi}{(u^2-w^2)^{(k+1)/2}}\,\mathcal{P}_k\left(\frac{u}{\sqrt{u^2-w^2}}\right), \qquad u \geq |w| \tag{51}$$

where $\mathcal{P}_k(\cdot)$ is the Legendre polynomial of order k [56, eq. (8.911.1), p. 1044], it can be shown that the average BER in Nakagami-m fading is given by (52), shown at the bottom of the page, where C_m and C_k are shown in (53) at the bottom of the page. Note that when $m = 1$, which corresponds to Rayleigh fading, (52) reduces, as expected, to (35) since $\mathcal{P}_0(x) = 1$.

Letting $a = 0$ and $b = 1$ ($b = \sqrt{2}$) in (50), it easy to see that $\overline{P}_b$ checks, as expected, with the expression attributed to Barrow [59] and reported in [60, eq. (11)] and in [61, eq. (B1)] for orthogonal binary FSK (binary DPSK), namely,

$$\overline{P}_b\left(\overline{\gamma},\, m;\, 0,\, \sqrt{2g},\, 1\right) = \frac{1}{2}\left(\frac{m}{m+g\overline{\gamma}}\right)^m \tag{54}$$

$$\overline{P}_b(\overline{\gamma},\, m;\, a,\, b,\, 1) = \frac{1}{2}\left[1+\sum_{k=1}^{m}\frac{(1-\beta^2)C_k}{\left[1+\dfrac{b^2\overline{\gamma}}{m}\,(1+\beta^2)+\left(\dfrac{b^2\overline{\gamma}}{2m}\right)^2(1-\beta^2)^2\right]^{k/2}}\quad \mathcal{P}_{k-1}\left(\frac{1+\dfrac{b^2\overline{\gamma}}{2m}\,(1+\beta^2)}{\left(1+\dfrac{b^2\overline{\gamma}}{m}\,(1+\beta^2)+\left(\dfrac{b^2\overline{\gamma}}{2m}\right)^2(1-\beta^2)^2\right)^{1/2}}\right)\right] \tag{52}$$

$$C_m = -\frac{b^2\overline{\gamma}}{2m}$$

$$C_k = \frac{1}{(m-k)!}\,\frac{d^{m-k}}{du^{m-k}}\left[\frac{1}{\left(1+\beta^2+\dfrac{2m}{b^2\overline{\gamma}}\,u\right)\left(1+\dfrac{b^2\overline{\gamma}}{2m}\,(1+\beta^2)+u\right)^m}\right]_{u=-1-((b^2\overline{\gamma})/2m)(1+\beta^2)}; \quad k = 1,\, \cdots,\, m-1. \tag{53}$$

$g = 1/2$ for orthogonal binary FSK and $g = 1$ for DPSK. The other special case of interest is that for $a = \sqrt{2-\sqrt{2}}$ and $b = \sqrt{2+\sqrt{2}}$. In this case, it can be easily shown that $\overline{P}_b(\overline{\gamma}, m; \sqrt{2-\sqrt{2}}, \sqrt{2+\sqrt{2}}, 1)$ is equivalent to the expression for DQPSK with Gray coding given in [10, eq. (7)], namely,

$$\overline{P}_b\left(\overline{\gamma}, m; \sqrt{2-\sqrt{2}}, \sqrt{2+\sqrt{2}}, 1\right) = \frac{1}{2\pi}\left(\frac{m}{m+2\overline{\gamma}}\right)^m \int_0^\pi \times \frac{1}{\left(\sqrt{2}-\cos\phi\right)\left(1-\frac{\sqrt{2}\overline{\gamma}}{m+2\overline{\gamma}}\cos\phi\right)^m}\,d\phi. \quad (55)$$

2) Log-Normal Shadowing: If the channel statistics follow a log-normal distribution, it is straightforward to show that $\mathcal{J}_\sigma(\mu, \sigma; a, b; \phi)$ can be accurately approximated by Gauss–Hermite integration yielding

$$\mathcal{J}_\sigma(\mu, \sigma; a, b; \phi) \simeq \frac{1}{\sqrt{\pi}}\sum_{n=1}^{N_p} H_{x_n} \exp\left(-\frac{b^2 10^{(\sqrt{2}\sigma x_n+\mu)/10}}{1+2\beta\sin\phi+\beta^2}\right) \quad (56)$$

where N_p is the order of the Hermite polynomial, $H_{N_p}(.)$. Setting N_p to 20 is typically sufficient for excellent accuracy. In (56), x_n are the zeros of the N_p-order Hermite polynomial, and H_{x_n} are the weight factors of the N_p-order Hermite polynomial and are given by

$$H_{x_n} = \frac{2^{N_p-1}N_p!\sqrt{\pi}}{N_p^2 H_{N_p-1}^2(x_n)}. \quad (57)$$

Both the zeros and the weights factors of the Hermite polynomial are tabulated in [58, Table (25.10), p. 924] for various polynomial orders N_p.

3) Composite Multipath/Shadowing: If the channel statistics follow a gamma/log-normal distribution, it is straightforward to show that $\mathcal{J}_{m\sigma}(\mu, m\sigma; a, b; \phi)$ can be accurately evaluated by using (49) followed by a Gauss–Hermite integration yielding

$$\mathcal{J}_{m\sigma}(\mu, m\sigma; a, b; \phi) \simeq \frac{1}{\sqrt{\pi}}\sum_{n=1}^{N_p} H_{x_n}\left(1+\frac{b^2 10^{(\sqrt{2}\sigma x_n+\mu)/10}}{m(1+2\beta\sin\phi+\beta^2)}\right)^{-m}. \quad (58)$$

4) Combined (Time-Shared) Shadowed/Unshadowed: If the channel statistics follow a combined Lutz *et al.* distribution, it is straightforward to show that $\mathcal{J}_{AK\sigma}(\overline{\gamma}^u, \mu^s, K, \sigma; a, b; \phi)$ can be broken into two terms, one which can be evaluated in closed-form and the other which can be accurately approximated by Gauss–Hermite integration yielding

$$\mathcal{J}_{AK\sigma}(\overline{\gamma}^u, \mu^s, K, \sigma; a, b; \phi) \simeq (1-A)\mathcal{J}_n\left(\overline{\gamma}^u, \sqrt{K}; a, b; \phi\right) + A\mathcal{J}_{m\sigma}(\mu^s, m\sigma; a, b; \phi) \quad (59)$$

with $n=\sqrt{K}$ in $\mathcal{J}_n(\overline{\gamma}^u, n; a, b; \phi)$ and $m=1$ in $\mathcal{J}_{m\sigma}(\mu^s, m\sigma; a, b; \phi)$.

IV. BER WITH MULTICHANNEL RECEPTION ($L > 1$)

A. Desired Product Form Representation of the Conditional BER (General Case)

Many problems dealing with the BER performance of multichannel reception of differentially coherent and noncoherent detection of PSK and FSK signals in AWGN channels have a decision variable which is a quadratic form in complex-valued Gaussian random variables. Almost three decades ago, Proakis [2] developed a general expression for evaluating the BER when the decision variable is in that particular form. Indeed, the development and results originally obtained in [2] later appeared in [3, Appendix 4B] and have become a classic in the annals of communication system performance literature. The most general form of the BER expression, i.e., [3, eq. (4B.21)] obtained by Proakis was given in terms of the first-order Marcum Q-function and modified Bessel functions of the first kind. Although implied but not explicitly given in [2] and [3], this general form can be rewritten in terms of the generalized Marcum Q-function, $Q_l(.,.)$, as

$$\begin{aligned} P_b(L, \gamma_t; a, b, \eta) &= Q_1(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - \left[1-\frac{\sum_{l=0}^{L-1}\binom{2L-1}{l}\eta^l}{(1+\eta)^{2L-1}}\right] \\ &\quad\times \exp\left[-\frac{(a^2+b^2)\gamma_t}{2}\right] I_0(ab\gamma_t) \\ &\quad+ \frac{1}{(1+\eta)^{2L-1}}\left[\sum_{l=2}^{L}\binom{2L-1}{L-l}\eta^{L-l}\right. \\ &\quad\left.\times\left[Q_l(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - Q_1(a\sqrt{\gamma_t}, b\sqrt{\gamma_t})\right]\right] \\ &\quad- \frac{1}{(1+\eta)^{2L-1}}\left[\sum_{l=2}^{L}\binom{2L-1}{L-l}\eta^{L-1+l}\right. \\ &\quad\left.\times\left[Q_l(b\sqrt{\gamma_t}, a\sqrt{\gamma_t}) - Q_1(b\sqrt{\gamma_t}, a\sqrt{\gamma_t})\right]\right] \end{aligned} \quad (60)$$

where

$$\binom{2L-1}{L-l} = (2L-1)!/((L-l)!(L+l-1)!)$$

denotes the binomial coefficient, and where all the modulation-dependent parameters have already been defined previously. As a check for $L = 1$, the latter two summations in (60) do not contribute, and hence one immediately obtains the result (16), as expected. Note that although the form in (60) does not give the appearance of being much simpler than [3, eq. (4B.21)], we shall see shortly that it does have particular advantage for obtaining the average BER performance over generalized fading channels.

As in the single channel reception case, the parameters a and b in (60) are typically independent of SNR, and furthermore $b > a$. Let us introduce again the modulation dependent parameter $\beta = a/b$ which is independent of SNR. With this in mind, we now show how the alternate integral representation of the generalized Marcum Q-function yields a desired product form representation of the conditional BER. In particular, it was shown in [13], or equivalently in [14], that

$$\begin{aligned} Q_l(u, w) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \\ &\times \frac{\beta^{-(l-1)}(\cos[(l-1)(\phi+\pi/2)] - \beta\cos[l(\phi+\pi/2)])}{1+2\beta\sin\phi+\beta^2} \\ &\times \exp\left[-\frac{w^2}{2}(1+2\beta\sin\phi+\beta^2)\right]d\phi; \\ &\quad 0^+ \le \beta = \frac{u}{w} < 1, \\ Q_l(u, w) &= 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \\ &\times \frac{\beta^{l}(\cos[l(\phi+\pi/2)] - \beta\cos[(l-1)(\phi+\pi/2)])}{1+2\beta\sin\phi+\beta^2} \\ &\times \exp\left[-\frac{u^2}{2}(1+2\beta\sin\phi+\beta^2)\right]d\phi; \\ &\quad 0 \le \beta = \frac{w}{u} < 1 \end{aligned} \tag{61}$$

with the special case of $l = 1$ being given in (22). Now, using (22), (24), and (61) in (60), it can be shown after tedious manipulations that *the entire conditional BER expression* (60) *can be written as a single integral with an integrand that contains a single exponential factor in* γ_t *of the form* $\exp[-((b^2\gamma_t)/2)(1+2\beta\sin\phi+\beta^2)]$, namely,

$$\begin{aligned} P_b(L, \gamma_t; a, b, \eta) = \frac{\eta^L}{2\pi(1+\eta)^{2L-1}}&\int_{-\pi}^{\pi}\frac{f(L;\beta,\eta;\phi)}{1+2\beta\sin\phi+\beta^2} \\ &\times \exp\left[-\frac{b^2\gamma_t}{2}(1+2\beta\sin\phi+\beta^2)\right]d\phi; \\ &\quad 0^+ \le \beta = \frac{a}{b} < 1 \end{aligned} \tag{62}$$

where

$$f(L;\beta,\eta;\phi) = f_0(L;\beta,\eta;\phi) + f_1(L;\beta,\eta;\phi)$$

with

$$\begin{aligned} f_0(L;\beta,\eta;\phi) &= \left[-\frac{(1+\eta)^{2L-1}}{\eta^L} + \sum_{l=1}^{L}\binom{2L-1}{L-l}(\eta^{-l}+\eta^{l-1})\right] \\ &\quad\times \beta(\beta+\sin\phi), \\ f_1(L;\beta,\eta;\phi) &= \sum_{l=1}^{L}\binom{2L-1}{L-l}\left[(\eta^{-l}\beta^{-l+1} - \eta^{l-1}\beta^{l+1})\right. \\ &\quad\times\cos((l-1)(\phi+\pi/2)) - (\eta^{-l}\beta^{-l+2} - \eta^{l-1}\beta^{l}) \\ &\quad\left.\times\cos(l(\phi+\pi/2))\right]. \end{aligned} \tag{63}$$

As a check, for the special case of $L = 1$, we obtain

$$\begin{aligned} f_0(1;\beta,\eta;\phi) &= 0, \\ f_1(1;\beta,\eta;\phi) &= \frac{1-\eta\beta^2+\beta(1-\eta)\sin\phi}{\eta} \end{aligned} \tag{64}$$

and hence (62) reduces to (25), as expected.

The form of the conditional BER in (62) has the advantage of being a single finite-range integral with limits independent of the conditional SNR and an integrand which can be written in a *product form,* such as

$$\begin{aligned} P_b(L, \gamma_t; a, b, \eta) &= \frac{\eta^L}{2\pi(1+\eta)^{2L-1}}\int_{-\pi}^{\pi}\frac{f(L;\beta,\eta;\phi)}{1+2\beta\sin\phi+\beta^2} \\ &\quad\prod_{l=1}^{L}\times\exp\left[-\frac{b^2\gamma_l}{2}(1+2\beta\sin\phi+\beta^2)\right]d\phi, \\ &\quad 0^+ \le \beta = \frac{a}{b} < 1. \end{aligned} \tag{65}$$

Furthermore, the form of (65) is desirable since we can first independently average over the individual statistical distributions of the γ_l's, and then perform the integral over ϕ, as described in more detail below (Section IV-C). Before showing this, however, we first offer some simplifications of (60) and (62) for some special cases of interest.

B. Desired Product Form Representation of the Conditional BER (Special Case ($\eta = 1$))

For $\eta = 1$, and any $L \ge 1$, which corresponds to the case of multichannel detection of equal energy correlated binary signals, the conditional BER expression (60) becomes

$$\begin{aligned} P_b(L, \gamma_t; a, b, 1) &= Q_1(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - \frac{1}{2}\exp\left[-\frac{(a^2+b^2)\gamma_t}{2}\right]I_0(ab\gamma_t) \\ &\quad+ \frac{1}{2^{2L-1}}\left[\sum_{l=1}^{L}\binom{2L-1}{L-l}\right. \\ &\quad\times[(Q_l(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - Q_l(b\sqrt{\gamma_t}, a\sqrt{\gamma_t})) \\ &\quad\left.-(Q_1(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - Q_1(b\sqrt{\gamma_t}, a\sqrt{\gamma_t}))]\right] \\ &= Q_1(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - \frac{1}{2}\exp\left(-\frac{(a^2+b^2)\gamma_t}{2}\right)I_0(ab\gamma_t) \\ &\quad+ \frac{1}{2^{2L-1}}\sum_{l=1}^{L}\binom{2L-1}{L-l} \\ &\quad\times[Q_l(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - Q_l(b\sqrt{\gamma_t}, a\sqrt{\gamma_t})] \\ &\quad-\frac{1}{2}[Q_1(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - Q_1(b\sqrt{\gamma_t}, a\sqrt{\gamma_t})] \end{aligned} \tag{66}$$

where we have added back the $l = 1$ term in the sums of (60) since they have zero value anyway. However, comparing eqs. (40) and (42) of [62],

$$\begin{aligned} Q_1(u, w) - \frac{1}{2}\exp\left(-\frac{u^2+w^2}{2}\right)I_0(uw) \\ = \frac{1}{2}[1 - Q_1(w, u) + Q_1(u, w)]. \end{aligned} \tag{67}$$

Thus, combining (66) and (67) gives the simplified expression

$$\begin{aligned} P_b(L, \gamma_t; a, b, 1) &= \frac{1}{2} + \frac{1}{2^{2L-1}}\sum_{l=1}^{L}\binom{2L-1}{L-l} \\ &\quad\times[Q_l(a\sqrt{\gamma_t}, b\sqrt{\gamma_t}) - Q_l(b\sqrt{\gamma_t}, a\sqrt{\gamma_t})] \end{aligned} \tag{68}$$

which appears not to be given in [2] and [3]. Setting $a=0$ and $b=1$ ($b=\sqrt{2}$) in (68), then using the relations [55, eq. (9)]

$$Q_l(0, w) = e^{-w^2/2} \sum_{k=0}^{l-1} \frac{(w^2/2)^k}{k!}$$
$$Q_l(u, 0) = 1$$

along with the identity $\sum_{l=1}^{L} \binom{2L-1}{L-l} = 2^{2(L-1)}$, it can be shown that (68) reduces to the well-known expression reported by Proakis for multichannel binary orthogonal FSK (binary DPSK) given by [3, eq. (4.4.13), p. 301], namely,

$$P_b\left(L, \gamma_t; 0, \sqrt{2g}, 1\right) = \frac{1}{2^{2L-1}} e^{-g\gamma_t} \sum_{l=0}^{L-1} c_l (g\gamma_t)^l \quad (69)$$

where

$$c_l = \frac{1}{l!} \sum_{k=0}^{L-1-l} \binom{2L-1}{k}$$

$g = 1/2$ for orthogonal binary FSK, and $g = 1$ for binary DPSK. Note that an alternate (equivalent) form to (69), involving the confluent hypergeometric function, ${}_1F_1(\cdot; \cdot; \cdot)$, and given by Charash [5, eq. (32)] as

$$P_b\left(L, \gamma_t; 0, \sqrt{2g}, 1\right) = \frac{e^{-2g\gamma_t}}{2^L \Gamma(L)} \sum_{l=0}^{L-1} \frac{\Gamma(L+l)}{2^l \Gamma(l+1)} {}_1F_1(L+l; L; g\gamma_t) \quad (70)$$

has also been used in the literature for the BER of multichannel binary orthogonal FSK and binary DPSK [15], [17].

The conditional BER expression (66) for the special case of $\eta = 1$ and any $L \geq 1$, can also be put in the desired product form. Indeed, it can be shown that in this particular case $f_0(L; \beta, 1; \phi) = 0$, and hence (62) reduces to

$$P_b(L, \gamma_t; a, b, 1) = \frac{1}{2^{2L}\pi} \int_{-\pi}^{\pi} \frac{f_1(L; \beta, 1; \phi)}{1 + 2\beta \sin\phi + \beta^2} \times \exp\left[-\frac{b^2 \gamma_t}{2}(1 + 2\beta \sin\phi + \beta^2)\right] d\phi, \quad 0^+ \leq \beta = \frac{a}{b} < 1 \quad (71)$$

where the functions $f_1(\cdot; \cdot, \cdot; \cdot)$ is now given by

$$f_1(L; \beta, 1; \phi) = \sum_{l=1}^{L} \binom{2L-1}{L-l} [(\beta^{-l+1} - \beta^{l+1}) \cos((l-1)(\phi + \pi/2)) - (\beta^{-l+2} - \beta^l) \cos(l(\phi + \pi/2))]. \quad (72)$$

Again, as a check for $L = 1$, we obtain

$$f_1(1; \beta, l; \phi) = 1 - \beta^2. \quad (73)$$

which when substituted in (71) gives an expression for the BER which agrees with (25) for $\eta = 1$, as expected. Note also that as $\beta \to 0$, (71) assumes an indeterminate form and thus an analytical expression for the limit is more easily obtained from (69) with g replaced by $b^2/2$. We further point out that the limit of (71) as $\beta \to 0$ converges smoothly to the exact BER expression of (69). For example, numerical evaluation of (71) setting $\beta = 10^{-3}$ ($a = 10^{-3}$, $b = 1$) gives an accuracy of 5 digits when compared with numerical evaluation of (69) for the same system parameters. The representation (71) is therefore useful even in this specific case. This is particularly true for the performance of binary orthogonal FSK and binary DPSK which cannot be obtained via the classical representation of (69) in the most general fading case, but which can be solved using the desirable conditional BER expression (71) as we will show next.

C. Average BER

To obtain the unconditional BER, $\overline{P}_b(L, \{\overline{\gamma}_l\}_{l=1}^L, \{i_l\}_{l=1}^L; a, b, \eta)$, we must average the conditional BER, $P_b(L, \gamma_t; a, b, \eta)$, over the joint PDF of the instantaneous SNR sequence $\{\gamma_l\}_{l=1}^L$, namely $p_{\gamma_1, \gamma_2, \cdots, \gamma_L}(\gamma_1, \gamma_2, \cdots, \gamma_L)$. Since the RV's $\{\gamma_l\}_{l=1}^L$ are assumed to be statistically independent, then $p_{\gamma_1, \gamma_2, \cdots, \gamma_L}(\gamma_1, \gamma_2, \cdots, \gamma_L) = \prod_{l=1}^L p_{\gamma_l}(\gamma_l; \overline{\gamma}_l, i_l)$, and the averaging procedure results in

$$\overline{P}_b(L, \{\overline{\gamma}_l\}_{l=1}^L, \{i_l\}_{l=1}^L; a, b, \eta) = \underbrace{\int_0^\infty \int_0^\infty \cdots \int_0^\infty}_{L\text{-fold}} P_b(L, \gamma_t; a, b, \eta) \times \left[\prod_{l=1}^L p_{\gamma_l}(\gamma_l; \overline{\gamma}_l, i_l)\right] d\gamma_1 \, d\gamma_2 \cdots d\gamma_L. \quad (74)$$

If the classical representation of $P_b(L, \gamma_t; a, b, \eta)$, as given by [3, eq. (4B.21)] or equivalently (60), were to be used, (74) would result in an $L+1$-fold integral with infinite limits (one of these integrals comes from the classical definition of the generalized Marcum Q-function in $P_b(L, \gamma_t; a, b, \eta)$), and an adequately efficient numerical integration method would not be available.

Using the desired product form representation of $P_b(L, \gamma_t; a, b, \eta)$, namely (65) in (74) yields

$$\overline{P}_b(L, \{\overline{\gamma}_l\}_{l=1}^L, \{i_l\}_{l=1}^L; a, b, \eta) = \frac{\eta^L}{2\pi(1+\eta)^{2L-1}} \underbrace{\int_0^\infty \int_0^\infty \cdots \int_0^\infty}_{L\text{-fold}} \int_{-\pi}^{\pi} \frac{f(L; \beta, \eta; \phi)}{1 + 2\beta \sin\phi + \beta^2} \times \left[\prod_{l=1}^L \exp\left[-\frac{b^2 \gamma_l}{2}(1 + 2\beta \sin\phi + \beta^2)\right]\right] \times \left[\prod_{l=1}^L p_{\gamma_l}(\gamma_l; \overline{\gamma}_l, i_l)\right] d\phi \, d\gamma_1 \, d\gamma_2 \cdots d\gamma_L. \quad (75)$$

The integrand in (75) is absolutely integrable, and the order of integration can therefore be interchanged. Thus, grouping like terms we have

$$\begin{aligned}\overline{P}_b(L, \{\overline{\gamma}_l\}_{l=1}^L, \{i_l\}_{l=1}^L; a, b, \eta) &= \frac{\eta^L}{2\pi(1+\eta)^{2L-1}} \int_{-\pi}^{\pi} \frac{f(L; \beta, \eta; \phi)}{1+2\beta \sin\phi + \beta^2} \\ &\quad \times \left[\prod_{l=1}^{L} \int_0^\infty \exp\left[-\frac{b^2\gamma_l}{2}(1+2\beta\sin\phi+\beta^2)\right] \right. \\ &\quad \left. \times\, p_{\gamma_l}(\gamma_l; \overline{\gamma}_l, i_l)\, d\gamma_l \right] d\phi \\ &= \frac{\eta^L}{2\pi(1+\eta)^{2L-1}} \int_{-\pi}^{\pi} \frac{f(L; \beta, \eta; \phi)}{1+2\beta \sin\phi + \beta^2} \\ &\quad \times \prod_{l=1}^{L} \mathcal{J}_{i_l}(\overline{\gamma}_l, i_l; a, b; \phi)\, d\phi \end{aligned} \tag{76}$$

where $\mathcal{J}_{i_l}(\overline{\gamma}_l, i_l;\ a, b;\ \phi)$ is given above for the various channel models associated with path l. Note that if the fading is identically distributed with the same fading parameter i and the same average SNR per bit $\overline{\gamma}$ for all L channels, then (76) reduces to

$$\begin{aligned}\overline{P}_b(L, \overline{\gamma}, i; a, b, \eta) &= \frac{\eta^L}{2\pi(1+\eta)^{2L-1}} \int_{-\pi}^{\pi} \frac{f(L; \beta, \eta; \phi)}{1+2\beta \sin\phi + \beta^2} \\ &\quad \times [\mathcal{J}_i(\overline{\gamma}, i; a, b; \phi)]^L\, d\phi. \end{aligned} \tag{77}$$

Hence, this approach reduces the $L+1$-fold integral with infinite limits of (74) to a single finite-range integral (76) whose integrand contains only elementary functions (i.e., no special functions) and which can therefore be easily evaluated numerically.

V. Conclusion

The myriad of results obtained by the cited authors for the error probability performance of noncoherent and differentially coherent modulations over generalized fading channels can now all be obtained as special cases of a unified approach to the problem. Aside from unifying the past results, the new approach also allows for a more general solution to the problem in that it includes many situations that in the past defied a simple solution. The best example of this occurs for multichannel reception where the fading on each channel need not be identically distributed nor even distributed according to the same family of distributions. It is now possible to obtain results for this case corresponding to a wide variety of modulation/fading channel combinations. Other situations of comparable complexity are also now solvable in the form of simple and elegant solutions.

References

[1] M. K. Simon, S. M. Hinedi, and W. C. Lindsey, *Digital Communication Techniques—Signal Design and Detection.* Englewood Cliffs, NJ: Prentice Hall, 1995.

[2] J. G. Proakis, "On the probability of error for multichannel reception of binary signals," *IEEE Trans. Commun. Technol.,* vol. COM-16, pp. 68–71, Feb. 1968.

[3] ——, *Digital Communications,* 2nd ed. New York: McGraw-Hill, 1989.

[4] W. C. Lindsey, "Error probabilities for Ricean fading multichannel reception of binary and N-ary signals," *IEEE Trans. Inform. Theory,* vol. IT-10, pp. 339–350, Oct. 1964.

[5] U. Charash, "Reception through Nakagami fading multipath channels with random delays," *IEEE Trans. Commun.,* vol. COM-27, pp. 657–670, Apr. 1979.

[6] J. F. Weng and S. H. Leung, "Analysis of DPSK with equal gain combining in Nakagami fading channels," *Electron. Lett.,* vol. 33, pp. 654–656, Apr. 1997.

[7] F. Patenaude, J. H. Lodge, and J.-Y. Chouinard, "Error probability expressions for noncoherent diversity in Nakagami fading channels," in *Proc. IEEE Vehicular Technology Conf. VTC'97,* Phoenix, AZ, May 1997, pp. 1484–1487.

[8] T. T. Tjhung, C. Loo, and N. P. Secord, "BER performance of DQPK in slow Rician fading," *Electron. Lett.,* vol. 28, pp. 1763–1765, Aug. 1992.

[9] M. Tanda, "Bit error rate of DQPSK signals in slow Nakagami fading," *Electron. Lett.,* vol. 29, pp. 431–432, Mar. 1993.

[10] C. Tellambura and V. K. Bhargava, "Unified error analysis of DQPSK in fading channels," *Electron. Lett.,* vol. 30, pp. 2110–2111, Dec. 1994.

[11] G. L. Stüber, *Principles of Mobile Communications.* Norwell, MA: Kluwer, 1996.

[12] M. K. Simon and M.-S. Alouini, "A unified approach to the performance analysis of digital communications over generalized fading channels," *Proc. IEEE,* vol. 86, pp. 1860–1877, Sept. 1998.

[13] C. W. Helstrom, *Elements of Signal Detection and Estimation.* Englewood Cliffs, NJ: Prentice Hall, 1995.

[14] M. K. Simon, "A new twist on the Marcum Q-function and its applications," *IEEE Commun. Lett.,* vol. 2, pp. 39–41, Feb. 1998.

[15] T. Eng and L. B. Milstein, "Comparison of hybrid FDMA/CDMA systems in frequency selective Rayleigh fading," *IEEE J. Select. Areas Commun.,* vol. 12, pp. 938–951, June 1994.

[16] R. Prasad, H. S. Misser, and A. Kegel, "Performance evaluation of direct-sequence spread spectrum multiple-access for indoor wireless communication in a Rician fading channel," *IEEE Trans. Commun.,* vol. 43, pp. 581–592, Feb.–Apr. 1995.

[17] G. P. Efthymoglou, V. A. Aalo, and H. Helmken, "Performance analysis of noncoherent binary DS/CDMA systems in a Nakagami multipath channel with arbitrary parameters," in *Proc. IEEE Global Communication Conf. GLOBECOM'96,* London, UK, Nov. 1996, pp. 1296–1300; full paper published in *IEE Proc. Commun.,* June 1997, vol. 144, pp. 166–172.

[18] M.-S. Alouini, M. K. Simon, and A. Goldsmith, "A unified performance analysis of DS–CDMA systems over generalized frequency-selective fading channels," in *Proc. IEEE Int. Symp. Information Theory ISIT'98,* Cambridge, MA, Aug. 1998; full paper in preparation for submission to *IEEE J. Select. Areas Commun. Special Issue on Spread Spectrum for Global Communications*.

[19] M. K. Simon and M.-S. Alouini, *Digital Communications Over Generalized Fading Channels: A Unified Approach to Performance Analysis.* New York: Wiley.

[20] H. B. James and P. I. Wells, "Some tropospheric scatter propagation measurements near the radio-horizon," *Proc. IRE,* pp. 1336–1340, Oct. 1955.

[21] G. R. Sugar, "Some fading characteristics of regular VHF ionospheric propagation," *Proc. IRE,* pp. 1432–1436, Oct. 1955.

[22] S. Basu, E. M. MacKenzie, S. Basu, E. Costa, P. F. Fougere, H. C. Carlson, and H. E. Whitney, "250 MHz/GHz scintillation parameters in the equatorial, polar, and aural environments," *IEEE J. Select. Areas Commun.,* vol. SAC-5, pp. 102–115, Feb. 1987.

[23] T. L. Staley, R. C. North, W. H. Ku, and J. R. Zeidler, "Performance of coherent MPSK on frequency selective slowly fading channels," in *Proc. IEEE Vehicular Technology Conf. VTC'96,* Atlanta, GA, Apr. 1996, pp. 784–788; full paper submitted to *IEEE Trans. Veh. Technol*.

[24] R. S. Hoyt, "Probability functions for the modulus and angle of the normal complex variate," *Bell Syst. Tech. J.,* vol. 26, pp. 318–359, Apr. 1947.

[25] M. Nakagami, "The m-distribution—A general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation.* Oxford, UK: Pergamon, 1960, pp. 3–36.

[26] B. Chytil, "The distribution of amplitude scintillation and the conversion of scintillation indices," *J. Atmos. Terr. Phys.,* vol. 29, pp. 1175–1177, Sept. 1967.

[27] K. Bischoff and B. Chytil, "A note on scintillaton indices," *Planet. Space Sci.,* vol. 17, pp. 1059–1066, 1969.
[28] S. O. Rice, "Statistical properties of a sine wave plus random noise," *Bell Syst. Tech. J.,* vol. 27, pp. 109–157, Jan. 1948.
[29] K. A. Stewart, G. P. Labedz, and K. Sohrabi, "Wideband channel measurements at 900 MHz," in *Proc. IEEE Vehicular Technology Conf. VTC'95,* Chicago, IL, July 1995, pp. 236–240.
[30] R. J. C. Bultitude, S. A. Mahmoud, and W. A. Sullivan, "A comparison of indoor radio propagation characteristics at 910 MHz and 1.75 GHz," *IEEE J. Select. Areas Commun.*, vol. 7, pp. 20–30, Jan. 1989.
[31] T. S. Rappaport and C. D. McGillem, "UHF fading in factories," *IEEE J. Select. Areas Commun.,* vol. 7, pp. 40–48, Jan. 1989.
[32] G. H. Munro, "Scintillation of radio signals from satellites," *J. Geophys. Res.,* vol. 68, Apr. 1963.
[33] P. D. Shaft, "On the relationship between scintillation index and Rician fading," *IEEE Trans. Commun.,* vol. COM-22, pp. 731–732, May 1974.
[34] H. Suzuki, "A statistical model for urban multipath propagation," *IEEE Trans. Commun.,* vol. COM-25, pp. 673–680, July 1977.
[35] T. Aulin, "Characteristics of a digital mobile radio channel," *IEEE Trans. Veh. Technol.,* vol. VT-30, pp. 45–53, May 1981.
[36] W. R. Braun and U. Dersch, "A physical mobile radio channel model," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 472–482, May 1991.
[37] A. U. Sheikh, M. Handforth, and M. Abdi, "Indoor mobile radio channel at 946 MHz: Measurements and modeling," in *Proc. IEEE Vehicular Technology Conf. VTC'93,* Secaucus, NJ, May 1993, pp. 73–76.
[38] E. J. Fremouw and H. F. Bates, "Worldwide behavior of average VHF–UHF scintillation," *Radio Sci.,* vol. 6, pp. 863–869, Oct. 1971.
[39] H. E. Whitney, J. Aarons, R. S. Allen, and D. R. Seeman, "Estimation of the cumulative probability distribution function of ionospheric scintillations," *Radio Sci.,* vol. 7, pp. 1095–1104, Dec. 1972.
[40] E. J. Fremouw, R. C. Livingston, and D. A. Miller, "On the statistics of scintillating signals," *J. Atmos. Terr. Phys.,* vol. 42, pp. 717–731, Aug. 1980.
[41] P. K. Banerjee, R. S. Dabas, and B. M. Reddy, "C-band and L-band transionospheric scintillation experiment—Some results for applications to satellite radio systems," *Radio Sci.,* vol. 27, pp. 955–969, June 1992.
[42] G. L. Turin, F. D. Clapp, T. L. Johnston, S. B. Fine, and D. Lavry, "A statistical model of urban multipath propagation," *IEEE Trans. Veh. Technol.,* vol. VT-21, pp. 1–9, Feb. 1972.
[43] H. Hashemi, "Simulation of the urban radio propagation channel," *IEEE Trans. Veh. Technol.,* vol. VT-28, pp. 213–225, Aug. 1979.
[44] T. S. Rappaport, S. Y. Seidel, and K. Takamizawa, "Statistical channel impulse response models for factory and open plan building radio communication system design," *IEEE Trans. Commun.,* vol. 39, pp. 794–807, May 1991.
[45] P. Yegani and C. McGlilem, "A statistical model for the factory radio channel," *IEEE Trans. Commun.,* vol. 39, pp. 1445–1454, Oct. 1991.
[46] H. Hashemi, "Impulse response modeling of indoor radio propagation channels," *IEEE J. Select. Areas Commun.,* vol. 11, pp. 967–978, Sept. 1993.
[47] F. Hansen and F. I. Meno, "Mobile fading—Rayleigh and lognormal superimposed," *IEEE Trans. Veh. Technol.,* vol. VT-26, pp. 332–335, Nov. 1977.
[48] M. J. Ho and G. L. Stüber, "Co-channel interference of microcellular systems on shadowed Nakagami fading channels," in *Proc. IEEE Vehicular Technology Conf. VTC'93,* Secaucus, NJ, May 1993, pp. 568–571.
[49] C. Loo, "A statistical model for a land-mobile satellite link," *IEEE Trans. Veh. Technol.,* vol. VT-34, pp. 122–127, Aug. 1985.
[50] G. Corazza and F. Vatalaro, "A statistical model for land mobile satellite channels and its application to nongeostationary orbit systems," *IEEE Trans. Veh. Technol.*, vol. 43, pp. 738–742, Aug. 1994.
[51] S.-H Hwang, K.-J. Kim, J.-Y. Ahn, and K.-C. Wang, "A channel model for nongeostationnary orbiting satellite system," in *Proc. IEEE Vehicular Technology Conf. VTC'97,* Phoenix, AZ, May 1997, pp. 41–45.
[52] E. Lutz, D. Cygan, M. Dippold, F. Dolainsky, and W. Papke, "The land mobile satellite communication channel—Recording, statistics, and channel model," *IEEE Trans. Veh. Technol.,* vol. 40, pp. 375–386, May 1991.
[53] M. Rice and B. Humpherys, "Statistical models for the ACTS K-band land mobile satellite channel," in *Proc. IEEE Vehicular Technology Conf. VTC'97,* Phoenix, AZ, May 1997, pp. 46–50.
[54] R. M. Barts and W. L. Stutzman, "Modeling and simulation of mobile satellite propagation," *IEEE Trans. Antennas Propagat.,* vol. 40, pp. 375–382, Apr. 1992.
[55] G. M. Dillard, "Recursive computation of the generalized Q-function," *IEEE Trans. Aerosp. Electron. Syst.,* vol. AES-9, pp. 614–615, July 1973.
[56] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products,* 5th ed. San Diego, CA: Academic, 1994.
[57] R. F. Pawula, "Relations between Rice I_e-function and Marcum Q-function with applications to error rate calculations," *Electron. Lett.,* vol. 31, pp. 1717–1719, Sept. 1995.
[58] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,* 9th ed. New York, NY: Dover, 1970.
[59] B. B. Barrow, "Error probabilities for data transmission over fading radio paths," SHAPE Air Defense Technical Center, Tech. Rep. TM-26, 1962.
[60] A. H. Wojnar, "Unkown bounds on performance in Nakagami channels," *IEEE Trans. Commun.,* vol. COM-34, pp. 22–24, Jan. 1986.
[61] P. J. Crepeau, "Uncoded and coded performance of MFSK and DPSK in Nakagami fading channels," *IEEE Trans. Commun.,* vol. 40, pp. 487–493, Mar. 1992.
[62] S. Stein, "Unified analysis of certain coherent and noncoherent binary communication systems," *IEEE Trans. Inform. Theory,* vol. IT-10, pp. 43–51, Jan. 1964.

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