Design Criteria of Two-Hop Based Wireless Networks with Non-Regenerative Relays in Arbitrary Fading Channels

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Abstract—In this paper we evaluate outage performances of multirelay amplify and forward (AF) transmission over fading channels. We focus on the uplink of a two-hop based network where N (with $N \geq 1$) single antenna relay stations (RSs) serve as non-regenerative repeaters for the message transmitted by a single antenna source node (or mobile station MS). Performances are derived at high SNR for arbitrary fading distributions over each cooperative link. In Rayleigh fading the high price of cooperation in terms of signalling (e.g. during MS-to-RSs transmissions and control messages broadcasts) causes a significant loss in spectral efficiency. Instead, provided that the RSs can be strategically positioned to benefit from a channel with marginal diffusive fading component compared to Rayleigh, this paper shows that this loss in efficiency can be significantly alleviated depending on the particular propagation environment. Closed form conditions on the fading statistics for the relayed links are derived to guarantee that collaborative transmission performs as if the MS node would benefit from the same diversity and bandwidth efficiency provided by multi-antenna transmission. The proposed conditions are computed for arbitrary distributed fading for the relays-to-BS link and are specialized for Rice faded RSs-to-BS links.

Index Terms—Cooperative diversity, amplify and forward relaying, relay deployment, Rice fading.

I. INTRODUCTION

HIGH demand of data rate envisioned for fourth generation wireless systems (4G) does not appear to be feasible with the conventional cellular architecture. Being the 4G spectrum located above the 2GHz band (i.e., see WiMAX specifications [1]), the radio propagation is significantly more vulnerable to non-line of sight (NLOS) and path loss [2].

Multiple antenna solutions at mobile stations (MS) are widely acknowledged as an effective technology (e.g., in conjunction with space-time (ST) encoding) to increase the link throughput by providing spatial redundancy (or diversity order). However, hardware, size and cost constraints limit the number of antennas that can be used in practice at the MS.

In recent years research interests have been focused on improving point-to-point networks by enabling multi-hop transmissions. This architecture can be considered as alternative to MSs with multiple antennas and it allows fixed or nomadic low cost and low power single (or multiple) antenna relay stations (RSs) to serve as repeaters (either regenerative or

non-regenerative) for a number of single antenna source nodes: the main purpose of this scheme is to guarantee the joint exploitation of the benefits of multihop diversity and cooperation. RSs can be fixed and positioned higher than the surrounding scatterings so that they experience propagation with meaningful line-of-sight (LOS) component that dominate, in some cases, over random NLOS components [2].

It is widely acknowledged that collaborative transmissions impaired by Rayleigh fading can potentially provide the same diversity benefits of multi-antenna transmission. However, an high price in terms of signalling and overhead (e.g., during MS-to-RSs transmissions and coordination messages) needs to be accounted for. Large overhead is shown to be responsible for meaningful reduction in the achievable spectral efficiency [3]. In this work we show that a careful planning when deploying the RSs that allows the relayed channel to benefit from milder fading compared to Rayleigh can counterbalance the efficiency loss from signalling. Accurate design of relay deployment (as well as the number of active relays) can potentially improve network coverage and per-user throughput by providing connections for MSs in shadowed or deep faded locations, as well as by extending the BS coverage beyond the regular cell boundaries [4].

In this paper we focus on the uplink scenario of a two-hop based network where N single antenna RSs surrounding the MS are capable of amplify and forwarding the received signal, and thus they are collaborating to serve as non-regenerative repeaters for source messages originating from (single antenna) terminals. The analysis here is limited to amplify and forward relays for their simplicity, low cost and large appeal to a practical implementation [5]. Each RS amplifies and forwards the received message by using orthogonal subchannels (using time division, see Fig. 1-(a)), thus avoiding mutual interference.

Consider the propagation setting in Fig. 1-(a): the source node (e.g., the MS station) can reach the BS with the potential help of relayed transmissions from $N = 2$ RSs. The RSs might experience arbitrary channel power distributions for the links towards the BS $f_{|h_{rd,i}|^2}(\cdot)$ (e.g., RSs might be fixed and placed higher than the surrounding scattering to reduce the fading impairments). For a given propagation environment, in this work we investigate the necessary conditions on the RSs-to-BS fading distributions that make collaborative transmission with N nodes to perform as if the MS station would employ multiple antennas (M) and ST coded transmission (see Fig. 1-(b)). By using this equivalence, we quantify the benefits from a careful choice of the relays placement with varying propagation settings reducing the analysis and design of cooperative systems to a comparative analysis with an equivalent
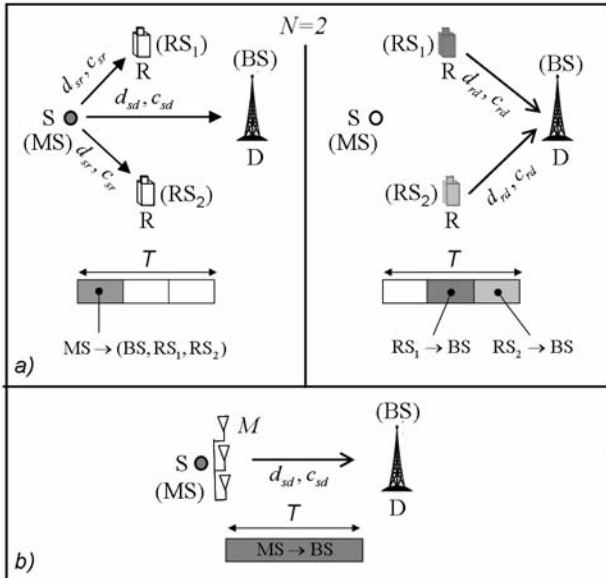
Fig. 1.   (a) Uplink two-hop based network (e.g. cellular based) with non-regenerative relays. (b) Non cooperative system with multiantenna source node. Links are impaired by arbitrary fading (with different diversity and coding gain pairs). Shaded nodes are transmitting terminals during the (reserved) time fraction.

multiantenna non-cooperative transmission used as reference system. Design rules for relayed links are valid for *arbitrarily distributed* fading (thus they do not depend on the particular channel or path loss model) and arbitrary number N of RSs. These rules can be used for improving the first selection of the best candidate sites for deploying the relays.

A. Related work

The performance analysis of (cooperative) transmissions embracing *arbitrary* fading distributions is dealt with in [6] for decode and forward relaying. Fading channels are described therein by the inherent (fractional) diversity (d) and coding gain (c) parameters [7] that are evaluated in a general way from the asymptotic behavior of the moment generating function (MGF) of the random fading power over each link. This approach to performance evaluation of communications over fading channels requires only knowledge of the MGF of the fading power distribution that is a standard concept either in statistical analysis and in communication field [8]. By considering an arbitrary propagation environment, [6] introduces the concept of *cooperative fading regions* as the collection of fading power distributions that make relayed transmission to perform as if the source node would be equipped with multiple antennas.

Relay deployment problem for regenerative (decode and forward) relaying has been investigated in [14] for sensor network applications. Numerical solutions therein suggest that the deployment of the relay nodes can be arranged for a specific probabilistic distribution rather than specifying the exact places.

For amplify and forward relaying, the ergodic and outage capacity are derived in [9] and [10] for Rayleigh fading and different amplify and forward protocols with different degree of interference at the receiver. In [11] the symbol error rate performances of multirelay amplify and forward protocols

are investigated for Rice fading. Performances for two-hop networks with variable gain non-regenerative RS (with the perfect channel knowledge of the MS-to-RS channel) are evaluated in [12] [13] for the Rayleigh fading. Other models for the relay gain are proposed in literature (see e.g. [10]). In [3] it is shown that, for Rayleigh fading, when transmission is constrained to support high spectral efficiencies, direct transmission becomes *always* preferable over a cooperative (decode or amplify and forward) system. In this paper we show that this conclusion is indeed limited to the Rayleigh fading case. Cooperative transmission might be instead beneficial even at high rates provided that the inefficiencies that are caused by large overheads can be counterbalanced by placing the relays in strategically selected locations so that the propagation can experience less severe fading (with smaller diffusive components, or larger LOS) compared to Rayleigh.

B. Paper organization and main contributions

In what follows we outline the original contributions of the paper as compared to the existing works.

i) The outage probability is the design criteria employed here and it can be approximated asymptotically (for large signal to noise ratio, SNR) as $\bar{P}_{out} = (c \cdot SNR)^{-d}$, or equivalently in terms of the "inherent" diversity order d and the coding gain c as outage parameters that are provided by the channel itself [7]. In Sect. III scaling laws for outage probability of multirelay amplify and forward cooperative protocols (AF) [9] in arbitrary fading are derived (Proposition 1) (an overview of existing results is given in Sect. II). The proposed method provides an analytic framework that allows for closed form high SNR performance evaluation of multirelay amplify and forward systems under arbitrary propagation environments (including Rayleigh fading as a special case). Scaling laws for outage probability are specialized to fit with Rice fading (Remark 2) and Nakagami-m (Remark 3) fading.

ii) *Cooperative diversity* provided by the amplify and forward transmission is shown to depend on the fading statistics and on the spatial redundancy from multiple transmissions. In Sect. III-B we extend the (asymptotic) *cooperative fading region* [6] to the amplify and forward case as the collections of fading power distributions that makes AF transmissions to provide a (cooperative) diversity that is larger than the diversity provided by non-cooperative multi-antenna MISO transmissions used as reference setting (Fig. 1-(b)). In the same Sect. III-B we evaluate the diversity-multiplexing tradeoff as compared to non-cooperative transmission.

iii) Using the large SNR outage analysis developed in Sect. III, in Sect. IV analysis focuses on the derivation of closed form design rules for the RSs-to-BS fading distributions (in terms of constraints) so that cooperative and non-cooperative systems can now exhibit comparable performances in terms of outage, spectral efficiency and power consumption. The analysis is limited to the RS-to-BS links (although problem can be readily extended to any case) by assuming the RSs be fixed or nomadic [1]. Other cooperative links are assumed to be impaired by Rayleigh fading[1]. In Sect. IV-A the design

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[1]In most cellular/mesh networks with mobile users only fixed/nomadic connections might experience milder fading compared to Rayleigh [2]. MS station is typically surrounded by local scattering so that Rayleigh distribution for MS-to-BS and MS-to-RS links can be assumed without leading in generality.

rules are further specialized to fit with Rice faded RSs-to-BS links. For this case, we show that for a given end-to-end transmission rate $\bar{R}$ necessary condition for cooperative system with N RSs to perform as if the MS node would be equipped with $M = N + 1$ antennas requires the Rice factor for the relayed link(s) K_{rd} to satisfy $K_{rd} \in \mathcal{O}\left(M \times \bar{R}\right)$ for large enough communication rate $\bar{R}$. Results are numerically corroborated in Sect. V.

II. OVERVIEW ON OUTAGE ANALYSIS OF AMPLIFY AND FORWARD PROTOCOL

For a direct transmission with duration T (here $T = 1$) let E_S be the transmitted symbol energy from a single antenna source node S (MS) node towards the destination D (e.g. the BS), the transmit power is $\rho = E_S/T$ and the signal to noise ratio (SNR) referred to the transmitting side is ρ/σ^2 with $\sigma^2 = N_0/T$ the additive white Gaussian noise (AWGN) noise power and N_0 the single sided AWG noise density. Here we assume that the intercell interference can be neglected as it is managed by proper design of the cellular reuse factor [15]. Moreover, without loss in generality, it is $N_0 = 1$ and $\sigma^2 = 1$ so that ρ refers equivalently to SNR or transmit power.

The baseband complex valued channel gain is h and term $\mathrm{E}[|h|^2] = g$ accounts for antenna gain, path loss and shadowing[2] so that the instantaneous SNR at the receiving side is $\gamma = \rho|h|^2$. Assuming a channel with static fading for the whole transmission duration, the maximum mutual information over the link is $I = \log_2(1 + \mathcal{A} \cdot \mu)$ where the gap $0 < \mathcal{A} \leq 1$ accounts for the selected modulation and coding format [16] (e.g., $\mathcal{A} = 1$ for Gaussian codebook). Given a target rate $\bar{R}$ (bandwidth efficiency) [b/s/Hz], the outage probability is $\Pr[I < \bar{R}] = \Pr\left[\gamma < \left(2^{\bar{R}} - 1\right)/\mathcal{A}\right]$.

For the multiantenna case (Fig. 1-(b)) used herein as reference for performance evaluation, an orthogonal ST coded direct transmission is employed for the whole transmission duration T by the MS node now equipped with M (uncorrelated) antennas. To simplify the analysis we assume that the ST codeword does not introduces any rate loss (this can be achieved in practice only for $M = 2$ with Alamouti coding [17]). Using subscript sd to indicate a direct (non-cooperative) multiantenna transmission (from source to destination), the outage probability at the BS is

$$\Pr[I_{sd}(\rho) < \bar{R}] = \Pr\left[\gamma_{sd}(\rho) < \left(2^{\bar{R}} - 1\right)/\mathcal{A}\right] \quad (1)$$

and, as in [18], the SNR at the decision variable $\gamma_{sd}(\rho) = (\rho/M)\sum_{i=1}^{M}|h_{sd,i}|^2$ depends on the fading power for each transmitter-receiver pair $|h_{sd,i}|^2$. Notice that transmit power of each antenna scales as ρ/M in order to highlight the benefits of diversity when constraining the overall transmit energy to $E_S = \rho T$.

Amplify and Forward (AF) [9]-[11] protocol is in Fig. 1-(a). It is based on a relayed transmission of same duration T, where N single antenna RSs amplify and forward the same information message from the single antenna MS. To constrain the same end-to-end rate (bandwidth efficiency) $\bar{R}$, as for

direct non-cooperative transmission, RS and MS terminals coordinate themselves by using time division (with perfect synchronization): the MS node broadcasts the message for a fixed time fraction $1/(N + 1)$, then each RS that is willing to cooperate (say $N = 2$ RSs from the example in Fig. 1-(a)) stores the received noisy symbols and forwards this amplified signal during the reserved time fraction of $1/(N + 1)$ with an increased rate $(N + 1)\bar{R}$[3]

At the BS, the receiver exploits the full Channel State Information (CSI) of each link from the RSs and the MS by optimally combining (using Maximum Ratio Combining, MRC) the $N + 1$ noisy replicas (to generalize, the copy of the MS is also included for combining). Notice that more complex transmission strategies might involve larger signalling by selectively activating RSs according to opportunistic scheduling policies [19].

Variable gain RSs [12] are chosen here so that amplification factor β_i is

$$\beta_i = 1/\sqrt{|h_{sr,i}|^2 + \sigma^2/\rho} \quad (2)$$

where subscript sr indicates the (i-th) MS-to-RS link, β_i is the amplification factor for i-th RS and varies with the channel power $|h_{sr,i}|^2$ to preserve the power constraint [9]. Models for the relay gain are proposed in literature, other than (2), see e.g. [10].

Each relay and MS use a transmit power of ρ for a time fraction of $1/(N + 1)$ to preserve the overall transmit energy E_S. The mutual information for multi-node amplify and forward relaying with factor β_i in (2) is found in [9] as $I_{AF}(\rho) = 1/(N + 1)\log_2\left[1 + \mathcal{A}_c \cdot \gamma_{AF}(\rho)\right]$ where

$$\gamma_{AF}(\rho) = \gamma_{sd} + \sum_{i=1}^{N} g(\gamma_{sr,i}, \gamma_{rd,i}), \quad (3)$$

function $g(x, y) = xy/(x + y + 1)$ and terms are $\gamma_{sd} = \rho|h_{sd}|^2$, $\gamma_{sr,i} = \rho|h_{sr,i}|^2$ and $\gamma_{rd,i} = \rho|h_{rd,i}|^2$, subscript rd is used to indicate the RS-to-BS link. $\mathcal{A}_c$ accounts for the selected coding and modulation format used for relayed transmissions. The outage probability is

$$\Pr[I_{AF}(\rho) < \bar{R}] = \Pr[\gamma_{AF}(\rho) < \left(2^{(N+1)\bar{R}} - 1\right)/\mathcal{A}_c]. \quad (4)$$

III. LARGE SNR DIVERSITY AND OUTAGE ANALYSIS IN FADING CHANNELS

In this section, the outage analysis of AF protocol is carried out for arbitrary fading statistics over each link. Fading channels (and outage probability) are described by inherent diversity (d) and coding gain (c) parameters that can be derived from the asymptotic behavior of the random fading power MGF [6]. To have the paper self-contained, herein we review the basic principles of diversity and coding gain derivations in arbitrary fading before tailoring to AF protocol (Sect. III-(A,B))

For direct transmission, the outage probability at high ρ can be approximated as [7]

$$\Pr[I < \bar{R}] = \Pr[\rho|h|^2 < \left(2^{\bar{R}} - 1\right)/\mathcal{A}] \approx \left(\frac{2^{\bar{R}} - 1}{c\rho \times \mathcal{A}}\right)^d, \quad (5)$$

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[2]Shadowing contribution is approximated here as a constant power loss. Other factors that impact on the path loss are the transmitter-receiver distance and the antenna heigh (e.g., for fixed RSs) and beamwidth (as for Hata-Okumura model [25])

[3]Each relay and source access to the wireless medium for a shorter fraction $(1/(N+1))$ of time if compared to direct multi-antenna transmission. To have same bandwith efficiency $\bar{R}$ ($T = 1$) as for direct transmission, transmission rate for each collaborative link is $(N + 1)\bar{R}$

TABLE I
DIVERSITY ORDERS AND CODING GAINS FOR RICE AND NAKAGAMI-M
FADING POWER DISTRIBUTIONS.

Diversity and coding gain parameters
(Rice) $F_{|h|^2}(s, K) = \frac{1+K}{1+K+sg} \exp(\frac{-Kgs}{1+K+gs})$
$d = 1$ ($\lim_{s\to\infty} sF_{|h|^2}(s, K) > 0$), $c = \frac{g\exp(K)}{K+1}$
(Nakagami-m) $F_{|h|^2}(s, m) = (1 + \frac{sg}{m})^{-m}$
$d = m$ ($\lim_{s\to\infty} s^m F_{|h|^2}(s, m) > 0$), $c = \frac{g}{m}[\Gamma(m+1)]^{\frac{1}{m}}$

where notation $\approx$ indicates that equality holds for asymptotically high SNR ρ. For any fading with generic power distribution $|h|^2 \sim f_{|h|^2}(w)$, d is the (fractional) diversity order $d \triangleq \lim_{\rho\to\infty} \log\Pr[I < \bar{R}]/\log\rho$ that is provided by the channel, while parameter c is the amount of coding gain.

For arbitrary fading channels, diversity d can be defined from the Laplace transform $\mathcal{F}_{|h|^2}(s)$ (or Moment Generating Function, MGF) of $f_{|h|^2}(w)$ as

$$d = \lim_{s\to\infty} \frac{-\log\mathcal{F}_{|h|^2}(s)}{\log s} > 0. \quad (6)$$

The main reasoning of the proof is in Appendix VII-A while the detailed derivation is in [6]. Coding gain c follows as:

$$c \triangleq [\Gamma(d+1)/\phi]^{1/d}, \quad (7)$$

with $\phi = \lim_{s\to\infty} s^d \mathcal{F}_{|h|^2}(s)$ and where $\Gamma(x) = \int_0^\infty y^{x-1}\exp(-y)dy$ is the complete Gamma function.

A. Multirelay amplify and forward transmission

Outage probability at high SNR is now derived for AF protocol and arbitrary fading distributions. The MS node is activating N RSs using N links to the BS with the same fading statistics (in general different from Rayleigh fading). These are described by the inherent diversity d_{sr} (6) and the coding gain c_{sr} (7) that are related to the specific fading power distribution, say $|h_{sr}|^2 \sim f_{|h_{sr}|^2}(\cdot)$. Similarly, the MS-to-BS link might suffer from a fading that is different from Rayleigh (and also from the one experienced by MS-to-RSs links) and it is characterized by the same diversity/coding gain pair d_{sd} and c_{sd}. The N relayed transmissions from the RSs toward the BS (see Fig. 1-(a)) are performed over channels with diversities $d_{rd,i}$ and coding gains $c_{rd,i}$, $(i = 1, ..., N)$ related to the fading power distributions $|h_{rd,i}|^2 \sim f_{|h_{rd,i}|^2}(\cdot)$. As a motivation for this general setting, notice that the RSs might be placed at different locations within the cell to experience different fading statistics over each relayed link (e.g., exhibiting different LOS factors).

Proposition 1: For ρ large enough, the outage probability of the AF scheme at the BS node is

$$\Pr[I_{AF}(\rho) < \bar{R}] \approx$$
$$\approx \frac{\Gamma(d_{sd}+1)\prod_{i=1}^{N}\mathcal{F}_i}{c_{sd}^{d_{sd}} \cdot \Gamma(d_{AF}+1)}\left(\frac{2^{(N+1)\bar{R}}-1}{\mathcal{A}_c\rho}\right)^{d_{AF}}. \quad (8)$$

The cooperative diversity

$$d_{AF} = \lim_{\rho\to\infty} \frac{-\log\Pr[I_{AF}(\rho) < \bar{R}]}{\log\rho} =$$
$$= \sum_{i=1}^{N}\min\{d_{sr}, d_{rd,i}\} + d_{sd} \quad (9)$$

is a function of the number of RSs N and of the inherent diversities d_{sr}, $d_{rd,i}$ that are provided by the links involved in the collaborative transmission. Factors $\mathcal{F}_i$ are

$$\mathcal{F}_i = \frac{u(d_{rd,i}-d_{sr})}{c_{sr}^{d_{sr}} \cdot \Gamma(d_{sr}+1)^{-1}} + \frac{u(d_{sr}-d_{rd,i})}{c_{rd,i}^{d_{rd,i}} \cdot \Gamma(d_{rd,i}+1)^{-1}}, \quad (10)$$

and $u(x)$ is the Heaviside step function ($u(x) = 1$ if $x \geq 0$ and $u(x) = 0$ elsewhere).

Proof: see Appendix VII-C. ∎

Large SNR outage probability approximation for Rice (or Nakagami-m) fading with different LOS factors K (or fading figures m) for each cooperative link is in (8) with substitutions in Table I. The following remarks specialize outage probability (8) for Rice and Nakagami-m fading environments.

Remark 1: Outage probability for Rice fading is

$$\Pr[I_{AF}(\rho) < \bar{R}] \approx \frac{K_{sd}+1}{\exp(K_{sd})g_{sd}(N+1)!} \times$$
$$\times \prod_{i=1}^{N}\left[\frac{K_{sr}+1}{\exp(K_{sr})g_{sr}} + \frac{K_{rd,i}+1}{\exp(K_{rd,i})g_{rd,i}}\right] \times$$
$$\times \left(\frac{2^{(N+1)\bar{R}}-1}{\mathcal{A}_c\rho}\right)^{N+1}, \quad (11)$$

where K_{sd}, K_{sr} and $K_{rd,i}$ are the LOS factors corresponding to each cooperative link, while g_{sd}, g_{sr} and $g_{rd,i}$ are the average fading powers for the same links. Notice that similar results were obtained in [11].

Remark 2: Outage probability for Nakagami-m is

$$\Pr[I_{AF}(\rho) < \bar{R}] \approx$$
$$\approx \left(\frac{m_{sd}}{g_{sd}}\right)^{m_{sd}} \frac{1}{\Gamma(N\min\{m_{sr}, m_{rd}\}+m_{sd}+1)} \times$$
$$\times \prod_{i=1}^{N}\left[\begin{array}{c}\left(\frac{m_{rd,i}}{g_{rd,i}}\right)^{m_{rd,i}}u(m_{sr}-m_{rd,i})+ \\ +\left(\frac{m_{sr}}{g_{sr}}\right)^{m_{sr}}u(m_{rd,i}-m_{sr})\end{array}\right] \times$$
$$\times \left(\frac{2^{(N+1)\bar{R}}-1}{\mathcal{A}_c\rho}\right)^{N\min\{m_{sr},m_{rd,i}\}+m_{sd}} . \quad (12)$$

where m_{sd}, m_{sr} and $m_{rd,i}$ are the Nakagami-m factors corresponding to each cooperative link, while g_{sd}, g_{sr} and $g_{rd,i}$ are the average fading powers for the same links.

B. Cooperative fading regions and diversity-multiplexing trade-offs

Diversity and multiplexing tradeoff for multiple antenna transmission arises when the spatial redundancy (diversity) that can be provided to the data stream can be traded for higher multiplexing gain. Gain r can be obtained when the transmission rate $\bar{R}$ increases with the SNR [21] as $\bar{R} = \bar{R}(\rho) = r \times \log(1 + \rho)$ and [20]

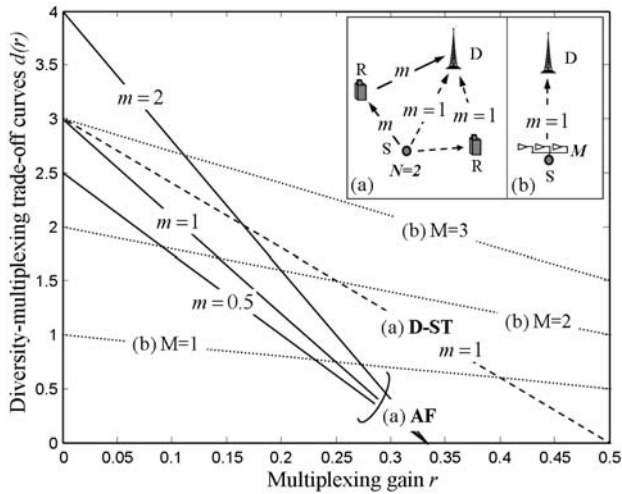$$r = \lim_{\rho\to\infty} \frac{\bar{R}(\rho)}{\log(\rho)} \quad (13)$$

Fig. 2. Diversity-multiplexing trade-off curves for non-cooperative ST coded transmission with $M = 2, 3$ antennas (and single antenna transmission $M = 1$) (dotted lines), AF (solid lines) and Distributed ST coding (D-ST) [21] (dashed lines) with $N = 2$ relays. For AF only *one* source-relay-destination path is impaired by Nakagami-m fading with (same) factor m ranging from $m = 0.5$ to $m = 2$, other links are impaired by Rayleigh fading.

Diversity-multiplexing trade-off curve $d(r)$ for AF transmission reveals the available (cooperative) diversity versus the achievable multiplexing gain r ($r < 1$ for MISO transmission, as done in [20] and [21]):

$$d(r) = \lim_{\rho \to \infty} \frac{-\log \Pr[I_{AF}(\rho) < \bar{R}(\rho)]}{\log \rho} = \\ = d_{AF} \times [1 - r(N+1)] \quad (14)$$

The proof is trivial as it follows from Proposition 1 and it is based on asymptotic outage analysis. Achievable tradeoff for non-cooperative direct transmission is $d(r) = Md_{sd}(1-r)$ (see Appendix VII-B for further details).

For a given multiplexing gain r, trade-off curve (14) shows that for $\{d_{sr}, d_{rd,1}, ..., d_{rd,N}, d_{sd}, M\} \in \mathcal{R}^\infty(r)$, with

$$\mathcal{R}^\infty(r) = \{d_{AF} \times [1 - (N+1)r] > M \cdot d_{sd}[1-r]\} \quad (15)$$

being the (asymptotic) *cooperative fading region* [6] the cooperation that uses AF protocol is beneficial in providing higher *diversity* compared to non-cooperative M-antenna ST coded transmission by constraining the *same* multiplexing gain r for both schemes. As a numerical example, in Fig 2 we compare the trade-off curves $d(r)$ for AF transmission, ST coded non-cooperative transmission with $M = 2, 3$ antennas (single antenna case $M = 1$ is also included) and Distributed ST coding protocol (D-ST) that uses decode and forward relays [21]. Cooperative transmissions exploit $N = 2$ relays. For AF transmission, *one* source-relay-destination path is impaired by Nakagami-m fading [8] with factor m that might range between $m = 0.5$ (one sided Gaussian fading) and $m = 2$, so that $d_{AF} = m + 2$ (see substitutions in Table I and (12)). Decode and forward relaying (for D-ST protocol) and non-cooperative transmission are employed over Rayleigh fading channels ($d_{sd} = d_{sr} = d_{rd} = 1$). The most promising propagation settings that can motivate the use of cooperative transmissions with non-regenerative relaying (e.g., that satisfy condition (15) and thus provides larger diversity for some $r < 1$) primarily arise when the Nakagami factor m for

the relayed links is larger than 1 (so that milder fading is experienced compared to direct source-to-destination link). Moderate fading (that is modeled by larger Nakagami factor m) provides larger diversity that can be traded for higher multiplexing gain (thus higher spectral efficiencies). Although AF protocol offers the same full diversity as for D-ST protocol (for $r \to 0$), to support higher multiplexing gain $r > 0$ (for same achievable diversity as for D-ST) AF requires at least one source-relay-destination path to be impaired by milder fading ($m > 1$) compared to Rayleigh. This is necessary to counterbalance the inefficiencies that are caused by the repetition based coding and by the signalling overhead (compared to non-cooperative transmission). Although these results are only valid at asymptotically large SNR they can be useful in providing additional insight to the problem at hand.

In the following Sections the problem is tackled now by constraining a fixed amount of power for transmission, a spectral efficiency and a reliability requirement at the receiver: design is thus focused on evaluating the conditions for which cooperative and non-cooperative transmissions exhibits the same diversity *and* coding gain.

IV. DESIGN RULES FOR RELAYS-TO-BS LINKS

Motivated by diversity/multiplexing analysis (see Fig. 2 and (15)) that reveal settings (in terms of fading statistics) where cooperative diversity provided by AF protocol is larger than that for non-cooperative transmission, the focus is now on the RS deployment design problem. To simplify the reasoning, the fading impairments for the MS-to-BS and MS-to-RSs links are Rayleigh distributed to model relevant propagation scenarios in cellular systems where mobile MS nodes are surrounded by local scattering (and, for example, the LOS component is typically not available). Outage performances of AF protocol are in (8) with substitutions $d_{sd} = d_{sr} = 1$, $c_{sd} = \mathcal{A}g_{sd}$ and $c_{sr} = \mathcal{A}g_{sr}$ with $g_{sr} = E[|h_{sr}|^2]$ (as for Rayleigh fading). The focus here is to derive closed form design rules on the required channel statistic (or equivalently, the required diversity (6) d_{rd} and coding gain pairs c_{rd} (7)) of the RSs-to-BS links (here assumed to have the same statistics as easily adopted from the general relationship (8)) that provides more advantageous outage performances for cooperative system than direct multiantenna transmission.

To guarantee a fair comparison between the cooperative and non-cooperative scheme, we constrain for both schemes the same amount of transmit energy $E_S = \bar{\rho}T$ (or power $\bar{\rho}$) and a specified outage probability at the destination $\bar{P}_{out}$ to guarantee the end-to-end rate (outage capacity for $\mathcal{A} = 1$) $\bar{R}$. For the multiantenna case, the available power $\bar{\rho}$ is equally split among the transmit antennas, while for amplify and forward based cooperative transmission (Fig. 1-(a)) the sum of transmit energy consumptions of each MS and RS nodes does not exceed the maximum budget E_S. This is simply done by allowing each RS and MS node to use the channel for the same time fraction $1/(N+1)$ with a transmit power of $\bar{\rho}$, transmit energy consumption of each node is thus $E_S/(N+1)$. Notice that power consumption due to amplification circuitry at the RS stations is herein neglected.

In what follows the design rules for the RS-to-BS link are derived for varying transmission rates $\bar{R}$, sum transmit power $\bar{\rho}$ (or energy budget), outage requirements $\bar{P}_{out}$ and

number of relays N and antennas M. Statistics of relayed links are constrained so that AF transmission can perform as non-cooperative transmission for the same end-to-end rate and power consumption:

$$\Pr[I_{AF}(\bar{\rho}) < \bar{R}] \leq \Pr[I_{sd}(\bar{\rho}) < \bar{R}] = \bar{P}_{out}. \quad (16)$$

Proposition 2: For any $N > 1$ and $M > 1$ pairs and transmission requirements $(\bar{\rho}, \bar{R}, \bar{P}_{out})$, where $\bar{\rho}$ is found to satisfy equation $\Pr[I_{sd}(\bar{\rho}) < \bar{R}] = \bar{P}_{out}$

$$\bar{\rho} \approx \frac{\left(2^{\bar{R}} - 1\right) M}{\mathcal{A} g_{sd} \cdot \left(M! \bar{P}_{out}\right)^{1/M}}, \quad (17)$$

the following condition

$$\prod_{i=1}^{N} \left[\frac{\mathcal{A}}{\mathcal{A}_c} \frac{u(d_{rd,i} - 1)}{g_{sr}} + \frac{\mathcal{A} \cdot u(1 - d_{rd,i})}{(\mathcal{A}_c c_{rd,i})^{d_{rd,i}} \cdot \Gamma(d_{rd,i} + 1)^{-1}} \right] \leq$$
$$\leq \frac{M^N \cdot \mathcal{C}(N, M)}{2^{N(N+1)\bar{R}} \cdot g_{sd}^N \cdot \bar{P}_{out}^{\frac{N+1-M}{M}}}, \quad (18)$$

with $\mathcal{C}(N, M) = \left[(N+1)!/(M)!^{(N+1)/M} \right]$ guarantees that (16) is satisfied at large SNR $\bar{\rho}$.

If condition (18) is met with strict equality then outage performances of the cooperative transmission system that uses a single antenna MS and N single antenna RSs can outperform those from M antenna ST coded direct transmission in Rayleigh fading (for the same power consumption).

Proof: Condition requires that $\Pr[I_{AF}(\bar{\rho}) < \bar{R}] < \bar{P}_{out}$ and $\bar{\rho}$ is in (17). It follows (18) by using outage derivation in (8) and substitutions $d_{sd}, d_{sr} = 1$, $c_{sd} = \mathcal{A} g_{sd}$ and $c_{sr} = \mathcal{A} g_{sr}$ (for MS-to-RSs and MS-to-BS impaired by Rayleigh fading). ∎

Notice that requirements (18) for the RS-to-BS link become more stringent in any case for an increasing end-to-end transmission rate $\bar{R}$.

The following corollary provides scaling laws for condition (18) at large end-to-end rates $\bar{R}$ (large bandwidth efficiencies).

Corollary 1: Under the following constraints: i) $N = M - 1$ (i.e., so that the number of transmitters $N + 1$ (including the MS node) equals the number of antennas M for the non-cooperative case); ii) $\forall i \ d_{rd,i} = d_{rd}$, $c_{rd,i} = c_{rd}$ (so that all RS-to-BS links have the same diversity and coding pair), then necessary condition (regardless of the outage requirement $\bar{P}_{out}$) for (18) to hold for large enough transmission rates $\bar{R} \gg 1$ is

$$\{G_{sr} \in \mathcal{O}\left(2^{M\bar{R}}\right) \wedge \log_2(c_{rd}) \in \mathcal{O}\left(\frac{M\bar{R}}{d_{rd}}\right)\} \text{ iff } d_{rd} \leq 1 \quad (19)$$

$$\{G_{sr} \in \mathcal{O}\left(2^{M\bar{R}}\right)\} \text{ iff } d_{rd} > 1 \quad (20)$$

$G_{sr} = g_{sr}/g_{sd}$ refers to the average path loss gain of the MS-to-RS link as compared to the path loss for direct MS-to-BS link.

In case diversity provided by RS-to-BS links is larger than that for Rayleigh case ($d_{rd} > 1$) the only requirement is on $G_{sr} \in \mathcal{O}(2^{M\bar{R}})$. Instead, if $d_{rd} \leq 1$ coding gain (c_{rd}) provided by the fading that impairs the RS-to-BS links should satisfy the condition $\log_2(c_{rd}) \in \mathcal{O}(M\bar{R}/d_{rd})$. The smaller the diversity d_{rd} is the larger the coding gain c_{rd}, as expected.
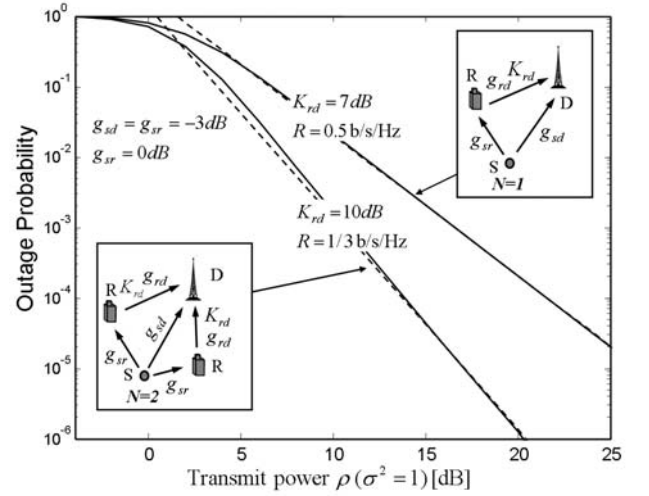

Fig. 3. Outage probability (solid lines) for AF system versus the transmit power ρ. Results are shown for varying LOS factors K_{rd} ($K_{rd} = 7, 10dB$) and number of collaborating relays N ($N = 1, 2$). Average channel powers are $g_{sr} = g_{sd} = -3dB$ and $g_{rd} = 0dB$. Dashed lines refer to outage approximation for large SNR.

A. Design criteria for Rice faded relayed links

The design rules (18) are specialized here assuming Rice fading for the RSs-to-BS links. Rice distribution is described in terms of the fading parameter K that accounts for the ratio of the power of the LOS component to the power in the diffusive (NLOS) components. The random fading power $|h|^2$ with $\mathrm{E}\left[|h|^2\right] = g$ has density $f_{|h|^2}(w; K)$ in [2] and MGF $\mathcal{F}_{|h|^2}(w; K)$ in Table I. In outdoor cellular environments it is shown that the LOS factor K that is experienced in (e.g., fixed) communications exhibits large variability with respect to the transmitter position. Moreover, for a given relay location, the Rice factor is shown to vary with the operational transmitting bandwidth by following a distribution that can be approximated by a lognormal density with average value being a simple function of deployment parameters such as antenna height, beamwidth and distance towards the receiver [22].

Relayed links exhibit a fading with LOS factor $K_{rd,i}$ and average power $\mathrm{E}[|h_{rd,i}|^2] = g_{rd,i}$ that are related to the diversities $d_{rd,i}$ and coding gains $c_{rd,i}$ as in Table I. Deployment rules are given in terms of the LOS factors by using outage probability (11) (extension of this work to Nakagami-m [2] is straightforward according to (12)). Notice that the proposed design is based on the estimation of the LOS parameter and of the average channel power at time of network deployment for each link (the reader should refer to [23] for further details on how to compute such estimates).

In Fig. 3 we compare the large SNR outage probability approximation (11) (dashed lines) of AF system with the simulated one (solid lines) for varying transmission settings and propagation environments: complete matching between the simulated and the analytical outage curves is met when the ratio $\rho/(2^{(N+1)\bar{R}} - 1)$ is above $8dB$ (or the outage falls below 10^{-2}).

Remark 4 (to Proposition 2) The condition (18) simplifies

for Rice fading as

$$\prod_{i=1}^{N} \left(\frac{K_{rd,i}+1}{G_{rd,i}\exp(K_{rd,i})} + \frac{1}{G_{sr}} \right) \leq$$
$$\leq \frac{M^N \mathcal{C}(N,M)}{2^{(N+1)N\bar{R}} \cdot \bar{P}_{out}^{\frac{N+1-M}{M}}} \times \frac{\mathcal{A}_c}{\mathcal{A}}, \quad (21)$$

with $G_{rd,i} = g_{rd,i}/g_{sd}$. Notice that $\mathcal{A}_c/\mathcal{A} \leq 1$ and it can be regarded as a penalty factor for condition (21); equality holds for i.i.d. Gaussian input symbol distribution.

Condition (21) gives the required set of LOS factors $K_{rd,i}$ for each (RS-to-BS) relayed link and channel powers ratios $G_{rd,i}$ and G_{sr} to guarantee that AF transmission with N RSs outperforms (or has same diversity performances if condition (21) is met with equality) M antenna ST coded non-cooperative transmission from the MS node.

Remark 5 (to Corollary 3) Conditions (19) and (20) for Rice fading ($d_{rd} = 1$) and $K_{rd,i} = K_{rd}$, $G_{rd,i} = G_{rd}$ $\forall i = 1,...,N$ reduce for large enough transmission rates $\bar{R}$ and $\bar{\rho}$ to

$$\left\{ G_{sr} \in \mathcal{O}(2^{M\bar{R}}) \right\} \wedge \left\{ K_{rd} \in \mathcal{O}(M\bar{R}) \vee G_{rd} \in \mathcal{O}(2^{M\bar{R}}) \right\}. \quad (22)$$

It is required from (22) that either the LOS factor K_{rd} is $\mathcal{O}(M\bar{R})$, or the average channel power ratio $G_{rd} = g_{rd}/g_{sd}$ is $\mathcal{O}(2^{M\bar{R}})$. Notice that a stringent requirement (but still reasonable as far as the MS is in short range with the chosen RS [4]) is needed for the ratio $G_{sr} = g_{sr}/g_{sd}$ as $G_{sr} = \mathcal{O}(2^{M\bar{R}})$ and it holds regardless of the RS-to-BS link statistic.

V. COMPARATIVE ANALYSIS OF NON-COOPERATIVE AND COOPERATIVE NETWORKS FOR RICE FADED LINKS

In this section, the design rules for deployment are numerically evaluated and analyzed for Rice fading. The reference power $\bar{\rho}$ in (17) is computed from the end-to-end (MS-to-BS) outage requirement $\bar{P}_{out} = 10^{-6}$ and for various rates $\bar{R}$ b/s/Hz (Gaussian codebook is assumed so that $\mathcal{A} = \mathcal{A}_c = 1$). As a practical assumption, the LOS component for the RS-to-BS link falls within the range $[-35dB, 10dB]$ according to [2] and [24]. Performance benefits of collaborative AF transmissions compared to direct multi-antenna transmission are studied in terms of the LOSs K and of the path loss ratios G ($G_{sr} = g_{sr}/g_{sd}$, $G_{rd} = g_{rd}/g_{sd}$) that measure the relative gain in average fading power of collaborative links (MS-to-RS and RS-to-BS) compared to the loss for the direct link. This is done to develop a generic framework for analysis that is not limited to a particular path loss model (e.g., see Hata-Okumura model [25]). The values for ratios G are typically ruled by transmitter-receiver distances, shadow fading and RS antenna height[4]. The focus here is thus to analyze the impact of the LOS factor and of the relative path loss gains G on AF transmission performances.

Figure 4 illustrates the condition (21) where relayed links are impaired by Rice fading, lines refer to the boundaries of region (21) (where link parameters are solution to (21) with strict equality). Performances of cooperative AF transmission

[4]As an example for Hata-Okumura model, assuming a RS with height h_{RS}, the path loss ratio g_{rd} experiences $3 \times h_{RS}$ [dB] gain [25] compared to the direct link g_{sd} (MS is at ground level, $h_{MS} = 0$), so that $G_{rd} \propto h_{RS}$.
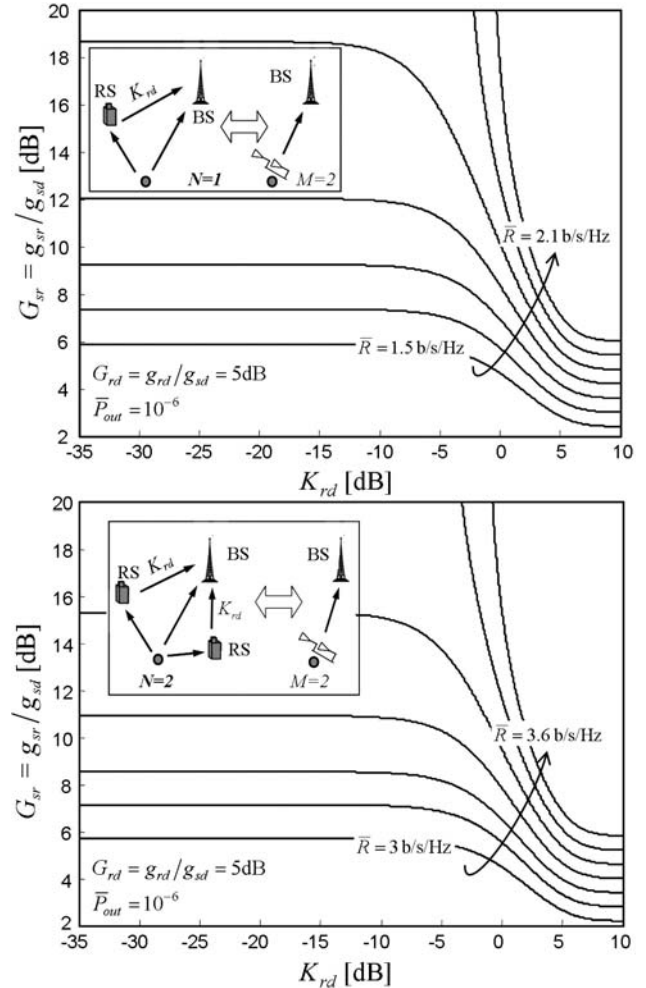

Fig. 4. Required Rice factor K_{rd} for the relay-to-BS links and channel power ratio G_{sr} (for $G_{rd} = 5dB$) for $N = 1$ (on top) and for $N = 2$ (at the bottom) amplify and foward RSs to have a diversity order of $M = 2$ (e.g., as for Alamouti ST coding) for varying rate requirements $\bar{R}$ of the end-to-end link. $\bar{R}$ is ranging from $\bar{R} = 1.5$ to $\bar{R} = 2.1$ b/s/Hz (on top) and from $\bar{R} = 3$ to $\bar{R} = 3.6$ b/s/Hz (at the bottom) with step 0.1 b/s/Hz. $\bar{P}_{out} = 10^{-6}$.

that takes advantage of one single ($N = 1$) amplify and forward RS (on the top) or two RSs (at the bottom) are compared with the same Alamouti ST coding ($M = 2$) transmission for various rate requirements $\bar{R}$ for the end-to-end link.

As an example, assume the required transmission rate $\bar{R}$ to be 1.5 b/s/Hz (with outage $\bar{P}_{out} = 10^{-6}$), on the top of Fig. 4 it is shown that if the MS node can find an RS station for which $G_{sr} > 5dB$ and $K_{rd} > 0dB$ ($G_{rd} = 5dB$) then transmission can reach the BS with the help of the relay fulfilling at the same time the rate, the outage and the power $\bar{\rho}$ (17) requirements, and thus having similar performances (and similar diversity order) as if the MS would be equipped with $M = 2$ antennas employing Alamouti ST coding. Instead, a RS station with higher LOS $K_{rd} > 5dB$ ($G_{rd} = 5dB$) would substantially ease the constraint on G_{sr} (now $G_{sr} > 2dB$) for the same rate requirement $\bar{R} = 1.5$ b/s/Hz, thus simplifying the relay selection phase. When the required transmission rate increases from 1.5 b/s/Hz to 2.1 b/s/Hz (Fig. 4, on the top), link requirements $\bar{\rho}$ (17) and $\bar{P}_{out}$ (to have comparable performances to Alamouti ST coding) can be met only if the source node can find a RS station with higher G_{sr} or
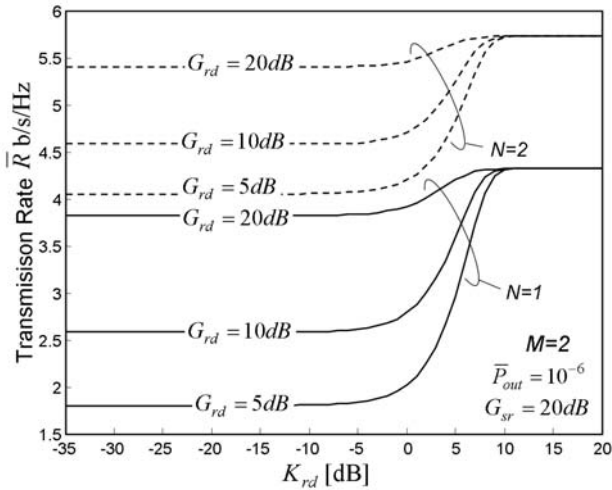
Fig. 5. Maximum supported transmission rate $\bar{R}$ for varying LOS factor K_{rd} of the relayed link for a cooperative AF system with $N = 1$ RS (solid lines) and $N = 2$ RSs (dashed lines). $G_{sr} = 20dB$ while G_{rd} ranges between $5dB$ to $20dB$. Both AF systems have same outage performances and overall energy consumption as that for reference Alamout ST coded transmission.

Fig. 6. Minimum LOS factor K_{rd} to satisfy condition (22) with strict equality (solid lines) versus the transmission rate $\bar{R}$ for $G_{rd} = 0dB$ and $G_{rd} = 7dB$ and varying diversity requirements (for $M = 2, 3, 4$) and for $G_{rd} = 0dB$ and $G_{rd} = 7dB$. Dashed lines show the asymptotic (for $\bar{R} \to \infty$) lowerbound $M\bar{R}\ln(2)$ for fading factor K_{rd}.

higher K_{rd} (thus requiring more careful relay deployment[5], as shown also in Remark 3). Higher transmission rates (higher bandwidth efficiencies) require more careful relay deployment so that (e.g., for Rice fading) the larger LOS can balance the inefficiency of repetition based AF transmission, this result will be further analyzed in Sect. V-A.

In Fig. 4 (at the bottom) we evaluate the same conditions on the RS-to-BS link fading statistics that make a cooperative AF system now with $N = 2$ RSs to perform as if the MS node would employ Alamouti ST coding (still $M = 2$). By allowing a second RS to collaborate with the MS, the cooperative system can support higher transmission rates. As an example, for the same fading requirements $G_{sr} > 5dB$ and $K_{rd} > 0dB$ ($G_{rd} = 5dB$), the cooperative system now can support $\bar{R} = 3$ b/s/Hz (that is 2 times higher compared to $\bar{R} = 1.5$ b/s/Hz on top of Fig. 4) and it still performs as if the MS would employ Alamouti ST coded transmission towards the BS. The performance loss, compared to ST coding, that is experienced by constraining the RSs stations to simply repeat the amplified signal without adding any coding gain is mitigated by increasing the provided cooperative diversity (9) of the AF scheme.

To get more insight into the interplay between the fading parameters for the relayed RS-to-BS links (K_{rd}, G_{rd}) and the number of collaborating RSs N, we now fix the ratio G_{sr} to a large value ($G_{sr} = 20dB$: in most settings this means that the MS needs to find an in-range RS[6]). In Fig. 5 we evaluate the maximum transmission rate $\bar{R}$ that can be supported by the AF system with $N = 1$ RS (solid lines) and $N = 2$ RSs (dashed lines). Achievable rates are plotted versus the LOS factor K_{rd} of the relayed link and for different G_{rd} ratios, outage requirement is $\bar{P}_{out} = 10^{-6}$ (power $\bar{\rho}$ in (17)) while reference non-cooperative transmission is still Alamouti

[5]A large Rice factor for the relayed link would tipically require transmit antenna to be positioned higher than the surrounding scattering and/or to exhibit strong directivity (small enough beamwidth).

[6]This turns out to be true when the path loss exponentially decays with transmitter-receiver distance and shadowing impact is negligible. Notice that to have G_{sr} large enough it might be sufficient that the direct MS-to-BS is in deep fade.
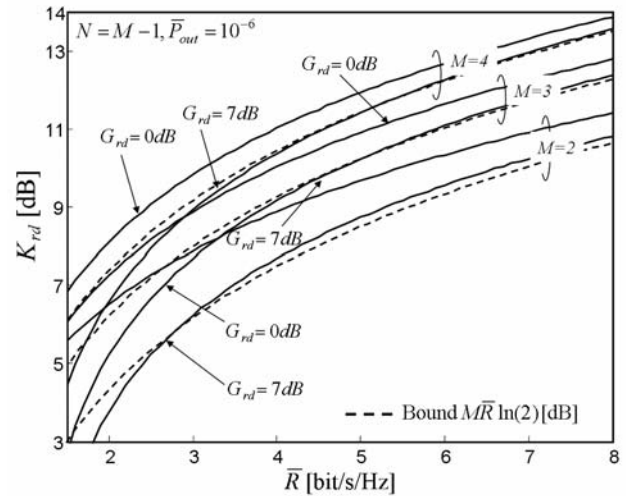
ST coding ($M = 2$). It is now interesting to analyze the interaction between the benefits that are provided by adding a RS as repeater for the MS information symbols ($N = 2$ case) and those that can be achieved by increasing the LOS factor of the single (e.g., fixed) RS link ($N = 1$). As an example, in Fig. 5 we show that an AF system with one single RS that benefits from a link with large Rice factor ($K_{rd} > 8dB$) and large G_{rd} ($G_{rd} > 10dB$) can support the same rate as if the system would allow $N = 2$ RSs that experience Rayleigh fading $K_{rd} \simeq 0$ and $G_{rd} = 5dB$. Proper design of RS location can therefore make the cooperative systems to perform *as if* it would benefit from an increased diversity order (e.g., for large number of collaborating RSs N).

A. Large transmission rate analysis

The necessary conditions from Corollary 3 (see Remark 3) that are valid for large enough transmission rate $\bar{R}$ and for $N = M - 1$ (to have the same number of transmitting elements) are evaluated herein for varying network settings. Impact of the RS-to-BS statistics on performances are analyzed in Figs. 6 and 7 where it is set $G_{sr} = 2^{M\bar{R}}$ to satisfy the condition $G_{sr} = \mathcal{O}(2^{M\bar{R}})$ (22).

In Fig. 6 we show the minimum required LOS factor K_{rd} that satisfy condition (21) with strict equality (solid lines) versus the transmission rate $\bar{R}$ (ranging from $\bar{R} = 1.5$ to $\bar{R} = 8$ b/s/Hz). LOS factors are evaluated to guarantee that the cooperative AF system with N relays performs as if the MS node is equipped with $M = N + 1$ antennas and employ ST coded transmission directly to the BS. Results are shown for various diversity requirements (for $M = 2, 3, 4$) and for $G_{rd} = 0dB$ and $G_{rd} = 7dB$. In accordance with Remark 3 that prescribes $K_{rd} \in \mathcal{O}(M\bar{R})$, the bounding curve $M\bar{R} \times \ln(2)$ in dB scale (dashed lines) shows that is tight in predicting the required LOS factor at least for $G_{rd} > 0dB$ and $\bar{R} > 4$ b/s/Hz. For large enough $\bar{R}$ and power $\bar{\rho}$, LOS factor K_{rd} should therefore scale *linearly* with the product of the required diversity M and the transmission rate $\bar{R}$.
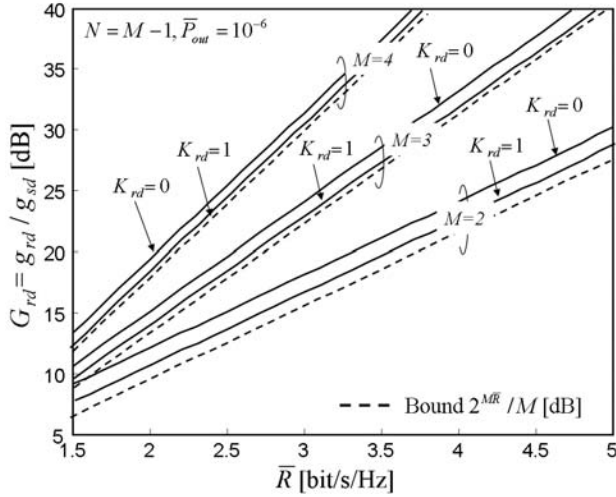
Fig. 7. Minimum average channel power ratio G_{rd} (solid lines) to satisfy condition (22) with strict equality versus the transmission rate $\bar{R}$ for varying diversity requirements (for $M = 2, 3, 4$) and for $K_{rd} = 0$ (Rayleigh faded RS-to-BS link) and $K_{rd} = 1$. Dashed lines show the lowerbound to the ratio $G_{rd} = 2^{M\bar{R}}/M$.

Figure 7 analyze the condition on the average channel power ratio G_{rd}, for which Remark 3 prescribes $G_{rd} \in \mathcal{O}(2^{M\bar{R}})$. Solid lines refer to the required ratio G_{rd} to satisfy (21) with strict equality for $M = 2, 3, 4$, $K_{rd} = 0$ (to model Rayleigh faded RS-to-BS links) and $K_{rd} = 1$. Dashed lines refer to the function $G_{rd} = 2^{M\bar{R}}/M$ that can be used as a scaling law (for large $\bar{R}$) for the required ratio $G_{rd} \in \mathcal{O}(2^{M\bar{R}})$. In Rayleigh fading (or quasi-LOS fading), the ratio $G_{rd} = g_{rd}/g_{sd}$ should therefore scale *exponentially* with the product of the required diversity M and the transmission rate $\bar{R}$, thus revealing a more stringent requirement with respect to that for the Rice parameter. An increase in the LOS factor for the relayed link would decrease the diffusive component (e.g., the variance) experienced by the fading power distribution: this cause larger performance benefits compared to simply increasing the average channel power.

VI. CONCLUDING REMARKS

In this paper we considered a multirelay AF transmission protocol where a number of Relay Stations (RS) are serving as non-regenerative repeaters for the single antenna mobile station. Under a generic framework for propagation environment, we explicitly investigated the necessary conditions on the RSs-to-BS fading distributions that make collaborative transmission to perform as if the MS station would be equipped with multiple (uncorrelated) antennas. This is done to provide an analytical tool for cooperative system analysis that is based on a performance comparison with multiantenna non-cooperative transmission used here as reference setting. We showed how the propagation settings can influence the performances of collaborative transmission: the main conclusion from the analysis is that even if RSs are simply amplifying and forwarding the same codeword from the MS node, a careful selection of RS location (that constrains the fading distribution) can make the performances of collaborative transmission comparable (in terms of provided diversity and spectral efficiency) to those obtained from multiantenna transmissions.

Closed form conditions are written in terms of constraints on the diversity/coding gain provided by the RSs-to-BS fading

density functions: design rules do not suggest a particular optimal set of RS positions, but only give necessary specifics that need to be satisfied by candidate sites for relay deployment. Of course, to develop sufficient conditions, the impact of interference from other cells (and shadow fading impairments) should be included as well. Design rules are adapted in this paper to fit with Rice fading for the RS-to-BS link and Rayleigh (NLOS) fading for the MS-to-BS and RS links. Higher bandwidth efficiencies $\bar{R}$ (higher end-to-end rates) require more careful relay deployment so that milder fading for the RS-to-BS links compared to Rayleigh (e.g., for larger LOS or smaller path loss) can balance the inefficiency of codeword repetition from non-regenerative relays. It is found that Rice factor for relayed link should linearly scale with the product of end-to-end rate $\bar{R}$ and the diversity order M ($K_{rd} \in \mathcal{O}(M\bar{R})$): the larger is the required bandwidth efficiency ($\bar{R}$) and the reliability of the data stream (the diversity M), the larger should be the LOS factor of the relayed links. On the contrary, when same links are Rayleigh faded, the average fading power ratio g_{rd}/g_{sd} shall scale *exponentially* with the product $M\bar{R}$ ($G_{rd} \in \mathcal{O}(2^{M\bar{R}})$), thus reveling more stringent condition (as also noticed by [3]) compared to that valid for Rice faded links. Requirements on fading statistics can be made less stringent by allowing an higher number of collaborating terminals $N > M - 1$ to provide larger diversity.

VII. APPENDIX

A. Diversity and coding gain for arbitrarily distributed fading - MGF based approach

In this section we show that the diversity and coding gain parameters that rule the outage performances of single antenna systems at high SNR can be found from the asymptotic behavior of the MGF of the (arbitrary) fading power, the reader might refer to [6] for more detailed derivations and proofs.

Outage performances in fading channels $\Pr[I(\rho) < \bar{R}] = \int_0^{(2^{\bar{R}}-1)/\mathcal{A}\rho} f_{|h|^2}(w)dw$ are primarily limited by the (outage) events that cause the SNR $\mu = \rho|h|^2$ to be small with probability that depends on the terms $f_{|h|^2}(w)|_{w \to 0^+}$. The probability density function $f_{|h|^2}(w)$ can be written through the integral expansion [26]:

$$f_{|h|^2}(w) = \int_0^\infty \Gamma(t+1)^{-1}\mathcal{D}^t\left[f_{|h|^2}(0)\right]w^t dt, \quad (23)$$

where w^t are the basis functions and $\Gamma(x) = \int_0^\infty y^{x-1}\exp(-y)dy$ is the complete Gamma function. The t-th order fractional derivative [26] of $f_{|h|^2}(w)$ in $w \to 0^+$ is $\mathcal{D}^t\left[f_{|h|^2}(0)\right]$. From (23) the probability density function can be written for w small enough as

$$f_{|h|^2}(w) = \Gamma(t^*+1)^{-1}\mathcal{D}^{t^*}\left[f_{|h|^2}(0)\right]w^{t^*} + o(w^{t^*}), \quad (24)$$

where t^* is the order of the first non-zero fractional derivative of $f_{|h|^2}(w)$ in $w \to 0^+$ that satisfies $\lim_{\varepsilon \to 0^+}\int_0^{t^*-\varepsilon}\Gamma(t+1)^{-1}\mathcal{D}^t\left[f_{|h|^2}(0)\right]w^t dt = 0$.

The outage probability is a function of t^* and scales with the power ρ as

$$\Pr[I(\rho) < \bar{R}] \approx \frac{\mathcal{D}^{t^*}\left[f_{|h|^2}(0)\right]}{\Gamma(t^*+2)}\left(\frac{2^{\bar{R}}-1}{\mathcal{A}\rho}\right)^{t^*+1} = \left(\frac{2^{\bar{R}}-1}{c\rho \times \mathcal{A}}\right)^d,$$
$$(25)$$

with diversity $d = t^* + 1$ and coding gain $c = \left(\mathcal{D}^{t^*}\left[f_{|h|^2}(0)\right]/\Gamma(t^*+2)\right)^{-1/(t^*+1)}$. We used the notation $\approx$ to indicate that the equality holds for asymptotically high power ρ.

For right-continuous distributions that can be adjusted to fit empirical measurements, the following proposition can be used for evaluating the order t^* of the first non-zero fractional derivative.

Proposition 3: A *necessary and sufficient* condition for calculating t^* is

$$\phi = \lim_{s\to\infty} s^{t^*+1} F_{|h|^2}(s) > 0 \qquad (26)$$

and finite.

Proof: Necessary condition comes from the fractional derivative definition and the initial value theorem [27]. Proving that the condition (6) is sufficient is trivial as

$$\forall \epsilon > 0 \text{ it is } \mathcal{D}^{t^*-\epsilon}\left[f_{|h|^2}(0)\right] = \lim_{s\to\infty} s^{t^*-\epsilon+1} \cdot F_{|h|^2}(s) = 0,$$
$$(27)$$

therefore t^* is the smallest fractional derivative order that satisfies (26) and it is unique. ∎

Remark 6: The value of t^* satisfies (26) iff

$$d = t^* + 1 = \lim_{s\to\infty}\left[\frac{-\log \mathcal{F}_{|h|^2}(s)}{\log s}\right] > 0 \qquad (28)$$

and finite. Proof is trivial.

Diversity d is thus related to the order t^* of the first non-zero fractional derivative.

The coding gain from (25) and (28) is $c \triangleq (\phi/\Gamma(d+1))^{-1/d}$ and ϕ is in (26). Extended results for multi-antenna transmission that include cases with unbalanced/heterogeneous fading can be found in [6].

B. Outage and diversity-multiplexing trade-offs for MISO transmission in fading channels

Here the outage performances of multiantenna ST coded transmission are extended for a channel that is impaired by a fading with an arbitrary power distribution, say $|h_{sd}|^2 \sim f_{|h_{sd}|^2}(\cdot)$ (and MGF $\mathcal{F}_{|h_{sd}|^2}(s)$). The outage probability at high SNR for a ST coded M antenna transmission can be written as a function of the diversity d_{sd} (6) and the coding gain c_{sd} (7) provided by the MS-to-BS channel h_{sd}:

$$\Pr[I_{sd}(\rho) < \bar{R}] \approx \frac{\Gamma(d_{sd}+1)^M}{\Gamma(Md_{sd}+1)}\left(\frac{\left(2^{\bar{R}}-1\right)M}{c_{sd}\rho\mathcal{A}}\right)^{Md_{sd}}.$$
$$(29)$$

Assuming uncorrelated fading over each antenna, the diversity-multiplexing tradeoff curve is

$$\lim_{\rho\to\infty}\frac{-\log\left[\Pr(I_{sd}(\rho) < \bar{R}(\rho))\right]}{\log(\rho)} = M \cdot d_{sd}(1-r), \quad (30)$$

Diversity scales with the number of antennas at the MS node M (as ST coding is used) and with the term d_{sd} that depends on the fading statistic, term $(1-r)$ reveal the tradeoff between spatial redundancy and multiplexing gain r (or normalized spectral efficiency). Notice that for Rayleigh fading $d_{sd} = 1$ and $c_{sd} = \mathcal{A}g_{sd}$ with $g_{sd} = \mathrm{E}[|h_{sd}|^2]$, as expected. Using mappings in Table I the parameters for outage probability under Rice and Nakagami-m (for this case see also [28]) fading are easily obtained.

C. Proposition 1

Proof: Let us define $\varsigma = \left(2^{(N+1)\bar{R}} - 1\right)$, the outage probability at high SNR is conveniently approximated by

$$\Pr[I_{AF} < \bar{R}] = \int_0^{\varsigma/\mathcal{A}_c} f_{\gamma_{AF}}(w)dw \approx \left[\frac{\varsigma}{c_{AF}\cdot\mathcal{A}_c\rho}\right]^{d_{AF}}$$
$$(31)$$

We now derive the factors d_{AF} and c_{AF}: for ρ large enough the amplification factor β_i (2) reduces to $|h_{sr,i}|^{-1}$, function $g(\gamma_{sr,i},\gamma_{rd,i}) \approx \rho\tilde{g}_i(|h_{sr,i}|^2, |h_{rd,i}|^2)$ [11] with $\tilde{g}_i(x,y) = xy/(x+y) \sim f_{\tilde{g}_i}(w)$, $x \sim f_{|h_{sr}|^2}(w)$ and $y \sim f_{|h_{rd}|^2}(w)$.

By letting

$$\gamma_{AF} \approx \rho \cdot \tilde{\gamma}_{AF} =$$
$$= \rho \times \left[|h_{sd}|^2 + \sum_{i=1}^{N}\tilde{g}_i(|h_{sr,i}|^2, |h_{rd,i}|^2)\right], \quad (32)$$

the outage probability can be therefore written as

$$\Pr[I_{AF} < \bar{R}] \approx \int_0^{\varsigma/\rho\mathcal{A}_c} f_{\tilde{\gamma}_{AF}}(w)dw \qquad (33)$$

Diversity and coding gain d_{AF} and c_{AF} come from the asymptotic behavior of the MGF of $\tilde{\gamma}_{AF}$: $\mathcal{F}_{\tilde{\gamma}_{AF}}(s) = \mathcal{F}_{|h_{sd}|^2}(s) \cdot \prod_{i=1}^{N}\mathcal{F}_{\tilde{g}_i}(s)$ with $\mathcal{F}_{\tilde{g}_i}(s)$ the MGF of $\tilde{g}_i(x,y) \sim f_{\tilde{g}_i}(w)$. From [6] diversity is found as

$$d_{AF} = \lim_{s\to\infty}\frac{-\log\mathcal{F}_{\tilde{\gamma}_{AF}}(s)}{\log s} = d_{sd} + \sum_{i=1}^{N}\lim_{s\to\infty}\frac{-\log\mathcal{F}_{\tilde{g}_i}(s)}{\log s}$$
$$(34)$$

and we thus need to evaluate the limit $\mu_i = \lim_{s\to\infty} -\log\mathcal{F}_{\tilde{g}_i}(s)/\log s$. From Proposition 3 and Remark 6 a necessary and sufficient condition for μ_i is

$$\lim_{s\to\infty} s^{\mu_i}\mathcal{F}_{\tilde{g}_i}(s) = \mathcal{D}^{\mu_i-1}\left[f_{\tilde{g}_i}(0)\right] > 0. \qquad (35)$$

We thus need to find *one* μ_i that satisfies (35). The $(\mu_i - 1)$-order fractional derivative $\mathcal{D}^{\mu_i-1}\left[f_{\tilde{g}_i}(0)\right]$ is

$$\phi_{srd,i}(\mu_i) = \mathcal{D}^{\mu_i-1}\left[f_{\tilde{g}_i}(0)\right] =$$
$$= \mathcal{D}^{\mu_i-1}\left[f_{|h_{sr,i}|^2}(0)\right]\int_0^\infty \frac{f_{|h_{rd,i}|^2}(y)}{\nabla_y\tilde{g}_i(0,y)}dy +$$
$$+ \mathcal{D}^{\mu_i-1}\left[f_{|h_{rd,i}|^2}(0)\right]\int_0^\infty \frac{f_{|h_{sr,i}|^2}(x)}{\nabla_x\tilde{g}_i(x,0)}dx =$$
$$= \mathcal{D}^{\mu_i-1}\left[f_{|h_{sr,i}|^2}(0)\right] + \mathcal{D}^{\mu_i-1}\left[f_{|h_{rd,i}|^2}(0)\right] =$$
$$= \lim_{s\to\infty} s^{\mu_i}\left[\mathcal{F}_{|h_{sr,i}|^2}(s) + \mathcal{F}_{|h_{rd,i}|^2}(s)\right], \quad (36)$$

where $\nabla_y\tilde{g}_i(0,y) = 1$ and $\nabla_x\tilde{g}_i(x,0) = 1$. Therefore

$$\mu_i = \lim_{s\to\infty}\frac{-\log\mathcal{F}_{\tilde{g}_i}(s)}{\log s} = \min\{d_{sr}, d_{rd,i}\}. \qquad (37)$$

and diversity d_{AF} (34) is in the main text (9). Coding gain is $c_{AF} = [\phi_{AF}/\Gamma(d_{AF}+1)]^{-1/d_{AF}}$ with

$$\phi_{AF} = \lim_{s \to \infty} s^{d_{AF}} \mathcal{F}_{\tilde{\gamma}_{AF}}(s) =$$
$$= \lim_{s \to \infty} s^{d_{sd}} \mathcal{F}_{|h_{sd}|^2}(s) \cdot \prod_{i=1}^{N} \phi_{srd,i}(\min\{d_{sr}, d_{rd,i}\}). \tag{38}$$

Since $\lim_{s \to \infty} s^d \mathcal{F}_{|h|^2}(s) = \Gamma(d+1)/c^d$ from (7), we can express ϕ_{AF} and thus c_{AF} using (36) and (37) in terms of the coding gains (and diversities) for each cooperative link as

$$\phi_{srd,i}(\min\{d_{sr}, d_{rd,i}\}) = u(d_{rd,i} - d_{sr})\frac{\Gamma(d_{sr}+1)}{c_{sr}^{d_{sr}}} +$$
$$+ u(d_{sr} - d_{rd,i})\frac{\Gamma(d_{rd,i}+1)}{c_{rd,i}^{d_{rd,i}}} \tag{39}$$

and $\lim_{s \to \infty} s^{d_{sd}} \mathcal{F}_{|h_{sd}|^2}(s) = \Gamma(d_{sd}+1)/c_{sd}^{d_{sd}}$. Outage probability (31) can now be restated as in the main text (8) after straightforward algebraic computations. ∎

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