TRANSMIT BEAMSPACE DESIGN FOR DIRECTION FINDING IN COLOCATED MIMO RADAR WITH ARBITRARY RECEIVE ARRAY AND EVEN NUMBER OF WAVEFORMS

Arash Khabbazibasmenj, Sergiy A. Vorobyov, Aboulnasr Hassanien, and Matthew W. Morency

Dept. Electrical and Computer Engineering
University of Alberta, Edmonton, AB, T6G 2V4, Canada
Emails: {khabbazi,svorobyo,hassanie,morency}@ualberta.ca

ABSTRACT

Colocated multiple-input multiple-output (MIMO) radar is used for direction-of-arrival (DOA) estimation. The case of even but otherwise arbitrary number of transmit waveforms is considered. In order to obtain a virtual array with a large number of virtual antenna elements and at the same time obtain a significant signal-to-noise ratio (SNR) gain, a proper beamspace is designed. Moreover, to allow for simple DOA estimation algorithms at the receive array, the rotational invariance property (RIP) for the virtual array is guaranteed at the transmit array by a proper beamspace design. The main idea of such beamspace design is to obtain the RIP by imposing a specific structure on the beamspace matrix and then designing the beamspace matrix to obtain a desired beampattern and a uniform power distribution across antenna elements. Simulation results demonstrate the advantages of the proposed DOA estimation method based on colocated MIMO radar with beamspace design.

Index Terms— Colocated MIMO radar, direction of arrival (DOA) estimation, transmit beamspace design

1. INTRODUCTION

Parameter estimation is a classical problem of applied statistics. In the application of array processing, the direction-of-arrival (DOA) parameter estimation problem is most fundamental [1]. Many DOA estimation techniques have been developed for the classical array processing single-input multiple output (SIMO) setup [1], [2]. The development of a novel array processing configuration that is best known as multiple-input multiple output (MIMO) radar [3] (with colocated transmit and colocated receive antenna elements) has opened new opportunities in parameter estimation.

A virtual array with a larger number of virtual antenna elements can be used for improved DOA estimation as compared to the SIMO configuration [4], [5] for relatively high signal-to-noise ratios (SNRs), i.e., when the benefits of increased virtual aperture start to show up. The SNR gain for fully MIMO radar, however, decreases as compared to the phased array radar where the transmit array transmits a single waveform coherently from all antenna elements [6], [7]. A trade-off between the phased-array and fully MIMO radar can be achieved [7], which gives the best of both configurations - the increased number of virtual antenna elements together with SNR gain due to coherent transmission.

The increased number of degrees of freedom for MIMO radar due to the possibility of transmitting multiple waveforms is used to design a transmit beampattern that is as close as possible to a desired one, such as, for example, a perfectly rectangular beampattern [3], [8]. However, one of the major motivations for designing transmit beampattern is realizing the possibility of achieving SNR gain together with increased aperture for improved DOA estimation in a wide range of SNRs [9], [10].

Remarkably, using MIMO radar with proper transmit beamspace design, it is possible to achieve and guarantee the satisfaction of such desired property for DOA estimation as rotational invariance property (RIP) at the receiver [10], [11]. This is somewhat similar in effect to the property of orthogonal space-time block codes in that the shape of the transmitted constellation does not change at the receiver independent on a channel. The latter allows for simple decoder [12]. Similarly, here the RIP allows for simple DOA estimation techniques at the receiver although the RIP is actually enforced at the transmitter, and the propagation media cannot break it thanks to the proper design of beamspace. Since the RIP holds at the receiver independent on the propagation media and receive antenna array configuration, the receive antenna array can be an arbitrary array. In [11], we have proposed such beamspace design technique for the special case of only two transmit waveforms.

In this paper, we consider the problem of beamspace design for DOA estimation in MIMO radar based on the RIP for the case of even but otherwise arbitrary number of transmit waveforms. We show that if the beamspace is designed in a way that in addition to obtaining a desired transmit beampattern and a uniform power distribution across the antenna elements, the RIP between two newly defined vectors is guar-

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anteed, and therefore simple DOA estimation techniques can be used at the receiver.

The rest of the paper is organized as follows. Section II introduces the system model for mono-static MIMO radar system with transmit beamspace. The transmit beamspace design problem for even but otherwise arbitrary number of transmit waveforms is considered in Section III. Section IV gives simulation examples for the proposed DOA estimation technique based on MIMO radar with transmit beamspace design.

2. SYSTEM MODEL

Consider a mono-static radar system with a transmit array being an M-antenna uniform linear array (ULA) and a receive array being an N-antenna array with arbitrary geometry. Let $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$ denote the steering vectors of the transmit and receive arrays, respectively. The transmit energy focusing can be achieved using the $M \times K$ transmit beamspace matrix $\mathbf{W}$, where $K \leq M$ is the number of orthonormal basis waveforms [9]. Then the $M \times 1$ vector of transmitted signals can be expressed as

$$\mathbf{s}(t) = \mathbf{W}_{M\times K}\boldsymbol{\phi}_{K\times 1}(t), \quad 0 \leq t \leq T \tag{1}$$

where $\boldsymbol{\phi}(t) = [\phi_1(t), \ldots, \phi_K(t)]$ is the set of orthonormal basis waveforms such that $\int_0^T \boldsymbol{\phi}(t)\boldsymbol{\phi}^H(t) = \mathbf{I}_K$, $(\cdot)^H$ stands for the Hermitian transpose, $\mathbf{I}_K$ is the identity matrix of size $K \times K$, and T is the radar pulse width.

Using (1), the array transmit beampattern can be written as

$$p(\theta) = \|\mathbf{W}^H\mathbf{d}(\theta)\|^2 \tag{2}$$

where $\|\cdot\|$ denotes the Euclidian norm of a vector, $\mathbf{d}(\theta) = \mathbf{a}^*(\theta)$, and $(\cdot)^*$ stands for the conjugation.

Assuming that L targets are present, the $N \times 1$ receive array observation vector can be written as [9]

$$\mathbf{x}(t,\tau) = \sum_{l=1}^{L}\beta_l(\tau)\mathbf{b}(\theta_l)\left(\mathbf{W}^H\mathbf{d}(\theta_l)\right)^H\boldsymbol{\phi}(t) + \mathbf{z}(t,\tau) \tag{3}$$

where t and τ are the fast and slow time indexes, respectively, $\beta(\theta_l)$ is the reflection coefficient of the target located at the angle θ_l with variance σ_β^2, and $\mathbf{z}(t,\tau)$ is the $N \times 1$ vector of zero-mean white Gaussian noise. By matched filtering $\mathbf{x}(t,\tau)$ to each of the orthonormal basis waveforms $\phi_k(t), k = 1, \ldots, K$, the $N \times 1$ virtual data vectors can be obtained as

$$\begin{aligned}\mathbf{y}_k(\tau) &= \int_{\mathrm{T}}\mathbf{x}(t,\tau)\phi_k^*(t)dt \\ &= \sum_{l=1}^{L}\beta_l(\tau)e^{j\psi_k(\theta_l)}\left|\mathbf{w}_k^H\mathbf{d}(\theta_l)\right|\mathbf{b}(\theta_l) + \mathbf{z}_k(\tau)\end{aligned} \tag{4}$$

where $\mathbf{w}_k$ is the kth column of $\mathbf{W}$, $\psi_k(\theta)$ is the phase of the inner product $\mathbf{d}^H(\theta)\mathbf{w}_k$, and $\mathbf{z}_k(\tau) \triangleq \int_{\mathrm{T}}\mathbf{z}(t,\tau)\phi_k^*(t)dt$ is the $N \times 1$ noise term whose covariance is $\sigma_z^2\mathbf{I}_N$. Note that $\mathbf{z}_k(\tau)$ and $\mathbf{z}_{k'}(\tau)$ $(k \neq k')$ are independent due to the orthogonality between $\phi_k(t)$ and $\phi_{k'}(t)$.

It is worth noting that if $\mathbf{w}_k,\ k = 1, \ldots, K$ are designed such that the equality $|\mathbf{w}_k^H\mathbf{d}(\theta)| = |\mathbf{w}_{k'}^H\mathbf{d}(\theta)|$ is satisfied, then the RIP between $\mathbf{y}_k$ and $\mathbf{y}_{k'}$ holds, i.e., the signal component of $\mathbf{y}_k$ associated with the lth target is the same as the corresponding signal component of $\mathbf{y}_{k'}$ up to a phase rotation that is given by $\psi_k(\theta_l) - \psi_{k'}(\theta_l)$. It is worth mentioning that the latter property, which can be exploited in search free direction finding methods, is exactly the RIP that we are interested in.

3. TRANSMIT BEAMSPACE DESIGN

3.1. Approach

In our recent work, we have developed a simple DOA estimation algorithm based on the transmit beamspace preprocessing for the special case of two orthonormal waveforms [11]. In this work, we consider a more general form of the problem. Specifically, let us consider the $M \times K$ transmit beamspace matrix $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_K]$ where $K \leq M$. As it was mentioned earlier, the virtual data vector matched to the basis waveform $\phi_k(t),\ k = 1, \cdots, K$ can be written as (4).

From (4), it can be easily concluded that the RIP between $\mathbf{y}_k$ and $\mathbf{y}_{k'}$ holds if the signal component of $\mathbf{y}_k$ associated with the lth target has the same magnitude as the corresponding signal component of $\mathbf{y}_{k'}$. Therefore, the RIP between $\mathbf{y}_k$ and $\mathbf{y}_{k'}, k \neq k'$ is equivalent to the following relationship

$$|\mathbf{w}_k^H\mathbf{d}(\theta)| = |\mathbf{w}_{k'}^H\mathbf{d}(\theta)|, \qquad \theta \in (-\pi/2, \pi/2). \tag{5}$$

Due to the fact that the number of equations in (5) is significantly larger than the number of the variables, this condition cannot be satisfied unless a specific structure on the beamforming matrix $\mathbf{W}$ is imposed. One possibility is to satisfy the equations (5) approximately, however, due to the fact that the performance of the proposed method is very sensitive to such approximation, this option is not considered here.

In [11], based on the observation that for any arbitrary steering vector $\mathbf{a}(\theta)$, which corresponds to a ULA, the following relations hold

$$|\mathbf{w}^H\mathbf{a}(\theta)| = |(\tilde{\mathbf{w}}^*)^H\mathbf{a}(\theta)|, \quad \theta \in (-\pi/2, \pi/2) \tag{6}$$

we obtained the RIP for the special case of two waveforms by restricting the transmit beamspace matrix to have the specific structure of $\mathbf{W} = [\mathbf{w}\ \tilde{\mathbf{w}}^*]$ where $\tilde{\mathbf{w}}$ denotes the flipped version of the vector $\mathbf{w}$, i.e., $\tilde{\mathbf{w}}(i) = \mathbf{w}(M - (i-1)),\ i = 1, \cdots, M$, and M is the size of the vector $\mathbf{w}$. Then based on the specific structure of $\mathbf{W} = [\mathbf{w}\ \tilde{\mathbf{w}}^*]$ which guarantees the RIP, we designed $\mathbf{w}$ to have the additional required properties.

In the following, we are interested in obtaining the RIP for the more general case of more than two waveforms which would provide more degrees of freedom for getting a better performance. For this goal, we show that if for some k' the following relation holds

$$\left|\sum_{i=1}^{k'} \mathbf{w}_i^H \mathbf{d}(\theta)\right| = \left|\sum_{i=k'+1}^{K} \mathbf{w}_i^H \mathbf{d}(\theta)\right|, \ \forall\theta \in (-\pi/2, \pi/2) \tag{7}$$

two new sets of vectors defined as the summation of the first k' of the data vectors $\mathbf{y}_i(\tau), \ i = 1, \cdots, k'$ (4) and the last $K - k'$ data vectors $\mathbf{y}_i(\tau), \ i = k' + 1, \cdots, K$ would satisfy the RIP. More specifically, by defining the following vectors

$$\begin{aligned}
\mathbf{a}_1 &\triangleq \sum_{i=1}^{k'} \mathbf{y}_i(\tau) \\
&= \sum_{l=1}^{L} \beta_l(\tau) \cdot \left(\sum_{i=1}^{k'} \mathbf{w}_i^H \mathbf{d}(\theta_l)\right) \cdot \mathbf{b}(\theta_l) + \sum_{i=1}^{k'} \mathbf{z}_i(\tau) \quad (8) \\
\mathbf{a}_2 &\triangleq \sum_{i=k'+1}^{K} \mathbf{y}_i(\tau) \\
&= \sum_{l=1}^{L} \beta_l(\tau) \cdot \left(\sum_{i=k'+1}^{K} \mathbf{w}_i^H \mathbf{d}(\theta_l)\right) \cdot \mathbf{b}(\theta_l) + \sum_{i=k'+1}^{K} \mathbf{z}_i(\tau)
\end{aligned} \tag{9}$$

the corresponding signal component of target l in the vector $\mathbf{a}_1$ has the same magnitude as in the vector $\mathbf{a}_2$. The only difference between the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ is the phase which can be used for DOA estimation. Based on this fact, in order to have the aforementioned RIP between the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$, equation (7) needs to be satisfied for every angle $\theta \in (-\pi/2, \pi/2)$. Based on the relations (6), it can be shown that the equation (7) holds for any arbitrary θ only if the following structure on the matrix $\mathbf{W}$ is imposed:

- K is an even number,
- k' equals to $K/2$,
- $\mathbf{w}_i = \tilde{\mathbf{w}}_{k'+i}^*, \quad i = 1, \cdots, K/2$.

More specifically, if the beamforming matrix is assumed to be as

$$\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_{K/2}, \tilde{\mathbf{w}}_1^*, \cdots, \tilde{\mathbf{w}}_{K/2}^*] \tag{10}$$

the signal component of $\mathbf{a}_1$ associated with the lth target is the same as the corresponding signal component of $\mathbf{a}_2$ up to phase rotation of

$$\angle \sum_{i=1}^{K/2} \mathbf{w}_i^H \mathbf{d}(\theta_l) - \angle \sum_{i=K/2+1}^{K} \mathbf{w}_i^H \mathbf{d}(\theta_l) \tag{11}$$

which can be used as look-up table for finding DOA of a target. By considering the aforementioned structure for the beamspace matrix $\mathbf{W}$, the RIP will be satisfied and other design requirements can be satisfied through the proper design of $\mathbf{w}_1, \cdots, \mathbf{w}_{K/2}$.

Since the DOA is estimated based on the phase difference between $\sum_{i=1}^{K/2} \mathbf{w}_i^H \mathbf{d}(\theta_l)$ and $\sum_{i=K/2+1}^{K} \mathbf{w}_i^H \mathbf{d}(\theta_l)$, to obtain the best performance $\mathbf{W}$ should be designed in a way that the magnitudes of the summations $\sum_{i=1}^{K/2} \mathbf{w}_i^H \mathbf{d}(\theta_l)$ and $\sum_{i=K/2+1}^{K} \mathbf{w}_i^H \mathbf{d}(\theta_l)$ take their largest possible values. Since the phase of the product term $\mathbf{w}_i^H \mathbf{d}(\theta_l)$ in $\sum_{i=1}^{K/2} \mathbf{w}_i^H \mathbf{d}(\theta_l)$ may be different for different waveforms, the terms in the summation $\sum_{i=1}^{K/2} \mathbf{w}_i^H \mathbf{d}(\theta_l)$ may add incoherently and therefore it may result in a small magnitude which in turn degrades the performance. In order to avoid this situation, we design the matrix $\mathbf{W}$ so that the beamforming vectors $\mathbf{w}_1, \cdots, \mathbf{w}_{K/2}$ corresponding to different transmit waveforms do not overlap in their corresponding transmit beampatterns.

Using (2), the corresponding transmit pattern of $\mathbf{W}$ can be expressed as

$$\begin{aligned}
p(\theta) &= \sum_{i=1}^{K} \mathbf{d}^H(\theta) \mathbf{w}_i \mathbf{w}_i^H \mathbf{d}(\theta) \\
&= \sum_{i=1}^{K} |\mathbf{d}^H(\theta) \mathbf{w}_i|^2
\end{aligned} \tag{12}$$

which is the summation of the transmit beampatterns of different waveforms in direction θ.

3.2. Problem Formulation

We are interested in designing $\mathbf{W}$ such that in addition to the RIP, the transmit beampattern is as close as possible to the desired beampattern and there is a uniform power distribution across the transmit antenna elements. Since as it was explained, the beamspace matrix $\mathbf{W}$ should be designed in a way that the beamforming vectors $\mathbf{w}_1, \cdots, \mathbf{w}_{K/2}$ corresponding to different transmit waveforms do not overlap in their corresponding transmit beampatterns, we consider different desired beampatterns denoted as $P_{d,k}(\theta)$ for the vectors $\mathbf{w}_k, k = 1, \cdots, K/2$.

Using the minimum error criteria [8], the beampattern design problem can be formulated as

$$\min_{\mathbf{w}_k} \max_{\theta_q} \sum_{k=1}^{K/2} \left| P_{d,k}(\theta_q) - \|[\mathbf{w}_k \, \tilde{\mathbf{w}_k}^*]^H \mathbf{d}(\theta_q)\|^2 \right| \tag{13}$$

$$\text{s.t.} \sum_{k=1}^{K/2} |\mathbf{w}_k(i)|^2 + |\tilde{\mathbf{w}_k}(i)|^2 = \frac{E}{M}, \quad i = 1, \ldots, M \tag{14}$$

where E is the total transmit power, and $\{\theta_q : q = 1, \ldots, Q\}$ is a uniform grid that approximates the interval $[-\pi/2, \pi/2]$

into Q number of directions. The constraint used in (14) ensures that the transmit power distribution across the antenna elements is uniform.

Problem (14) results in such beamspace matrix that the maximum sum of the errors between transmit beampattern of $\mathbf{w}_k$ and its corresponding desired beampattern $P_{d,k}(\theta_q)$ is minimized and also a uniform power distribution across antenna elements is guaranteed.

3.3. Solution

Using the facts that

$$\|[\mathbf{w}_k \, \tilde{\mathbf{w}_k}^*]^H \mathbf{d}(\theta_q)\|^2 = 2 \cdot |\mathbf{w}_k^H \mathbf{d}(\theta_q)|^2, \quad (15)$$

$$|\mathbf{w}_k^H \mathbf{d}(\theta_q)|^2 = Tr(\mathbf{d}(\theta_q)\mathbf{d}^H(\theta_q)\mathbf{w}_k \mathbf{w}_k^H), \quad (16)$$

$$|\mathbf{w}_k(i)|^2 + |\mathbf{w}_k(M-i+1)|^2 = Tr(\mathbf{w}_k \mathbf{w}_k^H \mathbf{A}_i), \quad i = 1, \ldots, M/2 \quad (17)$$

where Tr stands for the trace and $\mathbf{A}_i$ is an $M \times M$ matrix such that $\mathbf{A}_i(i,i) = \mathbf{A}_i(M-(i-1), M-(i-1)) = 1$ and the rest of the elements are equal to zero, the problem (14) can be cast as

$$\min_{\mathbf{w}_k} \max_{\theta_q} \sum_{k=1}^{K/2} \left| P_{d,k}(\theta_q)/2 - (\mathbf{w}_k^H \mathbf{d}(\theta_q))^2 \right| \quad (18)$$

$$\text{s.t.} \quad \sum_{k=1}^{K/2} Tr(\mathbf{w}_k \mathbf{w}_k^H \mathbf{A}_i) = \frac{E}{M}, \; i = 1, \ldots, \frac{M}{2} \quad (19)$$

Introducing the new variable $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$, the problem above can be equivalently rewritten as

$$\min_{\mathbf{w}} \max_{\theta_q} \sum_{k=1}^{K/2} \left| P_{d,k}(\theta_q)/2 - (\mathbf{w}_k^H \mathbf{d}(\theta_q))^2 \right| \quad (20)$$

$$\text{s.t.} \quad \sum_{k=1}^{K/2} Tr(\mathbf{X}_k \mathbf{A}_i) = \frac{E}{M}, \; i = 1, \ldots, \frac{M}{2} \quad (21)$$

$$\text{rank}(\mathbf{X}_k) = 1, \;\; k = 1, \cdots, K/2 \quad (22)$$

where $\mathbf{X}$ is the Hermitian matrix and $rank(\cdot)$ denotes the rank of a matrix. The problem above can be solved by dropping the rank one constraint and solving the resulting problem which is convex, and then obtaining a rank one solution based on the solution of the relaxed problem. The latter procedure is well known as semi-definite programming (SDP) relaxation [13], [14].

One easier but suboptimal alternative for designing $\mathbf{W}$ is to design different pairs of beamforming vectors $\mathbf{w}_k$, $\tilde{\mathbf{w}}_k{}^*$, $k = 1, \cdots, K/2$ separately so that their corresponding transmit powers from antennas would be the same and their transmit beampatterns would be as close as possible to their corresponding beampatterns $P_{d,k}(\theta)$, where the beampatterns $P_{d,k}(\theta)$ do not intersect. The design of such pair of beampatterns has been explained in [11].

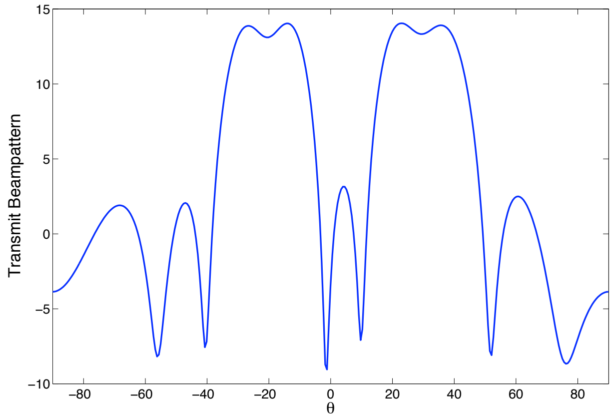

Fig. 1. Transmit Beampattern versus angle θ.

4. SIMULATIONS

The transmit array is modeled as a ULA of $M = 10$ omnidirectional antennas, spaced half wavelength apart. The receiver consists of $N = 10$ omnidirectional antennas randomly distributed from 0 to 9 wavelengths apart. The noise is assumed to be Gaussian zero-mean with variance of 1.

Two examples are considered for two different purposes. The first example is used to compare the root mean square error (RMSE) performance between the DOA estimation technique for the traditional MIMO radar with uniform transmit power distribution [4] and the proposed beamspace MIMO radar, while the second example is used to compare the probability of target resolution for these two methods.

In our first example, we use two targets located at $-20°$ and $30°$, respectively. Transmit energy for the proposed beamspace MIMO radar is focused in the spatial sectors $\theta = [-30° \;\; -10°]$ and $\theta = [20° \;\, 40°]$. Fig. 1 shows the transmit power distribution of the proposed beamspace MIMO radar. It is assumed here that the beamspace matrix has four columns that corresponds to four transmit waveforms. The RMSE is calculated based on 500 independent runs and is shown in Fig. 2. It is clear from this figure that the proposed DOA estimation method outperforms the DOA estimation method of [4] for the traditional MIMO radar.

In the second example, the transmit beamspace matrix remained unchanged from that in the first example. However, the locations of the two targets are changed to 30° and 32°. The probability of target resolution is calculated based on definition that the signal is resolved if $|\theta_i - \hat{\theta}_i| \leq (\theta_1 - \theta_2)/2$ where θ_i and $\hat{\theta}_i$ are respectively, the actual and the estimated DOAs of two close targets. If both estimates for the DOAs satisfy $|\theta_i - \hat{\theta}_i| \leq (\theta_1 - \theta_2)/2$, the angle is said to be resolved. Probability of target resolution is calculated based on 500 independent runs. Fig. 3 shows this probability versus SNR for the DOA estimations based on the proposed beamspace

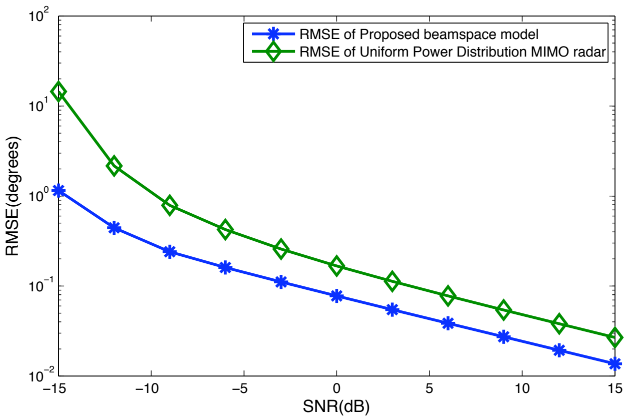

Fig. 2. RMSE versus SNR.

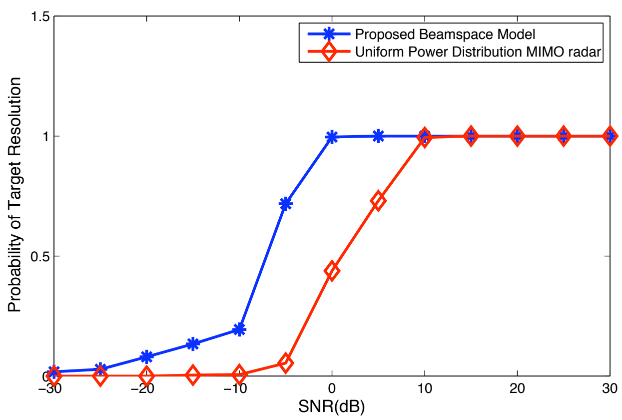

Fig. 3. Probability of resolution versus SNR

MIMO radar and the traditional MIMO radar. It can be seen that the SNR levels in which the transition of the target resolution probabilities from low to high values occurs are different for the two tested method and favorable for the proposed method as expected since the energy focussing and, thus, higher SNR gain is achieved for the proposed method.

5. CONCLUSION

The transmit beamspace design problem for colocated MIMO radar has been considered for the case of even but otherwise arbitrary number of transmit waveforms. It is guaranteed by our design that the RIP is satisfied at the receive array that enables us to use the search-free techniques for DOA estimation. It is a remarkable fact since different to the traditional DOA estimation techniques, the RIP is enforced at the transmit array and hold at the receive array of arbitrary geometry independent on the propagation media. It is shown that the DOA estimation performance is improved for our method due to the effect of transmit energy focusing in sectors were the targets are likely to be located.

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