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Physical-layer Network Coding in OFDM System: Analysis and Performance

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***Abstract*—The number of mobile terminals keeps a sustaining growth, and brings the importance of wireless network to a new level. Wireless networks are supposed to provide higher capacity for broadband services, as well as to be as robust and fast as wired network. The reason for the distinct gap between the two is that in wireless networks, the signal transmitted by a node may reach several other nodes, and a node may receive signals from several other nodes simultaneously. This broadcast nature is traditionally considered harmful, because it may cause interference or even collision. However, the novel concept of network coding turns this drawbacks into a capacity boosting potential advantage. Network coding is a technique where the nodes of a network take several packets and combine them together for transmission instead of simply relaying the packets they receive. In this paper, we proposed a physical-layer network coding scheme which is simple to implement for OFDM system. We apply this scheme in 802.11g system to demonstrate that with modified frame format, intentionally collided messages can be decoded and improve network throughput as well as spectrum efficiency.**

I. Introduction

OFDM has drawn great attention as a fundamental transmission technique of modern high-speed wireless networks. There are a lot of advantages of OFDM, such as low system rate, effective usage of large frequency band, and resistance to frequency-selective fading [1]. Therefore OFDM is gaining popularity with booming development of broadband wireless communication systems such as Wi-Fi, WiMax and Long-Term Evolution (LTE). Especially, 802.11g is becoming almost ubiquitously developed in densely populated neighborhood.

There are as many as 11 channels in 802.11g system, and the interval of center frequency between adjacent channels is 5MHz. To avoid interference, the frequency interval of channels in adjacent cells is required to be at least 25MHz. We can infer that only 3 channels are independent and orthogonal. Therefore, it is inevitable that many 802.11g access points in range of each other use overlapping channels. In [2], the authors show that partially overlapping channels may improve the network throughput even when the number of orthogonal channels is limited. However, collisions may also happen more easily due to the broadcast nature. Much progress has been made in handling collisions (e.g., Zigzag [3]).

Instead of avoiding collisions, schemes which make use of collisions are proposed recently, such as remap decoding [5] and network coding [6]. Remap decoding detects special collision pairs and then decodes the collided packages with the help of collision-free subcarriers. The idea of remap is to introduce structured permutation on the mapping from bits to subcarriers after each collision to create structured diversity. By using the diversity created by remapped packages, an 802.11g receiver can decode any package after 4 collisions with other transmissions in the adjacent channels. The number of collisions reduces to 2 if the other transmissions are in non-adjacent channels. Thus, decoding efficiency and wireless throughput are improved. Endeavor the same principle, network coding makes intended collisions strategically to increase throughput and decrease delay by taking advantage of the broadcast nature. Two packages are made to be transmitted on the same frequency band simultaneously. A collided package is received by the network coding node. Then, this network coding node performs some linear coding on received packets and broadcast the encoded packets to different recipients. By this way, time slots used for transmission can be reduced and meanwhile the spectrum efficiency is increased.

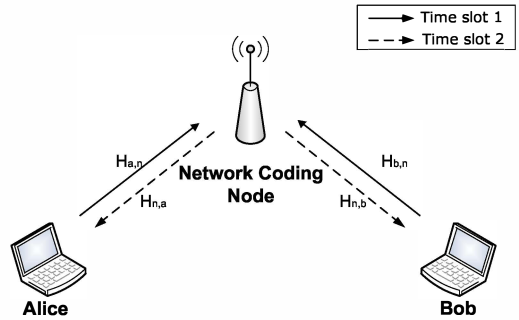

Fig. 1. Alice-Bob topology

In this paper, we apply the concept of network coding at the physical layer in OFDM system to turn the broadcast property into a capacity boosting advantage. We will illustrate our basic idea using the Alice-Bob topology shown in Fig.1. In this example, Alice and Bob want to send messages to each other, but the radio range does not allow them to communicate directly without a router. In traditional approach, explained in Fig. 2 (a), four time slots are needed.Considering that Alice has full knowledge of Pa and wants to get Pb while Bob has full knowledge of Pb and wants to get Pa, both Alice and Bob could get the message they want by canceling the known message from the combiner of Pa and Pb. Straightforward network coding shown in Fig.2 (b) is proposed [6].This scheme reduces time slots from four to three. Later, analog network

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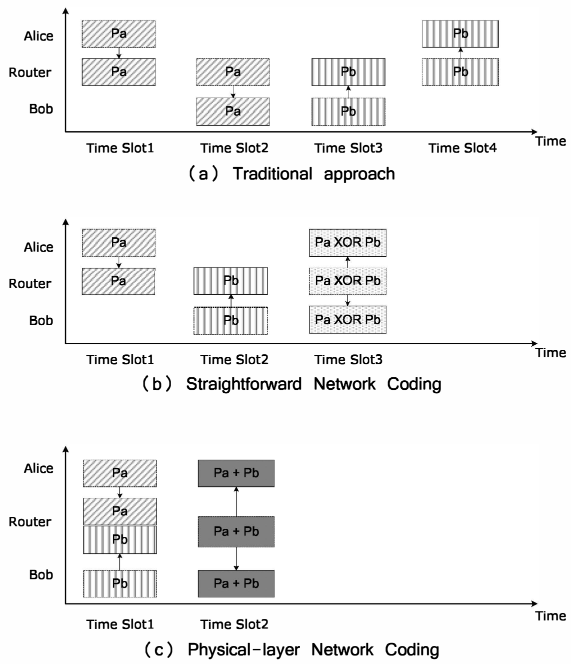

Fig. 2. Flows Intersecting at a Router

coding shown in Fig.2 (c) is proposed by S. Katti et al. [7]. This scheme further reduces the time slot to two while double increases the spectrum efficiency.

Based on the basic idea of network coding, we propose a physical-layer network coding scheme in OFDM system. In our scheme, only two time slots and one channel are needed for this package exchanging process. In the first time slot, Alice and Bob separately transmit Pa and Pb to the network coding node simultaneously on the same frequency band. Later in the second time slot, the network coding node decodes 'Pa + Pb' from the mixed samples, and broadcasts the added message to Alice and Bob. In this way the proposed scheme turns the broadcast property into a capacity boosting advantage as well as spectrum efficiency advantage. Additionally, this scheme is suitable for higher order modulations. In the former network coding schems, higher order constellations are not used, because of the unacceptable coding complexity as well as the BER performance. In our scheme, the decoding and encoding process at network coding node has almost nothing to do with the modulation type. Less complexity is additional introduced while using higher order modulations which can greatly increase the transmission rate.

The paper is organized as follows. Section II describes the preparations needed in the proposed physical-layer network coding scheme. Frame format design, package detection, and channel estimation are all introduced. Section III illustrates key steps of our scheme which are decoding and encoding processes at network coding node. We evaluate this scheme on both communication and energy saving performance, and the simulation results are presented in Section IV. The paper is concluded in Section V.

Preamble 12 symbols	SIGNAL 1 OFDM symbol	NULL 1 OFDM symbol	DATA Variable Number of OFDM Symbols

a) PPDU frame format of Alice

Preamble 12 symbols	NULL 1 OFDM symbol	SIGNAL 1 OFDM symbol	DATA Variable Number of OFDM Symbols

b) PPDU frame format of Bob

Preamble 12 symbols	SIGNAL 1 1 OFDM symbol	SIGNAL 2 1 OFDM symbol	DATA Variable Number of OFDM Symbols

c) PPDU frame format of Network Coding node

Fig. 3. PPDU frame formats

II. Preparations

To apply the proposed physical-layer network coding in 802.11g system, we slightly modify the standard frame format. The message sent by Alice and Bob are transmitted on the same frequency band at the same time. It is difficult for the network coding node to separate the information of two senders from the combined message in traditional manner. Therefore, we did some modification on the stranded frame format. The Presentation Protocol Data Unit (PPDU) structure is almost unchanged except that NULL is inserted to make sure that SIGNAL parts are not polluted. As for the preamble part, 10 short preambles which are used in frame synchronization keep unchanged. The package detection can also be done in the traditional way. The two long preambles are modified to help with the channel estimation of the two transmission channels. And the pilots insertion manner is modified to help with the residual frequency and timing offset estimation. Details of the modifications mentioned above are described in this section. This modified frame format is only used in the first time slot. As for the second time slot, in which the message is also transmitted on the same frequency band at the same time, broadcast function supported by standard 802.11g is used.

A. Frame Formats

We modify the PPDU frame format to make the SIGNAL part of two messages more easily separated. The modified PPDU frame format is shown in Fig.3. NULL stands for an OFDM symbol which consists of idle samples. We insert NULL after SIGNAL field at Alice side, before SIGNAL field at Bob side. This is illustrated in Fig.3 (a) and (b). Then, it is obvious that the frame format of message received by the network coding node is just like what is shown in Fig.3 (c). In this way, the SIGNAL part of the two messages will not be polluted by each other. Additionally, the transmitter needs two bit to encode the order of these two SIGNAL messages. These two bits are carried in the reserved bits in the SERVICE field of the PLCP header. For example, '01' stands for P_a's SIGNAL part is ahead of that of P_b's, and '10' for the opposite. When the message is not transmitted in the network coding manner, these two bits are set to be 0.

B. Package Detection

As we all know, 802.11g preamble consists of 10 identical short preambles, each of 16 samples; and 2 identical long preambles, each of 64 samples. The short preambles are as same as that in standard 802.11g which is used to detect when the transmission starts. An 802.11g receiver does auto-correlation on the first received 160 samples which is the sample length of all 10 short preambles. Then it slides one sample at a time and performs auto-correlation on those 160 samples. The peak of the auto-correlation Γ indicates the beginning of a new package, according to the repetitiveness of the short preamble and the independence of user data. Being a receiver in the first slot, network coding node seeks the beginning of a new package in the same way.

$$\Gamma(\Delta) = \sum_{k=0}^{L/2-1} y^*[k+\Delta]y[k+\Delta+L/2] \tag{1}$$

Specifically, let y be the received message at network coding node. This message is the sum of the message from Alice (y_a), the message from Bob (y_b), and the noise term w. Let T_s be sample interval, L be the auto-correlation length which is 160 here. Let r_a and r_b be the received signal without frequency offset ε_a and ε_b. The k-th sample of received message can be expressed as follows:

$$\begin{aligned} y[k] &= y_a[k] + y_b[k] + w[k] \\ &= r_a[k]e^{2\pi\varepsilon_a kT_s} + r_b[k]e^{2\pi\varepsilon_b kT_s} + w[k] \end{aligned} \tag{2}$$

And substitute (2) into (1), auto-correlation at the network coding node received signals would be :

$$\begin{aligned} \Gamma(\Delta) &= \sum_{k=0}^{L/2-1} y^*[k+\Delta]y[k+\Delta+L/2] \\ &= \sum_{k=0}^{L/2-1} \Big\{ r_a^*[k+\Delta]r_a[k+\Delta+\frac{L}{2}]e^{\pi\varepsilon_a LT_s} + \\ &\quad r_b^*[k+\Delta]r_b[k+\Delta+\frac{L}{2}]e^{\pi\varepsilon_b LT_s} + \\ &\quad r_a^*[k+\Delta]r_b[k+\Delta+\frac{L}{2}]e^{2\pi(\varepsilon_b-\varepsilon_a)kT_s}e^{\pi\varepsilon_b LT_s} + \\ &\quad r_b^*[k+\Delta]r_a[k+\Delta+\frac{L}{2}]e^{2\pi(\varepsilon_a-\varepsilon_b)kT_s}e^{\pi\varepsilon_a LT_s} \Big\} \end{aligned} \tag{3}$$

When Δ is not where the preamble begins, r_a is independent of r_b that their auto-correlation is approximate to 0. And r_a and r_b are both without repetitiveness because they are self-independent. The cross-correlation of them is also approximate to 0. Accordion to equation (3), the auto-correlation at the position where is not the beginning of a preamble is approximate to 0.

On the contrary, when Δ is where the preamble begins, $r_a[k] = r_b[k] = r[k]$ and $r[k]$ is repetitive. In this case, (3) can be rewritten as follows:

$$\begin{aligned} \Gamma(\Delta) = \sum_{k=0}^{L/2-1} |r[k]|^2 \Big\{ & e^{\pi\varepsilon_a LT_s}(1 + e^{2\pi(\varepsilon_a-\varepsilon_b)kT_s}) + \\ & e^{\pi\varepsilon_b LT_s}(1 + e^{2\pi(\varepsilon_b-\varepsilon_a)kT_s}) \Big\} \end{aligned} \tag{4}$$

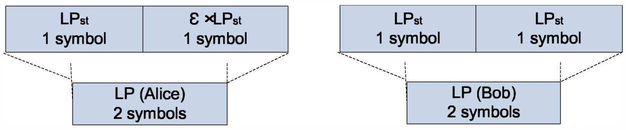

Fig. 4. Long Preamble Design

C. Channel Estimation

We estimate the two channels which are $H_a[l]$ and $H_b[l]$ for each subcarrier using the two symbols known in long preambles. These samples are used after correcting time and frequency offset which will be introduced afterward. Long preambles design for both Alice and Bob are shown in Fig.4. Let the transmitted symbols of Bob be $X_a[1,l]$ and $X_a[2,l]$, $1 \le l \le 64$. These two symbols are as same as the standard preamble symbol $LP_{st}[l]$. We shrink the second symbol in the long preambles Alice transmitted by ε amount, where $0 \le \varepsilon \le 1$. Let the FFT of the two symbols which network coding node receives be $Y_n[1,l]$ and $Y_n[2,l]$. Obviously, $Y_n[i,l]$ is the combiner of $Y_a[i,l]$ and $Y_b[i,l]$, then:

$$\begin{aligned} Y_n[1,l] &= Y_a[1,l] + Y_b[1,l] \\ &= H_a[l]X_a[1,l] + H_b[l]X_b[1,l] \\ &= LP_{st}[l](H_a[l] + H_b[l]) \end{aligned} \tag{5}$$

$$\begin{aligned} Y_n[2,l] &= Y_a[2,l] + Y_b[2,l] \\ &= H_a[l]X_a[2,l] + H_b[l]X_b[2,l] \\ &= LP_{st}[l](H_a[l] + \varepsilon H_b[l]) \end{aligned} \tag{6}$$

Solving the equations formed by (5) and (6), $H_a[l]$ and $H_b[l]$ can be estimated as:

$$H_a[l] = \frac{Y_n[2,l] - \varepsilon Y_n[1,l]}{(1-\varepsilon)LP_{st}[l]} \tag{7}$$

$$H_b[l] = \frac{Y_n[1,l] - Y_n[2,l]}{(1-\varepsilon)LP_{st}[l]} \tag{8}$$

Considering that 802.11g has unused subcarriers, we skip these when doing channel estimation. To estimate the frequency offset, we perform auto-correlation on the two long preamble symbols separately to get two equations similar to (4). From these two equations ε_a and ε_b can be obtained. We first correct the samples of fractional part, then estimate the integer part of the frequency offset refer to the [8] and [9].

D. Residual Frequency and Timing Offset

There are still residual frequency and timing errors even with timing acquisition and frequency offset corrections because of the inexact location of the beginning of OFDM symbols and multi-path effect. So, four known pilots are designed in each 802.11g OFDM symbol to track these residual errors shown as Fig.5. We modified this pilot insertion manner to keep away the pilots transmitted in two separate channels from being polluted by each other. In our scheme Alice use only *P1* and *P3* while Bob use only *P2* and *P4*. We use the pilot phase rotation to estimate the linear ramps caused by the frequency

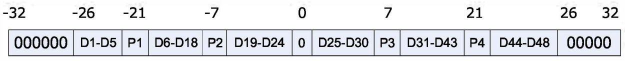

Fig. 5. Pilot Insertion Manner

and timing offset, though the accuracy of this estimation may decrease.

III. Decoding and Encoding

Linear network coding is shown to be sufficient to achieve the maximum flow bounds between the source-destination pairs in wired networks in [10]. Among the simplest coding schemes is linear coding, which regards a block of data as a vector over a certain base field and allows a node to apply a linear transformation to a vector before passing it on. Later, this result is extended in [11] that a random linear combination of packets is enough to achieve the capacity for multicast traffic. And the transmission between Alice and Bob happens randomly that the process which is done by network coding node can be considered as a random linear combination.

We assume that perfect knowledge of channel state information could be collected by accurate channel estimation introduced above. Unlike the previous network coding schemes [12][13], the coding method in the proposed scheme is not limited to some certain types. Thus, the transmission rate can be increased by using advanced modulations such as 16-QAM. The modulation process of 802.11g is illustrated in Fig.6, and things are almost the same in our scheme. On the network coding side it is similar to decode-and-forward process in relay networks. We decode $D_a + D_b$ from the combined message and then forward this to both Alice and Bob in a broadcasting way. Canceling the known message on Alice or Bob side, the two original messages could be decoded by the intended receiver.

Decoding process which aims at getting $D_a + D_b$ on the network coding node side is done as follows. Firstly, we perform FFT to each received symbol to get 64 samples in frequency domain. Let $r[l]$ be the sample on the l-th subcarrier, $1 \leq l \leq 64$. Each sample is expressed by a complex number which consists of real part and imaginary part. The sample $r[l]$ is the combiner of messages from Alice($D_a[l]$) and Bob($D_b[l]$) with noise term Z on a certain subcarrier (e.g. $r[l] = D_a[l]H_a[l] + D_b[l]H_b[l] + Z$). Then, we can estimate $D_a[l] + D_b[l]$ which is denoted as $S[l]$ according to the maximum likelihood method.

$$\hat{S} = \arg_{S_i} \max p(r|S_i) \tag{9}$$

After getting the approximate value of $D_a[l] + D_b[l]$, pilots are inserted in the manner defined in 802.11g standard. Then the network coding node performs IFFT, adds circular prefix, and broadcasts this message to both Alice and Bob simultaneously. We take decoding process at Alice side in the second time slot as an example. When Alice receives the broadcasting message, we can get $D_a[l] + D_b[l]$ after performing FFT and channel compensation. Then, $D_b[l]$ can be decoded by canceling $D_a[l]$ which is known by Alice. Rest of

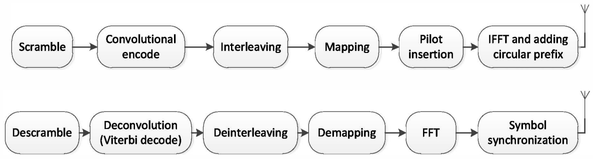

Fig. 6. Modulation process of 802.11g

the decoding process is the same as those in 802.11g standard. Viterbi decoding is preferred while doing deconvolution.

IV. Evaluation

Before transmitting we search for nodes which consist of a transmission pair that the proposed physical-layer network coding scheme could be applied. As we illustrated in the introduction part, if Alice and Bob want to send messages to each other, we call them a pair. At the beginning of this scheme, each transceiver searches for its partner in the transmission range of network coding node. Because it is not common that two transceivers want to send message to each other at the same time. We set a delay constraint which is the longest time a transceiver waits for a partner to appear. If no partner appears in the definite period, the transceiver transmits message in the traditional manner. A loosen time constraint tolerates a longer delay that a transceiver has more opportunity to find its partner. On the other hand, it is easier to form transmission pairs when there are fewer users under the same time constraint.

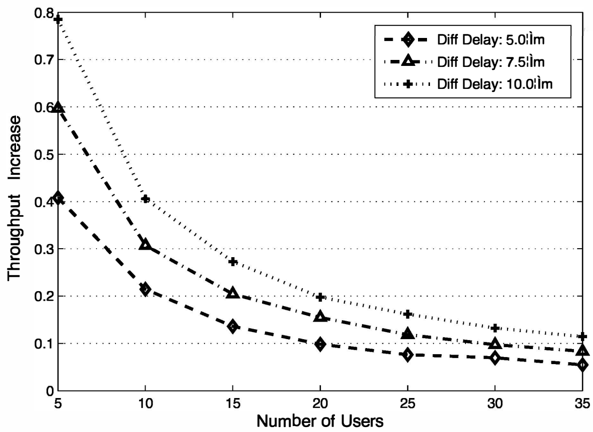

Fig. 7. Throughput under different delay constraint

As we analyzed above, network throughput can be improved by applying this scheme in 802.11g system. And the more the transmission pairs are the greater the network throughput improves. We simulate the increase in throughput under different delay constraint and different number of users. The simulation result is shown in Fig.7. It is obvious that network throughput increases when the number of users decreases or the time constraint goes loosen. This is in consonance with the theoretical analysis above. Especially, the throughput rises to be as high as 79% when there are only 5 users which tolerate

TABLE I
DECODING PERFORMANCE COMPARISON

SNR(Alice,Bob)	BER
(26.3dB, 26.1dB)	6.2×10^{-5}
(26.3dB, 21.2dB)	2.7×10^{-4}
(26.3dB, 16.5dB)	4.1×10^{-3}

a delay of $10\mu m$. Considering the worst case, the throughput only increase 5.5% when the system is area overloaded with a strict time constraint of $5\mu m$. So, one should balance between throughput and delay when applying this scheme.

Also, we evaluate the decoding performance for 16-QAM constellations under a variety of SNR settings performing our scheme on soft defined radio. The SNR of the Alice's is fixed to 26.3dB while the SNR of Bob's keeps changing from 26.3dB to 16.5dB. The table above shows the BER at three SNR settings. From the table, we observe that in the first case, when the SNR is almost the same, the BER in our scheme is as low as 6.2×10^{-5}. The BER increases to 2.7×10^{-4} when the SNR difference is 5dB. However, when the SNR difference is 10dB, the BER becomes large, achieving 4.1×10^{-3}. The signal of Bob is almost immersed by that of Alice. In other words the interference caused by Alice is too strong that it leads to worse BER performance for Bob. We can conclude that it is better for a transceiver to choose a partner with similar SNR value. Or the transceivers in a transmission pair is preferred to be configured to the same SNR.

V. Conclusion

In this paper, we proposed a physical-layer network coding scheme which enable two users transmit information to the same receiver on the same frequency band simultaneously. Such scheme is especially useful under Alice-Bob topology under which the network throughput can be greatly improved. And a corresponding frame format modified based on 802.11g is also proposed. The analysis and simulation results show that the network throughput is increased and the BER performance is still acceptable. But this scheme is more proper to use under the condition that the number of users is not too large in the certain area. Also, SNR difference results in performance degradation. It is better that the transmission channels for transceivers in a communication pair have similar SNR. Because BER increases as the SNR difference goes larger.

Cognitive radio is a booming technology to deal with the challenge brought by network coding. Cognitive radio with the help of soft defined radio is a typical technology with learning ability. Learning algorithms could be help with the balance between delay and throughput increase. And another character of cognitive radio, which is its awareness of current state, would be help a lot with the collaboration between the two transceivers in configuring the same constellation and SNR, as well as synchronizing the transmissions. After solving the problems mentioned above by using cognitive radio, the performance of this proposed scheme can be further improved.

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