# Robust Two-Way Amplify-and-Forward MIMO Relay Beamforming with Non-reciprocal and Reciprocal CSI

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***Abstract*—In this paper, we investigate robust beamforming design for two-way amplify-and-forward (AF) MIMO relaying system with Gaussian distributed channel errors. Take mean-square-error (MSE) of the detected data as the design criteria, beamforming matrix at relay is optimized under imperfect channel state information (CSI). Both frequency-division-duplex (FDD) system and time-division-duplex (TDD) system are considered. Then two algorithms are proposed based on a framework of quadratic programming. The effectiveness of the proposed robust design for different channel mode are finally demonstrated by the simulation results.**

***Index Terms*—Two-way, AF, MIMO, beamforming, channel error**

## I. Introduction

Recently, cooperative communication has received much attention as it can greatly improve communication quality and reliability. Two-way relay protocol is a new technique which exploits the idea of physical network coding to further improve the spectral efficiency [1]. In two-way relay systems, only two time slots are needed to exchange information between the two source nodes. In the first time slot, the two source nodes transmit signals to the relay node simultaneously; in the second time slot, the relay node broadcasts the processed signal received in previous time slot to the two nodes.At relay, the relay strategies can generally be classified as three types, i.e., amplify-and-forward (AF) [2] - [6], decode-and-forward (DF) [7], compress-and-forward (CF). Among different relay strategies, AF has the lowest complexity and is most suitable for practical implementation.

Multiple-Input-Multiple-Output (MIMO) is one of the key technologies of the advanced communication system and has been studied a lot in the past decades. The technology of MIMO can significantly increase the data rate and system capacity without the sacrifice of spectrum and time resources. Beamforming in MIMO systems can be exploited to reduce the interference. In order to achieve more performance gain, MIMO is introduced to the two-way relaying systems. There are a rich body of literatures on two-way MIMO relaying systems. An optimal relay beamforming matrix structure for single-pair two-way MIMO relaying systems is presented in [2]. And beamforming designs under SINR and MMSE criterias for multi-pair two-way AF MIMO relaying systems are investigated in [5]. The distributed beamforming designs for two-way relaying networks with multiple relays are discussed in [3] - [4].

In most of previous works, channel state information (CSI) is assumed to be perfectly known. Unfortunately, this assumption cannot be met in practice due to various reasons. Channel estimation errors are always inevitable and drastically degrades system performance. To mitigate the negative effect on the performance of AF relaying systems, such channel estimation errors should be taken into account in the beamforming design process [6], [8]. This is also the motivation of our work. On the other hand, there are two widespread channel modes (TDD and FDD) in next generation wireless communication, and whether the channel has reciprocity affects the design of beamforming matrix at a great extent. So it is of practical significance to investigate beanforming designs under these two kinds of channel.

In this paper, we consider a two-way AF MIMO relaying network in which there two single-antenna users relying one multi-antenna relay to exchange information. Notice that CSI is not perfectly known, the beamforming design should be considered to be robust against channel errors. The beamforming matrix at the relay is designed based on minimum mean square error (MMSE) criteria under the relay power constraint. Both FDD system and TDD system are discussed, and the optimization problems can be solved using the framework of quadratic programming [8]. Simulation results demonstrate the performance of the proposed robust designs.

**Notation**: Scalars are denoted by lower-case letters, e.g., $x$, and bold-face lower-case letters are used for vectors, e.g., $\mathbf{x}$, and bold-face upper-case letters for matrices, e.g., $\mathbf{X}$. In addition, $\mathrm{Tr}(\cdot), \mathbb{E}(\cdot)$, denote the trace, expectation operator respectively, and $(\cdot)^*, (\cdot)^{\mathrm{T}}, (\cdot)^{\mathrm{H}}$, denote the conjugate, transpose, and conjugate transpose, respectively. $|x|$ is the norm of a complex number $x$ and $\|\mathbf{x}\|$ is the Euclidean norm of a complex vector $\mathbf{x}$. $\mathrm{vec}(\mathbf{X})$ represents the vector-version of the matrix $\mathbf{X}$. $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product of matrix $\mathbf{A}$ and $\mathbf{B}$. $\mathbf{I}_N$ denotes the $N$-dimension identity matrix. $C^{m\times n}$ denotes the space of $m \times n$ matrices with complex-valued elements.

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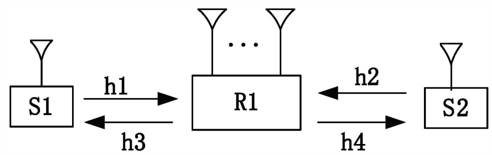


Fig. 1. Two-way MIMO Relay system model

## II. System Model

As shown in figure 1, in this paper a two-way AF relaying network is considered, in which there are two single-antenna source nodes and one multi-antenna relay station. The relay station is equipped with $N$ antennas. The two source nodes use two-way relaying strategy to communicate with each other. For two-way relaying, there are two time slots. In the first time slot which is called multiple access phase, the two source nodes transmit information to the relay station simultaneously. Then the relay station linearly processes the received signal and broadcasts the resulting signal to all source nodes in the second time slot which is termed as broadcast phase. Note that the direct link between the two source nodes is ignored due to the large path loss.

In the multiple access phase, the received signal at the relay station is

$$\mathbf{y}_{\mathrm{r}} = \sum\nolimits_{i=1}^{2} \mathbf{h}_i s_i + \mathbf{v} \tag{1}$$

where $\mathbf{y}_{\mathrm{r}}$ is an $N \times 1$ received signal vector, and $\mathbf{h}_i \in C^{N\times 1}$ is the channel from the $i^{\mathrm{th}}$ source node to the relay station. Furthermore, the scalar $s_i$ represents the transmitted signal from the $i^{\mathrm{th}}$ source node and with loss of generality $\mathbb{E}[s_i s_i^{\mathrm{H}}] = 1$. Finally, $\mathbf{v} \in C^{N\times 1}$ denotes the additive white Gaussian noise at the relay station, which satisfies the normal distribution $\mathbf{v} \sim N(\mathbf{0}, \mathbf{R}_{\mathbf{v}})$.

During the second phase, before broadcasting the signal, the relay station multiplies the received signal by a beamforming matrix $\mathbf{F} \in C^{N\times N}$ under a power constraint

$$\mathrm{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_{\mathrm{r}}}\mathbf{F}^{\mathrm{H}}) \leq P_{\mathrm{r}}^{\max} \tag{2}$$

where $\mathbf{R}_{\mathbf{y}_{\mathrm{r}}} = \mathbb{E}\{\mathbf{y}_{\mathrm{r}}\mathbf{y}_{\mathrm{r}}^{\mathrm{H}}\}$ is the covariance matrix of $\mathbf{y}_{\mathrm{r}}$, which equals to

$$\mathbf{R}_{\mathbf{y}_{\mathrm{r}}} = \sum\nolimits_{i=1}^{2} \mathbf{h}_i \mathbf{h}_i^{\mathrm{H}} + \mathbf{R}_{\mathbf{v}}. \tag{3}$$

After broadcasting the signal from the relay, the received signal at the $i^{\mathrm{th}}$ source node can be written as

$$\begin{aligned} y_1 &= \mathbf{h}_3 \mathbf{F} \mathbf{y}_{\mathrm{r}} + n_1 \\ y_2 &= \mathbf{h}_4 \mathbf{F} \mathbf{y}_{\mathrm{r}} + n_2 \end{aligned} \tag{4}$$

where $\mathbf{h}_3, \mathbf{h}_4 \in C^{1\times N}$ denote the channels from the relay station to the $1^{\mathrm{st}}$ and $2^{\mathrm{nd}}$ source node respectively. The scalar $n_i$ denotes the noise at the $i^{\mathrm{th}}$ source node and $\mathbb{E}[n_i n_i^{\mathrm{H}}] = \sigma_{n_i}^2$ for $i = \{1, 2\}$.

In real systems, due to various reasons, e.g. quantization errors, the limit length of training sequence and time varying nature of wireless channels, channel errors are inevitable. So channel errors are taken into account in this paper, channel can be modeled as following:

$$\mathbf{h}_i = \bar{\mathbf{h}}_i + \boldsymbol{\Delta}\mathbf{h}_i \tag{5}$$

where $\mathbf{h}_i$ is the real channel, $\boldsymbol{\Delta}\mathbf{h}_i$ is the error which satisfies the normal distribution $\boldsymbol{\Delta}\mathbf{h}_i \sim N(\mathbf{0}, \boldsymbol{\Psi}_i)$.

Taking $1^{\mathrm{st}}$ source node as an example, its received signal can be expressed as

$$\begin{aligned} y_1 =& (\bar{\mathbf{h}}_3 + \boldsymbol{\Delta}\mathbf{h}_3)\mathbf{F}[(\bar{\mathbf{h}}_2 + \boldsymbol{\Delta}\mathbf{h}_2)s_2 + (\bar{\mathbf{h}}_1 + \boldsymbol{\Delta}\mathbf{h}_1)s_1 \\ & + \mathbf{v}] + n_1 \end{aligned} \tag{6}$$

As we all know, there are two ubiquitous channel modes (TDD and FDD) in next generation wireless communication, and whether the channel has reciprocity affects the design of beamforming matrix at a great extent. Generally speaking, it is more difficult when the channel has reciprocity. In this paper, we will analyze the two cases in the following sections respectively.

## III. Non-reciprocal Channel

In this section, we assume the uplink channels from the source nodes to the relay is not reciprocal to the downlink channels from the relay to the source nodes, which may hold for frequency-division-duplex(FDD) system.

Taking source node 1 as an example, according to the received signal (6), the mean-square-error(MSE) of the $1^{\mathrm{st}}$ source node can be derived as

$$\begin{aligned} \mathrm{MSE}_1 =& \mathbb{E}[(y_1 - s_2)(y_1 - s_2)^{\mathrm{H}}] \\ =& 1 - \bar{\mathbf{h}}_3\mathbf{F}\bar{\mathbf{h}}_2 - (\bar{\mathbf{h}}_3\mathbf{F}\bar{\mathbf{h}}_2)^{\mathrm{H}} + |\bar{\mathbf{h}}_3\mathbf{F}\bar{\mathbf{h}}_1|^2 \\ & + |\bar{\mathbf{h}}_3\mathbf{F}\bar{\mathbf{h}}_2|^2 + \sigma_{n_1}^2 + \bar{\mathbf{h}}_3\mathbf{F}\boldsymbol{\Psi}_1(\bar{\mathbf{h}}_3\mathbf{F})^{\mathrm{H}} \\ & + \mathrm{tr}[(\mathbf{F}\bar{\mathbf{h}}_1)^*(\mathbf{F}\bar{\mathbf{h}}_1)^{\mathrm{T}}\boldsymbol{\Psi}_3] + \mathrm{tr}(\boldsymbol{\Psi}_1\mathbf{F}^{\mathrm{H}}\boldsymbol{\Psi}_3\mathbf{F}) \\ & + \bar{\mathbf{h}}_3\mathbf{F}\boldsymbol{\Psi}_2(\bar{\mathbf{h}}_3\mathbf{F})^{\mathrm{H}} + \mathrm{tr}[(\mathbf{F}\bar{\mathbf{h}}_2)^*(\mathbf{F}\bar{\mathbf{h}}_2)^{\mathrm{T}}\boldsymbol{\Psi}_3] \\ & + \mathrm{tr}(\boldsymbol{\Psi}_2\mathbf{F}^{\mathrm{H}}\boldsymbol{\Psi}_3\mathbf{F}) + \bar{\mathbf{h}}_3\mathbf{F}\boldsymbol{\Psi}_{\mathbf{v}}(\bar{\mathbf{h}}_3\mathbf{F})^{\mathrm{H}} \\ & + \mathrm{tr}(\mathbf{R}_{\mathbf{v}}\mathbf{F}^{\mathrm{H}}\mathbf{R}_3\mathbf{F}) \end{aligned} \tag{7}$$

Similarly, the MSE of the $2^{\mathrm{nd}}$ source node can be formulated as

$$\begin{aligned} \mathrm{MSE}_2 =& \mathbb{E}[(y_2 - s_1)(y_2 - s_1)^{\mathrm{H}}] \\ =& 1 - \bar{\mathbf{h}}_4\mathbf{F}\bar{\mathbf{h}}_1 - (\bar{\mathbf{h}}_4\mathbf{F}\bar{\mathbf{h}}_1)^{\mathrm{H}} + |\bar{\mathbf{h}}_4\mathbf{F}\bar{\mathbf{h}}_2|^2 \\ & + |\bar{\mathbf{h}}_4\mathbf{F}\bar{\mathbf{h}}_1|^2 + \sigma_{n_2}^2 + \bar{\mathbf{h}}_4\mathbf{F}\boldsymbol{\Psi}_2(\bar{\mathbf{h}}_4\mathbf{F})^{\mathrm{H}} \\ & + \mathrm{tr}[(\mathbf{F}\bar{\mathbf{h}}_2)^*(\mathbf{F}\bar{\mathbf{h}}_2)^{\mathrm{T}}\boldsymbol{\Psi}_4] + \mathrm{tr}(\boldsymbol{\Psi}_2\mathbf{F}^{\mathrm{H}}\boldsymbol{\Psi}_4\mathbf{F}) \\ & + \bar{\mathbf{h}}_4\mathbf{F}\boldsymbol{\Psi}_1(\bar{\mathbf{h}}_4\mathbf{F})^{\mathrm{H}} + \mathrm{tr}[(\mathbf{F}\bar{\mathbf{h}}_1)^*(\mathbf{F}\bar{\mathbf{h}}_1)^{\mathrm{T}}\boldsymbol{\Psi}_4] \\ & + \mathrm{tr}(\boldsymbol{\Psi}_1\mathbf{F}^{\mathrm{H}}\boldsymbol{\Psi}_4\mathbf{F}) + \bar{\mathbf{h}}_4\mathbf{F}\mathbf{R}_{\mathbf{v}}(\bar{\mathbf{h}}_4\mathbf{F})^{\mathrm{H}} \\ & + \mathrm{tr}(\mathbf{R}_{\mathbf{v}}\mathbf{F}^{\mathrm{H}}\boldsymbol{\Psi}_4\mathbf{F}) \end{aligned} \tag{8}$$

We aim at minimizing the weighted sum of MSE under the relay power constraint. So the optimization problem is formulated as

$$\begin{aligned} \min \quad & \sum\nolimits_{i=1}^{2} w_i \mathrm{MSE}_i \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_{\mathrm{r}}}\mathbf{F}^{\mathrm{H}}) \leq \mathrm{P}_{\mathrm{r}}^{\max} \end{aligned} \tag{9}$$

where $w_i$'s are the weighting factors, $w_i \geq 0$. $\mathbf{R}_{\mathbf{y}_{\mathrm{r}}}$ is the covariance of the signal $\mathbf{y}_{\mathrm{r}}$ received at the relay, which equals to

$$\mathbf{R}_{\mathbf{y}_{\mathrm{r}}} = \bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^{\mathrm{H}} + \bar{\mathbf{h}}_2\bar{\mathbf{h}}_2^{\mathrm{H}} + \boldsymbol{\Psi}_1 + \boldsymbol{\Psi}_2 + \boldsymbol{\Psi}_{\mathbf{v}} \tag{10}$$

In nature, the optimization problem (9) is a quadratic programming problem with one constraint, and it can be solved by searching a Lagrange multiplier.The Lagrangian dual function of the optimization problem (9) is

$$\mathcal{L}(\mathbf{F},\mu)=\sum_{i=1}^{2} w_i \text{MSE}_i+\mu[\text{tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})-\text{P}_\text{r}^{\max}] \quad (11)$$

where $\mu$ is the Lagrangian multiplier. Using the technique of complex matrix derivative, the derivative of the Lagrangian dual function (11) is

$$\begin{aligned}\partial\mathcal{L}/\partial\mathbf{F}^* =&-\mathcal{H}+\mathbf{A}_1\mathbf{F}\mathbf{B}_1+\mathbf{A}_1^\text{T}\mathbf{F}\mathbf{B}_2\\&+\mathbf{A}_2\mathbf{F}\mathbf{B}_3+\mu\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}=0\end{aligned} \quad (12)$$

where $\mathbf{A}_i$'s, $\mathbf{B}_i$'s and $\mathcal{H}$ are defined as follows:

$$\begin{aligned}\mathbf{A}_1 &\triangleq w_1\mathbf{\Psi}_3+w_2\mathbf{\Psi}_4\\ \mathbf{A}_2 &\triangleq w_1\bar{\mathbf{h}}_3^\text{H}\bar{\mathbf{h}}_3+w_2\bar{\mathbf{h}}_4^\text{H}\bar{\mathbf{h}}_4\\ \mathbf{B}_1 &\triangleq \mathbf{\Psi}_1+\mathbf{\Psi}_2+\mathbf{\Psi}_\mathbf{v}\\ \mathbf{B}_2 &\triangleq \bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^\text{H}+\bar{\mathbf{h}}_2\bar{\mathbf{h}}_2^\text{H}\\ \mathbf{B}_3 &\triangleq \mathbf{B}_1+\mathbf{B}_2=\mathbf{R}_{\mathbf{y}_\text{r}}\\ \mathcal{H} &\triangleq w_1(\bar{\mathbf{h}}_2\bar{\mathbf{h}}_3)^\text{H}+w_2(\bar{\mathbf{h}}_1\bar{\mathbf{h}}_4)^\text{H}\end{aligned} \quad (13)$$

Using $\text{vec}(\mathbf{ABC})=(\mathbf{C}^\text{T}\otimes\mathbf{A})\text{vec}(\mathbf{B})$, $\text{vec}(\mathbf{F})$ can be solved to be

$$\begin{aligned}\text{vec}(\mathbf{F})=&(\mathbf{B}_1^\text{T}\otimes\mathbf{A}_1+\mathbf{B}_2^\text{T}\otimes\mathbf{A}_1^\text{T}\\&+\mathbf{B}_3^\text{T}\otimes\mathbf{A}_2+\mathbf{R}_{\mathbf{y}_\text{r}}^\text{T}\otimes\mu\mathbf{I})^{-1}\text{vec}(\mathcal{H})\end{aligned} \quad (14)$$

According to (14), the only unknown parameter in the formulation of $\mathbf{F}$ is the Lagrange multiplier $\mu$. Based on KKT conditions, there are two possible cases for $\mu$: (1) $\mu=0$, (2) $\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})=P_\text{r}^{\max}$.

Notice that $\mu=0$ means that the constraint is not active. But the power constraint is always active from physical understanding, so case (1) can not be satisfied. Then the key problem becomes how to find $\mu$ that satisfies $\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})=P_\text{r}^{\max}$. Based on (14), $\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})$ is a function of $\mu$, which can be reformulated as

$$\begin{aligned}\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})&=\text{vec}^\text{H}(\mathbf{F})\left(\mathbf{R}_{\mathbf{y}_\text{r}}^\text{T}\otimes\mathbf{I}\right)\text{vec}(\mathbf{F})\\&=\text{Tr}\left[\left(\mathbf{R}_{\mathbf{y}_\text{r}}^\text{T}\otimes\mathbf{I}\right)\text{vec}(\mathbf{F})\text{vec}^\text{H}(\mathbf{F})\right]\\&\triangleq g(\mu).\end{aligned} \quad (15)$$

In the Appendix it is proved that $g(\mu)$ is a monotonically decreasing function of $\mu$.

Therefore, $\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})=P_\text{r}^{\max}$ can be solved via a bisection search. For a fixed $\mu$, a corresponding $\mathbf{F}$ can be got. And we can also easily get the relay transmitted power. If $\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})\le P_\text{r}^{\max}$, it shows that the relay power isn't used sufficiently. If $\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})\ge P_\text{r}^{\max}$, it shows that the relay transmitted power goes beyond the power constraint. By adjusting $\mu$, $\mathbf{F}$ can be solved. The algorithm is shown in the following table.

| Algorithm 1 |
| --- |
| **Input**: $\mathbf{h}_i,\mathbf{R}_\mathbf{v},\mathbf{\Psi}_i$.<br>**Output**: $\mathbf{F},\mu$ |
| **1. Initialize**: $\mu_{min}=0,\mu_{\max}=\mu^{\max}$<br>**2. While** $(\lvert\mu_{\max}-\mu_{\min}\rvert\ge\varepsilon)$<br>1). Set $\mu=\frac{1}{2}(\mu_{\max}+\mu_{\min})$;<br>2). Calculate $\mathbf{F}$ using (14);<br>flag $=\text{Tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\text{r}}\mathbf{F}^\text{H})-P_\text{r}^{\max}$;<br>3). Update $\mu_{\max},\mu_{\min}$:<br>(1)if flag $>0,\mu_{\min}=\mu$;<br>(2)if flag $\le 0,\mu_{\max}=\mu,\mathbf{F}_\text{opt}=\mathbf{F}$;<br>4). end while |

## IV. Reciprocal Channel

In this section, we consider the case that the channels are reciprocal, meaning $\mathbf{h}_3=\mathbf{h}_1^\text{T},\mathbf{h}_4=\mathbf{h}_2^\text{T}$, which usually holds for time-division-duplex(TDD) system.

Taking source node 1 as an example, the received signal (6) can be re-written as

$$\begin{aligned}y_1=&(\bar{\mathbf{h}}_1+\mathbf{\Delta h}_1)^\text{T}\mathbf{F}(\bar{\mathbf{h}}_2+\mathbf{\Delta h}_2)s_2\\&+(\bar{\mathbf{h}}_1+\mathbf{\Delta h}_1)^\text{T}\mathbf{F}[(\bar{\mathbf{h}}_1+\mathbf{\Delta h}_1)s_1+\mathbf{v}]+n_1\end{aligned} \quad (16)$$

According to (16), the 1st source node's MSE is

$$\begin{aligned}\text{MSE}_1=&\mathbb{E}[(y_1-\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\bar{\mathbf{h}}_1s_1-s_2)\\&\qquad(y_1-\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\bar{\mathbf{h}}_1s_1-s_2)^\text{H}]\\=&1-(\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\bar{\mathbf{h}}_2)^\text{H}-\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\bar{\mathbf{h}}_2+\sigma_{n_1}^2+\left|\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\bar{\mathbf{h}}_2\right|^2\\&+\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\mathbf{\Psi}_\mathbf{v}(\bar{\mathbf{h}}_1^\text{T}\mathbf{F})^\text{H}+\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\mathbf{\Psi}_1(\bar{\mathbf{h}}_1^\text{T}\mathbf{F})^\text{H}\\&+\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\mathbf{\Psi}_2(\bar{\mathbf{h}}_1^\text{T}\mathbf{F})^\text{H}+\text{tr}[\mathbf{\Psi}_1(\mathbf{F}\bar{\mathbf{h}}_2)^*(\mathbf{F}\bar{\mathbf{h}}_2)^\text{T}]\\&+\text{tr}[\mathbf{\Psi}_1(\mathbf{F}\bar{\mathbf{h}}_1)^*(\mathbf{F}\bar{\mathbf{h}}_1)^\text{T}]+\text{tr}(\mathbf{F}\mathbf{\Psi}_2\mathbf{F}^\text{H}\mathbf{\Psi}_1)\\&+\text{tr}(\mathbf{F}\mathbf{\Psi}_\mathbf{v}\mathbf{F}^\text{H}\mathbf{\Psi}_1)+\text{tr}(\mathbf{\Psi}_1\mathbf{F}^*\mathbf{\Psi}_1\mathbf{F}^\text{T})\\&+\bar{\mathbf{h}}_1^\text{T}\mathbf{F}\mathbb{E}[\mathbf{\Delta h}_1(\mathbf{F}\bar{\mathbf{h}}_1)^\text{H}\mathbf{\Delta h}_1^*]\\&+\mathbb{E}[\mathbf{\Delta h}_1^\text{T}\mathbf{F}\bar{\mathbf{h}}_1\mathbf{\Delta h}_1^\text{H}](\bar{\mathbf{h}}_1^\text{T}\mathbf{F})^\text{H}\\&+\mathbb{E}[\mathbf{\Delta h}_1^\text{T}\mathbf{F}\mathbf{\Delta h}_1\mathbf{\Delta h}_1^\text{H}\mathbf{F}^\text{H}\mathbf{\Delta h}_1^*]\end{aligned} \quad (17)$$

Similarly the mean square error of the 2nd source node can be formulated as

$$\begin{aligned}\text{MSE}_2=&\mathbb{E}[(y_2-\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\bar{\mathbf{h}}_2s_2-s_1)\\&\qquad(y_2-\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\bar{\mathbf{h}}_21s_2-s_1)^\text{H}]\\=&1-(\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\bar{\mathbf{h}}_1)^\text{H}-\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\bar{\mathbf{h}}_1+\sigma_{n_2}^2+\left|\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\bar{\mathbf{h}}_1\right|^2\\&+\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\mathbf{\Psi}_\mathbf{v}(\bar{\mathbf{h}}_2^\text{T}\mathbf{F})^\text{H}+\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\mathbf{\Psi}_2(\bar{\mathbf{h}}_2^\text{T}\mathbf{F})^\text{H}\\&+\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\mathbf{\Psi}_1(\bar{\mathbf{h}}_2^\text{T}\mathbf{F})^\text{H}+\text{tr}[\mathbf{\Psi}_2(\mathbf{F}\bar{\mathbf{h}}_1)^*(\mathbf{F}\bar{\mathbf{h}}_1)^\text{T}]\\&+\text{tr}[\mathbf{\Psi}_2(\mathbf{F}\bar{\mathbf{h}}_2)^*(\mathbf{F}\bar{\mathbf{h}}_2)^\text{T}]+\text{tr}(\mathbf{F}\mathbf{\Psi}_1\mathbf{F}^\text{H}\mathbf{\Psi}_2)\\&+\text{tr}(\mathbf{F}\mathbf{\Psi}_\mathbf{v}\mathbf{F}^\text{H}\mathbf{\Psi}_2)+\text{tr}(\mathbf{\Psi}_2\mathbf{F}^*\mathbf{\Psi}_2\mathbf{F}^\text{T})\\&+\bar{\mathbf{h}}_2^\text{T}\mathbf{F}\mathbb{E}[\mathbf{\Delta h}_2(\mathbf{F}\bar{\mathbf{h}}_2)^\text{H}\mathbf{\Delta h}_2^*]\\&+\mathbb{E}[\mathbf{\Delta h}_2^\text{T}\mathbf{F}\bar{\mathbf{h}}_2\mathbf{\Delta h}_2^\text{H}](\bar{\mathbf{h}}_2^\text{T}\mathbf{F})^\text{H}\\&+\mathbb{E}[\mathbf{\Delta h}_2^\text{T}\mathbf{F}\mathbf{\Delta h}_2\mathbf{\Delta h}_2^\text{H}\mathbf{F}^\text{H}\mathbf{\Delta h}_2^*]\end{aligned} \quad (18)$$

The beamforming design alogorithm used here is similar with that of non-reciprocal case. At first, the MMSE optimization problem can be formulated as

$$\begin{aligned} \min \quad & \sum_{i=1}^{2} w_i \mathrm{MSE}_i \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\mathrm{r}}\mathbf{F}^\mathrm{H}) \le \mathrm{P}_\mathrm{r}^{\max} \end{aligned} \tag{19}$$

Then the Lagrangian dual function of the optimization problem (19) is

$$\mathcal{L}(\mathbf{F},\mu) = \sum_{i=1}^{2} w_i \mathrm{MSE}_i + \lambda[\mathrm{tr}(\mathbf{F}\mathbf{R}_{\mathbf{y}_\mathrm{r}}\mathbf{F}^\mathrm{H}) - \mathrm{P}_\mathrm{r}^{\max}] \tag{20}$$

where $\lambda$ is the Lagrangian multiplier. Using the technique of complex matrix derivative, the derivative of the Lagrangian dual function (20) is

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathbf{F}^*} =& -\mathcal{M} + w_1 \bar{\mathbf{h}}_1^* \bar{\mathbf{h}}_1^\mathrm{T} \mathbf{F}\mathbf{C}_1 + w_1 \mathbf{\Psi}_1^\mathrm{T} \mathbf{F}\mathbf{C}_2 \\ &+ w_1 \mathbf{\Psi}_1 \mathbf{F}\mathbf{C}_3 + w_2 \bar{\mathbf{h}}_2^* \bar{\mathbf{h}}_2^\mathrm{T} \mathbf{F}\mathbf{D}_1 + w_2 \mathbf{\Psi}_2^\mathrm{T} \mathbf{F}\mathbf{D}_2 \\ &+ w_2 \mathbf{\Psi}_2 \mathbf{F}\mathbf{D}_3 + \lambda \mathbf{F}\mathbf{R}_{\mathbf{y}_\mathrm{r}} \\ &+ w_1 \mathbb{E}[\mathbf{\Delta h}_1^* \bar{\mathbf{h}}_1^\mathrm{T} \mathbf{F} \mathbf{\Delta h}_1 \bar{\mathbf{h}}_1^\mathrm{H}] \\ &+ w_1 \mathbb{E}(\bar{\mathbf{h}}_1^* \mathbf{\Delta h}_1^\mathrm{T} \mathbf{F} \bar{\mathbf{h}}_1 \mathbf{\Delta h}_1^\mathrm{H}) \\ &+ w_1 \mathbb{E}(\mathbf{\Delta h}_1^* \mathbf{\Delta h}_1^\mathrm{T} \mathbf{F} \mathbf{\Delta h}_1 \mathbf{\Delta h}_1^\mathrm{H}) \\ &+ w_2 \mathbb{E}(\mathbf{\Delta h}_2^* \mathbf{h}_2^\mathrm{T} \mathbf{F} \mathbf{\Delta h}_2 \bar{\mathbf{h}}_2^\mathrm{H}) \\ &+ w_2 \mathbb{E}(\bar{\mathbf{h}}_2^* \mathbf{\Delta h}_2^\mathrm{T} \mathbf{F} \bar{\mathbf{h}}_2 \mathbf{\Delta h}_2^\mathrm{H}) \\ &+ w_2 \mathbb{E}(\mathbf{\Delta h}_2^* \mathbf{\Delta h}_2^\mathrm{T} \mathbf{F} \mathbf{\Delta h}_2 \mathbf{\Delta h}_2^\mathrm{H}) = 0 \end{aligned} \tag{21}$$

where $\mathbf{A}_i$'s, $\mathbf{B}_i$'s and $\mathcal{H}$ are defined as follows:

$$\begin{aligned} \mathcal{M} &\triangleq w_1 \bar{\mathbf{h}}_1^* \bar{\mathbf{h}}_2^\mathrm{H} + w_2 \bar{\mathbf{h}}_2^* \bar{\mathbf{h}}_1^\mathrm{H} \\ \mathbf{C}_1 &\triangleq \bar{\mathbf{h}}_2 \bar{\mathbf{h}}_2^\mathrm{H} + \mathbf{\Psi}_1 + \mathbf{\Psi}_2 + \mathbf{\Psi}_\mathbf{v} \\ \mathbf{C}_2 &\triangleq \bar{\mathbf{h}}_1 \bar{\mathbf{h}}_1^\mathrm{H} + \bar{\mathbf{h}}_2 \bar{\mathbf{h}}_2^\mathrm{H} + \mathbf{\Psi}_1^\mathrm{T} \\ \mathbf{C}_3 &\triangleq \mathbf{\Psi}_1 + \mathbf{\Psi}_\mathbf{v} \\ \mathbf{D}_1 &\triangleq \bar{\mathbf{h}}_1 \bar{\mathbf{h}}_1^\mathrm{H} + \mathbf{\Psi}_1 + \mathbf{\Psi}_2 + \mathbf{\Psi}_\mathbf{v} \\ \mathbf{D}_2 &\triangleq \bar{\mathbf{h}}_1 \bar{\mathbf{h}}_1^\mathrm{H} + \bar{\mathbf{h}}_2 \bar{\mathbf{h}}_2^\mathrm{H} + \mathbf{\Psi}_2^\mathrm{T} \\ \mathbf{D}_3 &\triangleq \mathbf{\Psi}_2 + \mathbf{\Psi}_\mathbf{v} \end{aligned} \tag{22}$$

Using $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^\mathrm{T} \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$, $\mathrm{vec}(\mathbf{F})$ can be solved to be

$$\begin{aligned} \mathrm{vec}(\mathbf{F}) =& \{\mathbf{C}_1^\mathrm{T} \otimes (w_1 \bar{\mathbf{h}}_1^* \bar{\mathbf{h}}_1^\mathrm{T}) + \mathbf{C}_2^\mathrm{T} \otimes (w_1 \mathbf{\Psi}_1^\mathrm{T}) \\ &+ \mathbf{C}_3^\mathrm{T} \otimes (w_1 \mathbf{\Psi}_1) + \mathbf{D}_1^\mathrm{T} \otimes (w_2 \bar{\mathbf{h}}_2^* \bar{\mathbf{h}}_2^\mathrm{T}) \\ &+ \mathbf{D}_2^\mathrm{T} \otimes (w_2 \mathbf{\Psi}_2^\mathrm{T}) + \mathbf{D}_3^\mathrm{T} \otimes (w_2 \mathbf{\Psi}_2) \\ &+ \mathbf{R}_{\mathbf{y}_\mathrm{r}}^\mathrm{T} \otimes (\lambda \mathbf{I}) + \mathbb{E}[(\bar{\mathbf{h}}_1^* \mathbf{\Delta h}_1^\mathrm{T}) \otimes (\mathbf{\Delta h}_1^* \bar{\mathbf{h}}_1^\mathrm{T})] \\ &+ \mathbb{E}[(\mathbf{\Delta h}_1^* \bar{\mathbf{h}}_1^\mathrm{T}) \otimes (\bar{\mathbf{h}}_1^* \mathbf{\Delta h}_1^\mathrm{T}) \\ &+ \mathbb{E}[(\mathbf{\Delta h}_1^* \mathbf{\Delta h}_1^\mathrm{T}) \otimes (\mathbf{\Delta h}_1^* \mathbf{\Delta h}_1^\mathrm{T})] \\ &+ \mathbb{E}[(\bar{\mathbf{h}}_2^* \mathbf{\Delta h}_2^\mathrm{T}) \otimes (\mathbf{\Delta h}_2^* \bar{\mathbf{h}}_2^\mathrm{T})] \\ &+ \mathbb{E}[(\mathbf{\Delta h}_2^* \mathbf{h}_2^\mathrm{T}) \otimes (\bar{\mathbf{h}}_2^* \mathbf{\Delta h}_2^\mathrm{T}) \\ &+ \mathbb{E}[(\mathbf{\Delta h}_2^* \mathbf{\Delta h}_2^\mathrm{T}) \otimes (\mathbf{\Delta h}_2^* \mathbf{\Delta h}_2^\mathrm{T})]\}^{-1} \\ & \mathrm{vec}(\mathcal{M}) \end{aligned} \tag{23}$$

The beamforming matrix and the multiplier here can also be obtained through a bisection search, and search algorithm is almost same with algorithm 1 for non-reciprocal channel. The difference between them only lies in the formula of calculating $\mathbf{F}$, which is (14) for non-reciprocal case and is (23) for reciprocal case. Obviously, the calculating procedure for reciprocal channel is much more complicated.

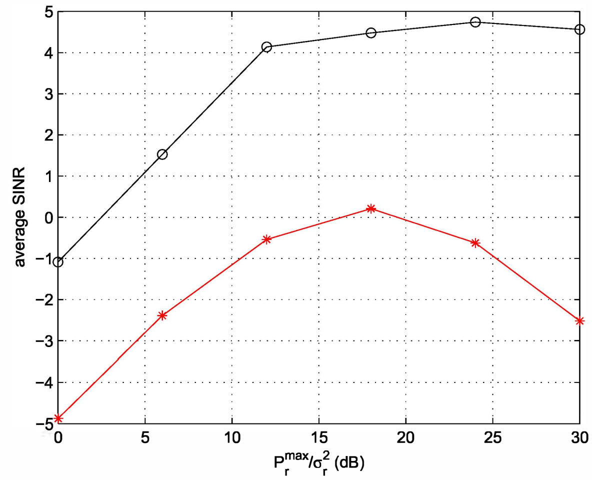


Fig. 2. Average SINR of different algorithms for non-reciprocal channel. robust(black), non-robust(red)

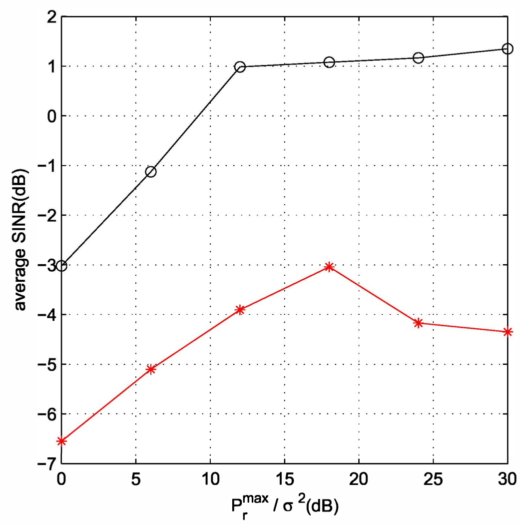


Fig. 3. Average SINR of different algorithm for reciprocal channel. robust(black), non-robust(red

## V. Simulation Results and Discussions

In this section, simulation results are presented to assess the performance of the proposed algorithm and the algorithm without taking the channel errors into account for the purpose of comparison. In the simulation setting, the relay has four antennas and each source node has single antenna. The noises at the relay and source nodes are assumed to be $\sigma_\mathrm{r}^2 = \sigma_{n_1}^2 = \sigma_{n_2}^2 = \sigma_{n_3}^2 = \sigma_{n_4}^2 = 0.01W$. The maximal power of the relay is set as dB. Furthermore, for simplicity $w_1 = w_2 = w_3 = w_4 = 1$. The real channels are randomly generated according to i.i.d. Gaussian distribution, and channel errors are generated from independent Gaussian random variables with zero mean and variance related to antenna spatial correlation. Each point in the following figure is an average of 10000 independent channel realizations.

Figure 2 and figure 3 show average SINR of different algorithms including the proposed robust design and its counterpart based on perfect CSI for non-reciprocal channel and reciprocal respectively. It can be seen that the performance of the proposed robust design is significantly better than that of the design without taking channel errors into consideration. It reveals the importance of rubus beamforming design for two-way AF MIMO relaying networks whenever channel has reciprocality or not. Furthermore, since the proposed algorithm has closed solution for computing beamforming matrix, it has some advantages in terms of implementing.

## VI. Conclusions

In this paper, robust beamforming design for two-way AF MIMO relaying networks is investigated. With Gaussian distributed channel errors, beamforming algorithms which aim at minimizing MSE of the detected signals at the source nodes are proposed, for FDD systems and TDD systems respectively. Both problems can be solved based on the fact that for quadratic programming problems, it can be solved by searching a Lagrange multiplier since there is only one constraint [8]. And from the given solution, it is apparent that the algorithm when channel has reciprocality is much more complicated. Finally, the simulation result demonstrated the performance advantage of our robust design.

## Appendix

Defining the following matrices

$$\begin{aligned}\mathbf{A} &\triangleq \sum_i \mathbf{A}_i \otimes \mathbf{h}_i^* \mathbf{h}_i^{\mathrm{T}},\\ \mathbf{B} &\triangleq \mathbf{R}_{\mathbf{y}_r}^{\mathrm{T}} \otimes \mathbf{I},\\ \mathbf{M} &\triangleq \mathrm{vec}(\boldsymbol{\mathcal{H}})\mathrm{vec}^{\mathrm{H}}(\boldsymbol{\mathcal{H}}),\end{aligned} \tag{24}$$

the function $g(\mu)$ can be written as

$$g(\mu) = \mathrm{Tr}[(\mathbf{A}+\mu\mathbf{B})^{-1}\mathbf{B}(\mathbf{A}+\mu\mathbf{B})^{-1}\mathbf{M}]. \tag{25}$$

As $\mathbf{B}$ is positive definite matrix, the following equality holds

$$\begin{aligned}&(\mathbf{A}+\mu\mathbf{B})^{-1}\mathbf{B}(\mathbf{A}+\mu\mathbf{B})^{-1}\\ =&\mathbf{B}^{-1/2}(\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu\mathbf{I})^{-2}\mathbf{B}^{-1/2},\end{aligned} \tag{26}$$

based on which $g(\mu)$ can be reformulated as

$$g(\mu) = \mathrm{Tr}[\mathbf{B}^{-1/2}(\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu\mathbf{I})^{-2}\mathbf{B}^{-1/2}\mathbf{M}]. \tag{27}$$

In order to prove that $g(\mu)$ is a monotonically decreasing function with respect to $\mu$, we assume that $\mu_1 \geq \mu_2$ and in the following we will prove that $g(\mu_1) \leq g(\mu_2)$.When $\mu_1 \geq \mu_2$, based on the fact that $\mathbf{A}$ is a positive semi-definite matrix we have

$$(\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu_1\mathbf{I}) \geq (\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu_2\mathbf{I}), \tag{28}$$

based on which, taking inversion of both sides, the following inequality holds

$$(\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu_1\mathbf{I})^{-1} \leq (\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu_2\mathbf{I})^{-1}. \tag{29}$$

Therefore, following a similar logic in [11] and together with the fact that $\mathbf{B}^{1/2}$ and $\mathbf{M}$ are Hermitian matrices, it is concluded that

$$\begin{aligned}&\mathrm{Tr}[\mathbf{B}^{-1/2}(\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu_1\mathbf{I})^{-2}\mathbf{B}^{-1/2}\mathbf{M}]\\ &\leq \mathrm{Tr}[\mathbf{B}^{-1/2}(\mathbf{B}^{-1/2}\mathbf{A}\mathbf{B}^{-1/2}+\mu_2\mathbf{I})^{-2}\mathbf{B}^{-1/2}\mathbf{M}].\end{aligned} \tag{30}$$

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