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A Robust Attitude Controller and its Application to Quadrotor Helicopters

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Abstract: In this paper, a novel nonlinear hierarchical controller for attitude control is proposed. This controller is obtained using Lyapunov methodology. Model uncertainties in the system are estimated on-line based on a time-delay control approach. The robustness of the flight controller is enhanced using an anti-windup integrator technique and semi-global asymptotical stability is proven. The control law obtained is simple enough for an implementation on a small microcontroller. Simulation results for a model of a quadrotor helicopter illustrate the performance of the proposed control algorithm.

Keywords: Nonlinear control, Hierarchical control, Attitude control, Lyapunov stability, Robust performance, Anti-windup integrator.

1. INTRODUCTION

Flight control design for quadrotors is currently very popular in the control and robotics community. A quadrotor has a compact form and is a hover-capable vehicle, which makes such a flying platform an ideal candidate for inspection and surveillance applications. However, it is also well known that a quadrotor is naturally an unstable system Pounds et al. (2010), Bristeau et al. (2009), Guenard (2007). Therefore, an active control structure has to be implemented for its attitude and motion control. This paper focuses on attitude control. A review of different attitude representations can be found in Shuster (1993). A wide variety of attitude controllers already exists in the literature. A control approach based on nested integrators is presented in Castillo et al. (2004). In Joshi et al. (1995), a system parameter independent control approach is presented whereas in Thienel and Sanner (2003) a coupled attitude estimation and control algorithm is presented. The work in Tayebi and McGilvray (2006) proposes a quaternion-based feedback scheme, which compensates the coriolis and gyroscopic torques in the attitude stabilization of a quadrotor. A model-independent PD controller is used and provides asymptotic stability. The work in Tayebi (2008) presents a quaternion-based dynamic output feedback for the attitude tracking problem of rigid body without velocity measurement. In that scheme, the control law is a pure quaternion feedback and the torques are bounded by the control gains. An almost global asymptotic stability is shown. In Pounds et al. (2007), a nonlinear attitude stabilization scheme is presented that combines attitude and gyroscopes bias estimation with attitude control. Although many linear and nonlinear attitude controllers have been investigated in the past, very few of them explicitly address the robustness issue with respect to physical parameter uncertainties and external disturbances.

The nonlinear controller of this paper is designed not only to provide almost-global asymptotic stability of the attitude of the UAV about its reference signal, but also to compensate for constant modeling uncertainties and external disturbances. Indeed, it is very difficult in practice to identify the physical parameters of the vehicle, like the inertia matrix. Moreover, aerodynamic effects are hard to model, and during the flight, there is no direct measurement of the aerodynamic torques acting on the dynamics of the unmanned aerial vehicle (UAV). Therefore, the robustness of the flight controller against parametric modeling uncertainties and external disturbances is addressed by the novel combination of the following two techniques :

- First, a time-delay approach is used to estimate on-line the parametric uncertainties and slow time-varying external disturbances. Time-delay techniques have been successfully implemented in the field of robotic arm manipulation Chang and Jung (2009), Youcef-Toumi and Ito (1990), but to the knowledge of the authors it has not been employed yet in the context of UAVs.
- Second, a novel bounded anti-windup integrator is added to deal with static errors in the attitude control loop. The attitude controller finally obtained is simple enough to be implemented on a small microcontroller.

In this paper, the attitude of the flying vehicle is described by the rotation matrix from body-fixed frame to inertial frame, thus removing the issue of singularities commonly encountered with Euler angles parametrization. This paper is structured into three sections. In section 2 the dynamical model of the system is introduced, and in section 3 the new control algorithm is designed. Finally, in section 4 simulation results show the performance of the proposed control algorithm.

2. SYSTEM MODELING

The quadrotor is considered to be a rigid-body vehicle, whose rotational dynamics satisfies

$$\dot{R} = Rsk(\omega) \tag{1}$$
$$J\dot{\omega} = -sk(\omega)J\omega + \Gamma_m + \Gamma_{ae} + d \tag{2}$$

where the attitude $R \in \mathrm{SO}(3)$ is a rotation matrix of the body-fixed frame $\mathcal{B}$ relative to the inertial frame $\mathcal{I}$; $\omega \in \mathbb{R}^3$ represents the angular velocity, expressed in $\mathcal{B}$, of the body-fixed frame $\mathcal{B}$ w.r.t. the inertial frame $\mathcal{I}$; J represents the vehicle's inertia matrix; the actuator moment control input vector is $\Gamma_m \in \mathbb{R}^3$; and the unknown aerodynamic moments and disturbances are $\Gamma_{ae} \in \mathbb{R}^3$ and $d \in \mathbb{R}^3$, respectively. The notation $sk(.)$ denotes the skew-symmetric matrix operator, i.e., $sk(u)v = u \times v, \forall u, v \in \mathbb{R}^3$.

Assume that only an estimate $\bar{J}$ of the inertia matrix J is known. The system (1)–(2) can be rewritten as

$$\dot{R} = Rsk(\omega) \tag{3}$$
$$\bar{J}\dot{\omega} = -sk(\omega)\bar{J}\omega + \Gamma_m + U(\omega, \Gamma_m, t) \tag{4}$$

with

$$\begin{aligned} U(\omega, \Gamma_m, t) :=& sk(\omega)\bar{J}\omega - \bar{J}J^{-1}sk(\omega)J\omega \\ &+ (\bar{J}J^{-1} - I)\Gamma_m + \bar{J}J^{-1}(\Gamma_{ae} + d)\,. \end{aligned} \tag{5}$$

The term $U(\omega, \Gamma_m, t)$ defined by (5) is generally unknown and acts as a disturbance to the dynamics of the vehicle's angular velocity.

3. CONTROL DESIGN

3.1 Control Objective

The control objective consists in stabilizing the vehicle attitude to a reference attitude R_r whose dynamics satisfies

$$\dot{R}_r = R_r sk(\omega_r), \tag{6}$$

where the reference angular velocity vector ω_r and its time-derivative $\dot{\omega}_r$ are known and bounded. The control objective can be interpreted as the stabilization the attitude error $\tilde{R} := R_r^\top R$ about the identity matrix I_3. From (3) and (6) one verifies that

$$\dot{\tilde{R}} = \tilde{R}sk(\omega) - sk(\omega_r)\tilde{R}\,. \tag{7}$$

3.2 Uncertainty Estimation via Time-delay Approach

The Time-Delay approach (see Youcef-Toumi and Ito (1990), Chang and Jung (2009)) is used to estimate the term $U(\omega, \Gamma_m, t)$. It is assumed that the derivative of the angular velocity $\dot{\omega}(t-l)$ at a small time delay l and the control input Γ_m are known. Using (4) the unknown term $U(\omega(t-l), \Gamma_m(t-l), t-l)$ at time $t-l$ can be calculated as

$$\begin{aligned} &U(\omega(t-l), \Gamma_m(t-l), t-l) = \\ &\quad \bar{J}\dot{\omega}(t-l) - \Gamma_m(t-l) + sk(\omega(t-l))\bar{J}\omega(t-l)\,. \end{aligned}$$

The term $\dot{\omega}(t-l)$ is calculated as

$$\dot{\omega}(t-l) = \frac{\omega(t-l) - \omega(t-2l)}{l}\,.$$

It is assumed that within the time period l, the unknown term has not changed much, and therefore, the following approximation holds

$$U(\omega(t-l), \Gamma_m(t-l), t-l) \approx U(\omega(t), \Gamma_m(t), t).$$

Since the derivative of the angular velocity is not measured directly, it has to be calculated out of the angular velocity measurement. Therefore, the unknown term is noisy and must be filtered. A simple first-order low-pass filter can be used and an estimate of the unknown term is obtained as follows

$$\begin{cases} \dot{\hat{U}} = \dfrac{1}{\tau}(\upsilon - \hat{U}) \\ \upsilon = \bar{J}\dot{\omega}_{t-l} + sk(\omega_{t-l})\bar{J}\omega_{t-l} - \Gamma_{m_{t-l}} \end{cases} \tag{8}$$

where τ denotes some positive filter constant.

Furthermore, the error between the unknown term U and its estimate $\hat{U}$ is defined by

$$\tilde{U} := U - \hat{U}. \tag{9}$$

From (8) and (9), it is straightforward to show that the norm of the estimation error defined in (9) is ultimately bounded by $|\tilde{U}| \leq \tau \sup|\dot{U}|$. The estimate $\hat{U}$ of the unknown term is then taken into account in the control moment so as to compensate for the unknown disturbance U, since the angular velocity dynamics are fully actuated.

3.3 The Control Law

Theorem 1. *Consider system (3)–(6). Assume that the unknown perturbation term $\tilde{U}$ defined by (9) is constant and bounded by a known value $\bar{\epsilon} > 0$. Define an anti-windup integrator z solution to the following differential equation*

$$\begin{cases} \dot{z} = -\kappa_{z_1} z + sat_{\Delta_1}(\kappa_{z_1} z + \kappa_{z_2} sat_{\Delta_2}(Q_{\bar{J}}(\omega - \omega_d))) \\ z(0) = 0 \end{cases} \tag{10}$$

where $\kappa_{z_1}, \kappa_{z_2}$ are some positive gains; Δ_1, Δ_2 are some positive constants associated with the classical saturation function $sat_\Delta(\cdot)$ defined by

$$sat_\Delta(x) := x \min(1, \Delta/|x|), \forall x \in \mathbb{R}^3;$$

$Q_{\bar{J}}$ satisfies $Q_{\bar{J}}^\top Q_{\bar{J}} = \bar{J}$; and

$$\omega_d := \omega_r - K_d \frac{vex(\Pi_a(\tilde{R}))}{(1 + tr(\tilde{R}))^2}, \tag{11}$$

with K_d a diagonal positive matrix gain, $\Pi_a(\tilde{R}) := \frac{\tilde{R} - \tilde{R}^T}{2}$ the anti-symmetric part of $\tilde{R}$, and $vex(.)$ the inverse operator of the $sk(.)$ operator. Apply the control law

$$\Gamma_m = \bar{\Gamma}_m - \hat{U}\,, \tag{12}$$
$$\bar{\Gamma}_m = sk(\omega_d)\bar{J}\omega + \bar{J}\dot{\omega}_d - \sigma(\omega - \omega_d) - k_z Q_{\bar{J}}^\top z\,, \tag{13}$$

with $\sigma(.)$ an increasing bounded function satisfying $\sigma(0) = 0$ and k_z a positive gain, to system (3)–(6). If

$$\Delta_1 \geq \frac{\kappa_{z_1}}{k_z} \|(Q_{\bar{J}}^\top)^{-1}\| \bar{\epsilon} + \kappa_{z_2}\Delta_2\,, \tag{14}$$

then the equilibrium

$$(\tilde{R}, \omega, z) = (I_3, \omega_r, \frac{1}{k_z}(Q_{\bar{J}}^\top)^{-1}\tilde{U})$$

of the composite system (7)+(4)+(10) is asymptotically stable, with domain of attraction equal to $\mathbb{U} \times \mathbb{R}^3 \times \mathbb{R}^3$ with

$$\mathbb{U} := \{\tilde{R} \in SO(3) \mid tr(\tilde{R}) \neq -1\}.$$

Proof: The system (4)–(7) can be rewritten with the control input defined in (12) as

$$\dot{\tilde{R}} = \tilde{R}sk(\omega) - sk(\omega_r)\tilde{R} \tag{15}$$
$$\bar{J}\dot{\omega} = -sk(\omega)\bar{J}\omega + \bar{\Gamma}_m + \tilde{U} \tag{16}$$

To prove the stability of the reformulated system (15)–(16) consider the following storage function

$$V_1 = \frac{1}{2} tr(I - \tilde{R}). \tag{17}$$

From (15) and (17) one verifies that

$$\begin{aligned}
\dot{V}_1 &= -\frac{1}{2} tr(\tilde{R} sk(\omega) - sk(\omega_r)\tilde{R}) \\
&= -\frac{1}{2} tr(\tilde{R} sk(\omega - \omega_r)) \\
&= -\frac{1}{2} tr(\Pi_a(\tilde{R}) sk(\omega - \omega_r)) \\
&= vex(\Pi_a(\tilde{R}))^\top (\omega - \omega_r) \\
&= vex(\Pi_a(\tilde{R}))^\top (\omega_d - \omega_r) + vex(\Pi_a(\tilde{R}))^\top \tilde{\omega} \\
&= -\frac{vex(\Pi_a(\tilde{R}))^\top K_d vex(\Pi_a(\tilde{R}))}{(1 + tr(\tilde{R}))^2} + vex(\Pi_a(\tilde{R}))^\top \tilde{\omega}
\end{aligned} \tag{18}$$

with $\tilde{\omega} = \omega - \omega_d$. In what follows, we will prove firstly that the control law (13) stabilizes $\tilde{\omega}$ about zero. Then, we will apply the singular perturbation theorem on equations (17)–(18) to prove that $\tilde{R}$ converges to I_3.

From (16), (13) and the definition of $\tilde{\omega}$ one obtains

$$\begin{aligned}
\bar{J}\dot{\tilde{\omega}} &= -sk(\tilde{\omega} + \omega_d)\bar{J}\omega - \bar{J}\dot{\omega}_d + \bar{\Gamma}_m + \tilde{U} \\
&= -sk(\tilde{\omega})\bar{J}\omega - \sigma(\tilde{\omega}) - k_z Q_{\bar{J}}^\top \tilde{z},
\end{aligned} \tag{19}$$

with $\tilde{z} := z - z^*$ and $z^* = \frac{1}{k_z}(Q_{\bar{J}}^\top)^{-1}\tilde{U}$. Using (10) one deduces

$$\dot{\tilde{z}} = -\kappa_{z_1}(\tilde{z} + z^*) + \mathrm{sat}_{\Delta_1}(\kappa_{z_1}(\tilde{z} + z^*) + \kappa_{z_2}\mathrm{sat}_{\Delta_2}(Q_{\bar{J}}\tilde{\omega})) \tag{20}$$

Introduce the following positive definite function

$$L(\tilde{\omega}) := \begin{cases} \frac{1}{2}\tilde{\omega}^\top \bar{J}\tilde{\omega}, & \text{if } |Q_{\bar{J}}\tilde{\omega}| \le \Delta_2, \\ |Q_{\bar{J}}\tilde{\omega}|\,\Delta_2 - \frac{1}{2}\Delta_2^2, & \text{otherwise.} \end{cases}$$

and consider the following candidate Lyapunov function

$$V_2 = \frac{\kappa_{z_2}}{k_z} L(\tilde{\omega}) + \frac{1}{2}|\tilde{z}|^2. \tag{21}$$

From (21), (19), and (20) one verifies that

$$\begin{aligned}
\dot{V}_2 &= \frac{\kappa_{z_2}}{k_z} \min\left(1, \frac{\Delta_2}{|Q_{\bar{J}}\tilde{\omega}|}\right) \tilde{\omega}^\top \bar{J}\dot{\tilde{\omega}} + \tilde{z}\dot{\tilde{z}} \\
&= -\frac{\kappa_{z_2}}{k_z} \min\left(1, \frac{\Delta_2}{|Q_{\bar{J}}\tilde{\omega}|}\right) \tilde{\omega}^\top \sigma(\tilde{\omega}) - \kappa_{z_1}|\tilde{z}|^2 \\
&\quad + \tilde{z}^\top(-\kappa_{z_1}z^* - \kappa_{z_2} sat_{\Delta_2}(Q_{\bar{J}}\tilde{\omega}) + sat_{\Delta_1}(\kappa_{z_1}(\tilde{z} + z^*) \\
&\quad + \kappa_{z_2} sat_{\Delta_2}(Q_{\bar{J}}\tilde{\omega}))) \\
&\le -\frac{\kappa_{z_2}}{k_z} \min\left(1, \frac{\Delta_2}{|Q_{\bar{J}}\tilde{\omega}|}\right) \tilde{\omega}^\top \sigma(\tilde{\omega}),
\end{aligned} \tag{22}$$

where the last inequality is obtained using condition (14) and the fact that $\forall a, b \in \mathbb{R}^3$, $a^\top b \le |a|\,|b|$, and that $|-a + sat_\Delta(b + a)| \le |b|$ if $|a| \le \Delta$ (see e.g. Hua et al. (2009), Hua (2009) for the proof). Clearly, $\dot{V}_2$ is negative semi-definite. However, since system (19)–(20) is not autonomous (due to the time-varying term ω), La Salle's principle does not apply. The next step of the proof consists in showing that $\dot{V}_2$ is uniformly continuous along every system's solution in order to deduce, by application of Barbalat's lemma, the convergence of $\tilde{\omega}$ to zero. To this purpose it suffices to show that $\ddot{V}_2$ is bounded. For instance, since $\dot{V}_2 \le 0$ one deduces from the definition of V_2 that $\tilde{\omega}$ and $\tilde{z}$ are bounded. From (20) one deduces also that $\dot{\tilde{z}}$ is bounded. Then, in view of (22), it suffices to show that $\dot{\tilde{\omega}}$ is bounded in order to show that $\ddot{V}_2$ is bounded. Besides, in view of (19) and the boundedness of $\tilde{\omega}$, the term $\dot{\tilde{\omega}}$ is bounded if ω_d and ω are bounded. Now we will use (17), (18) and the boundedness of $\tilde{\omega}$ in order to prove the boundedness of ω_d. Let us introduce some notations. Let $\tilde{\mathbf{q}} := (\tilde{q}_0, \tilde{\mathbf{q}}_\mathbf{v})^\top$, with $q_0 \in \mathbb{R}$ the real part and $\tilde{\mathbf{q}}_\mathbf{v} \in \mathbb{R}^3$ the pure part, denote the unit quaternion (i.e. $\tilde{q}_0^2 + |\tilde{\mathbf{q}}_\mathbf{v}|^2 = 1$) associated with the rotation matrix $\tilde{R}$. From Rodrigues' formula

$$\tilde{R} = I_3 + 2q_0 sk(\tilde{\mathbf{q}}_\mathbf{v}) + 2sk(\tilde{\mathbf{q}}_\mathbf{v})^2$$

one verifies that

$$\begin{aligned}
&\mathbf{tr}(\tilde{R}) = 3 - 4|\tilde{\mathbf{q}}_\mathbf{v}|^2, \\
&\Pi_a(\tilde{R}) = 2\tilde{q}_0 sk(\tilde{\mathbf{q}}_\mathbf{v}), \mathbf{vex}(\Pi_a(\tilde{R})) = 2\tilde{q}_0\tilde{\mathbf{q}}_\mathbf{v}.
\end{aligned}$$

One verifies also that the condition $\tilde{R}(0) \in \mathbb{U}$ is equivalent to $|\tilde{q}_0(0)| > 0$. Consequently, from (11), (17), and (18) one deduces that $\omega_d = \omega_r - \frac{K_d\tilde{\mathbf{q}}_\mathbf{v}}{8\tilde{q}_0^3}$, $V_1 = 2|\tilde{\mathbf{q}}_\mathbf{v}|^2$, and

$$\dot{V}_1 = -\frac{\tilde{\mathbf{q}}_\mathbf{v}^\top K_d \tilde{\mathbf{q}}_\mathbf{v}}{4\tilde{q}_0^2} + 2\tilde{q}_0\tilde{\mathbf{q}}_\mathbf{v}^\top\tilde{\omega}. \tag{23}$$

Since $\tilde{\omega}$ is bounded, in view of (23) there exists a constant $\varepsilon > 0$ such that

$$|\tilde{q}_0| < \varepsilon \Longrightarrow \dot{V}_1 < 0.$$

Therefore, $\forall t$ one has $|\tilde{q}_0(t)| \ge \underline{\varepsilon} := \min(\varepsilon, |\tilde{q}_0(0)|) > 0$. This and the fact that ω_r is bounded imply that ω_d remains also bounded. The boundedness of ω is then straightforwardly deduced from the definition of $\tilde{\omega}$ and the boundedness (proved previously) of $\tilde{\omega}$ and ω_d. This allows to deduce the boundedness of $\ddot{V}_2$ and, subsequently, the uniform continuity of $\dot{V}_2$. Finally, the application of Barbalat's lemma allows to conclude *the convergence of* $\tilde{\omega}$ *to zero.*

Next, we will again apply Barbalat's lemma to deduce the convergence of $\dot{\tilde{\omega}}$ to zero. It suffices to show that $\dot{\tilde{\omega}}$ is uniformly continuous by showing that $\ddot{\tilde{\omega}}$ remains bounded. For instance, from (19), (11), (7), the fact that $|\tilde{q}_0| \ge \underline{\varepsilon} > 0$, the boundedness of $\dot{\omega}_d$ proven in appendix A, and the boundedness of $\tilde{\omega}$, ω, ω_d (proved previously) one easily deduces the boundedness of $\ddot{\tilde{\omega}}$. Consequently, one deduces the convergence of $\dot{\tilde{\omega}}$ to zero. Then, in view of (19) *the convergence of* $\tilde{z}$ *to zero* directly follows.

Equation (23) and the fact that $|\tilde{q}_0| \ge \underline{\varepsilon} > 0$ implies the existence of some positive constants $\alpha_{1,2}$ such that

$$\dot{V}_1 \le -\alpha_1 V_1 + \alpha_2|\tilde{\omega}|. \tag{24}$$

This relation, the boundedness of $\tilde{\omega}$ and its convergence to zero imply the convergence of V_1 to zero (by application of the singular perturbation theorem). Consequently, the convergence of $\tilde{R}$ to I_3 directly follows. This and the convergence of $\tilde{\omega}$ to zero implies the convergence of ω to ω_r. As for the stability of the equilibrium $(\tilde{R}, \omega, \tilde{z}) = (I_3, \omega_r, 0)$, it is a direct consequence of (17), (21), (24), and (22). ∎

Remark 1. Recently, anti-windup integrator techniques have been increasingly investigated in the context of non-linear control theory, see e.g. Seshagiri and Khalil (2005), Hua et al. (2009), Hua and Samson (2011), Hua et al. (2011). Our controller can incorporate various forms of anti-windup integrators. For instance, instead of using z solution to (10), one may define

$$\begin{cases} \dot{z} = -\kappa_{z_1} z + \kappa_{z_1}\mathrm{sat}_{\Delta_1}(z) + \kappa_{z_2}\mathrm{sat}_{\Delta_2}(Q_{\bar{J}}(\omega - \omega_d)) \\ z(0) = 0 \end{cases}$$

4. SIMULATION RESULTS

The performance of the new control algorithm is evaluated in simulation using MATLAB. The controller is implemented to control a quadrotor model, see (Hamel et al. (2002)), under slow varying wind conditions. Furthermore, an additional Gaussian noise is applied as a second disturbance moment with a standard deviation of 0.03 Nm.

Further, it is assumed that the real inertia matrix of the quadrotor is not perfectly known. The chosen inertia matrix for the quadrotor model is

$$J = \begin{bmatrix} 0.008546944162550 & 0.00032439779874 & 0.00066871773367 \\ 0.00032439779874 & 0.008541970497699 & 0.00035198633017 \\ 0.00066871773367 & 0.00035198633017 & 0.017225615662632 \end{bmatrix}$$

For the controller, the inertia matrix is defined as

$$\bar{J} = \begin{bmatrix} 0.008 & 0 & 0 \\ 0 & 0.008 & 0 \\ 0 & 0 & 0.017 \end{bmatrix}.$$

The controller gains are listed in table 1 and are chosen based on a pole placement technique on a linearized closed-loop system of the model and controller ((4) -(7),(8)-(13)). More details on the gains tuning are given in Appendix B.

The performance of the controller proposed in Theorem 1 with the robustness terms ($\hat{U}$, z defined in Theorem 1) shown in Fig. 2 is compared to the same controller but without the robustness terms ($\hat{U} = 0$, $z = 0$) as shown in Fig. 1.

Both controllers ensure bounded tracking of the reference attitude. Clearly, the best performance is obtained for the controller, which includes the time-delay estimation of U and anti-windup integral term z. The results shown in Fig. 2 are very satisfactory, since the reference attitude can be tracked despite the uncertainties in the inertia matrix and external disturbances.

Table 1. Controller parameters

Controller Parameters	Value
τ	0.1
k_z	50
κ_{z_1}	1
κ_{z_2}	2
k_{d_1}	4
k_{d_2}	4
k_{d_3}	1
σ_1	6
σ_2	6
σ_3	6
Δ_1	3
Δ_2	1

Figure 3 shows the disturbance signal U which is a combination of model uncertainties and external disturbance, and its estimation $\hat{U}$.

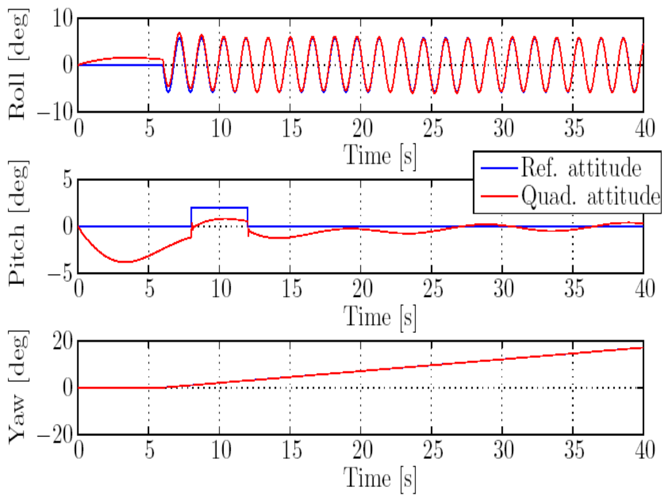

Fig. 1. Simulation of attitude control in the presence of model uncertainties and wind disturbance: without the robustness terms in the controller

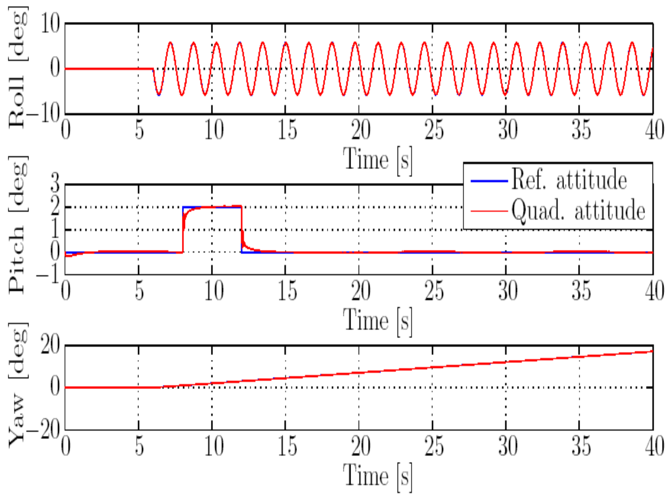

Fig. 2. Simulation of attitude control in the presence of model uncertainties and wind disturbance: with the robustness terms in the controller

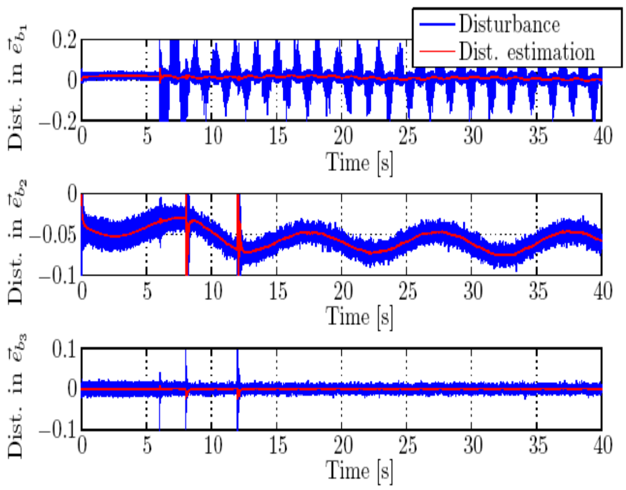

Fig. 3. Disturbance signal U and its estimation $\hat{U}$

5. CONCLUSION

In this paper, a nonlinear attitude controller is presented. The attitude of the vehicle is defined by the rotation matrix from the body frame to the inertial frame. The control objective is to track a reference attitude matrix. The proposed control law estimates model uncertainties and external disturbances using a time-delay estimation approach. Based thereon, the attitude control compensates for these disturbances. Last but not least, a novel anti-windup integrator is used to increase the robustness of the control system. A Lyapunov-based analysis proves the almost global stability of the vehicle's attitude about the reference attitude. Finally, simulation results demonstrate the effectiveness of the proposed method in the presence of model uncertainties and external disturbances.

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Appendix A. TIME DERIVATIVE OF THE DESIRED ANGULAR VELOCITY

Lemma 1. The time derivative of ω_d defined by (11) *is given by*

$$\begin{aligned}\dot{\omega}_d = \dot{\omega}_r &- \frac{3K_d(I_3 + \Pi_s(\tilde{R}))(\omega - \omega_r)}{2(1 + tr(\widetilde{R}))^2} \\ &+ \frac{K_d(\Pi_a(\tilde{R})(\omega + \omega_r) - (1 + tr(\widetilde{R}))(\omega - \omega_r))}{2(1 + tr(\widetilde{R}))^2},\end{aligned} \tag{A.1}$$

with $\Pi_s(\tilde{R}) = \frac{\tilde{R} + \tilde{R}^T}{2}$ *and* $\Pi_a(\tilde{R}) = \frac{\tilde{R} - \tilde{R}^T}{2}$.

Proof: In view of (7) the dynamics of the quaternion $\tilde{\mathbf{q}} := (\tilde{q}_0, \tilde{\mathbf{q}}_\mathbf{v})^\top$ associated with $\tilde{R}$ satisfies

$$\dot{\tilde{\mathbf{q}}} = \frac{1}{2}\tilde{\mathbf{q}} \otimes \mathbf{p}(\omega) - \frac{1}{2}\mathbf{p}(\omega_r) \otimes \tilde{\mathbf{q}},$$

with $\mathbf{p}(\omega) = (0, \omega)^\top$ or equivalently

$$\begin{cases} \dot{\tilde{q}}_0 &= -\frac{1}{2}\tilde{\mathbf{q}}_\mathbf{v}^\top(\omega - \omega_r) \\ \dot{\tilde{\mathbf{q}}}_\mathbf{v} &= \frac{1}{2}\tilde{q}_0(\omega - \omega_r) + \frac{1}{2}sk(\tilde{\mathbf{q}}_\mathbf{v})(\omega + \omega_r) \end{cases} \tag{A.2}$$

Note that (11) is equivalent to $\omega_d = \omega_r - \frac{K_d \tilde{\mathbf{q}}_\mathbf{v}}{8\tilde{q}_0^3}$. Then, one verifies

$$\dot{\omega}_d = \dot{\omega}_r - \frac{K_d(\dot{\tilde{\mathbf{q}}}_\mathbf{v}\tilde{q}_0 - 3\tilde{\mathbf{q}}_\mathbf{v}\dot{\tilde{q}}_0)}{8\tilde{q}_0^4},$$

with

$$\begin{aligned} &4(\dot{\tilde{\mathbf{q}}}_\mathbf{v}\tilde{q}_0 - 3\tilde{\mathbf{q}}_\mathbf{v}\dot{\tilde{q}}_0) \\ &= 2\tilde{q}_0{}^2(\omega - \omega_r) + 6\tilde{\mathbf{q}}_\mathbf{v}\tilde{\mathbf{q}}_\mathbf{v}^\top(\omega - \omega_r) + 2\tilde{q}_0 sk(\tilde{\mathbf{q}}_\mathbf{v})(\omega + \omega_r) \\ &= 3(I_3 + \Pi_s(\widetilde{R}))(\omega - \omega_r) - (1 + tr(\widetilde{R}))(\omega - \omega_r) \\ &\quad + \Pi_a(\tilde{R})(\omega + \omega_r) \end{aligned}$$

and

$$16\tilde{q}_0{}^4 = (1 + tr(\widetilde{R}))^2.$$

Finally, recalling (A.1), the fact that $|\tilde{q}_0| \geq \underline{\varepsilon} > 0$, and the bounded properties of ω and ω_r, it is straightforward to deduce that $\dot{\omega}_d$ is bounded.

Appendix B. GAIN SELECTION

Finding a set of gains for the controller is a tedious task. A selection of the gains by pole placement of the linearized closed-loop error system with constant reference attitude (i.e., $\omega_r = 0$) is proposed in this appendix. The dynamics of the error system is described by the quaternion equivalent of equations (15), (19) and (20) with the definition of the desired angular velocity given in (11) :

$$\begin{cases} \dot{\tilde{q}}_0 &= -\frac{1}{2}\tilde{\mathbf{q}}_{\mathbf{v}}^\top(\tilde{\omega} - \frac{K_d\tilde{\mathbf{q}}_{\mathbf{v}}}{8\tilde{q}_0^3}) \\ \dot{\tilde{\mathbf{q}}}_{\mathbf{v}} &= \frac{1}{2}\tilde{q}_0(\tilde{\omega} - \frac{K_d\tilde{\mathbf{q}}_{\mathbf{v}}}{8\tilde{q}_0^3}) + \frac{1}{2}\mathrm{sk}(\tilde{\mathbf{q}}_{\mathbf{v}})(\tilde{\omega} - \frac{K_d\tilde{\mathbf{q}}_{\mathbf{v}}}{8\tilde{q}_0^3} + 2\omega_r) \\ \bar{J}\dot{\tilde{\omega}} &= -\mathrm{sk}(\tilde{\omega})\bar{J}(\tilde{\omega} + \omega_r - \frac{K_d\tilde{\mathbf{q}}_{\mathbf{v}}}{8\tilde{q}_0^3}) - \sigma(\tilde{\omega}) - k_z Q_{\bar{J}}^T\tilde{z} \\ \dot{\tilde{z}} &= -\kappa_{z_1}(\tilde{z}+z^*)+\mathrm{sat}_{\Delta_1}(\kappa_{z_1}(\tilde{z}+z^*)+\kappa_{z_2}\mathrm{sat}_{\Delta_2}(Q_{\bar{J}}\tilde{\omega})) \end{cases}$$

The linearization of the above system around the equilibrium point $(\tilde{q}_0, \tilde{\mathbf{q}}_{\mathbf{v}}, \tilde{\omega}, \tilde{z}) = (1, 0, 0, 0)$, with $\omega_r = 0$, satisfies

$$\begin{bmatrix} \dot{\tilde{\mathbf{q}}}_{\mathbf{v}} \\ \dot{\tilde{\omega}} \\ \dot{\tilde{z}} \end{bmatrix} = \begin{bmatrix} -\frac{K_d}{16} & \frac{1}{2}I_3 & 0 \\ 0 & -\bar{J}^{-1}\frac{\partial\sigma(s)}{\partial s}\mid_{(s=0)} & -k_z\bar{J}^{-1}Q_{\bar{J}}^\top \\ 0 & \kappa_{z_2}Q_{\bar{J}} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}_{\mathbf{v}} \\ \tilde{\omega} \\ \tilde{z} \end{bmatrix} \tag{B.1}$$

Since the inertia matrix $\bar{J}$ for the quadrotor is defined as a diagonal matrix, one has $Q_{\bar{J}} = \mathrm{diag}(\sqrt{\bar{J}_{11}}, \sqrt{\bar{J}_{22}}, \sqrt{\bar{J}_{33}})$. In addition, the control matrix is chosen to be diagonal $K_d = \mathrm{diag}(k_{d_1}, k_{d_2}, k_{d_3})$, and the derivative of the bounded function with respect to its argument at zero is $\frac{\partial\sigma(s)}{\partial s}\mid_{(s=0)} = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3)$. Thus, the linearized system (B.1) can be decomposed into three independent subsystems Σ_i, $(i = 1, 2, 3)$, corresponding to the roll, pitch, and yaw channels :

$$\Sigma_i : \begin{bmatrix} \dot{\tilde{\mathbf{q}}}_{\mathbf{v_i}} \\ \dot{\tilde{\omega}}_i \\ \dot{\tilde{z}}_i \end{bmatrix} = \begin{bmatrix} -\frac{k_{d_i}}{16} & \frac{1}{2} & 0 \\ 0 & -\frac{\sigma_i}{\bar{J}_{ii}} & -\frac{k_z}{\sqrt{\bar{J}_{ii}}} \\ 0 & \kappa_{z_2}\sqrt{\bar{J}_{ii}} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}_{\mathbf{v_i}} \\ \tilde{\omega}_i^b \\ \tilde{z}_i \end{bmatrix} \tag{B.2}$$

whose characteristic polynomials are given by

$$P_i(\lambda) = \left(\lambda + \frac{k_{d_i}}{16}\right)\left(\lambda^2 + \frac{\sigma_i}{\bar{J}_{ii}}\lambda + k_z\kappa_{z_2}\right).$$

Since in the inertia matrix $\bar{J}_{11} = \bar{J}_{22}$, the gains for the roll and pitch subsystems are chosen equal, i.e., $k_{d_1} = k_{d_2}$ and $\sigma_1 = \sigma_2$ in order that the roll and pitch dynamics are the same. Then, for the roll and pitch subsystem, the three parameters $(k_{d_1}, \sigma_1, k_z\kappa_{z_2})$ can be defined according to the desired poles. The two parameters (k_{d_3}, σ_3) are chosen according to the desired poles of the yaw subsystem. Note that just two parameters can be defined to place the three poles of the yaw subsystem. Only the multiplication $k_z\kappa_{z_2}$ can be assigned with the pole placement method. Set a small value for κ_{z_2} and calculate k_z according to the defined multiplication. A small value for κ_{z_2} is desirable in order that the bound of the first saturation Δ_1 stays close to the upper bound of the unknown disturbance. It remains to assign the parameters $\kappa_{z_1}, \Delta_2, \tau$. The negative feedback gain of the bounded integrator is κ_{z_1}, and a high gain has to be chosen in order to have a fast desaturation rate. An upper bound on the integration speed of the angular velocity error is defined by Δ_2. A small value avoids undesired transient behavior. Finally, the time constant τ of the low-pass filter has to be chosen for the disturbance estimation in order to filter the noise in the calculated derivative of the angular velocity. Finally, a function σ has to be specified. Since it is proven that $\tilde{\omega}$ is bounded, even without the use of the bounded property of the function σ, the simplest way to define σ is

$$\sigma(\tilde{\omega}) = K_\sigma\tilde{\omega}\,,$$

with $K_\sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3)$. A drawback of this definition is that the upper bound of this function cannot be specified in advance. In contrast, other definitions could be used, like e.g.

$$\sigma(\tilde{\omega}) = \frac{K_\sigma\tilde{\omega}}{\sqrt{1 + \sigma_b^2\tilde{\omega}^\top\tilde{\omega}}}\,,$$

whose upper bound is given by $\frac{\|K_\sigma\|}{\sigma_b}$.