

When does a firm disclose product information?*

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Abstract

A firm chooses a price and how much product information to disclose to a consumer whose tastes are unknown to the firm. We provide a necessary and sufficient condition on the match function for full disclosure to be the unique equilibrium outcome whatever the costs and prior beliefs about product and consumer types. That condition encompasses the case where all consumer types agree on the ranking of product types' quality as in standard persuasion games, but it also allows for different consumer types to have different rankings of the potential product types. When product and consumer types are independently distributed, a necessary and sufficient condition on equilibrium payoffs is that they are at least as high as those under full disclosure for all product types; in particular, full disclosure is always an equilibrium with independent types.

KEYWORDS: Consumer heterogeneity; information certification; persuasion game; unraveling of information.

JEL CLASSIFICATION: C72; D82; L15.

1 Introduction

Much attention has been devoted to the transmission of quality information from firms to consumers. Product quality is however a very special kind of product information, since all consumers agree that higher quality products are better. Much of the relevant product information pertains to characteristics that appeal differently to different types of consumers and thus concerns the horizontal match between the buyer and the product. For instance,

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video game users usually prefer higher quality graphics, but most casual users' will not find such a higher quality so appealing if it is associated with high required skills to play the game. This paper analyzes the disclosure of certified information, allowing for horizontal match information, and investigates under what circumstances the firm will voluntarily and fully disclose product information.

We consider a monopoly seller and a buyer with unit demand. The match function indicates the consumer's valuation as a function of his own type (his privately known taste) and the type of the firm (the product characteristics). A special case is the standard "persuasion game" (Grossman, 1981, Grossman and Hart, 1980, Milgrom, 1981, Milgrom and Roberts, 1986) where all consumer types have an identical ranking of valuations for the different product types. In other words, product types may be ranked in terms of quality (see Milgrom, 2008, for a recent literature review). As is well known, if the firm may perfectly certify product quality at no cost, full quality disclosure is the unique equilibrium outcome. The argument runs as follows. The top quality product type would never pool with any other product type since it can certify that it is the highest quality and sell with the same probability at a higher price. The argument unravels down to the lowest quality type.

We generalize the unraveling result by providing a necessary and sufficient condition on the match function for full revelation of the product's type to be the unique equilibrium, regardless of the firm's costs and the prior on the agents' types. We call this condition *pairwise monotonicity*. It requires that, for every pair of types of the firm and every pair of types of the consumer the matches can be ordered with respect to the firm's types or with respect to the consumer's types. Equivalently, for each pair of product types, consumer types can be partitioned into subgroups such that (i) for any two subgroups all consumer types in one subgroup are willing to pay more for the two products than all consumer types in the other subgroup, and (ii) all consumer types within a subgroup rank the two product types identically.

The standard persuasion game is clearly a special case where pairwise monotonicity trivially holds. But pairwise monotonicity can also accommodate products differentiated in terms of their horizontal match with different consumer types. For example, it holds whenever consumer types can be ranked in terms of willingness to pay in the same manner for all product types. As a more elaborate illustration, consider a firm selling a video game that may be one of three possible product types, A , B and C . The situation depicted in Figure 1 is consistent with pairwise monotonicity. All gamers value C less than A and B . Furthermore, all types of hard-core gamers are willing to pay more than casual gamers for A and B , while hard-core (respectively casual) gamers order A and B in the same way.

Pairwise monotonicity is the weakest possible sufficient condition for uniqueness of the fully revealing equilibrium outcome in the sense that, if it does not hold, there are some

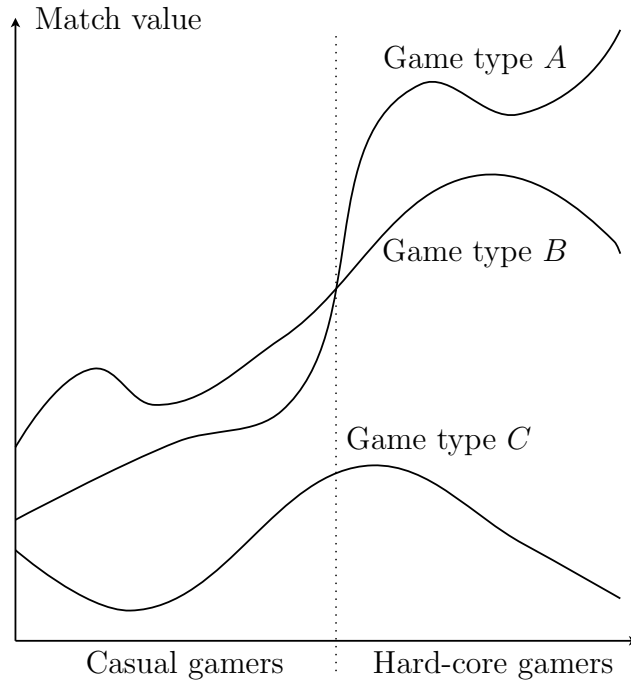


Figure 1: A match value satisfying the sufficient condition for full disclosure.

prior beliefs and costs such that an equilibrium exists where information is not fully revealed. However, if the prior is restricted so that product types and consumer types are independently distributed, then full disclosure of the product's type is always an equilibrium, even when pairwise monotonicity fails. But other (partially and non) revealing equilibria may exist. We show that when types are independent a necessary and sufficient condition for an outcome to be supported by an equilibrium is that it yields a profit at least as large as full revelation for all product types.

Finally, we explore the implications of our results in terms of the full information demands for the various product types. Interestingly, pairwise monotonicity puts no restriction on the shape of these demand curves: it holds as long as consumer types are ranked in the same manner along all the curves. We finally show that, if firm and consumer types are independently distributed, pairwise monotonicity is equivalent to the following requirement: for any two product types, the set of consumer types with willingness to pay between two crossing points of the demand curves should be the same for both product types and all consumer types in that set rank the two products identically. This condition ensures that for any purchase probability, the inverse demand for one product type lies above the inverse demand if the two types pool. This allows for a deviation from a pooling equilibrium that is profitable for one of the two types of the firm.

Related Literature. Recently, Anderson and Renault (2006, 2009), Johnson and Myatt

(2006) and Sun (2011) have analyzed disclosure of *horizontal* match information in specific examples and find that it may be profit maximizing for the firm to disclose no information or partial information. Anderson and Renault (2006) and Johnson and Myatt (2006) assume that all product types are symmetric, so that the profit maximizing solution is not type dependent. Johnson and Myatt (2006) consider a set of signals about the consumer’s valuation that may be ranked unambiguously in terms of informativeness. They find that the monopolist’s profit is maximized by using either the least informative or the most informative signal. By allowing for general informative signals, Anderson and Renault (2006) show that it is optimal for the firm to provide partial information that takes the form of a threshold on the consumer’s valuation, where those consumers with valuations above the threshold learn this information but no more.¹ Sun (2011) considers a monopolist selling a product on the Hotelling line and finds that there are equilibria where product types that are close enough to the end points pool on not disclosing product information.²

We present the model in Section 2 and then turn in Section 3 to establishing that pairwise monotonicity is sufficient for existence and uniqueness of a fully revealing equilibrium. Section 4 provides a general condition for existence of a fully revealing equilibrium and explores the possibility that other equilibria also exist. Some discussion of our results is presented in Sections 5 and 6 by interpreting our analysis in terms of demand curves and then discussing some possible extensions. Section 7 concludes.

2 Model

A monopolist sells a single unit of its product to a consumer. The firm has perfect and private information about the product’s characteristics while the consumer has perfect and private information about his tastes. The *match* between the characteristic of the firm’s product and the consumer’s tastes may therefore be written as

$$r(s, t) \in \mathbb{R}_+,$$

¹Although Anderson and Renault derive the result while allowing the consumer to acquire information through search, the result still holds if search is ruled out (see Saak, 2006).

²Other explanations in the literature for partial information disclosure include competition (Board, 2009, Levin, Peck, and Ye, 2009), unsophisticated buyers (Hirshleifer, Lim, and Teoh, 2004, Mullainathan, Schwartzstein, and Shleifer, 2008), costly communication (Jovanovic, 1982, Verrecchia, 1983) and partial certifiability (Shin, 1994). Recently, Chakraborty and Harbaugh (2011) also study informative advertising in a cheap talk framework, assuming that firms can only provide “soft information” to the consumers. Other papers study optimal disclosure while assuming that the seller can commit to a disclosure rule (see, e.g., Ottaviani and Prat, 2001, Rayo and Segal, 2010, Kamenica and Gentzkow, 2011); by contrast, we assume that the seller chooses which information to disclose at the interim stage.

where $s \in S$ is the firm's type and $t \in T$ is the consumer's type. A consumer's type (respectively a firm's type) describes his private information about his tastes (respectively its private information about the product's characteristics) as well as any information he may have about the other party's information. For technical simplicity the sets S and T are assumed to be finite. Let $\mu \in \Delta(S \times T)$ be the strictly positive prior probability distribution over the profile of types. The firm's type and the consumer's type are allowed to be correlated. This could for instance reflect a difference in information across consumer types about the product's characteristics or difference in information across firm types about the consumer's tastes.

The firm has a constant marginal cost $\gamma(s) \geq 0$ when its type is s . We assume that, for every $s \in S$, there exists $t \in T$ such that $r(s, t) > \gamma(s)$, meaning that each type of the firm can potentially make a strictly positive profit with at least one type of the consumer.

The timing of the game is as follows.

- (i) *Information stage.* The firm learns $s \in S$ and the consumer learns $t \in T$;
- (ii) *Pricing and disclosure stage.* The firm commits to an observable price $p \in \mathbb{R}$ and sends a message $m \in M(s)$, where $M(s)$ is a nonempty type dependent set;
- (iii) *Decision stage.* The consumer observes the price p and the message m , and chooses whether to buy the good or not;
- (iv) *Payoffs.* Players' payoffs are zero when the consumer does not buy the good; if the consumer buys the good his payoff is $r(s, t) - p$, and the firm's payoff is $p - \gamma(s)$.

Denote by $M = \bigcup_{s \in S} M(s)$ the set of all possible messages that can be sent in the disclosure stage. Every subset of types of the firm is certifiable in the sense that for every $\bar{S} \subseteq S$, there is a message $m_{\bar{S}} \in M$ such that $m_{\bar{S}} \in M(s)$ if and only if $s \in \bar{S}$. Notice that under this assumption, any information that could be signaled through prices can also be transmitted at no cost with direct information disclosure. Hence, although the price may depend on the firm's type, it has no real signaling role along the equilibrium path.

For the most part, the analysis only requires that full disclosure of the firm's type s (message m_s) is possible. The assumption that disclosed information is certifiable is especially justified for advertising because of laws on misleading advertising, but is also justified if the firm can provide hard evidence about the product characteristic, or if disclosed information can be verified by the consumer at no cost.

A (pure) strategy of the firm is a mapping $\varphi_F : S \rightarrow \mathbb{R} \times M$ such that $\varphi_F(s) \in \mathbb{R} \times M(s)$ for every $s \in S$. A (pure) strategy of the consumer is a mapping $\varphi_C : T \times \mathbb{R} \times M \rightarrow \{Buy, NotBuy\}$. A belief function of the consumer is a mapping $\beta : T \times \mathbb{R} \times M \rightarrow \Delta(S)$.

As a solution concept for this game, we use the sequential equilibrium. The additional restrictions imposed on a sequential equilibrium, on top of those on a perfect Bayesian equilibrium, are especially relevant in this context, where beliefs are defined for a receiver (the consumer) who may have different types (see, for instance, Footnote 10).³ Our results focus on the existence of particular pure strategy equilibria, but our uniqueness results are proved in the class of all equilibria (pure and mixed).

After the pricing and disclosure stage, the optimal decision of the consumer of type t is to buy the product if and only if⁴

$$E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t} = t, (p, m)] \geq p.$$

In equilibrium, a consumer necessarily buys when indifferent.⁵ Hence, the optimal choice (p^*, m^*) of the firm of type s should satisfy

$$(p^*, m^*) \in \arg \max_{(p, m) \in \mathbb{R} \times M(s)} (p - \gamma(s))D(p, m, s),$$

where

$$D(p, m, s) \equiv \Pr [E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t}, (p, m)] \geq p \mid \mathbf{s} = s],$$

is the expected demand of the firm when its type is s and it sends the signal (p, m) . Notice that $E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t}, (p, m)]$ only depends on the realization of \mathbf{s} through \mathbf{t} , and only if types are not independent.

3 Generalized unraveling

We now provide a general condition for a fully revealing equilibrium to exist and be unique whatever the costs and the prior. We say that the match is *statewise monotonic with respect to the firm's type*, if, for every $s, s' \in S$, $s \neq s'$, either $r(s', t) > r(s, t)$ for all $t \in T$ or $r(s', t) < r(s, t)$ for all $t \in T$. There is then a quality ranking of product types as in the

³Strictly speaking, since the set of possible signals is infinite (any price $p \in \mathbb{R}$ is possible), a strictly positive perturbed strategy cannot be defined as in Kreps and Wilson (1982) (they consider finite games). However, it is easy to avoid this problem by assuming that the set of possible prices is finite but fine enough (all our results and examples apply with a fine enough set of prices).

⁴Bold letters denote random variables when there may be a risk of confusion. Expectations and probabilities are defined w.r.t. the consumer's belief β .

⁵Otherwise, the firm would brake the indifference by slightly lowering its price.

standard persuasion game or the following example:

$$r_A = \begin{array}{|c|c|c|c|} \hline & t_1 & t_2 & t_3 \\ \hline s_1 & 9 & 5 & 7 \\ \hline s_2 & 8 & 3 & 6 \\ \hline s_3 & 6 & 2 & 0 \\ \hline \end{array}$$

It is easy to show that existence and uniqueness of the fully revealing equilibrium is guaranteed in this case. Existence is obtained by considering beliefs that put probability one on the following price independent worst case type:

$$wct(p, m) = \arg \min_{s \in M^{-1}(m)} r(s, t),$$

for some $t \in T$. Thanks to statewise monotonicity with respect to the firm's type, the above worst case type is independent of the selected consumer type t and pins down the "lowest quality" product. Throughout the paper we resort to the idea of a worst case type. It can be readily established that the consistency requirement in the definition of a sequential equilibrium is satisfied whatever the prior as long as the worst case type is not consumer type dependent.

Deviation from full disclosure, that involves selecting $m \neq m_s$ is not profitable since $D(p, m, s) \leq D(p, m_s, s)$ for all $m \in M(s)$ and $p \in \mathbb{R}_+$. To show that there is no other equilibrium outcome assume by way of contradiction that types in a subset with at least two elements $\bar{S} \subseteq S$, send the same signal (p, m) with strictly positive probability. Let

$$\bar{s} = \arg \max_{s \in \bar{S}} r(s, t),$$

for any $t \in T$, be the highest quality product in \bar{S} . If \bar{s} 's demand is zero, then by certifying its type it can apply a price higher than $\gamma(\bar{s})$ and earn a strictly positive profit. If \bar{s} 's demand is strictly positive, then by statewise monotonicity with respect to the firm's type the willingness to pay with full revelation of type \bar{s} , $r(\bar{s}, t)$ exceeds the willingness to pay if \bar{s} pools, for all $t \in T$, so the type \bar{s} firm can increase its price when it certifies its type, while keeping the same demand as when it pools with types in \bar{S} . Hence pooling can never be sustained in equilibrium.

We next show by means of examples that statewise monotonicity with respect to product types is much too strong a condition for existence and uniqueness of a fully revealing

equilibrium. Consider first the following match:

$$r_B = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 1 & 4 \\ \hline s_2 & 2 & 3 \\ \hline \hline \end{array}$$

This match function is not statewise monotonic with respect to the firm's type, but it is statewise monotonic *with respect to the consumer's type* in the sense that for every $t, t' \in T$, $t \neq t'$, either $r(s, t') > r(s, t)$ for all $s \in S$ or $r(s, t') < r(s, t)$ for all $s \in S$.⁶ It is easy to see that whatever the prior the unique equilibrium is fully revealing. To get existence it suffices to consider the following worst case type for $m \in M(s_1) \cup M(s_2)$:

$$wct(p, m) = \begin{cases} s_2 & \text{if } p > 2, \\ s_1 & \text{if } p \leq 2. \end{cases}$$

In other words, the worst case type is the product type that the marginal consumer type likes the least. To get uniqueness, suppose on the contrary that s_1 and s_2 pool. Then the firm type selling the product that suits best the marginal consumer will deviate. If both t_1 and t_2 buy, then the price is given by $p \in (1, 2)$ and type s_2 can profitably deviate by revealing its type and charging price $p' = 2 > p$, in which case both types of the consumer still buy. Similarly, if only t_2 buys, then the price is given by $p \in (3, 4)$ and type s_1 can profitably deviate by revealing its type and charging price $p' = 4 > p$, in which case consumer t_2 still buys.

Consider now the following match function:

$$r_C = \begin{array}{|c|c|c|c|} \hline & t_1 & t_2 & t_3 \\ \hline s_1 & 5 & 4 & 3 \\ \hline s_2 & 6 & 1 & 2 \\ \hline \hline \end{array}$$

This match function is neither statewise monotonic with respect to the firm's type nor statewise monotonic with respect to the consumer's type. Nevertheless, the unique equilibrium is fully revealing whatever the prior. To see that there is a fully revealing equilibrium it

⁶The following application from Chakraborty and Harbaugh (2010) also satisfies statewise monotonicity with respect to the consumer's type but not with the respect of the firm's type: the firm's characteristic is multidimensional, $s = (s^1, s^2) \in \mathbb{R}_+^2$, where s^1 is an horizontal aspect of the product, s^2 a vertical aspect, the firm is located at 0, and the valuation of a consumer located at $t \in [0, 1]$ is $r(s^1, s^2, t) = s^2 - s^1 t$. In Chakraborty and Harbaugh (2010) full disclosure is not an equilibrium because they consider the case of cheap talk (i.e., information is not certifiable).

suffices to consider the following worst case type for $m \in M(s_1) \cup M(s_2)$:

$$wct(p, m) = \begin{cases} s_2 & \text{if } p \leq 4, \\ s_1 & \text{if } p > 4. \end{cases}$$

Again, it is the worst product for the marginal consumer type: for $p \leq 4$, type t_1 cannot be marginal since the firm would otherwise charge a price of at least 5. To see that this equilibrium outcome is unique, suppose on the contrary that s_1 and s_2 pool. If t_3 is the marginal type, then to prevent s_1 from revealing its type the price should not be smaller than 3, a contradiction with the fact t_3 buys. If t_2 is the marginal consumer, then to prevent s_1 from revealing its type the price should not be smaller than 4, a contradiction with the fact t_2 buys. Finally if only t_1 buys, then s_2 would deviate by revealing its type and choosing a price equal to 6.

The three match functions that we have considered thus far share a common feature that turns out to be a key property for establishing existence and uniqueness of a fully revealing equilibrium. It runs as follows. For any non empty subset of firm types, $\bar{S} \subseteq S$, and any non empty subset of consumer types, $\bar{T} \subseteq T$, there exists an ordered pair $(\bar{s}, \bar{t}) \in \bar{S} \times \bar{T}$ that has the following ‘‘saddle point’’ property: (i) $r(\bar{s}, \bar{t}) = \max_{s \in \bar{S}} r(s, \bar{t})$; (ii) $r(\bar{s}, \bar{t}) = \min_{t \in \bar{T}} r(\bar{s}, t)$. In other words, product type \bar{s} is the best product in \bar{S} for consumer type \bar{t} and consumer type \bar{t} is willing to pay the least among all consumer types in \bar{T} for product type \bar{s} . For instance, such a saddle point is reached for the whole 3×3 matrix in the match r_A , which satisfies statewise monotonicity with respect to the firm’s type, at $(\bar{s}, \bar{t}) = (s_1, t_2)$. Similarly, the 2×2 matrix has a saddle point at (s_2, t_1) in the match r_B , which that satisfies statewise monotonicity with respect to consumer types, and at (s_1, t_3) in our last 2×3 example, r_C . If we drop t_3 in that same last example then the saddle point is reached at (s_1, t_2) .

We now show that if the above property holds, then there cannot be an equilibrium with several firm types pooling with some positive probability, as long as the match function is generic.⁷ Consider some pooling of several firm types in \bar{S} , where the price is such that the set of consumer types who buy is \bar{T} . Then the price is less than consumer \bar{t} ’s expectation of the match, which, if the match function is generic, is strictly less than $r(\bar{s}, \bar{t})$. Hence, firm type \bar{s} could increase its price to $r(\bar{s}, \bar{t})$ and still sell to all types in \bar{T} by part (ii) of the saddle point property above.

⁷Formally, we only require that $r(s, t) \neq r(s', t)$ for every $s \neq s'$ and $t \in T$. This assumption rules out situations where two firm types correspond to the same product and differ only in terms of the available information about the other party. If we allow for different firm types selling the same product, so that the match would be constant across those firm types, then it is possible to adapt the argument to show that product information is revealed in equilibrium. This is the case if product information is certifiable and the consumer is never indifferent between two different products.

If we consider the restriction of the match to two firm types and two consumer types, then the saddle point property is equivalent to statewise monotonicity with respect to the firm's type or the product's type. This property holds for all pairs of product types and consumer types in all three match functions above. We now state this property formally.

Definition 1 The match function $r(\cdot, \cdot)$ is *pairwise monotonic* if for every pair of types of the firm $(s, s') \in S^2$, $s \neq s'$, and every pair of types of the consumer $(t, t') \in T^2$, $t \neq t'$, one of the following conditions hold:

$$\begin{array}{ll} (i) \begin{cases} r(s, t) > r(s', t) \\ r(s, t') > r(s', t'); \end{cases} & (ii) \begin{cases} r(s', t) > r(s, t) \\ r(s', t') > r(s, t'); \end{cases} \\ (iii) \begin{cases} r(s, t) > r(s, t') \\ r(s', t) > r(s', t'); \end{cases} & (iv) \begin{cases} r(s, t') > r(s, t) \\ r(s', t') > r(s', t). \end{cases} \end{array}$$

The proof of the next theorem establishes that pairwise monotonicity is equivalent to the existence of a saddle point in all sub-matrices of the match function. From our discussion above, this establishes that there cannot be an equilibrium that is not fully revealing.

Remark 1 The equivalence of pairwise monotonicity and the existence of a saddle point for all sub-matrices of the match function has an interesting alternative interpretation. It says that if a zero-sum game is pairwise monotonic, then every sub-matrix has a saddle point. Hence, every sub-matrix, interpreted as a zero-sum game, is strictly determined in the sense that the pure maxmin coincides with the pure minmax. Coincidentally, this is related to Shapley (1964) who shows that a zero-sum game has a (pure) saddle point if every 2×2 sub-matrix of the game has a saddle point.

Theorem 1 *If the match is generic and pairwise monotonic, then there is a unique sequential equilibrium outcome, which is fully revealing.*

Proof. Uniqueness. Assume that there exists an equilibrium where a set of types $\bar{S} \subseteq S$, with $|\bar{S}| \geq 2$, pool (i.e., choose the same signal (p, m) with strictly positive probability). Let $\bar{T} \subseteq T$ be the set of consumer's types that buy the good after the signal (p, m) . Since for every $s \in S$ there exists $t \in T$ such that $r(s, t) > \gamma(s)$, we necessarily have $\bar{T} \neq \emptyset$ and $p > \gamma(s)$ for all $s \in \bar{S}$. When $|\bar{T}| = 1$, the standard unraveling argument shows that pooling is impossible, so let $|\bar{T}| \geq 2$. Denote by $\bar{R} = (r(s, t))_{(s, t) \in \bar{S} \times \bar{T}}$ the matrix of matches restricted to types in $\bar{S} \times \bar{T}$. Notice that the price applied by firms in \bar{S} is such that

$$p \leq \min_{t \in \bar{T}} E(r(s, t) \mid (p, m)).$$

For every $s \in \bar{S}$ and $t \in \bar{T}$ let

$$\underline{t}(s) \in \arg \min_{t \in \bar{T}} r(s, t) \quad \text{and} \quad \bar{s}(t) = \arg \max_{s \in \bar{S}} r(s, t).$$

That is, $\underline{t}(s)$ is (possibly a selection of) the smallest match in the s line of \bar{R} (call it a white cell) and $\bar{s}(t)$ is the highest match in the t column of \bar{R} (call it a gray cell). Equilibrium conditions imply that those (gray and white) cells cannot be confounded; that is, there is no pair (s, t) such that $(s, t) = (\bar{s}(t), \underline{t}(s))$. Otherwise, firm s can profitably deviate by revealing its type and applying the price $r(s, t) = r(\bar{s}(t), t) = \max_{s' \in \bar{S}} r(s', t) > p$, for which all consumers in \bar{T} buy since $r(s, t) = r(s, \underline{t}(s)) \leq r(s, t')$ for every $t' \in \bar{T}$.

Now, in the matrix \bar{R} we delete iteratively any line without a gray cell, i.e., any line s such that $s \neq \bar{s}(t)$ for every $t \in \bar{T}$, and any column without a white cell, i.e., any column t such that $t \neq \underline{t}(s)$ for every $s \in \bar{S}$. Denote by R^* the remaining matrix of matches, and S^* and T^* the corresponding sets of types of the firm and of the consumer. Notice that this matrix has at least two lines and two columns because gray and white cells cannot be confounded. This procedure is illustrated below.

$$\bar{R} = \begin{pmatrix} \blacksquare & \square & \cdot & \cdot \\ \cdot & \blacksquare & \square & \cdot \\ \cdot & \cdot & \blacksquare & \square \\ \cdot & \square & \cdot & \blacksquare \end{pmatrix} \rightarrow \begin{pmatrix} \square & \cdot & \cdot \\ \blacksquare & \square & \cdot \\ \cdot & \blacksquare & \square \\ \square & \cdot & \blacksquare \end{pmatrix} \rightarrow \begin{pmatrix} \blacksquare & \square & \cdot \\ \cdot & \blacksquare & \square \\ \square & \cdot & \blacksquare \end{pmatrix} = R^*$$

Let

$$r^* = \max_{(s,t) \in S^* \times T^*} r(s, t),$$

be the highest match in the matrix R^* . By definition r^* necessarily corresponds to a gray cell, i.e., $r^* = r(\bar{s}(t^*), t^*)$ for some $t^* \in T^*$. Let $s^* = \bar{s}(t^*)$ so that $r^* = r(s^*, t^*)$. By the construction of R^* (any column having a white cell and any line having a gray cell) there exists $s' \in S^*$ such that $\underline{t}(s') = t^*$ and $t' \in T^*$ such that $\bar{s}(t') = s'$ as illustrated below:

$$\begin{pmatrix} \boxed{r^* = r(s^*, t^*)} & \square & r(s^*, t') \\ \cdot & \blacksquare & \square \\ \boxed{r(s', t^*)} & \cdot & \boxed{r(s', t')} \end{pmatrix}$$

Finally, $r(s^*, t') < r(s^*, t^*) \equiv \max_{(s,t) \in S^* \times T^*} r(s, t)$, so the match is not pairwise monotonic.

Existence. Consider a complete disclosure strategy of the firm such that a message $m_s \in M(s)$ is sent by each type of the firm, with $m_s \notin M(s')$ for all $s' \neq s$. We construct a worst case type function such that no firm's type has an incentive to deviate from this

complete disclosure strategy. Consider a signal (p, m) off the equilibrium path, and consider the matrix of matches restricted to firm types in $M^{-1}(m)$, with $|M^{-1}(m)| \geq 2$. By the first part of the proof we know that if the match is pairwise monotonic, then the minimum match of some firm's type coincides with the maximum match of some consumer's type (for every sub-matrix of matches); that is, at least one white cell coincides with some gray cell. Denote by (s_1, t_1) the corresponding cell, and $r_1 = r(s_1, t_1)$ the corresponding value. If $p > r_1$ then type t_1 never buys whatever his beliefs, and we apply the same construction to the matrix of matches without t_1 . If $p \leq r_1$ then type s_1 has no incentive to deviate from full disclosure to (p, m) and we let $wct(p, m) \neq s_1$ (i.e., we consider a belief system such that $\beta(s_1 | t, m, p) = 0$ for every t). If $|M^{-1}(m) \setminus \{s_1\}| = 1$ the argument is complete. Otherwise we consider the new matrix of matches without the s_1 line. We apply the same reasoning as above to this new matrix: let (s_2, t_2) be a gray and white cell of this matrix, let $r_2 = r(s_2, t_2)$ be the corresponding value, and construct the worst case types as above. If $p > r_2$ then type t_2 never buys whatever his beliefs, and we apply the same construction to the matrix of matches without t_2 . If $p \leq r_2$ then type s_2 has no incentive to deviate from full disclosure to (p, m) and we let $wct(p, m) \neq s_2$. If $|M^{-1}(m) \setminus \{s_1, s_2\}| = 1$ the argument is complete. Otherwise, we apply the same construction up to (s_k, t_k) such that $|M^{-1}(m) \setminus \{s_1, s_2, \dots, s_k\}| = 1$. ■

As will be seen in Proposition 3 below, pairwise monotonicity is the weakest possible sufficient condition on the match function that ensures uniqueness, while keeping prior beliefs and costs unrestricted.

For the sake of interpretation, we now provide the following alternative definition of pairwise monotonicity, which is useful to relate the properties of match functions to inverse demand functions (see Section 5). A match function is pairwise monotonic if for every pair of product types, consumer types can be partitioned into subgroups such that (i) there is monotonicity with respect to consumer types between subgroups (ii) all consumers in a subgroup rank the two product types identically.⁸ For instance, in the video game example in the introduction, there are two groups of consumer types, the casual users and the hard-core users. The ranking between the two high quality products is the same within subgroups, although it differs across subgroups. An illustration with four subgroups is depicted in Figure 2, assuming for clarity a continuum of consumer types: consumer types are on the horizontal axis while the vertical axis measures the match. There is no loss of generality in taking the match for s_1 to be increasing since consumer types may always be rearranged. Pairwise monotonicity, which is satisfied in Figure 2 (a), is equivalent to requiring that, for any crossing points between the two matches, the match with product s_2 lies below the crossing point for consumer types on the left and above the crossing point for consumer types to the right. For a pair of consumer types between two crossing points, statewise

⁸Of course, this interpretation also works by switching “product” and “consumer” types.

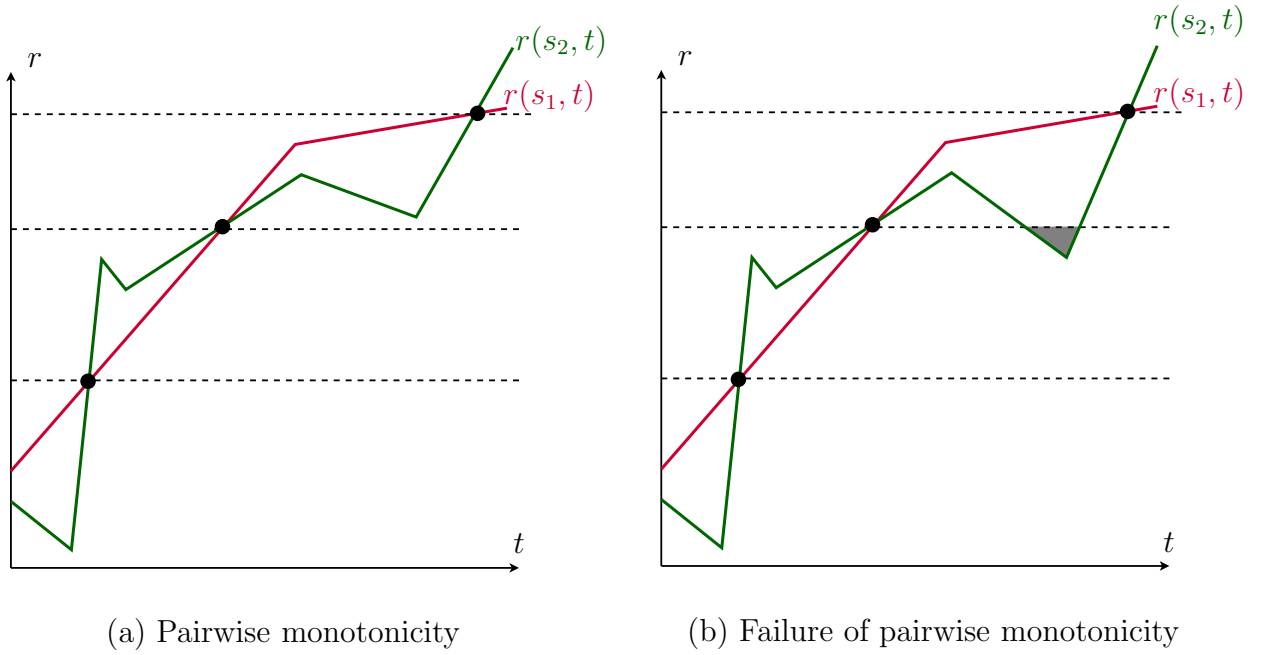


Figure 2:

monotonicity with respect to product types holds, while for a pair of consumer types on either side of a crossing point, statewise monotonicity with respect to consumer types holds. If the match for product s_2 loops up before a crossing point or loops down after a crossing point, then pairwise monotonicity fails as in Figure 2 (b).

We next consider some equilibrium properties when pairwise monotonicity does not necessarily hold.

4 Equilibria without unraveling

The next proposition shows that, even if pairwise monotonicity fails, there still exists a fully revealing equilibrium, whatever the prior as long as there is no correlation between the firm's and the consumer's type. To understand why, notice that if the type of the firm is independent of the type of the consumer, then the demand of the firm given p and m does not depend on its actual type, i.e.,

$$D(p, m, s) = D(p, m) = \Pr [E[r(\mathbf{s}, \mathbf{t}) \mid \mathbf{t}, (p, m)] \geq p].$$

It follows that the firm's preference over the consumer's belief is, conditionally on the price and the message sent, independent of its real type because the demand only depends on the price and the consumer's belief. Hence, starting from a strategy of full disclosure it is easy to prevent the firm from deviating by constructing, for each price p , a worst case type

that minimizes the firm's demand at price p .⁹ This can be done independently of the firm's real type.

Proposition 1 *Assume that the consumer's type and the firm's type are independent. Then, there exists a fully revealing equilibrium.*

Proof. Consider a complete disclosure strategy of the firm such that a message $m_s \in M(s)$ is sent by each type of the firm, with $m_s \notin M(s')$ for all $s' \neq s$. When types are independent, the demand for such a message at price p is given by

$$D(p, m_s) = \Pr[r(s, \mathbf{t}) \geq p].$$

For every $m \in M$ and $p \in \mathbb{R}$, let

$$wct(p, m) \in \arg \min_{s \in M^{-1}(m)} D(p, m_s),$$

be the (worst case) type of the firm whose demand at price p when it reveals its type to the consumer is the lowest one among all types that can send message m . For any signal $(p, m) \in \mathbb{R} \times M$ of the firm, consider the belief of the consumer that puts probability one on $wct(p, m)$ whatever the consumer's type. Along the equilibrium path, the firm gets

$$\max_{p_s} (p_s - \gamma(s)) D(p_s, m_s).$$

This profit is larger than what it gets by deviating to $(p, m) \in \mathbb{R} \times M(s)$, which by construction is equal to

$$(p - \gamma(s)) \min_{s' \in M^{-1}(m)} D(p, m_{s'}).$$

This completes the proof of Proposition 1. ■

The appendix describes an example with correlated types (but without pairwise monotonicity) where there is no fully revealing equilibrium.¹⁰ Proposition 1 shows that as long as types are independent, pairwise monotonicity is not necessary for existence of a fully revealing equilibrium. As we show below, pairwise monotonicity is however necessary to ensure uniqueness of the fully revealing outcome whatever the prior distribution of types and costs. We start by providing a useful characterization of equilibria with independent types.

⁹When the cost of the firm is not type dependent, the proof of Proposition 1 can be simplified by considering price-independent beliefs off the equilibrium path.

¹⁰In the example, it is important to require strong belief consistency (in the sense of Kreps and Wilson, 1982). If we allow arbitrary beliefs off the equilibrium path, existence of a fully revealing equilibrium is immediate even with correlated types. It suffices to consider beliefs off the equilibrium path that put probability one on $\arg \min_{s \in M^{-1}(m)} r(s, t)$, which are inconsistent when they depend on t .

The next proposition provides a simple method for characterizing all equilibrium outcomes when types are independent. To this end we define canonical disclosure strategies: a strategy $\varphi_F(\cdot)$ is canonical if, for every price and message (p, m) in the range of $\varphi_F(\cdot)$, if type $s \in S$ does not send (p, m) (i.e., $\varphi_F(s) \neq (p, m)$), then the message m cannot be sent by type s (i.e., $m \notin M(s)$). Notice that, under our assumption that every subset of types is certifiable, considering only canonical strategies does not restrict the set of possible equilibrium outcomes.¹¹ The proposition says that a (canonical) disclosure strategy induces an equilibrium outcome of the game if and only if the (interim) payoff for the firm is no smaller than the payoff the firm would earn at the fully revealing equilibrium whatever its type.¹²

Proposition 2 *Assume that the consumer's type and the firm's type are independent. A canonical disclosure strategy induces an equilibrium iff whatever the firm's type the induced interim expected payoff for the firm is not smaller than its fully revealing equilibrium payoff.*

Proof. Necessity. This part is obvious.¹³ If the disclosure strategy of the firm is such that $\varphi_F(s) = (p, m)$ with

$$(p - \gamma(s))D(p, m) < (p_s - \gamma(s))D(p_s, m_s),$$

where (p_s, m_s) is the signal sent by the firm at the fully revealing equilibrium, then (simply by subgame perfection) the firm can profitably deviate from φ_F at s by sending the signal (p_s, m_s) .

Sufficiency. Consider a canonical disclosure strategy φ_F of the firm such that $\varphi_F(s) = (p, m)$ and

$$(p - \gamma(s))D(p, m) \geq (p_s - \gamma(s))D(p_s, m_s), \quad (1)$$

where (p_s, m_s) is the signal sent by the firm of type s at the fully revealing equilibrium. Notice that any deviation from a canonical disclosure strategy is observable by the consumer. Consider the same beliefs for the consumer off the equilibrium as in the proof of Proposition 1.

¹¹For example, if $S = \{s_1, s_2\}$, $M(s_1) = \{m_0, m_1\}$, $M(s_2) = \{m_0, m_2\}$, then the disclosure strategy $\varphi_F(s_1) = (p, m_1)$ and $\varphi_F(s_2) = (p, m_0)$ is not canonical, but is equivalent to the canonical strategy $\varphi'_F(s_1) = (p, m_1)$ and $\varphi'_F(s_2) = (p, m_2)$ in terms of information that is transmitted to the consumer.

¹²The proposition does not extend to correlated types since we already observed in the example in the Appendix that full revelation may not be an equilibrium when types are correlated.

¹³Notice that this part also applies with correlated types and non-canonical disclosure strategies.

If the firm deviates from $\varphi_F(s) = (p, m)$ to (p', m') it gets

$$\begin{aligned} (p' - \gamma(s)) \min_{s' \in M^{-1}(m')} D(p', m_{s'}) &\leq \max_{p_s} (p_s - \gamma(s)) \min_{s' \in M^{-1}(m')} D(p_s, m_{s'}) \\ &\leq \max_{p_s} (p_s - \gamma(s)) D(p_s, m_s) \text{ because } s \in M^{-1}(m'), \end{aligned}$$

which by (1) is smaller than the payoff it gets without deviating from $\varphi_F(s)$. ■

Using Proposition 2 we may now show that the converse of Theorem 1 is also true in the sense that pairwise monotonicity is the weakest sufficient condition that ensures that the fully revealing outcome is unique, independent of the specification of the prior and production costs.

Proposition 3 *Assume that the match is generic but not pairwise monotonic. Then, there exists an open set of costs and priors such that the game has an equilibrium outcome, which is not fully revealing, and which is strictly better for the firm than the fully revealing equilibrium.*

Proof. If the match is generic but not pairwise monotonic then there exist two pairs $(s_1, s_2) \in S^2$ and $(t_1, t_2) \in T^2$, where $r(s_1, t_1) = a$, $r(s_1, t_2) = b$, $r(s_2, t_1) = c$ and $r(s_2, t_2) = d$ such that $a \geq b$, $a > c$, $d \geq c$ and $d > b$, as illustrated below:

$$\begin{pmatrix} a & \geq & b \\ \vee & & \wedge \\ c & \leq & d. \end{pmatrix}$$

Assume zero costs. Also assume for now that the prior puts all the weight on this 2×2 matrix. Because no consumer type has a higher willingness to pay for both firm types than the other, (here $a \geq b$ and $d \geq c$), we may specify a probability for firm type s_1 , σ , such that the two consumer types have the same expected willingness to pay if the firm's type is not revealed: $\sigma a + (1 - \sigma)c = \sigma b + (1 - \sigma)d$ (the left hand side is larger for $\sigma = 1$ while the reverse inequality holds for $\sigma = 0$). Hence, if the firm does not disclose, it can sell with probability 1 at a price equal to the common expected match.

If the firm wishes to sell with probability 1 while revealing its type, it must charge a price which is at most the lowest match for its product, b for type s_1 and c for type s_2 . However, these values also correspond to the lowest match for one of the consumer types (there would otherwise be a saddle point), and hence are strictly less than the expected match under no disclosure. The firm therefore earns strictly less profit than with no disclosure by revealing its type and selling with probability 1.

Finally, if the firm reveals while only selling to one consumer type, the best it can do is sell to the consumer type with the highest match with its product type and charge the

corresponding price, a for type s_1 and d for types s_2 . Assume that type t_1 has probability σ and type t_2 has probability $1 - \sigma$. Then the corresponding revenue with such a strategy is still strictly less than the no disclosure revenue.

Let us now alter probabilities slightly so that all firm types and consumer types other than s_1 , s_2 , t_1 , and t_2 have a negligible but strictly positive weight, and consider strictly positive but small enough costs. Then it is still true that types s_1 and s_2 are strictly better off pooling than they would be with full disclosure. It follows from Proposition 2 that there is an equilibrium where s_1 and s_2 pool and all the other firm types fully reveal. ■

We illustrate the multiplicity of equilibria when pairwise monotonicity is violated with an example where it is easy to identify the equilibrium that is most favorable to the firm. Consider the following match function, which is inspired by Table 1 in Anderson and Renault (2006), with a uniform prior and the same cost $\gamma(s) = \gamma \in [0, 5]$ for every product type.

$$r_D = \begin{array}{|c|c|c|c|} \hline & t_1 & t_2 & t_3 \\ \hline s_1 & 5 & 4 & 1 \\ s_2 & 5 & 1 & 4 \\ s_3 & 4 & 5 & 1 \\ s_4 & 1 & 5 & 4 \\ s_5 & 4 & 1 & 5 \\ s_6 & 1 & 4 & 5 \\ \hline \end{array}$$

The fully revealing equilibrium leads to the profit $\max\{\frac{8-2\gamma}{3}, \frac{5-\gamma}{3}\}$ whatever the firm's type. No information revelation yields the profit $\max\{0, \frac{10-3\gamma}{3}\}$, so it is an equilibrium if and only if $\max\{0, \frac{10-3\gamma}{3}\} \geq \max\{\frac{8-2\gamma}{3}, \frac{5-\gamma}{3}\}$, i.e., $\gamma \leq 2$. However, when $\gamma \in [1, 4]$ the best equilibrium for the firm is partially revealing: the firm's disclosure strategy yields the partition $\{\{s_1, s_3\}, \{s_2, s_5\}, \{s_4, s_6\}\}$, the price is equal to $9/2$ and the profit is equal to $\max\{0, \frac{9-2\gamma}{3}\}$. Note that this partially revealing equilibrium implements a threshold of 4 along the lines of the optimal solution described in Anderson and Renault (2006): all consumer types with willingness to pay above 4 learn this with no additional information.¹⁴

Also note that, for any cost γ , the profit maximizing equilibrium among the three considered above implements a socially first-best outcome since a consumer buys if and only if his match exceeds marginal cost. This is also the profit maximizing solution since the firm extracts all of the consumer's expected surplus through its price. Hence, as the unit cost increases, the profit maximizing solution is first, non disclosure, then partial disclosure, and

¹⁴In Anderson and Renault (2006) where consumers may acquire full product information through costly search before buying, the coincidence of profit maximization and the first-best socially optimum outcome only arises if search costs are large enough.

finally full disclosure.

5 Demand curves and information disclosure

As illustrated by Johnson and Myatt (2006), the strategic choice by a firm to reveal or not reveal product information amounts to choosing among different demand curves. For instance, the classic unraveling result may be loosely described as follows: high quality firms prefer to reveal product information because it puts them on a demand curve that dominates the one they would face by pooling with lower quality firms. We here explore how pairwise monotonicity relates to demand curve properties.

For each product type, s , we may define the *perfect information inverse demand* as

$$P(q, s) \equiv \max\{p : D(p, m_s, s) \geq q\} = \max\{p : \Pr[r(\mathbf{s}, \mathbf{t}) \geq p \mid \mathbf{s} = s] \geq q\}, \quad (2)$$

for any $q \in (0, 1]$. The inverse demand gives the highest price that the firm selling product s can charge while being able to sell with a probability of at least q .

A first question is whether there are properties of inverse demands for the various product types that are sufficient to guarantee that the match is pairwise monotonic. One situation where we might expect this to happen is when full information demands do not cross. Such a property holds for instance when products may be ranked in terms of quality so that the match function is statewise monotonic with respect to firm types. A generalization of the persuasion game unraveling result would be that the no crossing point property of demand curves is enough to guarantee full revelation of product information. The following example however shows that this is not the case:

$$r = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 5 & 3 \\ \hline s_2 & 2 & 4 \\ \hline \hline \end{array}$$

Assume that the prior is uniform. The inverse demand for product type s_1 always lies above the inverse demand for product s_2 : that is, the largest price at which product s_1 may be sold with a given probability is always at least as large as the largest price at which product s_2 may be sold with the same probability. The match however is not pairwise monotonic and non revelation may be sustained as an equilibrium, for instance with zero production costs. It yields a profit of 3.5 against only 3 for the largest full information profit. Non disclosure is therefore an equilibrium by Proposition 2.

The above example may be enriched to illustrate an interesting property of quality dis-

closure. Consider now the match function below, involving four products.

$$r = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 5 & 3 \\ \hline s_2 & 2 & 4 \\ \hline s_3 & 4 & 2 \\ \hline s_4 & 3 & 5 \\ \hline \end{array}$$

Note that product type s_1 dominates product type s_3 in terms of demand curve but also in terms of quality ranking, since it is preferred by both consumer types. The same is true regarding product type s_4 *vis-a-vis* product type s_2 . The two superior quality firm types s_1 and s_4 would like to separate themselves from the two other types. Still, assuming again a uniform prior and zero costs, non disclosure may be sustained as an equilibrium, as a direct application of Proposition 2. Hence, quality information is not necessarily revealed in equilibrium when the relevant product information also pertains to the match. Sun (2011) derives related results in a setting where the set of product types and consumer types are represented by the Hotelling line and product types may also differ in terms of quality.

In order to obtain a demand dominance condition that implies pairwise monotonicity, we need to require that for any pair of products, the lowest point on one demand curve is above the highest point on the other demand curve. Formally, for any $s, s' \in S$, we have $P(1, s) > P(0, s')$ or $P(1, s') > P(0, s)$.¹⁵ This is a very strong restriction that actually implies statewise monotonicity with respect to the firm's type.

Conversely, pairwise monotonicity, and hence uniqueness of a fully revealing outcome may be ensured for any set of perfect information demand curves. Recall that a special case of pairwise monotonicity is statewise monotonicity with respect to consumer types. Now consider a finite set of step inverse demand functions defined on $(0, 1]$. Then it is not too difficult to see that all consumers can be ranked identically along all demand curves and the prior can be specified appropriately so that statewise monotonicity with respect to consumer types is satisfied and the resulting inverse demands are as desired.

We illustrate this idea with the following simple example. Consider the two following inverse demand functions: $P(q, s_1) = 3$ for $q \in (0, .4]$, $P(q, s_1) = 2$ for $q \in (.4, 1]$, $P(q, s_2) = 4$ for $q \in (0, .3]$, $P(q, s_2) = 3$ for $q \in (.3, .7]$ and $P(q, s_2) = 1$ for $q \in (.7, 1]$. Such inverse demands may be generated, for example, with three consumer types and the following match

¹⁵The inverse demand P is not defined for $q = 0$ and $P(0, s)$ should be thought of as the limit as q tends to zero, which always exists.

function

$$r = \begin{array}{|c|c|c|c|} \hline & t_1 & t_2 & t_3 \\ \hline s_1 & 3 & 2 & 2 \\ \hline s_2 & 4 & 3 & 1 \\ \hline \end{array}$$

which is statewise monotonic with respect to the consumer's type. Then assume that the prior is such that the conditional probability of consumer types t_1 , t_2 and t_3 are respectively, .4, .3 and .3, conditional on $s = s_1$ and .3, .4 and .3, conditional on $s = s_2$. Such a procedure may clearly be adapted to any finite set of step inverse demand functions so that pairwise monotonicity puts no restriction on the full information demand curves for the various product types.

We now show that pairwise monotonicity imposes some restrictions as to the allocation of different consumer types along the various demand curves. This in turn provides a simple demand curve intuition on the sufficiency of pairwise monotonicity for uniqueness of the fully revealing outcome. Let us now assume that consumer and product types are independent. To illustrate the general idea, consider the example depicted in Figure 3. The match function for two product types s_1 and s_2 with a continuum of consumer types is shown in panel (a). It satisfies pairwise monotonicity. The corresponding demand curves are derived in panel (b), assuming that consumer types are uniformly distributed.

Because of independence and pairwise monotonicity, crossing points for the match functions in Figure 3 (a) exactly translate into crossing points between inverse demand curves in Figure 3 (b). Take for instance consumer types to the left of the first crossing point between matches. They are all willing to pay less than the match value at the crossing point and all other consumer types are willing to pay more. Hence the probability of selling at that price is given by the probability measure associated to these types which is the same for both products, thanks to our independence assumption. Hence demand curves must cross at this price and this corresponds to the farthest right crossing point in Figure 3 (b). Repeating the argument iteratively shows that all crossing points coincide. From this analysis we conclude that the consumer type population may be partitioned into subgroups such that the willingness to pay for both products within a given subgroup lies between two consecutive crossing points of the demand curves and all consumers within a subgroup agree on the ranking of the two products.

Figure 3 also shows as a dashed curve the expected match $E[r(\mathbf{s}, t)]$ for all consumer types in Panel (a) (assuming product types have equal probabilities) from which is derived the no information inverse demand in Panel (b), again as a dashed curve. The expected demand lies strictly in between the two demand curves between two crossing points as long as the probability defined on product types is non degenerate. Hence, if we consider an equilibrium where the two products pool and face the no information demand curve, there

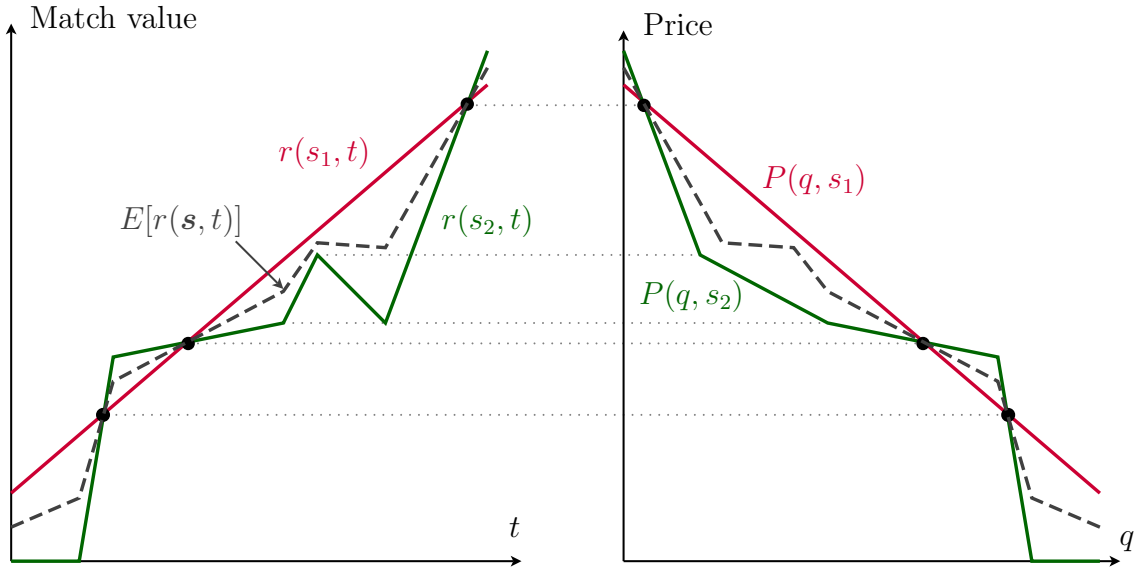


Figure 3:

is always one firm type that is better off deviating to full information thus selling at a higher price with the same probability.¹⁶

6 Discussion

This section discusses some possible extensions of the model.

More general communication. We have assumed that the consumer is not able to communicate information about his type to the firm before the pricing and disclosure stage. This is without loss of generality when our condition for the existence and uniqueness of a fully revealing equilibrium is satisfied; that is, the fully revealing equilibrium exists and remains unique under pairwise monotonicity even with two-way communication. But other interesting equilibria could be obtained when pairwise monotonicity is not satisfied, by adding communication from the consumer to the seller. To see this, consider the following match function with a uniform prior and no cost:

$$r = \begin{array}{c|ccc} & t_1 & t_2 & t_3 \\ \hline s_1 & 3 & 0 & 2 \\ \hline s_2 & 0 & 3 & 2 \end{array}$$

¹⁶This argument works as long as the price in the candidate pooling equilibrium does not coincide with a crossing point. If it does, then a standard argument using first order conditions shows that the firm type with the less elastic demand is better off deviating to full disclosure and increasing its price.

There is a fully revealing equilibrium with profit $4/3$ and a non revealing equilibrium with profit $3/2$. This profit can be improved upon if the consumer can first reveal whether his type is t_3 or not (this can even be a cheap talk claim). If the consumer reveals that his type is t_3 the price offered by the seller is 2 and he discloses no information; otherwise the price is 3 and product information is revealed by the seller. This yields profit $5/3$ which is strictly higher than any equilibrium profit without information transmission from the consumer.¹⁷

Competition. There is limited literature on product information disclosure and competition. The existing work such as Meurer and Stahl (1994), Board (2009) or Anderson and Renault (2009), considers a two stage game where firms first disclose information and then choose prices. Results in these models are often in contradiction with the predictions of the monopoly setting. Board (2009) considers Mussa and Rosen preferences (Mussa and Rosen, 1978) where consumers are heterogeneous in their willingness to pay for quality, and finds that the firm that draws the lower quality in a duopoly may choose not to disclose it. Note that the match induced by such preferences is both statewise monotonic with respect to the firm's type and with respect to the consumer's type. In contrast, Meurer and Stahl (1994) and Anderson and Renault (2009) consider information that pertains to the horizontal match between the firm's type and the consumer's type and find that, in equilibrium, product information is revealed. Monopoly models in similar settings would predict that the equilibria that maximize the firm's profit involve no disclosure or partial disclosure, as in the example based on match r_D at the end of Section 4.

The source of the difference however is not so much competition *per se*, but rather the strategic effect of information revelation in the first stage on the competitor's pricing in the second stage. In both cases, firms choose not to reveal information, or on the contrary, to reveal it, in order to avoid a Bertrand type situation in the second stage that would wipe out profits. If we think of product information disclosure and pricing as selected simultaneously, such a strategic effect disappears. Then if we consider quality disclosure as in Board (2009), a standard unraveling argument applies to each firm's reaction function and there cannot be an equilibrium where quality is not revealed. Whether and how the argument can be extended to a situation where the match between the consumer type and each firm's type is pairwise monotonic remains an open question. The relevant condition might be a joint condition on the two match functions associated with the two competing firms.

¹⁷We thank V. Bhaskar for suggesting this example.

7 Concluding remarks

Our analysis considerably extends the well known unraveling argument according to which certified quality information is always revealed: we allow the relevant information to pertain to the horizontal match along with quality. We show that pairwise monotonicity is sufficient for existence and uniqueness of a fully revealing equilibrium outcome, and is also necessary in order for uniqueness to hold independent of the prior and costs. However, a fully revealing equilibrium always exists if the firm's type and the consumer's type are independent. We also find that pairwise monotonicity imposes no restriction on the shape of the full information demand curves for the different product types but rather, on the allocation of consumer types along the demand curves. We use this property to provide some intuition in terms of demand curves as to how pairwise monotonicity guarantees full disclosure of product information.

Pairwise monotonicity is a condition that applies to the match alone independent of production costs or the prior on the agents' types. It may still be the case that, when it does not hold, there are production costs and prior distributions of types for which full revelation of product information is the unique equilibrium outcome. For instance, in the example at the end of Section 4, the fully revealing equilibrium is unique when costs are high enough, so that the firm wishes to only serve the consumer type with the highest valuation. Regarding the prior distribution of types, we have shown in Section 4 that if pairwise monotonicity fails, there is a prior for which two firm types pool in some equilibrium. However, for a given match function that does not satisfy pairwise monotonicity, there are typically priors for which there is necessarily full revelation. Our ongoing research aims at exploring these issues further as well as considering extensions to two sided information transmission and competition.

A Appendix

Non-existence of a fully revealing equilibrium due to correlated types. Assume that costs are zero ($\gamma(s_1) = \gamma(s_2) = 0$) and consider the following match function and correlation matrix, where we assume $0 < \rho < 2\varepsilon < 2$:¹⁸

$$r = \begin{array}{|c|} \hline \begin{array}{cc} t_1 & t_2 \\ s_1 & \rho & 2 \\ s_2 & 2 & \rho \end{array} \\ \hline \end{array} \quad \mu = \begin{array}{|c|} \hline \begin{array}{cc} t_1 & t_2 \\ s_1 & \frac{1-\varepsilon}{2} & \frac{\varepsilon}{2} \\ s_2 & \frac{\varepsilon}{2} & \frac{1-\varepsilon}{2} \end{array} \\ \hline \end{array}$$

¹⁸Notice that this match is not pairwise monotonic. Otherwise, a fully revealing equilibrium would always exist by our theorem.

If the firm fully reveals its type to the consumer, then it sets the price $p = 2$ and gets a profit equal to 2ε whatever its type. We show that if ε is small enough, then the firm has an incentive to deviate from full revelation. We have to show that whatever the consumer's belief after a deviation by the firm, s_1 or s_2 gets a profit which is strictly larger than 2ε .

Notice that the consumer's belief off the equilibrium path may depend on the observed price. This allows a large flexibility to punish the firm if it deviates from complete information disclosure. In this example, however, a deviation to the same price $p = 2\varepsilon + (1 - \varepsilon)\rho$ will be profitable for at least one of the firm's type whatever the consistent belief of the consumer. The idea is that this price is accepted by one type of the consumer with large probability $(1 - \varepsilon)$ whenever a type s_i of the firm makes the consumer believe that it is the other type s_{-i} , for $i = 1, 2$. Under some conditions on the game, the fact that two types want to imitate each others is sufficient to prevent full revelation of information (see, e.g., the "single crossing" property in Giovannoni and Seidmann, 2007), but is not sufficient in our framework since we also have to consider non-degenerated beliefs off the equilibrium path. For example, when $\rho = 0$, by setting the price to $2(1 - \varepsilon)$, each type of the firm would be strictly better off when the consumer believes that it is the other type, but a fully revealing equilibrium can be constructed by setting the consumer's belief off the equilibrium path to his prior belief.

We first observe that if t_1 or t_2 buys the good at price $p = 2\varepsilon + (1 - \varepsilon)\rho$, then the expected profit of one of the two types of the firm is at least

$$\Pi = (1 - \varepsilon)(2\varepsilon + (1 - \varepsilon)\rho),$$

so the deviation would be profitable for one of those types whenever $\Pi > 2\varepsilon$, i.e., $\rho > \frac{2\varepsilon^2}{(1 - \varepsilon)^2}$. This is possible under the assumption that $\rho < 2\varepsilon$ whenever ε is small enough (take $\frac{\varepsilon}{(1 - \varepsilon)^2} < 1$).

It remains to check that at least one of the consumer's type t_1 or t_2 always accepts to buy at price p off the equilibrium path. Let $m \in M(s_1) \cap M(s_2)$ be a message available to the firm whatever its type, and let μ_i be the consumer's belief that the firm's type is s_1 when the consumer's type is t_i and he observes the signal (p, m) off the equilibrium path. The maximum price under which the consumer accepts to buy the good is $\bar{p}_1 = 2(1 - \mu_1) + \rho\mu_1$ when his type is t_1 and $\bar{p}_2 = 2\mu_2 + \rho(1 - \mu_2)$ when his type is t_2 . The firm does not deviate from full revelation by sending (p, m) only if \bar{p}_1 and \bar{p}_2 are both smaller than p , which yields $\mu_1 > 1 - \varepsilon$ and $\mu_2 < \varepsilon$. However this belief system is not consistent since it cannot be obtained by Bayes' rule whatever the strategy of the firm.

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