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**Abstract:**

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the simulated, yet realistic, forced-vibration testing of a simple framed structure using its frequency response function as input data. By comparing the results of BVLSrc with those obtained *via* (the competing) pure BVLS and regularization methods, we show that BVLSrc and regularization methods yield approximate solutions with similar and sufficiently high accuracy, while pure BVLS method yields physically inadmissible solutions. We further demonstrate that BVLSrc is computationally more efficient, because, unlike regularization methods, it does not require the laborious *a priori* calculations to determine an optimal penalty parameter, and its results are far less sensitive to the initial estimates of the updating parameters. Copyright © 2006 John Wiley & Sons, Ltd.

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# Parameter Identification of Framed Structures Using An Improved Finite Element Model Updating Method—Part I: Formulation & Validation

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## ABSTRACT

In this study, we formulate an improved finite element model updating method to address the numerical difficulties associated with ill conditioning and rank-deficiency. These complications are frequently encountered model updating problems, and occur when the identification of a larger number of physical parameters is attempted than that warranted by the information content of the experimental data. Based on the standard Bounded Variables Least-squares (BVLS) method, which incorporates the usual upper/lower-bound constraints, the proposed method (henceforth referred to as BVLSrc) is equipped with novel sensitivity-based relative constraints. The relative constraints are automatically constructed using the correlation coefficients between the sensitivity vectors of updating parameters. The veracity and effectiveness of BVLSrc is investigated through the simulated, yet realistic, forced vibration testing of a simple framed structure using its frequency response function as input data. By comparing the results of BVLSrc with those obtained via (the competing) pure BVLS and regularization methods, we show that BVLSrc and regularization methods yield approximate solutions with similar and sufficiently high accuracy, while pure BVLS method yields physically inadmissible solutions. We further demonstrate that BVLSrc is computationally more efficient, because, unlike regularization methods, it does not require the laborious *a priori* calculations to determine an optimal penalty parameter, and its results are far less sensitive to the initial estimates of the updating parameters.

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## 1. INTRODUCTION

The true dynamic response of a structure obtained from vibration measurements generally displays significant discrepancies with the responses estimated using a highly idealized parametric model. “Finite Element (FE) model updating” is an analytical procedure for minimizing this discrepancy by adjusting, for example, the stiffness, mass, and/or damping parameters of the discrete numerical model. As such, model updating techniques are useful tools for assessing the health or performance of a structure, and for validating or improving the modeling assumptions.

There are two main categories of FE model-updating methods proposed to date. In methods belonging to the first category, the coefficients of the system (e.g., stiffness and/or mass) matrices that comprise the analytical model are directly updated, generally using modal data [1-3]. These are called the “*direct methods*,” since no iteration is needed in the updating computations. Although direct methods can reproduce the measured response quite accurately, they have the drawback of being based on updated quantities that are not directly related to the design parameters (e.g., the stiffness values of individual members) of the FE model. Therefore, it is often difficult to assign physical meanings to the changes made to the initial FE model matrices. The methods in the second category are referred to as “*iterative methods*,” whereby the system matrices of the structure are defined as explicit functions of a selected set of parameters (called “updating parameters”), and subsequently, their optimal values are sought by minimizing an objective function [4]. The basic form of an objective function is an error norm, defined as the sum of squares of the differences between estimated and measured response quantities. When the updating parameters are selected to represent physical (material or geometric) properties of the structure, their final (converged) values can be directly interpreted using the updated FE model.

The quantities used for constructing the objective function can be modal or frequency response function (FRF) data or both. Although modal data (i.e., the natural frequencies and mode shapes) are frequently used in updating procedures [5-7], the use of FRF data has several distinct advantages. To wit, (i) identification errors can be avoided through the use of FRF data, as system identification analysis is circumvented; and (ii) the FRF data contains information from all modes of the structure. The use of modal data is further complicated when a mode has high damping and/or modal density or when more than two modes are closely spaced, because identification of modal properties in such cases is extremely difficult [8].

Even for iterative methods, physically admissible solutions are not always attainable; and the results strongly depend on the selection of updating parameters. This phenomenon manifests itself as the “rank deficiency” or “ill conditioning” of the sensitivity matrix used in the updating equations. Rank deficiency occurs when information is insufficient to yield a unique solution to the model-updating problem, whereas ill conditioning occurs when the system response is insensitive to some of the updating parameters or when they have similar effects on the system’s response [4, 9]. Consequently, it is generally not possible to obtain accurate parameter values in ill-conditioned or rank-deficient problems using basic model updating techniques.

There are two major approaches to avoid the aforementioned numerical difficulties. In the first approach, a *sufficiently small* number of updating parameters are selected in order to reduce the possibility (or, at least, the order) of rank deficiency and ill conditioning. However, it is generally difficult to determine the suitable (optimal or pareto-optimal) subset of parameters from numerous potential candidates—requiring engineering judgment, and/or trial-and-error. When the number of chosen parameters is too small, it often becomes very difficult to make plausible physical explanations to their updates, and as such, the main advantage of iterative

methods is significantly weakened. The second approach is to obtain an approximate solution in a “minimum-norm sense,” using a regularization technique. This alternative is preferred when the set of candidate updating parameters is large. Nonetheless, as the solutions depend on a regularization factor and the initial guess of the updating parameters—both of which are unknowns—evaluation of solutions from repetitive computations (by varying the two sets of unknowns) is needed to attain results with sufficient confidence. This additional computational task renders the model-updating problem intractable for even moderately complex structures.

In this study, we propose an improved iterative FE model updating method based on the Boundary Variable Least Square Method (BVLS) [10]. In addition to (conventional) absolute bounds for the updating parameters, we impose relative constraints to any pair of parameters that have similar effects on the system response using the correlation coefficient between their sensitivity vectors. The proposed updating method is iterative, and thus, enables the direct identification of the *properties of individual members* of building structures. In what follows, we first present the formulation of this novel method—capable of using FRF and/or modal data as input—followed by its validation using a simulated, yet realistic, forced-vibration testing of a simple multi-degree-of-freedom structure. We demonstrate the advantages of the proposed method by comparing its predictions with those of existing methods. The application of the method—developed and validated in this article—to data obtained from forced-vibration tests of a four-story reinforced concrete building that we have recently conducted is presented in the companion manuscript [11].

## 2. FORMULATION

### 2.1 Sensitivity based updating equation

The equation of motion of a system with  $n$  degrees-of-freedom can be expressed in the frequency

domain as,

$$\mathbf{B}(\omega) \mathbf{x}(\omega) = \mathbf{f}(\omega) \quad (1)$$

where

$$\mathbf{B}(\omega) = -\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}, \quad (2)$$

$$\mathbf{f}(\omega) = \mathbf{l} f(\omega). \quad (3)$$

Here,  $\mathbf{B}(\omega)$  is the dynamic stiffness matrix, and  $\mathbf{M}$ ,  $\mathbf{C}$  and,  $\mathbf{K}$  are  $(n \times n)$  mass, damping and stiffness matrices of the system, respectively;  $\mathbf{x}(\omega)$  is the  $(n \times 1)$  frequency response vector, and  $\mathbf{f}(\omega)$  is the forcing function vector in the frequency domain described by the scalar external force  $f(\omega)$  and the  $(n \times 1)$  influence vector  $\mathbf{l}$  indicating the DOF being excited by  $f(\omega)$ . In Eq. (1), input  $\mathbf{f}(\omega)$  and output  $\mathbf{x}(\omega)$  may be replaced with the influence vector  $\mathbf{l}$  and transfer function vector  $\mathbf{H}(\omega)$ , respectively, since transfer functions are commonly used to express the dynamic properties obtained from a test, instead of using input and outputs individually. By definition, transfer function vectors are equivalent to the outputs (responses) of system from the unit (whitenoise) input to the excitation DOF specified by  $\mathbf{l}$ . Thus, Eq. (1) can be converted to

$$\mathbf{B}(\omega) \mathbf{H}(\omega) = \mathbf{l}. \quad (4)$$

The dynamic stiffness matrix  $\mathbf{B}(\omega)$  can be replaced with its analytical counterpart  $\mathbf{B}(\mathbf{p}, \omega)$ , which is an explicit function of a set of (updating) parameters  $\mathbf{p} = [p_1, p_2, \dots, p_k]^T$  that are used in constructing the structural matrices  $\mathbf{M}$ ,  $\mathbf{C}$ , and  $\mathbf{K}$  through a finite element model. Similarly, the transfer function  $\mathbf{H}(\omega)$  can be replaced with  $\tilde{\mathbf{H}}(\omega)$  that is obtained experimentally (from measured input and outputs). As such, we can define the FRF error vector as

$$\boldsymbol{\varepsilon}_F(\mathbf{p}, \omega) = \mathbf{l} - \mathbf{B}(\mathbf{p}, \omega) \tilde{\mathbf{H}}(\omega). \quad (5)$$

Upon choosing a set of frequency points  $\{\omega_1, \omega_2, \dots, \omega_m\}$  where we sample the error vector in Eq. (5), we can define the overall FRF error vector as

$$\mathbf{e}_F(\mathbf{p}) \equiv \left[ \boldsymbol{\varepsilon}_F^T(\mathbf{p}, \omega_1), \boldsymbol{\varepsilon}_F^T(\mathbf{p}, \omega_2), \dots, \boldsymbol{\varepsilon}_F^T(\mathbf{p}, \omega_m) \right]^T. \quad (6)$$

We then seek an optimal parameter vector that minimizes an objective function that is constructed using an appropriate norm (Euclidean norm in this study) of the error vector. The formal problem statement then becomes

$$e_F(\mathbf{p}) \equiv \min_{\mathbf{p}} \frac{1}{2} \|\mathbf{e}_F(\mathbf{p})\|^2. \quad (7)$$

Notwithstanding the fact that this statement can be augmented with constraints (which shall be discussed later), the optimal solution is obtained by determining the root(s) of the gradient of the objective function in Eq. (7) with respect to the updating parameters  $\mathbf{p}$ , to wit,

$$\nabla_{\mathbf{p}} e_F(\mathbf{p}) = \nabla_{\mathbf{p}} \mathbf{e}_F(\mathbf{p}) \mathbf{e}_F(\mathbf{p}) = \mathbf{0}. \quad (8)$$

Depending on the finite element model, the preceding set of equations is generally nonlinear with respect to the updating parameters, and a solution can be obtained via Newton's method by linearizing them. To simplify the presentation, let us first rename

$$\mathbf{d}(\mathbf{p}, \omega_j) \equiv \mathbf{B}(\mathbf{p}, \omega_j) \tilde{\mathbf{H}}(\omega_j) \quad (9)$$

from which it follows that,

$$\nabla_{\mathbf{p}} \mathbf{e}_F(\mathbf{p}) \equiv - \left[ \nabla_{\mathbf{p}} \mathbf{d}^H(\mathbf{p}, \omega_1), \nabla_{\mathbf{p}} \mathbf{d}^H(\mathbf{p}, \omega_2), \dots, \nabla_{\mathbf{p}} \mathbf{d}^H(\mathbf{p}, \omega_m) \right]^H, \quad (10)$$



and thus

$$\nabla_{\mathbf{p}} e_F(\mathbf{p}) \equiv - \begin{bmatrix} \nabla_{\mathbf{p}}^H \mathbf{d}(\mathbf{p}, \omega_1) (\mathbf{I} - \mathbf{d}(\mathbf{p}, \omega_1)) \\ \nabla_{\mathbf{p}}^H \mathbf{d}(\mathbf{p}, \omega_2) (\mathbf{I} - \mathbf{d}(\mathbf{p}, \omega_2)) \\ \vdots \\ \nabla_{\mathbf{p}}^H \mathbf{d}(\mathbf{p}, \omega_m) (\mathbf{I} - \mathbf{d}(\mathbf{p}, \omega_m)) \end{bmatrix} = \mathbf{0} \quad (11)$$

where the superscript  $(.)^H$  denotes a complex conjugate transpose. By linearizing a typical term  $j \in \{1, 2, \dots, m\}$  in this vector, about the parameter vector at the current iteration  $\mathbf{p}^{(v)}$ , and by neglecting the second-order derivatives—thereby adopting a Gauss-Newton approach—we arrive at

$$\nabla_{\mathbf{p}}^H \mathbf{d}(\mathbf{p}^{(v)}, \omega_j) [\mathbf{I} - \mathbf{d}(\mathbf{p}^{(v)}, \omega_j)] - \nabla_{\mathbf{p}}^H \mathbf{d}(\mathbf{p}^{(v)}, \omega_j) \nabla_{\mathbf{p}} \mathbf{d}(\mathbf{p}^{(v)}, \omega_j) \Delta \mathbf{p} = \mathbf{0}. \quad (12)$$

Thus, the combined linearized equations (12) become

$$\mathbf{S}_F^{(v)H} \mathbf{S}_F^{(v)} \Delta \mathbf{p} = \mathbf{S}_F^{(v)H} \mathbf{r}_F^{(v)} \quad (13)$$

where we have defined the complex-valued FRF sensitivity matrix, and the residual vector for the full set of frequencies as

$$\mathbf{S}_F^v \equiv \begin{bmatrix} \mathbf{B}_{,1}^{(v)}(\omega_1) \tilde{\mathbf{H}}(\omega_1) & \mathbf{B}_{,2}^{(v)}(\omega_1) \tilde{\mathbf{H}}(\omega_1) & \cdots & \mathbf{B}_{,k}^{(v)}(\omega_1) \tilde{\mathbf{H}}(\omega_1) \\ \mathbf{B}_{,1}^{(v)}(\omega_2) \tilde{\mathbf{H}}(\omega_2) & \mathbf{B}_{,2}^{(v)}(\omega_2) \tilde{\mathbf{H}}(\omega_2) & \cdots & \mathbf{B}_{,k}^{(v)}(\omega_2) \tilde{\mathbf{H}}(\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{,1}^{(v)}(\omega_m) \tilde{\mathbf{H}}(\omega_m) & \mathbf{B}_{,2}^{(v)}(\omega_m) \tilde{\mathbf{H}}(\omega_m) & \cdots & \mathbf{B}_{,k}^{(v)}(\omega_m) \tilde{\mathbf{H}}(\omega_m) \end{bmatrix}, \mathbf{r}_F^v = \begin{Bmatrix} \mathbf{I} - \mathbf{B}^{(v)}(\omega_1) \tilde{\mathbf{H}}(\omega_1) \\ \mathbf{I} - \mathbf{B}^{(v)}(\omega_2) \tilde{\mathbf{H}}(\omega_2) \\ \vdots \\ \mathbf{I} - \mathbf{B}^{(v)}(\omega_m) \tilde{\mathbf{H}}(\omega_m) \end{Bmatrix}. \quad (14)$$

Note that each of the terms in Eq. (14) are defined as Eq. (15) using a frequency point  $\omega_j$  within

$$\{\omega_1, \omega_2, \dots, \omega_m\}$$

$$\mathbf{B}^{(\nu)}(\omega_j) \equiv \mathbf{B}(\mathbf{p}^{(\nu)}, \omega_j), \quad \mathbf{B}_{,i}^{(\nu)}(\omega_j) \equiv \left. \frac{\partial \mathbf{B}(\mathbf{p}, \omega_j)}{\partial p_i} \right|_{\mathbf{p}=\mathbf{p}^{(\nu)}} \quad (15)$$

with

$$\mathbf{B}_{,i}^{(\nu)}(\omega_j) = -\omega_j^2 \mathbf{M}_{,i}^{(\nu)} + i \omega_j \mathbf{C}_{,i}^{(\nu)} + \mathbf{K}_{,i}^{(\nu)}. \quad (16)$$

A second objective function—in addition to that given in Eq.(13)—can be constructed for model updating, which is a measure of the discrepancy between the natural frequencies obtained via modal system identification, and those of the finite element model. Omitting the intermediate steps for brevity, the resulting iterative equations (again, in a Gauss-Newton sense) can be stated as,

$$\mathbf{S}_M^{(\nu)T} \mathbf{S}_M^{(\nu)} \Delta \mathbf{p} = \mathbf{S}_M^{(\nu)T} \mathbf{r}_M^{(\nu)} \quad (17)$$

where

$$\mathbf{S}_M^{(\nu)} \equiv \left[ \boldsymbol{\Omega}_{,1}^{(\nu)}, \boldsymbol{\Omega}_{,2}^{(\nu)}, \dots, \boldsymbol{\Omega}_{,k}^{(\nu)} \right], \quad \mathbf{r}_M^{(\nu)} = \tilde{\boldsymbol{\Omega}} - \boldsymbol{\Omega}^{(\nu)}. \quad (18)$$

The vectors  $\tilde{\boldsymbol{\Omega}}$  and  $\boldsymbol{\Omega}^{(\nu)}$  denote the natural frequencies obtained from the experimental data (modal system identification) and those of the analytical model, evaluated at the iterative value of the updating parameter vector  $\mathbf{p}^{(\nu)}$ , respectively. The vector  $\boldsymbol{\Omega}_{,i}^{(\nu)}$  represents the sensitivity (partial derivative) of the analytical natural frequencies with respect to  $p_i$ , and  $\mathbf{r}_M^{(\nu)}$  is the residual vector between the measured and analytical natural frequencies for the current estimate. Because it is not generally possible to identify the natural frequencies of all modes accurately from measured data, the analytical natural frequency vector is typically truncated to contain only the counterparts of experimental natural frequencies that were determined with sufficient

confidence.

Both of the aforementioned (FRF and modal) objective functions can be combined to improve the accuracy of the model updating solutions. To wit, we have

$$\mathbf{S}^{(\nu)T} \mathbf{S}^{(\nu)} \Delta \mathbf{p} = \mathbf{S}^{(\nu)T} \mathbf{r}^{(\nu)} \quad (19)$$

where the combined  $(2mn+q) \times k$  sensitivity matrix  $\mathbf{S}^{(\nu)}$ , and the  $(2mn+q) \times 1$  residual vector  $\mathbf{r}^{(\nu)}$  are defined as

$$\mathbf{S}^{(\nu)} \equiv \begin{bmatrix} \text{Re}(\mathbf{S}_F^{(\nu)}) \\ \text{Im}(\mathbf{S}_F^{(\nu)}) \\ \mathbf{S}_M^{(\nu)} \end{bmatrix}, \quad \mathbf{r}^{(\nu)} \equiv \begin{Bmatrix} \text{Re}(\mathbf{r}_F^{(\nu)}) \\ \text{Im}(\mathbf{r}_F^{(\nu)}) \\ \mathbf{r}_M^{(\nu)} \end{Bmatrix} \quad (20)$$

with  $m$ ,  $n$ ,  $q$ , and  $k$  denoting the numbers of frequency points, degrees-of-freedom, selected natural frequencies, and updating parameters, respectively. Note that, we separated the real and imaginary parts in Eq. (13) in order to get rid of the complex conjugate transpose operation, and combined them with Eq. (17) to obtain the overall objective function given in Eq. (19).

The relative contributions of the two objective functions (based on FRF and modal data) can be adjusted using a weighting matrix. Since the terms from FRF data occupy a vast majority of the elements of the sensitivity matrix, larger weighting factors should be assigned to the modal data (otherwise, the modal data may not contribute to the updating process). A weighting matrix also allows the analyst to assign different weighting factors to each DOF depending on their relative perceived importance. The weighted version of Eq. (19) is given by,

$$\mathbf{S}^{(\nu)T} \mathbf{W} \mathbf{S}^{(\nu)} \Delta \mathbf{p} = \mathbf{S}^{(\nu)T} \mathbf{W} \mathbf{r}^{(\nu)} \quad (21)$$

where  $\mathbf{W}$  is a diagonal matrix containing the weighting factors. It follows that Eq. (21) is iterated

using the sequence of approximations

$$\mathbf{p}^{(\nu+1)} = \mathbf{p}^{(\nu)} + \Delta \mathbf{p} \quad (22)$$

until convergence is achieved.

## 2.2 Selection of the updating parameters for building structures

In order to expect a direct relationship between the updating parameters and the analytical model, the relevant parameter set must be consistent with the assumptions under which the analytical model is established. Here, we shall adopt the conventional assumptions used to model building structures; i.e., (i) mass is lumped at the floor levels; (ii) floor diaphragms are rigid in their plane; and (iii) the damping is classical (mode-proportional). Given these assumptions, system matrices are typically constructed through a static condensation of full matrices ( $\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{K}$ ) based on the aforementioned assumptions. Parameters that influence the condensed matrices of a typical (three-dimensional) building structure are discussed next.

### 2.2.1 Mass parameters

Based on the assumption of a rigid diaphragm, the in-plane motion of all points within the diaphragm is defined using two horizontal translations and the rotation about the vertical axis. The mass matrix of a story which has three dynamic degrees-of-freedom can be expressed in terms of the translational story mass ( $m_t$ ) and the story mass moment of inertia ( $I_\theta$ ) lumped at the center of mass ( $CM$ ) of a given floor level as

$$\mathbf{M}_1 = \begin{bmatrix} m_{xx} & m_{xy} & m_{xr} \\ m_{yx} & m_{yy} & m_{yr} \\ m_{rx} & m_{yr} & m_{rr} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -y_{CM} \\ 0 & 1 & x_{CM} \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} m_t & 0 & 0 \\ 0 & m_t & 0 \\ 0 & 0 & m_t r^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -y_{CM} \\ 0 & 1 & x_{CM} \\ 0 & 0 & 1 \end{bmatrix} \quad (23)$$

where  $r = \sqrt{I_0/m_t}$  is the radius of gyration of story mass at its  $CM$  and  $x_{CM}$ ,  $y_{CM}$  denote the

distance to the *CM* from the reference point. It follows from Equation (23) that four distinct parameters are needed to describe the terms of the mass matrix of a given floor, and thus, the associated (mass) updating parameter vector is

$$\mathbf{p}_M = [m_t, r, x_{CM}, y_{CM}]^T \quad (24)$$

Since the connectivity relationships between the mass parameters and the coefficients of the global mass matrix are known, as in Eq. (23), the sensitivity terms are easily calculated, i.e.

$$\frac{\partial m_{xx}}{\partial m_t} = 1, \quad \frac{\partial m_{xr}}{\partial m_t} = -y_{CM}, \quad \frac{\partial m_{rr}}{\partial m_t} = x_{CM}^2 + y_{CM}^2 + r^2, \quad \text{etc.} \quad (25)$$

These presented results (for a single story building) can easily be extended to obtain the mass matrix of an *n*-story building, i.e.,

$$\mathbf{M} = \text{diag}[\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_n] \quad (26)$$

and its associated sensitivities.

### 2.2.2 Stiffness parameters

In a “condensed model” approach, whereby the beams and slabs are assumed to be rigid along the in-plane directions but not along the out-of-plane direction, a global stiffness matrix incorporating the vertical displacement and two (out-of-plane) rotations of each joint, in addition to the three rigid-body motions (in-plane) at each floor level, is formed first, and subsequently reduced to a condensed matrix for dynamic analysis. The flexural, shear, torsional and axial stiffness values of each structural member are all independent variables influencing the stiffness matrix of the system, and thus may be selected as updating parameters. However, assigning a single set of parameters to a particular type of structural members in a plane frame is deemed a

reasonable approach, as it simultaneously reduces the problem size and enhances numerical conditioning of updating matrices. For example, the stiffness contributions of two columns with equal dimensions and similar boundary conditions in a plane frame will be nearly identical. Clearly, assigning different parameters to two such columns will yield rank-deficient or, at best, ill-conditioned updating matrices. Thus, it is appropriate to select stiffness updating parameters as

$$\mathbf{p}_K = \left[ \mathbf{p}_{k\_flexual}, \mathbf{p}_{k\_shear}, \mathbf{p}_{k\_torsional}, \mathbf{p}_{k\_axial} \right]^T, \quad (27)$$

which represents the combined stiffnesses of a single type (or class) of member in each plane frame. In practice, only the major contributors to the stiffness (i.e., the flexural stiffness of columns or slender beams, or shear and flexural stiffnesses of short shear walls) are selected to be the updating parameters, while the effect of others terms that are considered negligible in comparison are ignored.

As per the preceding discussion, the global stiffness matrix,  $\mathbf{K}_G$ , is partitioned into four sub-matrices, and condensed using the transformation matrix derived from the appropriate operations on its sub-matrices as in

$$\mathbf{K}_G = \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{mo} \\ \mathbf{K}_{om} & \mathbf{K}_{oo} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{oo}^{-1}\mathbf{K}_{mo}^T \end{bmatrix}^T \mathbf{K}_G \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{oo}^{-1}\mathbf{K}_{mo}^T \end{bmatrix}. \quad (28)$$

All of the sub-matrices in  $\mathbf{K}_G$  are functions of stiffness of structural members (i.e., columns, beams, etc.) Therefore, if the stiffnesses of the structural members are selected as updating parameters, the terms of the sensitivity matrix (i.e., the derivative of the condensed stiffness matrix with respect to the updating parameters) will be highly nonlinear functions because the

simple connectivity rule that existed in the global stiffness matrix is destroyed due to condensation. Given this, expressing the sensitivity of each term with an analytical expression, as it was the case for the mass matrix, is nontrivial. As such, the stiffness sensitivity matrix has to be computed numerically, typically using a FE analysis program.

### 2.2.3 Damping parameters

The (classical) damping matrix  $\mathbf{C}$  is defined as

$$\mathbf{C} \equiv \mathbf{\Phi}^{-T} [2\mathbf{\Xi}\mathbf{\Omega}] \mathbf{\Phi}^{-1} \quad (29)$$

where  $\mathbf{\Xi}$  is the diagonal matrix containing modal damping ratios. The modal ( $\mathbf{\Phi}$ ) and spectral ( $\mathbf{\Omega}$ ) matrices contain the mass normalized mode shapes and natural frequencies, respectively, and are functions of the mass and stiffness matrices. Thus, the modal damping ratios comprise the only independent variables in Eq. (29), and thus, may be chosen as the updating parameters

$$\mathbf{p}_D = [\zeta_1, \zeta_2, \dots, \zeta_n]^T. \quad (30)$$

## 2.3 Rank deficiency and ill-conditioning

As previously mentioned, the dynamic behavior of a real structure obtained from vibration measurements and that estimated from a highly idealized FE model usually display significant discrepancies. These discrepancies mainly occur as a result of (i) measurement errors associated with sensor noise, (ii) numerical errors in data processing or calculations, (iii) modeling errors due to inappropriate idealization (e.g., inexact boundary conditions) of the structure, and (iv) parameter errors due to inaccurate material properties and geometric data for elements in the FE model. To a certain degree, the measurement and numerical errors are inevitable when collecting and processing experimental data (e.g., decimation, smoothing, filtering, etc.); and addressing these types of errors are not considered in this study.

However, updated parameters are not always physically admissible, even when the structure is modeled based on an appropriate idealization (modeling assumptions) and the measurement and numerical errors are believed to be reasonably small. This is because the optimal parameter set that minimizes the objective function within a normal range of the residual error is non-unique. This non-uniqueness happens when the number of selected updating parameters is larger than the number of measured response quantities [12]. Numerically, this aforementioned spatial incompleteness renders the updating procedure to be equivalent to a rank-deficient (or, at best, an ill-conditioned) least-squares problem as the sensitivity matrix becomes singular or nearly singular.

One approach to avoid this type of numerical difficulty is to select a sufficiently small number of updating parameters [4]. The selected parameters should have *significant* and, to the largest extent possible, *distinct* effects on the system response. While this information can be deduced from a separate sensitivity analysis, it is not easy to decide the specific subset of suitable parameters (from among numerous candidates). Furthermore, reasonable estimates for the values of unselected parameters are still needed to obtain a good solution to the updating problem. The said choice is generally made through a process of subset selection, whereby response residuals, corresponding to candidate subsets of parameters, are evaluated and compared against each other [13]. The drawback of this approach is that the number of candidate parameter sets, that need to be tested, grows rapidly as the size/detail of the initial finite element model is increased.

Alternative approaches to addressing the numerical difficulties are seeking (i) “minimum-norm” solutions using singular value decompositions, or (ii) approximate solutions using an appropriate regularization technique. The regularization approach is a generalized form of the minimum-norm approach that has been widely used for model updating [5-7]. Simply put, a



“minimum-norm” solution is a minimizer of the objective function (among all of the non-unique minimizers) that is closest to the initial guess of the updating parameters. Unfortunately, there is no guarantee that a minimum-norm solution will be physically more meaningful than the other minimizers of the objective function. In the more general—yet essentially identical—regularization approach, any drastic deviation of updating parameters from their initial (estimated) values is suppressed by a penalty function—comprising a regularization parameter, and a set of side constraints—under the assumption that the initial model is already a reasonable approximation. To wit, the original objective function  $e(\mathbf{p})$  is augmented as in

$$J(\mathbf{p}) = e(\mathbf{p}) + \lambda^2 \|\mathbf{B}_\lambda \mathbf{p}\|^2 \quad (31)$$

where,  $\lambda$  is the regularization parameter, and  $\mathbf{B}_\lambda$  is a matrix with a rank that is equal to the number of updating parameters. As Eq. (31) indicates, solutions using the regularization are dependent on the regularization parameter and the initial estimates of updating parameters. The value of the regularization parameter is chosen to give a suitable balance between the original objective function and the side constraints. A number of regularization methods are suggested (and demonstrated to be useful) in the literature for the solution of ill-conditioned least-squares problems [14-17]. We shall make use of these methods later in our validation studies.

In a fundamental sense, the minimum-norm, and the regularized solutions suffer from the same issues as the first approach does. Specifically, repetitive analyses are needed (so that the minimizer that is closest to the initial guess is confidently identified) in case the minimum-norm approach is adopted; and, the side constraints as well as the value of the regularization parameter have to be based on judgment (and hence the experience of the analyst) and/or determined through laborious *a priori* computations in case the regularization approach is adopted.

In the following section, we propose sensitivity-based relative constraints to the objective function to obtain solutions to the updating problem that are as reasonable as those obtained using a regularization approach; however, the proposed constraints do not require any *a priori* assessment of the relative sensitivity of the structural response with respect the updating parameters by the analyst to maintain the well-conditioning of the updating matrices.

#### 2.4 Solution of the updating equation using a novel type of constraint

As mentioned earlier, the conventional least-squares solution produces unbounded results to ill conditioned model-updating problems. Usually, ill conditioning causes big changes in a few parameters from their initial values when they have similar effects on the system response. Consequently, the obtained parameters have large variations, and their extreme values cannot be explained in a physical sense. Here, we add two kinds of constraint conditions to the least-squares problem in order to overcome this problem. The first constraints are the, usual, upper and lower bounds that restrict the optimal solution within a “feasible domain.” The solution to such constrained least-squares problems is achieved through the Bounded Variables Least-Squares (BVLS) technique [10], which determines the solutions that satisfy the two-sided inequality constraints for each parameter given as

$$\underline{\mathbf{p}} \leq \mathbf{p}^{(v)} + \Delta\mathbf{p} \leq \hat{\mathbf{p}} \quad (32)$$

where  $\underline{\mathbf{p}}$  and  $\hat{\mathbf{p}}$  denote the lower and upper bounds vectors, respectively, which are usually based on physical considerations. We propose additional constraints that restrict the deviation of two parameters that have similar effects on the system response, i.e.,

$$|p_i - p_j| < 1 - \text{cor}(\mathbf{S}_i, \mathbf{S}_j), \quad \text{if } \text{cor}(\mathbf{S}_i, \mathbf{S}_j) > c_{lim} . \quad (33)$$

Eq. (33) represents the constraints placed on the relative variations between *any* two parameters, where  $\mathbf{S}_i$  and  $\mathbf{S}_j$  are the sensitivity vectors (i.e., the  $i$ -th and  $j$ -th columns of the sensitivity matrix  $\mathbf{S}$ ) with respect to the updating parameters  $p_i$  and  $p_j$ , respectively. The term  $\text{cor}(\mathbf{S}_i, \mathbf{S}_j)$  denotes the coefficient of correlation between the vectors  $\mathbf{S}_i$  and  $\mathbf{S}_j$ . Therefore, if two parameters have similar effects on the response (i.e., the coefficient of correlation is larger than a given limit  $c_{lim}$ ), then the difference between the two updated parameters is restricted to remain within a range that depends on their degree of similarity. This requirement forces the updating parameters to change similarly (between iterations) if they have similar effects on the structural response. The least-squares solution of Eq.(21), subject to the constraints provided in Eqs. (32) and (33), can be achieved using the standard Bounded Variables Least Squares (BVLS) technique. As such, we shall refer to the proposed iterative method for model updating as the BVLSrc method, where “rc” stands for “relative constraints.”

### 3. VALIDATION OF BVLSrc

In this section, we investigate the effectiveness and the veracity of BVLSrc through a simulated forced-vibration test of a two-story frame building with an L-shaped floor plan, as shown in Figure 1.a. We arbitrarily vary the stiffness properties of structural members, and attempt to estimate their values using three types of FE model updating techniques: namely, the BVLSrc (proposed), regularization, and pure BVLS methods. We then compare the accuracy of the estimations of these methods. For brevity, we shall carry out model updating using FRF data only, and by selecting the updating parameters to be only the stiffness factors associated with the flexural stiffness of columns and beams in the building. Other (e.g., shear, axial, and torsional) stiffness properties, mass properties, and modal damping ratios can also be selected as updating

parameters, but their values will be fixed in this numerical investigation to simplify the comparison of methods. The use of the more general form of the proposed model updating technique (BVLSrc) is deferred to the companion paper [11] in which a larger set of updating parameters are included, and the model updating is performed using both the FRF and modal data of a real building, simultaneously.

### **3.1 Description of the simulated forced vibration test**

Figure 1.b displays the roof floor plan of the simulated structure, the location where the excitation (input) is applied and its direction, and the reference point where the response (output) is measured. The building has an asymmetrical plan, and thus, the input generates a reasonably general dynamic response (i.e., two lateral translations as well as rotation about the  $z$ -axis).

Simulation of the forced-vibration tests of the example building is performed using the proprietary finite element analysis program, SAP2000 [18]. Stiffnesses of the slabs are not included in the model, but their masses are accounted for. The modulus of elasticity ( $E$ ) of the material comprising the beams and columns is assumed to be 24821 MPa. All beams are 46 cm deep and 25 cm wide, and all columns are 46 cm by 30 cm. Flexural stiffnesses of members are given by the product of the gross section properties and varied flexural stiffness factors.

The effective flexural stiffness factors (i.e., the multipliers to  $EI_g$  for the beams and columns) are chosen as updating parameters. Following nominal assumptions, we group them by member type, by story, and by the plane frames to which they belong. To wit, as illustrated in Figure 2, the beams and the columns are divided into 12 groups each (i.e., 3 plane frames  $\times$  2 directions ( $x$  and  $y$ )  $\times$  2 stories). Arbitrary values ranging from 0.3 to 1.15, as shown in Table 1, are assigned to be the actual (“true”) effective stiffness factors (multipliers) for each group. These variations

may be thought of as a consequence of uncertainty in the values of material properties, construction tolerances, and/or the degree of concrete cracking. The actual stiffness values for a real building may be scattered even within a group of members; however, this is a reasonable simplifying assumption to assess the capability of the three updating methods by comparing the converged updating parameters that they yield with the true values.

Adopting the “lumped mass” and “rigid in-plane diaphragm” assumptions discussed earlier, the building is represented by a six degree-of-freedom system—i.e., two translational and one rotational degree-of-freedom per floor—for dynamic analysis. The mass matrix with respect to the reference point is evaluated using the assumed self-weights of the building and the superimposed loads. The undamped natural frequencies of the building—calculated using the mass and stiffness matrices—are provided in Table 2.

Transfer functions (i.e., experimental FRF data for model updating), were evaluated from story displacements (outputs) due to a whitenoise-type forcing function (input) via SAP2000. Six channels of output (i.e., one rotational and two translational components of story displacements in each floor) were obtained at the reference points, and the input force was transformed to an equivalent one at the reference point. The whitenoise-type forcing function, comprising 16384 samples in 0.02 sec intervals, was obtained by bandpass-filtering of random numbers with cut-off frequencies 1Hz and 10Hz. This type of broadband excitation can be generated using a linear inertial shaker [11]. The input force was applied at the B1 column on the roof, with an angle of 45° with respect to the  $x$ -axis, as shown in Figure 1.b.

A damping ratio of 5% was assumed for all modes. To simulate measurement errors, a random noise corresponding to 5% of the maximum amplitude of the signal was added to both the input

and output streams. Transfer functions for each direction were evaluated as the ratio of the cross-spectral density ( $S_{xf}$ ) between input and output to the auto-spectral density ( $S_{ff}$ ) of input as in

$$H(\omega) = \frac{S_{xf}}{S_{ff}} = \frac{x(\omega)\bar{f}(\omega)}{f(\omega)\bar{f}(\omega)}. \quad (34)$$

The Matlab [19] command `tfestimate` was used for calculating the transfer functions. Evaluations were made at 1024 frequency points ranging from 0Hz to 25Hz at 0.0488 Hz intervals, and the curves were smoothed using a Hanning window [20] and signal averaging. From the obtained curve (henceforth referred to as the “experimental curve”), 164 frequency points between 1Hz to 9Hz were selected for updating. Figure 3 shows the experimental FRF data points and the theoretical FRF curves calculated using the system matrices.

As discussed earlier, when the system matrices are obtained by static condensation, the condensed stiffness matrix is a nonlinear function of the updating parameters—in this particular case, the effective stiffness factors of structural members. Consequently, it is not trivial to express their relationships as closed-form functions. Thus, sensitivities of the system matrices with respect to the updating parameters, given in Eq. (16), were evaluated numerically using finite differences (i.e., using their rates of change to small perturbations in their values). For this purpose, a separate Matlab code was implemented that generates the three-dimensional condensed stiffness matrix of the example building, and calculates the terms of the sensitivity matrix using perturbations. This code was integrated with the Matlab code used for the updating computations to create an efficient solution process.

### 3.2 Normalization of data and the updating parameters

The initial values of the updating parameters were all assumed to be equal to one, and the

updating calculations were performed with normalized (by the flexural stiffness based on gross section,  $EI_g$ ) stiffness quantities. This type of normalization is commonly used to avoid ill-conditioning due to the differences in the relative magnitudes (units) of parameters [4]. Additionally, the translational components of story displacements were divided by a factor of 7.5 before the evaluation of experimental FRF curves from measured data, so that they have similar spectral amplitudes with the rotational components. A similar scaling was applied to the analytical model by dividing and multiplying the said factor to translational and torsional coefficients of system matrices, respectively. The condition number of initial sensitivity matrix in this example was reduced from  $2.46 \times 10^{14}$  to  $3.85 \times 10^{13}$  by the scaling as stated above.

The aforementioned normalization of updating parameters, scaling of experimental data and the system matrices model are not only essential for improving the *initial conditioning* of the model updating problem, but also for the efficient use of relative constraints given in Eq. (33). Of course, the aforementioned initial normalization operations are not a cure for the ill conditioning that results from spatial incompleteness. As it will be demonstrated in the following sub-sections, the proposed relative constraints in Eq. (33) alleviate the problem of ill-conditioning in the updating computations.

### **3.3 Constraints, weights, and regularization**

For all (i.e., pure BVLS, BVLSrc, and regularization) methods, the weighting matrix in Eq. (21) was set to identity, and the standard Matlab command `lsqlin` was used as the solver. For BVLS and BVLSrc methods, the absolute lower and upper bounds for all of the (normalized) updating parameters were chosen to be 0.15, and 1.5, respectively. For BVLSrc, the value for  $c_{lim}$  in Eq. (33) was set to 0.5. Because the coefficient of correlation varies from -1 to 1, and the

feasible domain is set to be  $[0.15, 1.5]$ , the range of relative constraints is  $[0.0, 2.0]$  (i.e., wider than the absolute constraints). Thus, the possibility of constraining two parameters with low correlation is eliminated.

For the regularization method, the iterative model updating equations—obtained by linearizing Eq. (31) with respect to the updating parameter vector  $\mathbf{p}$ —are

$$\left[ \mathbf{S}^T \mathbf{S} + \lambda^2 \mathbf{B}_\lambda^T \mathbf{B}_\lambda \right] \Delta \mathbf{p} = \mathbf{S}^T \mathbf{r}. \quad (35)$$

We set the side-constraint matrix  $\mathbf{B}_\lambda$  to identity as per Nakte [17], and tested various regularization parameters ranging from 0.134 to 13.4 to determine its optimal value. It is well known that the relationship between the norm of the residual and the parameter update vectors (i.e.,  $\|\mathbf{r}\|$  and  $\|\Delta \mathbf{p}\|$ ) is generally an L-shaped curve; and the optimal value of the regularization parameter can be selected as the corner point of this L-shaped curve [21]. Following this approach, the regularization parameter was set to 1.34.

### 3.4 Comparison of the three model updating methods

For all methods, model updating was performed using the experimental FRF data described in §3.1. Figures 4.a and 4.b display the analytical FRF before and after the updating, respectively. The amplitude spectra (of the lateral and rotational responses) in these figures were arbitrarily scaled to distinguish each component. Figure 4.b represents the results from BVLSrc only, and the updated FRF curves using the other methods are almost identical to (i.e., as accurate as) the results from BVLSrc, and thus, are omitted for brevity. Similarly, the natural frequencies of the original model were recovered accurately by all (three) methods as displayed in Table 3. However, as it will be demonstrated later, the accuracy of updated FRF and modal data does not



guarantee the accuracy of converged updating parameters.

Figure 5 displays the convergence plots indicating the change of norms of the residual, parameter update, and estimation error vectors for each iteration. The norm of the residual vector  $\|\mathbf{r}\|$  was decreased from its initial value 33.1 to 5.8 after updating. Noting that these residuals are due to the differences between the analytical FRF (of initial or updated parameters) and experimental FRF evaluated using noisy response, the residuals do not diminish to zero due to the scatter of experimental FRF points, which, in turn, are brought on by the numerical errors associated with the evaluation of the transfer functions from measured data containing sensor noise (measurement errors). The dashed line in Figure 5.a represents combined measurement and numerical errors, which is the difference between the analytical FRF using actual (true) parameters and the experimental FRF using noisy response. When noise-free response was used, this value dropped to 0.8. Therefore, the initial residual norm (33.1) can roughly be broken into parameter, measurement, and numerical errors as 27.3, 5.0, and 0.8, respectively. Of course, these values are valid only for this example, and for another problem, measurement and numerical errors will depend on the amount of noise, method of evaluation, frequency range, and the smoothing method employed.

The norms of parameter update vectors  $\|\Delta\mathbf{p}\|$  obtained from the three methods are compared in Figure 5.b. In this particular simulation, ten iterations were performed for all methods (in practice, a convergence criteria based on the norm of parameter update vector can also be established to terminate the iterations). The norms of estimation errors  $\|\mathbf{p}^v - \mathbf{p}^*\|$  obtained from the three methods are plotted in Figure 5.c where  $\mathbf{p}^*$  denotes the vector of true parameter values. As indicated by this figure, BVLSrc and the regularization methods have similar accuracy, while

the pure BVLS method does not appear to be capable of decreasing the parameter error from its initial value.

As indicated by Figure 6—where the true and the converged values of the updating parameters are displayed—updated parameters obtained from the three methods show substantial discrepancies, even though no significant differences were found in their updated FRF curves and residual vectors. These discrepancies confirm that there are indeed multiple solutions that minimize the objective function, and that the identification of true parameters from an ill-conditioned problem (i.e., with incomplete data) is, in general, not possible. As previously mentioned, this inherent limitation of model updating is due to the use of more updating parameters than the amount of information contained in the measurements. To estimate all parameters accurately, additional information (such as curvature distributions throughout the members) is needed. However, the rationale of model updating is that this incompleteness can be circumvented by taking the missing information from an *a priori* model instead of taking them from experimental data. This strategy distinguishes model updating from direct system identification [22].

Nevertheless, in general, the BVLSrc and the regularization methods yielded more accurate parameters than the pure BVLS method. In more detail, Figure 6.a displays the estimated parameters for beams on the second floor and columns between the second floor and the base (1<sup>st</sup> story), whereas Figure 6.b displays the parameters for beams on the roof and columns between the roof and the second floor (2<sup>nd</sup> story). Figure 6.b indicates that estimates from BVLS for the 2<sup>nd</sup> story members are substantially deviated from their true values (in fact, a number of these parameters hit the upper-bound constraint). This is because the parameters for the members in the second story are closely related to each other. The coefficients of correlation (COR) between

the pairs of closely related parameters are shown in Figure 6.b, indicating the potential source of ill conditioning for the simulated structure.

In Table 4, the coefficients of correlation between the sensitivity vectors in the final iteration (of the BVLSrc) are listed starting from the highest value. Deviation of estimates between each parameter pair was restricted by the given relative constraints. The relative constraints have the effect of reducing the number of parameters by forcing the closely related parameters to be updated similarly during iterations. Updated values for these parameters are located between the two true values. It was previously noted that one approach to address the ill conditioning is to reduce the number of parameters through the process of subset selection. BVLSrc performs this process automatically through the proposed relative constraints without requiring trial-and-error computations, and thus, minimizes the effort of choosing the “proper” set of updating parameters and assigning suitable values to the unselected ones.

Results from the BVLSrc and the regularization method display similar accuracy (Figures 5.c and 6). However, the regularization method requires a series of calculations to construct the L-shaped curve described in §3.3, so that the optimal value of the regularization parameter for a given initial parameter set can be determined. Additionally, the converged results of the regularization method are sensitive to the initial guess. Figure 7 compares the converged results of the BVLSrc and regularization methods when two different sets of initial guess (i.e., when  $\mathbf{p}^{(0)}$  are all 1.0 and when  $\mathbf{p}^{(0)}$  are all 1.4) are used. The regularization method yields somewhat different solutions for the different initial guesses, especially for closely correlated parameter pairs. On the other hand, BVLSrc yields almost identical solutions regardless of the initial guess even for the closely related pairs (for which the converged values always remained between the

true values of the pair).

#### 4. CONCLUSIONS

In this manuscript, we developed an improved finite element model updating method in order to address the numerical difficulties associated with ill conditioning and rank deficiency associated with the selection of a proper set of updating parameters. Often, it is not possible to determine *a priori* which subset of the candidate updating parameters are free of redundancies; or, at best, repetitive computations as well as engineering judgment are required to deduce such information for a given problem.

The proposed method is based on the standard Bounded Variables Least-squares (BVLS) method, and incorporates the usual upper/lower-bound constraints that are based on physical considerations, as well as *novel* sensitivity-based relative constraints (consequently, we named the proposed method BVLSrc). The relative constraints are constructed using the correlation coefficients between the sensitivity vectors (i.e., columns of the sensitivity matrix) of updating parameters, and thus, this approach does not require *a priori* calculations.

We investigated the veracity and the effectiveness of BVLSrc through the simulated forced vibration tests of a simple, albeit sufficiently general, structure using the structure's frequency response function as data. Through this numerical study, we compared the results obtained from BVLSrc with those from two other competing and prevalent, namely pure BVLS and regularization, methods. We have shown that pure BVLS yields physically inadmissible solutions (especially for parameter pairs, which have high coefficients of correlation due to ill conditioning), and that BVLSrc and regularization methods yield approximate solutions with similar and sufficiently high accuracy (to the extent possible with spatially incomplete data).

However, the BVLSrc is shown to be computationally more efficient than the regularization method. In the companion paper [11], we apply BVLSrc, developed and validated here, to the model updating of an actual, reinforced concrete office building that was subjected to a range of forced-vibration tests during the summer of 2004.

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### TABLES (4)

Table 1. True parameter values (actual stiffness factors) of the simulated structure.

	Line A	Line B	Line C	Line 1	Line 2	Line 3
Beams @2F	0.457	0.954	0.966	0.955	0.426	0.922
Columns @2F	0.805	0.973	0.905	0.597	1.017	0.385
Beams @1F	0.872	0.844	0.572	0.888	1.046	0.653
Columns @1F	0.427	0.646	0.819	0.621	0.888	0.348

Table 2. The undamped natural frequencies of the simulated structure.

Mode	1st	2nd	3rd	4th	5th	6th
Type	y-trans.	rotation	x-trans.	y-trans.	rotation	x-trans.
Natural frequency	1.66 Hz	1.99 Hz	2.25 Hz	4.81 Hz	5.84 Hz	6.61 Hz



Table 3. Natural frequencies of updated models normalized with their true values.

Mode	1st	2nd	3rd	4th	5th	6th
BVLS/True	1.000	1.004	1.001	1.000	1.000	0.998
BVLSrc/True	1.000	1.001	1.002	1.000	0.999	0.999
Regularization/True	1.000	1.002	1.002	1.000	1.000	0.999

Table 4. Coefficients of correlation between two closely related parameters.

Parameters			True		BVLSrc		Diff.
$P_i$	$P_j$	1-COR	$P_i$	$P_j$	$P_i$	$P_j$	
Col-3-2F	Bm-3-2F	<i>0.013</i>	0.653	0.922	0.712	0.699	<i>0.013</i>
Col-2-2F	Bm-2-2F	<i>0.016</i>	1.046	0.426	0.773	0.757	<i>0.016</i>
Col-1-2F	Bm-1-2F	<i>0.030</i>	0.888	0.955	0.946	0.916	<i>0.030</i>
Col-B-2F	Bm-B-2F	<i>0.082</i>	0.844	0.954	0.961	0.879	<i>0.082</i>
Col-A-2F	Bm-A-2F	<i>0.084</i>	0.872	0.457	0.683	0.599	<i>0.084</i>
Col-C-2F	Bm-C-2F	<i>0.125</i>	0.572	0.966	0.612	0.737	<i>0.125</i>
Bm-3-1F	Bm-3-2F	0.251	0.385	0.922	0.448	0.699	0.251
Bm-A-1F	Bm-A-2F	0.292	0.805	0.457	0.671	0.599	0.072
Bm-2-2F	Bm-3-2F	0.312	0.426	0.922	0.757	0.699	0.058

### FIGURES (7)

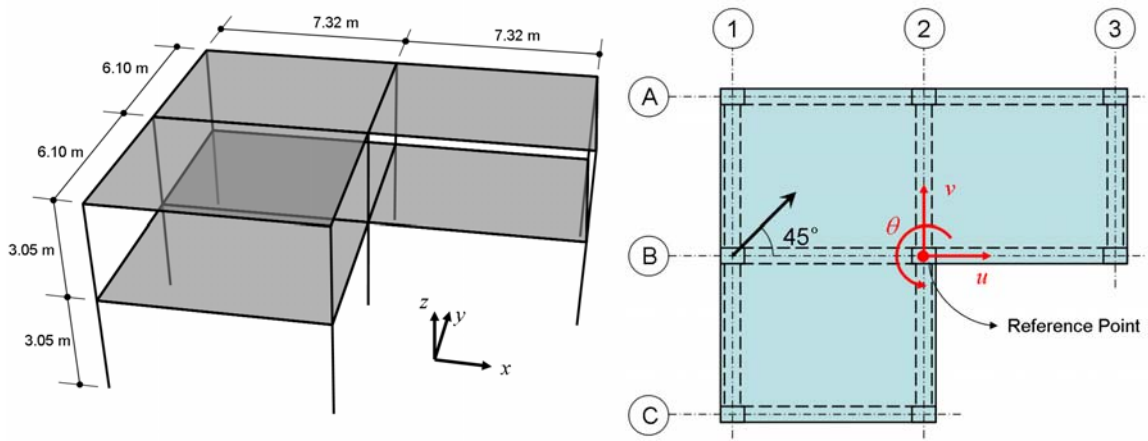


Figure 1. 3-D view of the simulated structure (left), its roof plan, and the applied excitation (right).

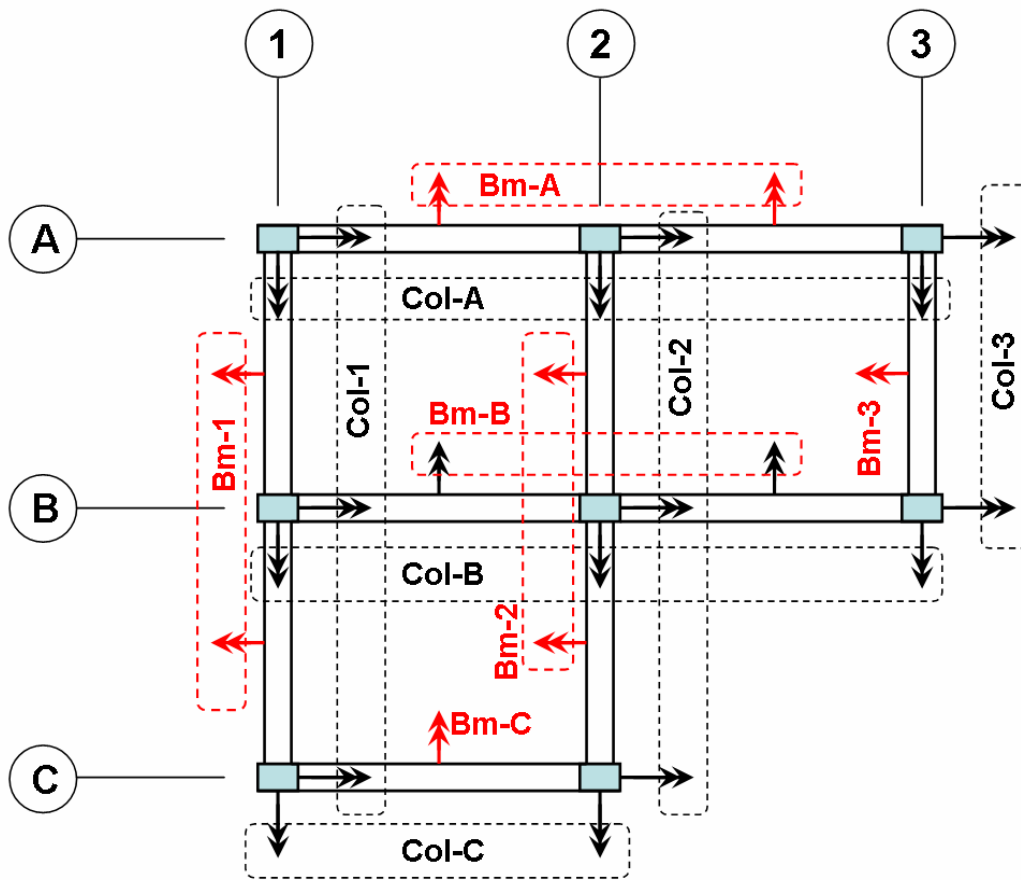


Figure 2. Grouping of member stiffnesses for updating.

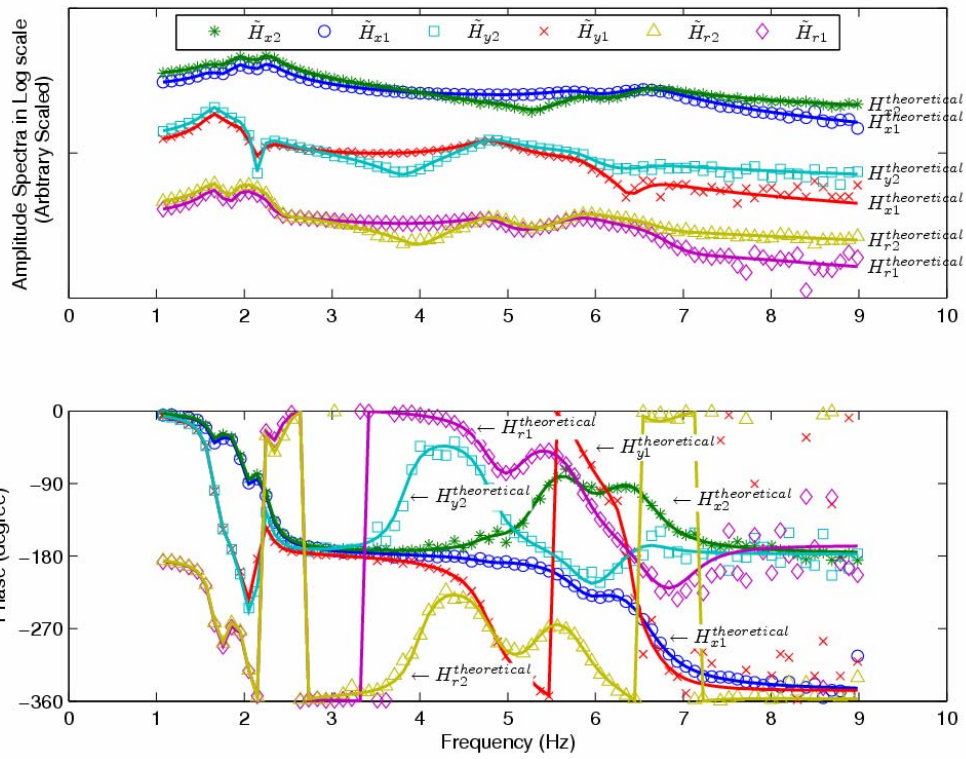
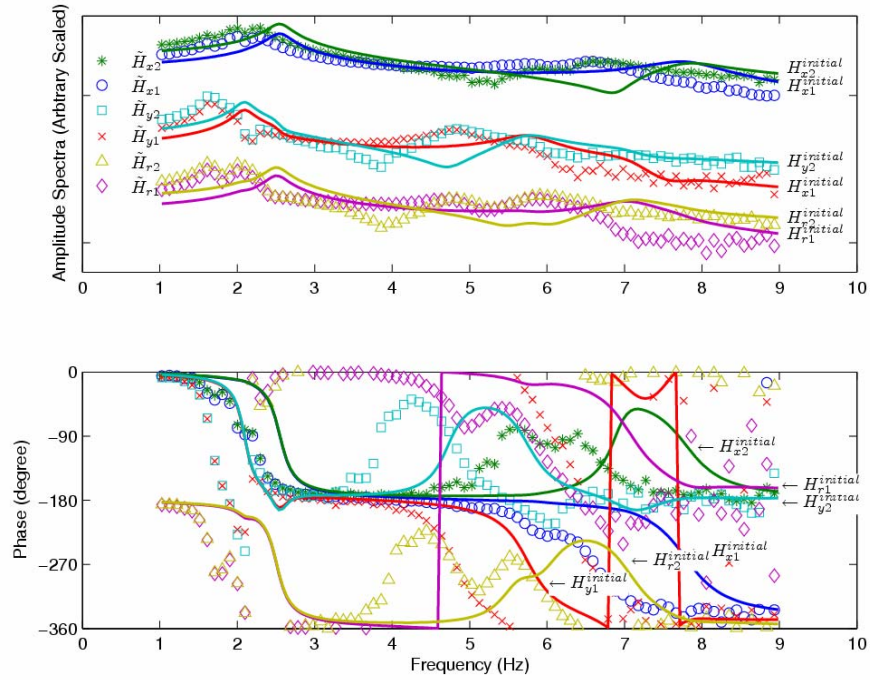
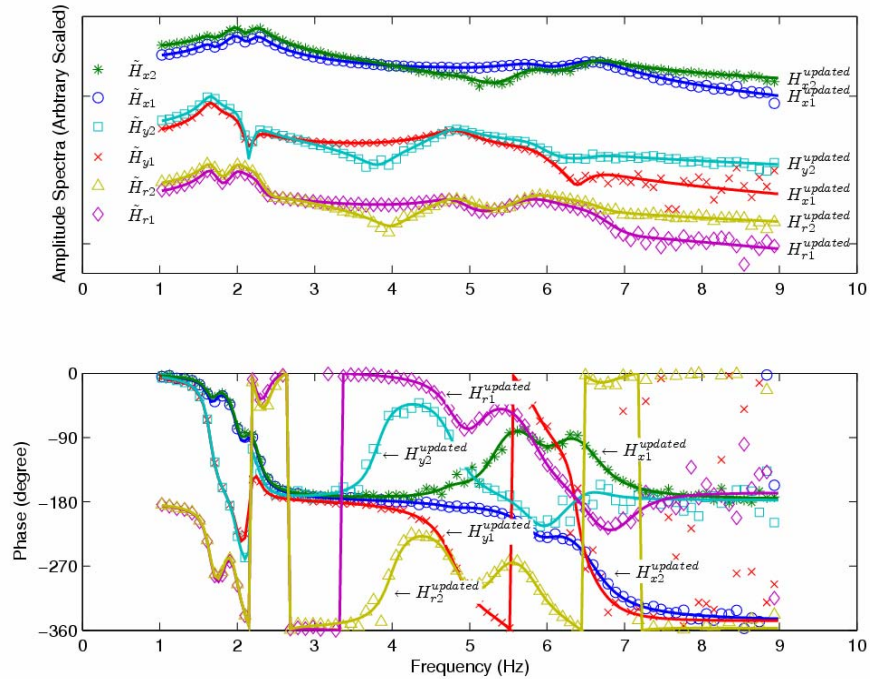


Figure 3. Theoretical (lines) and experimental (symbols) frequency response functions.



(a) Initial FRF spectra.



(b) Updated FRF spectra (BVLsrc).

Figure 4. Frequency response function curves before and after updating.

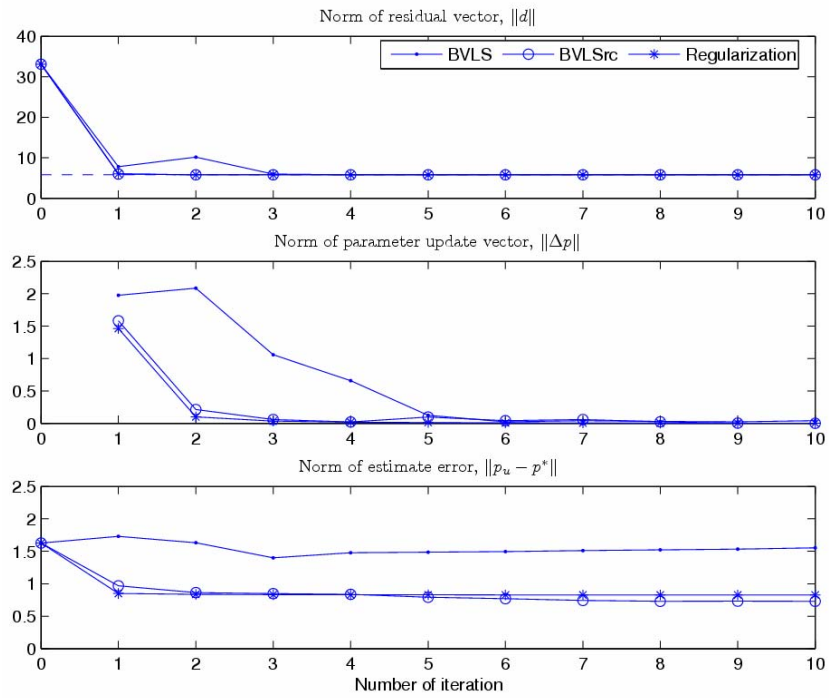


Figure 5. Change of error norms during successive model updating iterations.

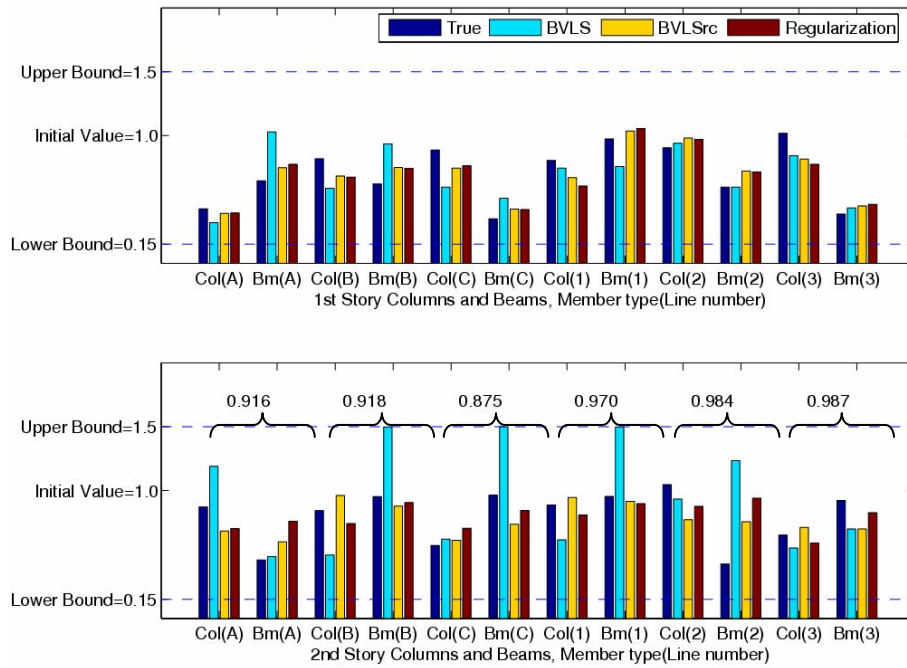


Figure 6. Comparison of true and updated parameter values.



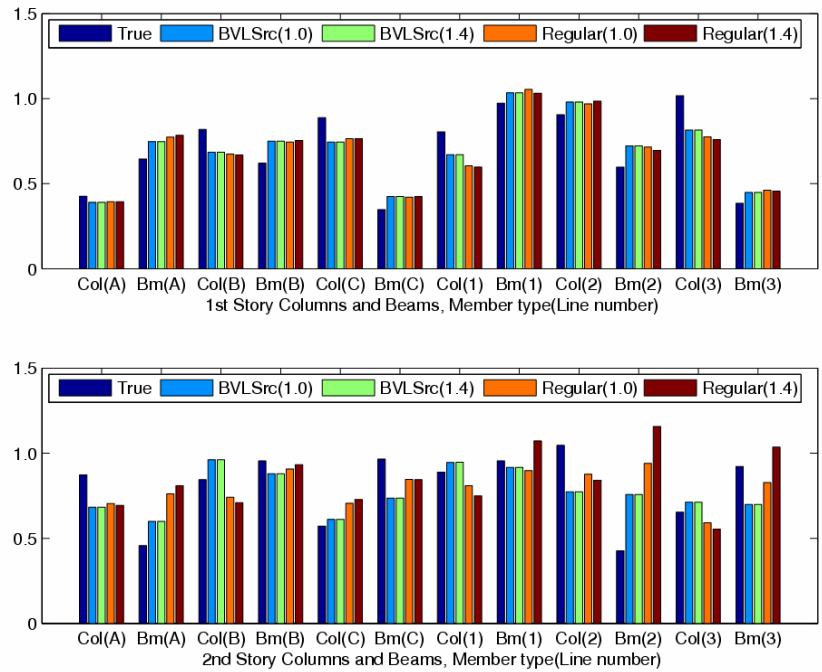


Figure 7. Updated parameters starting from different initial guesses.