Proceedings of the International Conference on Logic, Information, Control & Computation, ICLICC 2011, 25-27 Febraury, Gandhigram Rural Institute - DU, Tamil Nadu, India.

Optimal control for Navier-Stokes Takagi-Sugeno fuzzy equations using Simulink

Kumaresan Nallasamy[1], Kuru Ratnavelu[1], Bernardine R. Wong[1]

[1] Institute of Mathematical Sciences,
Faculty of Science, University of Malaya,
Kuala Lumpur 50603, Malaysia,
drnk2008@gmail.com

Abstract. In this paper, the unsteady Navier-Stokes Takagi-Sugeno (T-S) fuzzy equations (UNSTSFEs) are represented as a differential algebraic system of strangeness index one by applying any spatial discretization. Since such differential algebraic systems have a difficulty to solve in their original form, most approaches use some kind of index reduction. While processing this index reduction, it is important to take care of the manifolds contained in the differential algebraic equation (DAE) /singular system (SS) for each fuzzy rule. The Navier-Stokes equations are investigated along the lines of the theoretically best index reduction by using several discretization schemes. Applying this technique, the UNSTSFEs can be reduced into DAE. Optimal control for Navier-Stokes T-S fuzzy system with quadratic performance is obtained by finding the optimal control of singular T-S fuzzy system using Simulink. To obtain the optimal control, the solution of matrix Riccati differential equation (MRDE) is found by solving differential algebraic equation (DAE) using Simulink approach. The solution of Simulink approach is equivalent or very close to the exact solution of the problem. An illustrative numerical example is presented for the proposed method.

Keywords: Differential algebraic equation, Matrix Riccati differential Equation, Navier-Stokes equation, Optimal control and Simulink.

1 Introduction

A fuzzy system consists of linguistic IF-THEN rules that have fuzzy antecedent and consequent parts. It is a static nonlinear mapping from the input space to the output space. The inputs and outputs are crisp real numbers and not fuzzy sets. The fuzzification block converts the crisp inputs to fuzzy sets and then the inference mechanism uses the fuzzy rules in the rule-base to produce fuzzy conclusions or fuzzy aggregations and finally the defuzzification block converts these fuzzy conclusions into the crisp outputs. The fuzzy system with singleton fuzzifier, product inference engine, center average defuzzifier and Gaussian membership functions is called as standard fuzzy system [18].

Two main advantages of fuzzy systems for the control and modelling applications are (i) uncertain or approximate reasoning, especially difficult to express a mathematical model (ii) decision making problems with the estimated values under incomplete or uncertain information [20]. Stability and optimality are the most important requirements in any control system. For optimality, it seems that the field of optimal fuzzy control is totally open.

Singular systems contain a mixture of algebraic and differential equations. In that sense, the algebraic equations represent the constraints to the solution of the differential part. These systems are also known as degenerate, descriptor or semi-state and generalized state-space systems. The system arises naturally as a linear approximation of system models or linear system models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large-scale systems, robotics, biology, etc., see [3,4,5,11].

As the theory of optimal control of linear systems with quadratic performance criteria is well developed, the results are most complete and close to use in many practical designing problems. The theory of the quadratic cost control problem has been treated as a more interesting problem and the optimal feedback with minimum cost control has been characterized by the solution of a Riccati equation. Da Prato and Ichikawa [6] showed that the optimal feedback control and the minimum cost are characterized by the solution of a Riccati equation. Solving the Matrix Riccati Differential Equation (MRDE) is a central issue in optimal control theory. The needs for solving such equations often arise in analysis and synthesis such as linear quadratic optimal control systems, robust control systems with H_2 and H_∞-control [22] performance criteria, stochastic filtering and control systems, model reduction, differential games etc. One of the most intensely studied nonlinear matrix equations arising in Mathematics and Engineering is the Riccati equation. This equation, in one form or another, has an important role in optimal control problems, multivariable and large scale systems, scattering theory, estimation, detection, transportation and radiative transfer [7]. The solution of this equation is difficult to obtain from two points of view. One is nonlinear and the other is in matrix form. Most general methods to solve MRDE with a terminal boundary condition are obtained on transforming MRDE into an equivalent linear differential Hamiltonian system [8]. By using this approach, the solution of MRDE is obtained by partitioning the transition matrix of the associated Hamiltonian system [17]. Another class of methods is based on transforming MRDE into a linear matrix differential equation and then solving MRDE analytically or computationally [12,15,16]. However, the method in [14] is restricted for cases when certain coefficients of MRDE are non-singular. In [8], an analytic procedure of solving the MRDE of the linear quadratic control problem for homing missile systems is presented. The solution K(t) of MRDE is obtained by using K(t)=p(t)/f(t), where f(t) and p(t) are solutions of certain first order ordinary linear differential equations. However, the given technique is restricted to single input.

Simulink is a MATLAB add-on package that many professional engineers use to model dynamical processes in control systems. Simulink allows creating a block diagram representation of a system and running simulations very easily. Simulink is really translating block diagram into a system of ordinary differential equations. Simulink is the tool of choice for control system design, digital signal processing

(DSP) design, communication system design and other simulation applications [1]. This paper focuses upon the implementation of Simulink approach to compute optimal control for linear singular fuzzy system.

Although parallel algorithms can compute the solutions faster than sequential algorithms, there has been no report on Simulink solutions for MRDE. This paper focuses upon the implementation of Simulink approach for solving MRDE in order to get the optimal solution. This paper is organized as follows. In section 2, the statement of the problem is given. In section 3, solution of the MRDE is presented. In section 4, numerical example is discussed. The final conclusion section demonstrates the efficiency of the method.

2 Statement of the Problem

In computational fluid dynamics, a typical example of the equations of gas dynamics under the assumptions of incompressibility is the Navier Stokes equation [19]. It consists of as many differential equations as the dimension of the model indicates and the condition of incompressibility, see e.g.[13]:

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= -u.\nabla + \nu\, \Delta u - \nabla p + f \\ 0 &= \nabla . u \end{aligned}\right\} \quad (1)$$

These equations, together with the appropriate initial and boundary conditions, are yet to be solved in Ωx [0,T], where Ω is a bounded open domain in $\mathbf{R}^d$ [d = (2 or 3) dimension of the model] and T the endpoint of the time interval. Two dimensional cases are considered here for easy computation and simplification. The results are valid for a three dimensional model as well. The domain of reference is assumed to be rectangular. This is indeed a restriction. But, at the exact places, it will be pointed out, whether some techniques may be generalized to other domains or not.

After applying the method of lines (MOL), i.e. carrying out a spatial discretization by finite difference or finite element techniques, the equations in (1) can be written as a differential algebraic system.

$$\left.\begin{aligned} Mu'(t) &= K(u)u(t) - Bp(t) + f(t) \\ 0 &= B^T u(t) \end{aligned}\right\} \quad (2)$$

See [2]. Here u(t), p(t) and f(t) are approximations to the time and space dependent quantities u, p and f of (4). The matrix M is symmetric and positive definite. Quantity B stands for discrete gradient operator, while K(u) represents linear and nonlinear velocity terms.

The DAE (2) is of a higher index (i.e. non-decoupled), since pressure p does not appear in the algebraic condition. If we assume that B is a full column rank, then the differentiation index is two. Since p is only determined up to an additive constant, B has, in general, a rank deficiency which causes the undeterminedness of at least one solution component. The concept of the differentiation index can not be applied to such systems. Kunkel and Mehrmann [9] have generalized the index concept to the case of over and underdetermined DAEs. Their so called strangeness index (or s-index) μ is the number of additional block columns needed in the derivative array [10] that can filter out a strangeness free system by transformations from the left. This system then represents a DAE of differentiation index one with possibly undetermined components or a system of ordinary differential equations. Therefore, μ is one lower than the differentiation index if the system is a DAE of at least differentiation index one without undeterminedness. For ordinary differential equations (differentiation index zero), μ is defined as zero.

The system (2) is of a higher index, namely s-index 1. Such systems have a difficulty to solve in their original form because of the differential and algebraic components and strangeness [9]. It would be appropriate to remove this strangeness before solving the DAE. In most of the Navier-Stokes solution techniques, this is done without explicitly mentioning that an index reduction is carried out. If the index reduction is omitted, the results may become unsatisfactory, especially in the unsteady case. In [21], examples are computed when a steady state is reached and it is stated that satisfactory smoothness is achieved. But, this is completely impractical, if long time computations are carried out.

A characterization is also guaranteed by the concept of s-index when the DAE has an unique solution. However, this is not the main advantage of this approach over the usual concept of the differentiation index. The biggest progress seems to be that [9] provides a way to reformulate the higher index DAE as a strangeness free system with the same dimension and same solution structure as the original system. In other words, it is possible to rewrite a DAE of higher index in a so called normal form of *s* index zero. This form not only reflects the manifold included in the original system but also all the hidden manifolds. Thus, using strangeness free normal form, a consideration of all manifolds is ensured which makes this approach superior over other index reduction variants. Moreover, the derivative term is not transformed so that no errors in time are caused by the index reduction strategy for Navier Stokes equations. By this index reduction process, the linear differential algebraic system of arbitrary s-index μ,

$$E\, x'(t)=A(t)x(t)+f(t),$$

under suitable assumptions [10] can be transformed into strangeness free normal form by means of utmost $\mu.3+2$ rank decisions. This procedure is described in [9].

Consider the two dimensional Navier-Stokes (T-S) fuzzy equations

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= -u_i.\nabla+\nu\, \Delta u_i - \nabla p + f_i(t) \\ 0 &= \nabla.u_i \end{aligned}\right\} \qquad (3)$$

The above equation can be transformed into singular T-S fuzzy system using the method described in [9]. The singular time-invariant system by taking $f_i(t) = 0$ in (3)

$$E_i\, x'(t)=A_i x(t)+B_i u(t), \quad x(0)=x_0 \qquad (4)$$

where the matrix E_i is singular, $x(t) \in \mathbf{R}^n$ is a generalized state space vector and $u(t) \in \mathbf{R}^m$ is a control variable. $A_i \in \mathbf{R}^{nxn}$ and $B_i \in \mathbf{R}^{nxm}$ are known coefficient matrices associated with $x(t)$ and $u(t)$ respectively, x_0 is given initial state vector and $m \leq n$.

In order to minimize both state and control signals of the feedback control system, a quadratic performance index is usually minimized:

$$J= \tfrac{1}{2}\; x^T(t_f)E_i^T S E_i x(t_f)+ \tfrac{1}{2} \int^{t_f} [x^T(t)Qx(t)+u^T(t)Ru(t)]dt,$$

where the superscript T denotes the transpose operator, $S\in\mathbf{R}^{nxn}$ and $Q\in\mathbf{R}^{nxn}$ are symmetric and positive definite (or semidefinite) weighting matrices for x(t), $R\in\mathbf{R}^{m\,xm}$ is a symmetric and positive definite weighting matrix for u(t). It will be assumed that $|sE_i-A_i| \neq 0$ for some s. This assumption guarantees that any input u(t) will generate one and only one state trajectory x(t).

If all state variables are measurable, then a linear state feedback control law

$$u(t)= -R^{-1} B_i^T K_i(t)E_ix(t),$$

where $K_i(t) \in\mathbf{R}^{nxn}$ is a symmetric matrix and the solution of MRDE.

The relative MRDE for the linear singular fuzzy system (4)

$$E_i^TK_i'(t)E_i + E_i^TK_i(t)A_i + A_i^TK_i(t)E_i +Q - E_i^TK_i(t) B_iR^{-1}B_i^TK_i(t)E_i = 0 \qquad (5)$$

with the terminal condition $K_i(t_f)= E_i^TSE_i$. In the following section, the MRDE (5) is going to be solved for $K_i(t)$ in order to get the optimal solution.

3 Simulink Solution of MRDE

Simulink is an interactive tool for modelling, simulating and analyzing dynamic systems. It enables engineers to build graphical block diagrams, evaluate system performance and refine their designs. Simulink integrates seamlessly with MATLAB and is tightly integrated with state flow for modelling event driven behavior. Simulink is built on top of MATLAB. A Simulink model for the given problem can be constructed using building blocks from the Simulink library. The solution curves can be obtained from the model without writing any codes.

As soon as the model is constructed, the Simulink parameters can be changed according to the problem. The solution of the system of differential equation can be obtained in the display block by running the model.

3.1. Procedure for Simulink Solution

Step 1. Select the required number of blocks from the Simulink Library.
Step 2. Connect the appropriate blocks.
Step 3. Make the required changes in the simulation parameters.
Step 4. Run the Simulink model to obtain the solution.

4 Numerical Example

Consider the optimal control problem:
Minimize

$$J= \tfrac{1}{2}\, x^T(t_f)E_i^TSE_ix(t_f)+ \tfrac{1}{2} \int_0^{t_f}[x^T(t)Qx(t)+u^T(t)Ru(t)]dt,$$

subject to the linear singular fuzzy system R^i : If x_j is $T_{ji}(m_{ji},\sigma_{ji})$, i = 1, 2 and j = 1, 2,3, then

$$E_i\, x'(t)=A_i x(t)+B_i u(t), \quad x(0)=x_0 ,$$

where

$$S=\begin{pmatrix} 1.1517 & 0.1517 \\ 0.1517 & 1 \end{pmatrix}, \quad E_i = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_1= \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix},$$

$$A_2= \begin{pmatrix} -2 & 2 \\ 0 & -4 \end{pmatrix}, \quad B_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad R=1, \quad Q = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

The numerical implementation could be adapted by taking t_f =2 for solving the related MRDE of the above linear singular fuzzy system with the matrix A_1 . The appropriate matrices are substituted in MRDE. The MRDE is transformed into differentia algebraic equation (DAE) in k_{11} and k_{12}. The DAE can be changed into a system of differential equations by differentiating the algebraic equation. In this problem, the value of k_{22} of the symmetric matrix K(t) is free and let k_{22}=0. Then the optimal control of the system can be found out by the solution of MRDE.

4.1. Solution Obtained Using Simulink

The Simulink model is constructed for MRDE. The Simulink model is shown in Figure 1. The numerical solution of MRDE is calculated by Simulink and displayed in Table 1. The numerical solution curve of MRDE by Simulink is illustrated in Figure 2.

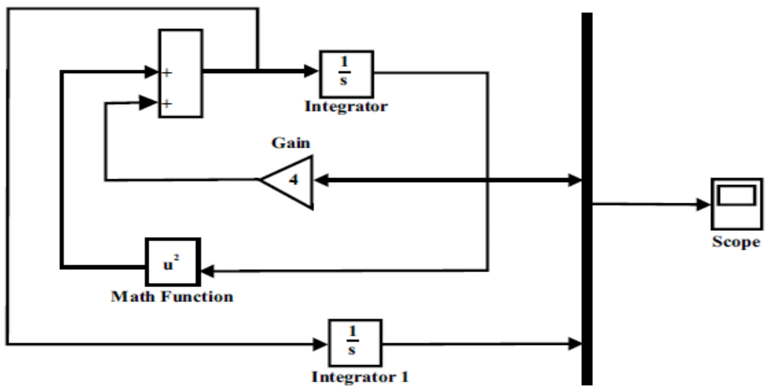

Figure 1 : Simulink Model for MRDE

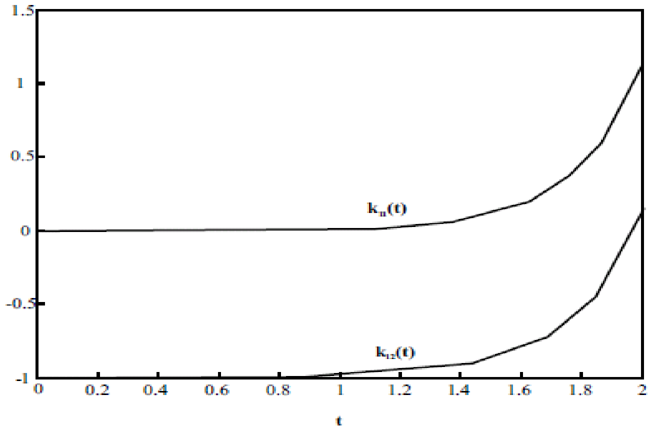

Figure 2: Simulink Curve for MRDE

Table 1: Simulink Solution of MRDE

t	k_{11}	k_{12}
0.0	0.0003	-0.9997
0.2	0.0007	-0.9993
0.4	0.0015	-0.9985
0.6	0.0033	-0.9967
0.8	0.0074	-0.9926
1.0	0.0164	-0.9836
1.2	0.0368	-0.9632
1.4	0.0828	-0.9172
1.6	0.1891	-0.8109
1.8	0.4467	-0.5533
2.0	1.1517	0.1517

Similarly the solution of the above system with the matrix A_2 can be found out using Simulink.

5 Conclusion

The optimal control for Navier-Stokes fuzzy equations can be obtained by finding the optimal control for DAE. The optimal control is found out by solving MRDE using Simulink. To obtain the optimal control, the solution of MRDE is computed by solving Differential algebraic equation (DAE) using Simulink. The Simulink solution is equivalent to the exact solution of the problem. Accuracy of the solution computed by Simulink approach to the problem is qualitatively better. A numerical example is given to illustrate the proposed method.

Acknowledgements: The funding of this work by the UMRG grant (Account No: RG099/10AFR) is gratefully acknowledged.

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