# State of the Art in Vehicle Active Suspension Adaptive Control Systems Based on Intelligent Methodologies

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*Abstract*—This paper reviews computational-intelligenceinvolved approaches in active vehicle suspension control systems with a focus on the problems raised in practical implementations by their nonlinear and uncertain properties. After a brief introduction on active suspension models, the paper explores the state of the art in fuzzy inference systems, neural networks, genetic algorithms, and their combination for suspension control issues. Discussions and comments are provided based on the reviewed simulation and experimental results. The paper is concluded with remarks and future directions.

*Index Terms*—Active suspension systems, adaptive control, computational intelligence, intelligent control.

# I. INTRODUCTION

SUSPENSION system is one of the important components of a vehicle, which plays a crucial role in handling the performance and ride comfort characteristics of a vehicle. A suspension system acts as a bridge between the occupants of a vehicle and the road on which it rides. It has two main functionalities. One is to isolate the vehicle body with its passengers from external disturbance inputs, which mainly come from irregular road surfaces. It always relates to riding quality. The other is to maintain a firm contact between the road and the tires to provide guidance along the track. This is called handling performance. In a conventional passive suspension system, which is composed of only springs and dampers, a tradeoff is needed to resolve the conflicting requirements of ride comfort and good handling performance. The reason is that stiff suspension is required to support the weight of the vehicle and to follow the track; on the other hand, soft suspension is needed to isolate the disturbance from the road. Hence, there exists a significantly growing interest in the design and control of active suspension systems for automotive engineers and researchers over the past three decades. An active suspension system is characterized by

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employing certain kinds of suspension force generation, such as pneumatic, magnetorheological, or hydraulic actuators. Practical applications of active suspension systems have been facilitated by the development of microprocessors and electronics since the middle 1980s [1]–[3]. Related surveys on theories and applications of active suspension control systems (ASCSs) were provided in 1997 [4], [5], with fast-growing computational intelligence methodologies significantly driving recent advances in this research area over the past decade.

The design of a vehicle ASCS is a long-standing control engineering problem, which is rooted in multiple-parameter optimization at real-time requirements. This includes ride comfort, body motion, road handling, and suspension travel [6]. Ride comfort directly relates to the acceleration sensed by passengers; body motion means that bounce, pitch, and roll of sprung mass are created by cornering, acceleration, or deceleration; road handling is associated with the contact forces of tires and the road surface; and suspension travel refers to the displacement between a sprung mass and an unsprung mass. It is a challenging issue for one active suspension system to simultaneously optimize all four sets of parameters. Hence, how to handle related tradeoffs is crucial for the successful design of an ASCS. Research over the past three decades has shown that a linear optimal control scheme provides an efficient way to design an active suspension system that can improve both vehicle ride and handling performance [4], [5]. This is based on the assumption that there exists a perfect (broad-bandwidth) actuator, which can generate the required force fast enough, and the system can be linearized within some opera regions. However, a real vehicle suspension system is inherently nonlinear, even with some uncertainties. Therefore, adaptive control schemes have to undertake the role of providing self-tuning feedback gains and to take the aforementioned four sets of parameters into account to ensure optimal operation of the system in different driving conditions and road surfaces [7]–[11].

A classical form of the adaptive scheme for a vehicle active suspension system was introduced in the late 1980s by Hac [7]. This is the starting point of the adaptive control scheme, in which a set of feedback gains are varied by the change of power spectral density (PSD) of terrain roughness obtained by processing the measurement data. Another comparison of adaptive linear quadratic Gaussian (LQG) and nonlinear controllers for active suspensions was presented by Gordon *et al.* [9]. A model reference adaptive control scheme was proposed by Alleyne *et al.* [12], which resulted in better performance than

	Mathematical	Learning Data	Operator	Real Time	Knowledge	Non-linearity	Optimisation
	Model	_	Knowledge		Representation		
Control	Good or	Unsuitable	Needs other	Good or	Unsuitable	Unsuitable	Unsuitable
Theory	Suitable		methods	Suitable			
Neural	Unsuitable	Good or	Unsuitable	Good or	Unsuitable	Good or	Fair
Network		Suitable		Suitable		Suitable	
Fuzzy Logic	Fair	Unsuitable	Good or	Good or	Needs other	Good or	Unsuitable
			Suitable	Suitable	methods	Suitable	
ther Arti cial	Needs other	Unsuitable	Good or	Unsuitable	Good or	Needs other	Unsuitable
Intelligence	methods		Suitable		Suitable	methods	
Genetic	Unsuitable	Good or	Unsuitable	Needs other	Unsuitable	Good or	Good or Suitable
Algorithms		Suitable		methods		Suitable	

 TABLE
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 COMPARISON OF CAPABILITIES OF DIFFERENT ADAPTIVE METHODOLOGIES [21]

the active suspension system with a nonadaptive controller and a passive suspension system. Furthermore, in this paper, 10%-30% variances of sprung mass and stiffness coefficients were examined to check the adaptation capability based on a single degree-of-freedom (DOF) quarter-vehicle model. Sunwoo and Cheok proposed an explicit adaptive control for an active suspension system that is based on a self-tuning controller design [8]. It consisted of online low-order recursive parameter estimation, closed-form algebraic gain computation, and manipulation of the control parameters. Some other works on adaptive control of active suspension systems can be found in [13]-[15]. Up to this point, most researchers have dealt with a linear model to develop control laws or use an adaptive control scheme to conquer the limited nonlinear properties of suspension systems. However, if the system is highly nonlinear over the range of operation, its adaptive schemes may show severe limitations. For instance, if a wheel stroke is so strong that the stiffness of a suspension is beyond the linear range, it might be practically impossible to identify parameters through ordinary identification [15]-[17]. In the early 1990s, many studies began to consider nonlinearities, uncertainties, and unmodeled parts of a real suspension system, which requires the use of a nonlinear model and some nonlinear forms of control scheme [12], [18]. In practice, these nonlinear models made ASCSs so complex and too challenging to employ.

In industrial applications, control engineers often have to deal with complex systems, having multiple-variable and multiple-parameter models with, perhaps, nonlinear coupling. The conventional approaches for understanding and predicting the behavior of such systems based on analytical techniques can be proved to be inadequate, even at the initial stages of establishing an appropriate mathematical model [19]. The computational environment used in such an analytical approach is perhaps too categorical and inflexible to cope with the intricacy and the complexity of real-world industrial systems. It turns out that in dealing with such systems, one has to face a high degree of uncertainty and tolerate imprecision. Trying to increase precision can be very costly. Thanks to the significant development of soft computing or computational intelligence over the past decades, it has provided alternative ways to nonlinear system modeling and control. Generally speaking, the principal constituents of computing intelligence include fuzzy logic (FL), artificial neural networks (ANNs), and evolutionary computing (EC). FL is mainly concerned with imprecision and approximate reasoning, ANNs mainly with

learning and curve fitting, and EC mainly with global optimization based on natural selection and genetics. These intelligent computing methodologies have resulted in the development of the "intelligent control" field, which consists of novel control approaches based on FL, ANNs, EC, other techniques induced from artificial intelligence and their combination. These methods provide an extensive freedom for control engineers to deal with practical problems of vagueness, uncertainty, or imprecision. Convincingly, these intelligent methods are good candidates to alleviate the problems associated with ASCSs [20]. Although, in hard computing, imprecision and uncertainty are undesirable properties, computationally intelligent approaches are also known, as soft computing provides the tolerance for imprecision and uncertainty, which is exploited to achieve a practically acceptable solution at a reasonable cost, tractability, and high machine intelligence quotient. Zadeh argued that soft computing, rather than hard computing, should be viewed as the foundation of machine intelligence. A complete comparison of their capabilities in different application fields was constructed by Fukuda and Shimojima in Table I, together with those of control theory and artificial intelligence [21].

This paper reviews recent intelligent control approaches for active suspension systems. The paper is organized as follows. Section II gives a revisit on the modeling of an active suspension system. Section III reviews adaptive fuzzy control methods. Section IV presents adaptive fuzzy sliding-mode control (SMC) approaches. Section V revisits neural-network-based control systems, and Section VI presents adaptive genetic algorithm (GA) control methods. Section VII describes combination methods based on neural networks (NNs), fuzzy inference, and GAs. Finally, we conclude the paper in Section VIII with discussions and future work.

# II. BACKGROUND

A vehicle body is generally a rigid body with six-DOF motions shown in Fig. 1 [5]; it consists of longitudinal, lateral, heave, roll, pitch, and yaw motions. These motions are restricted by suspension geometries in vehicles and are more or less coupled with one another. Moreover, as the suspensions have a mechanical structure with unsprung mass, coupling also occurs between the sprung and unsprung masses. Regardless of such coupling problems, the reduced-order mathematical model is useful for designing an ASCS. Therefore, a quarter-vehicle model or a half-vehicle model is often used



Fig. 1. Six-DOF vehicle model [5].

for the theoretical analysis and design of active suspension systems [4], [5].

In this section, a linear quarter-vehicle model and a linear half-vehicle model of an active suspension system are introduced. Their linear quadratic (LQ) controllers are designed based on the models; practical active suspension system models are also analyzed in terms of nonlinear properties and uncertain dynamic disturbances.

# A. Active Suspension System Linear Models and Control

1) Quarter-Vehicle Active Suspension System Modeling and LQ Control Design: The quarter-vehicle model was initially developed to explore active suspension capabilities and gave birth to the concepts of skyhook damping and fast load leveling, which are now being developed toward actual large-scale production applications. In this paper, we define

- $m_b$  quarter body mass (or sprung mass) (in kilograms);
- $m_w$  wheel mass (or unsprung mass) (in kilograms);
- $K_s$  suspension spring stiffness (in newtons per meter);
- $K_t$  tire stiffness (in newtons per meter);
- *c* damping coefficient (in newton seconds per meter);
- $G_0$  road roughness coefficient (in cubic meters per cycle);
- $U_0$  vehicle original forward velocity (in meters per second);
- $f_0$  low cutoff frequency (in hertz);
- $z_0$  road displacement (in meters);
- $z_w$  wheel displacement (in meters);
- $z_b$  body displacement (in meters);
- $f_a$  actuator force (in newtons).

The quarter-vehicle model is shown in Fig. 2. The dynamic differential equations of this suspension system can be represented as

$$m_b \ddot{z}_b = f_a + c(\dot{z}_w - \dot{z}_b) + K_s(z_w - z_b) \tag{1}$$

$$m_w \ddot{z}_w = -f_a - c(\dot{z}_w - \dot{z}_b) - K_s(z_w - z_b) - K_t(z_w - z_0).$$
(2)

The road surface is a natural changing condition for a vehicle. For better riding comfort, a perfect road surface model is necessary to design vehicle ASCS. There are many possible ways to analytically describe the road inputs, which can be classified as shock or vibration [4]. Shocks are the discrete events of relatively short duration and high intensity, e.g., a pronounced



Fig. 2. Two-DOF quarter-vehicle model.

TABLE  $\,$  II Road Roughness Values Classified by ISO (Degree of Roughness  $S(\Omega)\times 10^{-6})$ 

Road Class	Range	Geometric mean
A(very good)	< 8	4
B(good)	8-32	16
C(Average)	32-128	64
D(Poor)	128-512	256
E(very poor)	512-2048	1024

bump or pothole on an otherwise smooth road. Vibrations, on the other hand, are characterized by prolonged and consistent excitations that are called "rough" roads. In this section, the rough road is considered. The International Organization for Standardization (ISO) has proposed a series of standards of road roughness classification using PSD values (ISO 1982), as shown in Table II. Due to the ISO, the road displacement PSD can be described as

$$G(n) = G(n_0) \left(\frac{n}{n_0}\right)^{-w}.$$
(3)

Here, n is the space frequency  $(m^{-1})$ , and time frequency f is f = nv (v is the vehicle speed),  $n_0$  is the reference space frequency, G(n) is the road displacement PSD,  $G(n_0)$  is the road roughness coefficient shown in Table II, and w is the linear fitting coefficient, which is always w = 2. Then, based on the standard road surface description, the road surface input model has been built through an inform filter by Gaussian white noise and successfully used in many presented works [6], [22]. The equation of road surface input is

$$\dot{z}_0 = -2\pi f_0 z_0 + 2\pi \sqrt{G_0 U_0 w_0} \tag{4}$$

where  $f_0$  is the low cutoff frequency,  $G_0$  is the road roughness coefficient, and  $w_0$  is a Gaussian white noise.

Equations (1), (2), and (4) are combined to give the state space representation of the quarter-vehicle model

Y = CX + DU

$$X = AX + BU + FW \tag{5}$$

where

$$X = \begin{bmatrix} \dot{z}_b & \dot{z}_w & z_b & z_w & z_0 \end{bmatrix}$$
(7)

$$Y = \begin{bmatrix} \ddot{z}_b & \ddot{z}_w & z_w - z_b & z_w - z_0 \end{bmatrix}$$
(8)

$$U = [f_a], W = [w_0].$$
(9)

Based on the proposed model, linear optimal control theory is used to design the active suspension controller here. To obtain better handling performance and riding comfort, the performance index can be written as a weighted sum of mean-square values of output performance variables, including body acceleration, wheel-to-body displacement, and dynamic tire deflection. The weight coefficients are  $q_1$ ,  $q_2$ , and  $q_3$ . Therefore, we have

$$J = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} \left\{ q_1 (z_w - z_b)^2 + q_2 (z_w - z_0)^2 + q_3 \ddot{z}_b^2 \right\} dt.$$
(10)

Changing (10) into a general matrix format, it becomes

$$J = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} [X^{\mathrm{T}}QX + U^{\mathrm{T}}RU + 2X^{\mathrm{T}}NU]dt \qquad (11)$$

where Q, R, and N can be obtained from (1), (2), and (4). Assuming that an optimal state observer, i.e., a Kalman filter, is available to get a satisfactory estimation of state vector  $\hat{X}$ , based on the separation theorem, an optimal control force is

$$U = -R^{-1}B^{\mathrm{T}}P\hat{X} = -K\hat{X} \tag{12}$$

where K represents the gain matrix, and P is the solution of the following classical algebraic Riccati equation:

$$PA + A^{\mathrm{T}}P - (PB + N)R^{-1}(B^{\mathrm{T}}P + N^{\mathrm{T}}) = -Q.$$
 (13)

2) Half-Vehicle Active Suspension System Modeling and LQ Control Design: The half-vehicle model including pitch and heave modes was represented to simulate the ride characteristics of a simplified whole vehicle, which leads to a significant improvement in ride and handling [23]. Letting f and r denote the front and rear and x and z be the longitudinal forward direction and vertical up direction in this paper, we have the following definitions:

- $d_f$  distance from the front axle to the center of gravity (in meters);
- $d_r$  distance from the rear axle to the center of gravity (in meters);
- $I_b$  pitch inertia (in kilogram square meters);
- $z_{f0}$  road displacement at the front wheel (in meters);
- $z_{r0}$  road displacement at the rear wheel (in meters);
- $z_{wf}$  front-wheel displacement (in meters);
- $z_{bf}$  front-body displacement (in meters);
- $z_{wr}$  rear-wheel displacement (in meters);
- $z_{br}$  rear-body displacement (in meters);
- $f_{af}$  front-actuator force (in newtons);
- $f_{ar}$  rear-actuator force (in newtons).

The half-vehicle model is shown in Fig. 3. With the assumption of a small pitch angle, the following equations are obtained:

$$z_{bf} = z_b - d_f \cdot \theta \quad z_{br} = z_b + d_r \cdot \theta. \tag{14}$$



Fig. 3. Half-vehicle suspension model.

From (14), the pitch angle can be written as

$$\theta = \frac{z_{br} - z_{bf}}{d_f + d_r} \tag{15}$$

and hence, the model equations of motion can be written as follows:

$$\ddot{z}_{wf}m_{wf} = -K_{tf}(z_{wf} - z_{f0}) - [f_{af} + c_f(\dot{z}_{wf} - \dot{z}_{bf}) + K_{sf}(z_{wf} - z_{bf})]$$
(16a)  
$$\ddot{z}_{wr}m_{wr} = -K_{tr}(z_{wr} - z_{r0})$$

$$m_{wr} = -K_{tr}(z_{wr} - z_{r0})$$
  
- $[f_{ar} + c_r(\dot{z}_{wr} - \dot{z}_{br}) + K_{sr}(z_{wr} - z_{br})]$  (16b)

$$\begin{aligned} \ddot{z}_{b}m_{b} = & f_{af} + c_{f}(\dot{z}_{wf} - \dot{z}_{bf}) + K_{sf}(z_{wf} - z_{bf}) \\ & + f_{ar} + c_{r}(\dot{z}_{wr} - \dot{z}_{br}) + K_{sr}(z_{wr} - z_{br}) \end{aligned} \tag{16c} \\ \ddot{\theta}I_{b} = & -d_{f}\left[f_{af} + c_{f}(\dot{z}_{wf} - \dot{z}_{bf}) + K_{sf}(z_{wf} - z_{bf})\right] \\ & + d_{r}\left[f_{ar} + c_{r}(\dot{z}_{wr} - \dot{z}_{br}) + K_{sr}(z_{wr} - z_{br})\right]. \end{aligned} \tag{16d}$$

Substituting (14) into (16c) and (16d), we have the following:

$$\begin{split} \ddot{z}_{bf} &= \left(\frac{1}{m_b} + \frac{d_f^2}{I_b}\right) [f_{af} + c_f (\dot{z}_{wf} - \dot{z}_{bf}) + K_{sf} (z_{wf} - z_{bf})] \\ &+ \left(\frac{1}{m_b} - \frac{d_f d_r}{I_b}\right) [f_{ar} + c_r (\dot{z}_{wr} - \dot{z}_{br}) + K_{sr} (z_{wr} - z_{br})] \end{split}$$
(17a)

$$\ddot{z}_{br} = \left(\frac{1}{m_b} - \frac{d_f d_r}{I_b}\right) [f_{af} + c_f (\dot{z}_{wf} - \dot{z}_{bf}) + K_{sf} (z_{wf} - z_{bf})] \\ + \left(\frac{1}{m_b} + \frac{d_r^2}{I_b}\right) [f_{ar} + c_r (\dot{z}_{wr} - \dot{z}_{br}) + K_{sr} (z_{wr} - z_{br})].$$
(17b)

Using filtered white noise  $w_1$  and  $w_2$  as the road inputs, the road input equations for the front and rear wheels, respectively, are

$$\dot{z}_{f0} = -2\pi f_0 z_{f0} + 2\pi \sqrt{G_0 U_0} w_1 \tag{18a}$$

$$\dot{z}_{r0} = -2\pi f_0 z_{r0} + 2\pi \sqrt{G_0 U_0} w_2.$$
 (18b)

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So far, we have a state vector as given in (19), shown at the bottom of the page. Combining vehicle model equations of motion equations (15), (16a), (17a), and (17b) and road input equations (18a) and (18b), the system model and output equation in state space form are obtained as

$$\dot{X}_{\text{half}} = \widetilde{A}X_{\text{half}} + \widetilde{B}U_{\text{half}} + \widetilde{F}w_{\text{half}}$$
 (20a)

$$Y_{\text{half}} = CX_{\text{half}} + DU_{\text{half}} + v_{\text{half}}$$
(20b)

where  $\widetilde{A}$ ,  $\widetilde{B}$ ,  $\widetilde{C}$ ,  $\widetilde{D}$ , and  $\widetilde{F}$  are differential equation coefficient matrices,  $X_{\text{half}}$  is the state vector,  $Y_{\text{half}}$  is the output vector,  $U_{\text{half}}$  is control input matrix,  $w_{\text{half}}$  is the road inputs, and  $V_{\text{half}}$ is measurement noise. Here,  $Y_{\text{half}}$  is defined in (21), shown at the bottom of the page, and  $U_{\text{half}}$  and  $w_{\text{half}}$  are defined in (22), shown at the bottom of the page. Based on the proposed model, linear optimal control theory is used here to design the active suspension controller. To obtain better handling and riding comfort, the performance index can be written as a weighted sum of mean-square values of output performance variables, including body acceleration, wheel-to-body displacement, and dynamic tire deflection. The weight coefficients are  $\rho_1$ ,  $\rho_2$ ,  $q_1$ ,  $q_2$ ,  $q_3$ , and  $q_4$ . Therefore, we have

$$J = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} \left[ q_1 (z_{wf} - z_{f0})^2 + q_2 (z_{bf} - z_{wf})^2 + \rho_1 \ddot{z}_{bf} + q_3 (z_{wr} - z_{r0})^2 + q_4 (z_{br} - z_{wr})^2 + \rho_2 \ddot{z}_{br} \right] dt.$$
(23)

Similar to the quarter vehicle, the optimal LQ control can be solved from the Riccati equation.

# *B.* Nonlinearity and Unmodeling Dynamic Description of an Active Suspension System

Many researchers have dealt with a linear model in developing control laws. However, considering the inherent nonlinearities and uncertainties, it is not sufficient to represent the real system with a linear model, as in Sections II-A1 and A2. In the early 1990s, many studies began to consider nonlinearities, uncertainties, and unmodeled parts of a real suspension system, which required the use of a nonlinear model and some adaptive or robust form of control scheme [4], [8], [9], [12], [13], [15], [24], [25]. In this section, the nonlinear properties are introduced, and the general nonlinear models of suspension systems are carried out.

As Hrovat remarked, for many operations, the linear system approximation was appropriate; however, there were some situations that amplify the nonlinear effects [4]. One is created by discrete-event disturbances such as single bumps or potholes, which can cause a highly nonlinear phenomenon. Another is dry friction. Based on the quarter-vehicle model shown in Section II-A1, Kim and Ro modeled the connecting forces (e.g., spring force and damping force) as nonlinear functions using measured data [15]. In Kim and Ro's paper, the nonlinear spring properties mainly have two aspects. One is the bump stop that restricts the wheel travel within a given range and prevents the tire from contacting the vehicle body. The other is the strut bushing that connects the strut with the body structure and reduces harshness from the road input. These two nonlinear effects can be included in the spring force  $f_s$  with nonlinear characteristics versus suspension rattle space  $(z_w - z_b)$ . Based on the measured data in [15], Kim and Ro modeled the spring force  $f_s$  and the damping force by the high-order polynomial functions. The spring force was described as a third-order polynomial function as follows:

$$f_s = f_{sl} + f_{sn} = k_1 \Delta x + (k_0 + k_2 \Delta x^2 + k_3 \Delta x^3) \quad (24)$$

where  $f_{sl}$  is the linear part of the spring force, and  $f_{sn}$  is the nonlinear part of the spring force. The coefficients can be obtained by fitting the experimental data.

Furthermore, the damping force  $f_d$  was modeled as a secondorder polynomial function by fitting the measured data, which is shown as follows:

$$f_d = f_{dl} + f_{dn} = c_1 \Delta \dot{x} + c_2 \Delta \dot{x}^2 \tag{25}$$

where the  $f_{dl}$  is the linear part, and the  $f_{dn}$  is the nonlinear part of the damper force; the coefficients can be obtained by fitting the experimental data.

Except for the nonlinear properties presented by the spring force and damping force, the vertical tire force was highly nonlinear, particularly when the load condition seriously changed. Even the vertical tire force became zero when the tire lost contact with the road. Kim and Ro modeled the tire force as

$$f_{tl} = k_t(z_0 - z_w),$$
 when  $(z_0 - z_w) > 0$   
 $f_{tn} = 0,$  when  $(z_0 - z_w) < 0$ 

where  $f_{tl}$  denotes the linear tire force, and  $f_{tn}$  denotes the nonlinear tire force.

$$X_{\text{half}} = \begin{bmatrix} \dot{z}_{br} & \dot{z}_{wr} & \dot{z}_{bf} & \dot{z}_{wf} & z_{br} & z_{wr} & z_{bf} & z_{wf} & z_{r0} & z_{f0} \end{bmatrix}^{\text{T}}$$
(19)

$$Y_{\text{half}} = \begin{bmatrix} \ddot{z}_{bf} & z_{bf} - z_{wf} & z_{wf} - z_{f0} & \ddot{z}_{br} & z_{br} - z_{wr} & z_{wr} - z_{r0} \end{bmatrix}^{\text{T}}$$
(21)

$$U_{\text{half}} = \begin{bmatrix} f_{af} \\ f_{ar} \end{bmatrix}, w_{\text{half}} = \begin{bmatrix} w_2 \\ w_1 \end{bmatrix}$$
(22)



Fig. 4. Adaptive FLC scheme in [29].

To show the effect of the asymmetric tire stiffness on the response of the quarter-car model, some simulation results were shown to investigate the effect of nonlinear tire force under the different amplitudes of road input [15]. From the results, it was clear that vehicle nonlinearities should be considered in developing a more accurate system model, from which a more reliable control algorithm can be developed.

In this paper, two kinds of suspension system nonlinear models are provided for controller design and performance analysis. Considering the nonlinearity shown by (24) and (25), the active suspension system can be written as a multipleinput–multiple-output (MIMO) nonlinear model

$$\dot{X} = F(X) + BU + d \tag{26}$$

where F(X) is a nonlinear function, including the nonlinear force  $f_s$ ,  $f_t$ , and  $f_d$ , U is the input of the suspension system, and d is the unknown external disturbances.

The other nonlinear model can be described as a hybrid model with a linear part and a nonlinear part

$$\dot{X} = AX + BU + d \tag{27}$$

where AX + BU is the linear model of the suspension system based on  $f_{sl}$ ,  $f_{dl}$ , and  $f_{tl}$ , and  $\tilde{d}$  represents the nonlinear and uncertain model of the suspension system.

# III. ADAPTIVE FUZZY CONTROL

The control performance of a traditional controller greatly depends on the accuracy of the known system dynamic model, according to Section II-A1. To meet practical requirements in an active suspension system, it is crucial to derive or to identify an appropriate model for the traditional controller design. Estimating uncertain effects is even more challenging due to the random noise occurred by road inputs. Hence, some model-free intelligent controllers were introduced to solve these problems, e.g., the FL controller (FLC) [26]–[30]. The FLC is credited with being an adequate methodology for designing robust controllers that are capable of delivering satisfactory performance in the face of uncertainty and imprecision. As a result, the FLC has become a popular approach to nonlinear and uncertain system control in recent years.

There are different ways to construct FLCs for vehicle suspension control, with the most common method being to construct the FLCs by eliciting the fuzzy rules and its membership functions based on experts' knowledge or experience. The common problem that occurs then is that they cannot fully handle or accommodate for the linguistic and numerical uncertainties associated with changing and dynamic natural changing road inputs as they use precise fuzzy sets. To overcome this weakness, adaptive FLCs were designed to self-tune the fuzzy rules or membership functions [26]–[29], [31], [32]. Recently, with the development of type-2 fuzzy reasoning and control theory [33]–[35], Cao *et al.* [36] has studied the adaptive type-2 fuzzy control and optimization on an active suspension system. In this section, the adaptive FLC designs and applications on active suspension systems are reviewed.

The key components of an FLC are a set of linguistic fuzzy control rules and an inference engine to digest these rules. These fuzzy rules offer a transformation between the linguistic control knowledge of an expert and automatic control strategies of an actuator. Every fuzzy control rule is composed of an antecedent and a consequent. The structures and parameters of control rules dominate the performance of fuzzy control. From the control point of view, it is crucial that related parameters or structures are modified automatically by evaluating the results of fuzzy control. For instance, Huang et al. [29] proposed an adaptive FLC for an active suspension system. This adaptive FLC scheme is shown in Fig. 4. The inputs of FLC are the vertical position error and error change of the vehicle sprung mass. Its output is the control voltage increment. The antecedent membership functions consist of 11 equal triangular-type functions. The voltage increment membership function is a set of 15 equal triangular-type functions. Its self-tuning property is implemented by adjusted scaling factors  $S_1$ ,  $S_2$ , and  $S_3$ . That is to say that the membership functions are adapted to improve the FLC performance. Its 121 fuzzy rules are employed to suppress the sprung mass vibration amplitude due to road inputs.

To evaluate the fuzzy control system, a two-DOF quartervehicle suspension model was established. The suspension mechanism includes a spring mass and a hydraulic control loop. A hydraulic servo system is used to generate various road surfaces, and an optical linear scale and a linear potentiometer were employed to measure the sprung mass and road surface vertical displacements, respectively. Based on this realistic suspension model, the dynamic response of the active suspension system was provided for vehicle ride performance on a rough concave–convex road with 25-mm obstacles. The maximum displacement of the vehicle body is less than 5 mm, and it converges within 0.5 s. The control signal was very smooth and easy to employ in the practical vehicle. However, its adjusted scaling factors were chosen by experiments and many simulations, which limit the flexible and adaptive abilities of the adaptive FLC. To overcome this problem, researchers have compensated these types of adaptive FLCs by employing nonlinear optimal algorithms; they employed a GA and/or ANNs to self-tune the parameters of their membership functions and fuzzy rules. These kinds of adaptive FLC will be covered in Section VII.

# IV. ADAPTIVE FUZZY SMC

SMC currently enjoys a wide variety of application areas such as general motion control applications and robotics, process control, aerospace applications, and vehicle active suspension systems. The main reason for this popularity is its attractive properties, including good control performance for nonlinear systems, applicability to MIMO systems, and wellestablished design criteria for discrete-time systems. Note that its most significant property should be its robustness. Loosely speaking, when a system is in a sliding mode, it is insensitive to parameter changes or external disturbances [37]. However, SMCs also suffer from the following disadvantages in practical applications. First, SMCs suffer from the problem of chattering, which is the high-frequency oscillations of the controller output that is brought by the high-speed switching for the establishment of a sliding mode. Chattering is very undesirable and dangerous in practice because it may excite unmodeled high-frequency dynamics, resulting in unforeseen instabilities. Second, an SMC is extremely vulnerable to measure noise since its input depends on the sign of a measured variable that is very close to zero. Third, the SMC may employ unnecessarily large control signals to overcome the parametric uncertainties. Last, there is difficulty with the calculation of what is known as the equivalent control. The integration of an FL system in an SMC has been witnessed in many successful applications where an attempt to relieve the implementation difficulties of the SMC are made via the addition of the FL system [37]–[39]. On the other hand, some significant research has originated due to different difficulties, i.e., the difficulties in carrying out a rigorous stability analysis of FLCs.

The design of an SMC involves two steps. The first step is to select switching hyperplanes, called sliding surfaces, to describe the desired dynamic characteristics of a controlled system. The second step is to design discontinuous control such that the system enters a sliding surface and remains in it. Regarding the system given by (26), the sliding surface S is selected generally as

$$S(X) = GX = 0 \tag{28}$$

where S(X) denotes a set of switching hyperplanes, and G is a constant  $q \times n$  matrix to be determined.



Fig. 5. Effects of parameters G and K [37].

The main object in an SMC is to force the system states to the sliding surface. Once the states are on the sliding surface, the system errors converge to zero with error dynamics dictated by the matrix G. More details about the sliding-mode controller design can be found in [37]. Here, the total control of SMC is given as

$$U = U_{eq} + U_{sw} \tag{29}$$

where  $U_{eq}$  is the equivalent control, and  $U_{sw}$  is always called the switch control.

Generally speaking, two steps are required for an SMC design: 1) One is to select an approximation model such that the system trajectory exhibits desirable behavior when confined to the model, and 2) the other is to find feedback gains so that the system trajectory intersects and stays on the approximation model. In practical systems, these conditions will be constrained. Over the last two decades, FL has been employed to improve SMC in terms of efficient and practical issues. Two types of fuzzy SMC are introduced in this section. They are employed to solve two SMC weaknesses, i.e., alleviating SMC chattering and modeling the nonlinear or uncertain characteristics of practical systems.

# A. Alleviating SMC Chattering

FL is employed to self-tune the discontinuous switching control law to overcome the chattering phenomenon in SMC. Consider the switching control law in terms of (29), which has two parameters G and K to be optimized [37]. Their effects on system performance are shown in Fig. 5. Parameter G determines the slope of the sliding line, which means that the larger the G, the faster the system response. Due to the fact that an overlarge value of G can cause overshoot or instability, it would be advantageous to adaptively vary its slope in such a way that the slope is increased as the magnitude of its error gets smaller. The curve labeled "1" corresponds to the case when Kis large. The system states reach the sliding line in a short time but overshoot it by a considerable amount. The curve labeled "2" reflects the case with a small K parameter. Neither curve 1 nor 2 is desired. Curve "3" can be obtained via fuzzy adaptive algorithms in which parameter "K" is increased only when the states are close to its sliding line.

For instance, Chen-Sheng *et al.* proposed a fuzzy adaptive sliding-mode controller for an active suspension system [40]. The proposed quarter-car active suspension model was defined as (27). For the design of the SMC, a reduced-order dynamic model was used, and the state variables were  $x_1$  (the suspension

deflection) and  $x_2$  (the sprung mass velocity). Its sliding surface was defined as

$$S(X) = GX = x_2 + \lambda x_1 = 0, \qquad \lambda > 0.$$
 (30)

Likewise, the SMC  $U_{eq}$  and  $U_N$  were chosen as follows:

$$U_{eq} = b^{-1} \left[ -a_1 x_1 - (a_2 + \lambda) x_2 \right], \quad U_N = b^{-1} K \operatorname{sgn}(S).$$
(31)

Note that the actual inputs of the proposed fuzzy adaptive SMC controller are S, and its derivation is  $\dot{S}$ . The output was the hitting control. Fuzzification and defuzzification stood for an interface between the crisp values of the reality and the linguistic values of the inference. The controller was organized at two levels. At the basic level, the conventional fuzzy control rule sets and inference mechanism were constructed to generate a fuzzy control scheme. At the supervising level, the control performance was evaluated to modify system parameters, particularly to adaptively tune its scaling factors. The proposed fuzzy control rules were outlined in [40].

To investigate an active suspension performance based on the aforementioned fuzzy SMC, a pseudorandom disturbance road input was employed to test the robustness of the controller under the condition that the spring mass disturbance was increased by 30% and that the damping coefficient and the spring stiffness were decreased by 30% from the nominal values. The simulation results demonstrated that the controlled suspension deflection was smaller than its counterpart of a linear quadratic regulator optimal control but larger than that of a conventional SMC. Regarding the riding quality, the fuzzy SMC achieved the best performance of sprung mass acceleration. The simulation results also illustrated that the road-handling ability maintained by the fuzzy SMC outperformed that of an LQ controller and a conventional SMC. Similar conclusions were also drawn for the perturbed conditions.

Additionally, Zhang *et al.* also proposed a fuzzy adaptive sliding-mode controller for an active suspension system [20]. The main difference from Chen-Sheng *et al.*'s research is the way in which a sliding surface is constructed. In Zhang *et al.*'s paper, the sliding surface was constructed on the basis of conventional sliding surface *s* and its derivative  $\dot{s}$  as follows:

$$\sigma = \dot{s} + \lambda s \tag{32}$$

where  $\lambda$  was a positive value, and its Lyapunov stability condition must be satisfied as follows:

$$\dot{V} = \sigma \dot{\sigma} < 0. \tag{33}$$

The equivalent control can be obtained as

$$\dot{U}_{eq} = -(GB)^{-1} \left[ (GA + \lambda G)AZ + (GA + \lambda G)BU \right] \quad (34)$$

$$\dot{U}_N = -(GB)^{-1}\varepsilon \operatorname{sgn}(\sigma).$$
 (35)

Then, the SMC control output was achieved as

$$\dot{U} = \dot{U}_{eq} + \dot{U}_N. \tag{36}$$



Fig. 6. Fuzzy adaptive controller scheme in [20].

Finally, this led to the controller output

$$U(n) = U(n-1) + U(n).$$
 (37)

The scheme of fuzzy adaptive tuning controller is shown in Fig. 6. The simulations in the time and the frequency domains were carried out on a quarter-car active suspension system. In the time-domain analysis, the comparison between an LQG controller and the fuzzy adaptive SMC controller showed that the proposed controller can significantly decrease its sprung mass acceleration from the peak value to zero. However, the proposed fuzzy adaptive SMC simultaneously needed higher active forces than the LQG controller. In the frequency-domain analysis, the fuzzy adaptive SMC improved the frequency response from the road input to the sprung mass acceleration, particularly in the frequency range of 4-8 Hz. Based on ISO 2361, the human body is very sensitive to vertical vibration in the frequency range of 4-8 Hz. That is to say that the proposed controller can significantly improve the ride quality. Considering the existence of uncertain parameters, the sprung mass was assumed to change in a bounded range of  $\pm 50\%$ . The simulation results demonstrated that the maximum acceleration of the active suspension using the proposed controller was, on average, 54% smaller than a passive suspension system.

# *B. FL* Controller Complementary to SMC for System Nonlinearity and Uncertainty

Referring to a traditional SMC design, the equivalent control law always depends on its system model, due to the fact that it is very expensive to achieve an exact system model for a more complex nonlinear system. A practical method for a nonlinear problem is linearized around given operation points such that the well-developed linear control theory can be applied to the local region with apparent ease. However, this leads to the new problem of how to aggregate each locally linearized model into a global model that represents the corresponding nonlinear system. FL offers a solution to the problem without the need for a mathematical model and constant gain limitation [41]. Huang et al. [42] proposed an adaptive fuzzy sliding-mode controller (AFSMC) for an active vehicle suspension system. FL control was employed to approximate the nonlinear function of equivalent control law  $U_{eq}$ . The voltage output of an actuator in each sampling step was derived from fuzzy inference, instead of from the nominal model at the sliding surface. It significantly diminished the chattering phenomenon of the traditional SMC.

The input signal of this type of FL control was sliding surface variable S, in terms of its sprung mass position and velocity deviations. Its output signal was control voltage U, which was the output of the hydraulic servo actuator. Its fuzzy input variable S consisted of 11 equal-span triangular membership functions, which were employed for the fuzzy output variable U through 11 fuzzy inference rules. The tunable consequent parameters of those peaks of the triangular membership functions were initialized with zero by default. A novel online parameter tuning algorithm was proposed to adjust the consequent parameters to monitor the system control performance. A quarter-car, two-DOF active suspension system was designed and built to investigate its dynamic performance and control effect. The suspension system was tested under three different conditions. One was a rough road with a 40-mm amplitude sinusoidal wave; the other two were a rough concave-convex road with a randomly dynamic 40-mm height and a rough road with a random amplitude. The experimental results showed that the proposed AFSMC had significantly suppressed the sprung mass position oscillation amplitude. In addition, the control voltage was smooth, and the converging speed was fast.

Additionally, Kucukdemiral et al. proposed an FL method to handle the nonlinear system model and uncertain disturbance for an active suspension system [43]. The control U was given by  $u = u_{fz} + u_{vs}$ .  $u_{vs}$ , denoting the switching control, which improved by a boundary layer, alleviating the chattering;  $u_{fz}$ was obtained from the FL controller with the input S. To evaluate the proposed controller, the simulation environment was controlled as follows: Vehicle speed was 72 km/h, and two types of road surfaces were employed for controller performance evaluation, including a standard bump-type surface profile with 10-cm length  $\times$  10-cm height and a random road profile generated to simulate stabilized road with  $1 \text{ cm} \times 1 \text{ cm}$ pebbles. Four types of controllers were employed on the active suspension system. When the standard bump-type surface profile was used, the proposed controller clearly produced the shortest response time of 0.85 s and the lowest peak value of 0.4 cm. Under the condition of random road input, the AFSMC had overwhelming success over other controllers. Furthermore, since it has a single input FLC as the main controller, the rule base of FLC drastically decreased when it was compared with traditional FLCs.

#### V. ADAPTIVE NN CONTROL

Due to its nonlinear mapping and learning ability, NNs have been one of dominant methods for designing robust, adaptive, and intelligent control systems [44]. For further information on NN control systems, see [45] and [46].

An adaptive nonlinear controller is required for the nonlinearity and uncertainty during operation in an active suspension system. For instance, Guo *et al.* [47] designed an adaptive controller with an NN-based identifier to control a semiactive suspension with a magnetorheological damper based on a quarter-vehicle model. The NN control system scheme is shown in Fig. 7. In principle, the direct NN control takes the error between the ideal reference signal and the system response as the error of backpropagation. However, this error does not offer



Fig. 7. Scheme of indirect adaptive control based on NNs in [47].

good information for updating the weights of NNs because of potential uncertainty on the nonlinear model with natural and random disturbance. In Guo *et al.*'s paper, an indirect adaptive NN control strategy was proposed to approximate the input error. The structure of the NN controller is shown in Fig. 7. The NNC was the NN controller, the NNI was the NN identifier, and the TDL was tapped delay. Due to the unavailability of the inverse model of the nonlinear dynamic system, not only did the NNI trace the system response, but it also calculated the backpropagation error for the NNC. The topological structure of the NNC consisted of three layers with  $4 \times 9 \times 1$  nodes, including one hidden layer. The NNI structure was the same as the NNC. The sigmoid function served as the activation function for both the hidden and output layers; clearly, the backpropagation algorithm was used to update the weights.

To evaluate the adaptive NN-based control system, numerical simulations and experiments were carried out for the quartervehicle rig equipped with a magnetorheological damper. The fundamental natural frequency of the quarter-vehicle model was chosen as 1.8 Hz, and the road profile was given, based on the road classification of the ISO database. The numerical simulation and experiment results convincingly showed the vertical acceleration of the vehicle body to be considerably reduced with the indirect NNC than the traditional NNC. For example, the root-mean-square acceleration of the vehicle body subject to the random road disturbance of C grade was reduced by 38.2% when the direct NNC was used and by 55% when the indirect adaptive NNC was implemented in the numerical simulation. In the comparison with passive suspension, the semiactive suspension with indirect adaptive NNC reduced the acceleration of the vehicle body under the sinusoidal road excitation of C grade by 41% in the experiment. On the other hand, the indirect adaptive NNC worked very quickly since the NNs included only a single hidden layer, and the NNI received good training before the experiments took place.

# VI. GA-BASED ADAPTIVE OPTIMIZATION AND CONTROL

GAs, which are one kind of stochastic global optimization technique, have been successfully applied in a variety of research and industrial fields, particularly in optimization and control [48]–[56]. For instance, GAs have demonstrated their effectiveness in multipeak problems with local optimum solutions with approval in robust search around complex spaces. The main difference between GAs and conventional optimization and search procedures are the following: 1) They work with a coding set of the parameters and not the parameters themselves; 2) they search from a population of points and not a single point and are capable of handling large search spaces; and 3) they use probabilistic transition rules, rather than deterministic ones [37]. However, it also needs to be pointed out that the main disadvantage of GAs is that the optimal speed is too slow to use in real-time applications.

Considering the control strategy in active suspension systems, Baumal et al. [52] utilized the GA in a five-DOF halfvehicle model. In their research, all the involved parameters were comprised into one constraint optimal description with eight unknown parameters and seven constraints, which means that the active control and passive mechanical parameters, respectively, were the designed variables to be optimized. Two active elements provided forces proportional to the absolute vertical velocity of the points on the car body directly above the rear and front wheels. These devices, which are characterized by proportionality constants  $c_f$  and  $c_r$ , were known as skyhook dampers. The design variables were the set  $\{x\} =$  $\{k_1, c_1, k_3, c_3, c_r, k_4, c_4, c_f\}$ . Moreover, the constraints were obtained from the three performances of vehicle suspension systems: 1) ride comfort; 2) road-holding ability; and 3) the suspension working space. Two constraints were for the body acceleration and the seat acceleration. The other five constraints were for the seat, suspension, and tire deflections. Given the optimized initial set, there were three steps to implement the GA. The algorithm stopped when the maximum fitness design comprised at least 30% of a newly created generation. The reproduction stage itself was a simulation of the survival-ofthe-fittest designs. Moreover, to improve the efficiency of the GA, the binary strings and fitness values for each unique design of the current generation were stored in a linear search lookup table. If a design string in the next generation matched one in the table, then the fitness did not have to be recalculated. This significantly avoided GAs' weakness by improving computing time, particularly for expensive fitness evaluations. With five independent runs of the GA, the optimal values were obtained and compared with the local optimization search technique and the passive suspension design. The results showed that the proposed GA can carry out the best parameters with the least computing time among the three methods. The active and passive suspension system seat acceleration responses were compared to evaluate its dynamics performance. The response of the active system showed that the road disturbance had little effect on the seat acceleration and indicated that GAs had strong potential to incorporate global optimization methods for suspension system design.

Tsao and Chen [55] also proposed an active suspension force controller using GAs with maximum stroke constraints based on their former research [53], [54]. In contrast to the traditional approach, the maximum absolute values of suspension strokes were employed in the objective function to achieve better ride comfort within the stroke limitation. GA was employed to search for the parameters of damping ratios and spring constants to achieve an optimum tradeoff among ride comfort, handling quality, and suspension stroke limitation, simultaneously. Two driving conditions were tested on the active force controller. One was a steep ramp road with forward speed V = 10 m/s, and the other was a sinusoidal bump road with V = 40 m/s. The simulations were carried out for the three cases in each driving condition. Each case was ended after 500 generation runs. The comparisons of the performance among these cases showed that the proposed force controller using GA achieved great ride and handling quality, while the suspension stroke was restricted to be less than or equal to the passive system. In terms of the dynamic performance, the heave and pitch angle motions of the suspension system were shown and compared with the passive suspension system. In particular, considering the comparison of the suspension displacement, the summation of the quadratic values of the suspension displacements in the active system was larger than that of the passive suspension. However, the maximum displacement was smaller, and the vibration had been absorbed during the transient period. These results can explain why the maximum absolute value, instead of the summation quadratic form of suspension displacement, can achieve better performance.

# VII. ADAPTIVE CONTROL INTEGRATION

Control strategies are reviewed in this section based on the combination of presented methodologies in previous sections.

#### A. Adaptive Neuro-Fuzzy Control

Much attention has been paid to the combination of NNs and fuzzy systems [57] with a focus on combining fuzzy systems with NN learning techniques, particularly for the NN–fuzzy controller. The advantage is that the fuzzy systems can compensate the tuning ability of their rules by using the learning algorithms of NNs; on the other hand, the NN system can also improve the transparency and interpretability by rule-based fuzzy reasoning construction. Generally speaking, an NN–fuzzy system can be viewed as a special three-layer feedforward NN, and the fuzzy rules are trained by an NN algorithm. With both advantages of NN and FL, the neuro-fuzzy system had been successfully employed to solve a wide range of industry problems, particularly on nonlinear and uncertain systems.

For instance, Dong et al. employed an adaptive NN-fuzzy controller for a quarter-vehicle magnetorheological suspension system [58]. This controller consisted of a fuzzy neural network controller (FNNC) and a time-delay controller (TDC). The FNNC calculated the control force according to the error and the change of the error; the TDC was an NN model that predicted compensation for the suspension's time delay. For the quarter-vehicle model, the input was the damper force, the output was the sprung mass vertical acceleration, and the road input was treated as a disturbance. In the FNNC scheme, where two linguistic variables were input into the network and seven fuzzy sets were defined for each input as NB, NM, ZE, PS, PM, and PB in the first layer, the second layer included 14 neurons to correspond to all the fuzzy sets, and the third layer contained 49 neurons to do the fuzzy reasoning based on the defined fuzzy rules. The simulation and experimental results showed that the proposed FNNC with TDC can significantly reduce the acceleration peak value and decreased by 42.3% in comparison to a passive suspension system.

Additionally, Wu et al. proposed a fuzzy controller based on the neuro-fuzzy model for a half-vehicle active suspension system [59]. The half-vehicle active suspension was modeled as a nonlinear system including heave, pitch, and motion of the front and rear wheels. The proposed neuro-fuzzy network was a self-organizing inference network with six layers to derive the corresponding Takagi-Sugeno (T-S) fuzzy model. The learning structure included both precondition and consequence identification of fuzzy IF-THEN rules. Based on supervised learning algorithms, the parameters of linear equations in the consequent parts were adjusted by recursive least squares algorithms, and the parameters in the precondition part were adjusted by a backpropagation algorithm to minimize a given cost function. Based on the T-S fuzzy model, a fuzzy controller was designed to get the optimal active force. The simulation results showed that the proposed optimal fuzzy controller can improve the ride comfort by minimizing both the displacements and accelerations of the vehicle center and the pitch angle simultaneously.

#### B. Adaptive Genetic-Based Optimal Fuzzy Control

Due to the fact that IF–THEN rules in a fuzzy inference system are not always available, automatic design methods and rule acquisition procedures for fuzzy systems are required and have been proposed mostly based on GA and/or NNs over the past four decades. The key advantage of the hybrid system combining GA and FL is that almost all the tasks of the fuzzy system design can be accomplished automatically. Thanks to the global optimal ability, FL parameters of inference rules and membership functions are able to be determined by a hybrid system itself. For GA–fuzzy control systems, see [60]–[63].

Nawa et al. studied a GA-fuzzy control system with the aid of pseudobacterial GAs (PBGAs) and employed this controller to an active suspension system. Its encoding method is demonstrated in Fig. 8. Differing from the traditional canonical binary encoding, the parameters were put into the chromosome, each of which encoded the rules of the fuzzy system. Since every rule contained the information of antecedent and consequent variables, each chromosome encoded the parameters of the membership functions. Triangular-type membership functions were employed so that the parameters of the membership function were in pairs of center and width, as shown in Fig. 8. This encoding method gave a high degree of freedom for the GA, which can optimize the variables to be employed in the rules, the rules themselves, and the parameters of membership functions. Therefore, this encoding was desirable to simultaneously evolve the rules and the membership functions, minimizing the probability of arriving at a local optimal point.

The GA algorithm can be briefly described as follows: 1) generation of the initial population; 2) genetic operations mutation, evaluation, selection, and replacement; 3) crossover and production of the new generation. An adaptive method was used in a crossover operation instead of randomly deciding the chromosomes' cutting points. The adaptive crossover operator took into account the moving average of the degrees of truth



Fig. 8. Example of the fuzzy system encoded in a chromosome in [64].

values of the fuzzy rules when deciding where to cut the chromosome. The moving average was defined as the average of the accumulated truth values of the rules. The accumulated truth value of a fuzzy rule was the sum of the truth values for each one of the entries in the training data, which was a measure of quality. If a rule possessed a high value of accumulated truth, it meant that the rule was intensively and frequently triggered during the evaluation process. Consequently, this was an indication of the utility and possible effectiveness of that rule. On the other hand, if a rule possessed a low value of accumulated truth, this was an indication that the rule did not play an important role in the system. Four approaches were employed on the semiactive suspension control system. The first method was a GA with fixed membership functions, as defined in [49]. The second method is a GA with the possibility of defining the membership functions and rules of a fuzzy controller simultaneously. The third approach used the PBGA with a traditional crossover operator, and the fourth approach was the PBGA with adaptive crossover operation. The simulation results showed the proposed adaptive PBGA fuzzy controller worked well to find out better rules and obtained the best performance of these four control strategies. The results also indicated that this PBGA fuzzy controller focused more on the actuation, but the encoding methods increased the total number of membership functions in the system.

# C. GA-NN Combined Control

A combination of a GA and an NN was employed to design an active suspension controller by Tang and Zhang [65]. The GA searched for the optimal acceleration of the vehicle body, which served as the objective output of the NN control system. The NN had two hidden layers, and the input, hidden, and output neurons were 1, 10, 3, and 1, respectively. An adaptive leaning rate was applied to decrease the training by keeping the learning reasonably high, while ensuring stable learning. The input of the NN was the time response of the acceleration of the sprung mass; the objective output was the optimized suspension control force. The proposed GA-NN combined controller and an LQG controller were employed to evaluate the control performance. The simulation results demonstrated that the NNC with optimal acceleration parameters computed by the GA-based optimization provided better ride comfort in the time domain.

#### VIII. CONCLUDING REMARKS

Computational-intelligence-based adaptive control approaches are required due to the real-time, nonlinear, and

disadvantages are concluded based on theoretical analysis, analyzing simulations, and the experimental results of the reviewed systems. In summary, the fuzzy control systems with learning and adaptive capability can be used to solve most modeling problems and the uncertain disturbance of active suspension systems. However, the control stability analysis is also a bottleneck for the application of fuzzy control systems. A sliding-mode controller with an FL system has been studied to integrate the advantages of transferring human expert knowledge and stability verification. However, these designs are always complex, and the tuning parameters are not easily operated by the engineers. From the point of adaptive ability, the NN and GA also have shown many advantages in suspension systems by simulations and applications. In addition, the combination of these methods hopes to bring better performance. Simultaneously, these hybrid systems have shown poor interpreting ability and are difficult to evaluate in the same test case.

In what follows, we enumerate some open questions and scientific problems that suggest future research.

- Employing intelligent control based on the full-vehicle model or 3-D model: Most of the reported research on active suspension intelligent control has studied the suspension performance under the quarter-vehicle or half-vehicle model. A comprehensive consideration of a full-car 3-D model will bring further distinct functional and safety-related benefits. Furthermore, a full-vehicle model will be convenient for the integration of other control subsystems such as brake control, steering control, and antiroll control to a hybrid intelligent system and will benefit the analysis of a unit vehicle performance.
- 2) Integration of multiobjective optimization methods and FL reasoning: Considering the tradeoff between riding comfort and road-handling quality, the optimal objective will be changed with the requirements of different road surfaces. Then, the adaptive multiobjective optimization methods with high real-time computing efficiency need to be researched, particularly for the application of racevehicle active suspension systems.
- 3) Pursuing the balance of accuracy and interpreting ability in a hybrid ASCS such as NN–GA–fuzzy controllers, GA-NN controllers, or NN–GA–fuzzy sliding-mode controller: Although hybrid intelligent systems have been widely investigated in many domains, their future will lie in the careful integration of the best constituent technologies beyond simply combining individual methods.
- 4) Evaluating hybrid intelligent control methodology from the perspective of practical applications: It is necessary to build an evaluating system to compare the different intelligent systems according to application requirements such as computing cost, the number of tuning parameters, and the interface to faulty diagnosis. The comparison results will be beneficial to hybrid intelligent control system applications and the direction of future research.

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