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BER Reduction in OFDM Systems using ZCT Matrix Transform

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ABSTRACT

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) is preferred for transmission over a dispersive channel. It is because of its several advantages such as high spectral efficiency, low implementation complexity, less vulnerability to echoes and non–linear distortion. However, it has few limitations such as peak-to-average-power ratio (PAPR) and bit error rate (BER), which determines the system's power efficiency. High PAPR is one of the major drawbacks in OFDM systems, which causes a significant level of signal distortions, when the modulated signals are amplified through high power amplifiers (HPAs). In particularly, high PAPR increases the complexity of Analogue to Digital (A/D) and Digital to Analogue (D/A) converters and also reduces the efficiency of HPAs. A high PAPR and high noise (e.g. AWGN) may significantly distort the signal, resulting in high BER on demodulation of the received signal. Therefore, PAPR & BER reduction is very essential. For this purpose, a Zadoff-Chu matrix Transform (ZCT) based precoding and postcoding techniques are quantitatively evaluated in this paper in terms of its parameters q and r. From the analysis, it is inferred that an optimum selection of q and r can lead to a significant level of reduction in BER.

Keywords: Zadoff-Chu matrix Transform; OFDM conventional; HPA; Peak to Average Power Ratio and BER.

1. Introduction

OFDM is a key multicarrier modulation technique [1], which provides high spectral efficiency, low implementation complexity [2], less vulnerability to echoes and non–linear distortion [3]. Due to these advantages of the OFDM system, it is vastly used in various communication systems. However, it has a practical limitation of high bit error rate occurring due to high PAPR or noise [4]. A large PAPR increases the complexity of the analog–to–digital and digital–to–analog converter and reduces the efficiency of the radio – frequency (RF) power amplifier [5]. There are a number of techniques dealing with the problem of BER & PAPR. Some of these include: constellation shaping, nonlinear companding transforms [6], tone reservation [7] and tone injection (TI), clipping and filtering [8], partial transmit sequence [9] and precoding based techniques. However, these techniques do PAPR reduction at the expense of increase in transmit signal power, bit error rate [7], data rate loss and computational complexity. The reduction in BER is very essential. Therefore, two novel reduction techniques, namely, Zadoff-Chu matrix Transform (ZCT) precoding based BER reduction technique and ZCT postcoding based BER reduction technique for OFDM systems were proposed in this paper. In the proposed schemes, the reshaping of the ZCT is carried out one way for precoding and another way for postcoding. For precoding, the reshaping of the ZCT matrix is row wise and is applied before the IFFT; however, for postcoding, the reshaping of the ZCT matrix is done column wise and it is applied after the IFFT.

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2. System Model

2.1 OFDM System Model

To design the proposed ZCT precoding and postcoding system, consider the block diagram of an OFDM system shown in Figure 1. In Figure 1, an OFDM signal consists of N subcarriers that are modulated by N complex symbols selected from a particular QAM constellation.

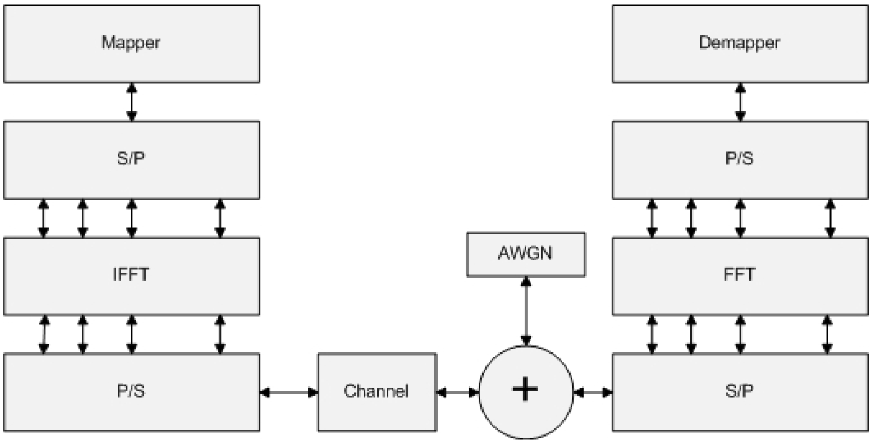

Figure 1: Block diagram of general OFDM system.

In Figure 1, the baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. This complex vector of size N can be expressed as

$$\mathbf{X} = [\mathbf{X}_0, \mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3 \ldots \mathbf{X}_{N-1}] \quad (1)$$

X is then passed through the IFFT block of size $N \times N$ IFFT matrix. The resulted complex baseband OFDM signal with N subcarriers can then be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} dX_k e^{\frac{j2\pi kn}{N}} \qquad n = 0, 1, \ldots, N-1. \quad (2)$$

After parallel-to-serial conversion, a cyclic prefix (CP) with a length of N_g samples is appended before the IFFT output to form the time-domain OFDM symbol, $s = [s_0, \ldots, s_{N+Ng-1}]$. The useful part of OFDM symbol does not include the N_g prefix samples and has duration of T_u seconds. The samples (s) are then amplified, with the amplifier characteristics is given by function F and the output of amplifier produces a set of samples denoted by y. At the receiver front end, the received signal is applied to a matched filter and then sampled at a rate $T_s = T_u/N$. After dropping the CP samples (N_g), the received sequence z, assuming an additive white Gaussian noise (AWGN) channel, can be expressed as

$$z = F(Wd) + \eta \quad (3)$$

Where, the noise vector η consists of N independent and normally distributed complex random variables with zero mean. Subsequently, the sequence z is fed to the fast Fourier transform (FFT), which produces the frequency-domain sequence r as

$$r = W^H z \quad (4)$$

Where, k_{th} element of r is given by

$$r_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} z_n e^{\frac{-j2\pi kn}{N}} \qquad k=0, 1, 2, \ldots, N-1 \tag{5}$$

Finally, the estimated symbols vector $\hat{d}$ can be obtained from r. It is to be noted that ideally, the demodulation is performed based on the assumption of perfect symbol timing, carrier frequency, and phase synchronization. However it is not possible to achieve in practice; therefore, the BER is introduced and which can be defined as the difference of the obtained demodulated signal r_k from the input signal.

2.2 ZADOFF-CHU SEQUENCES

In general, a Zadoff–Chu sequence is a complex-valued sequence which, when applied to radio signals, gives rise to an electromagnetic signal of constant amplitude, whereby cyclically shifted versions of the sequence imposed on a signal result in zero cross-correlation with one another at the receiver. Mathematically, Zadoff–Chu sequences can be defined for a sequence of length L as:

$$z(k) = \begin{cases} e^{\frac{j2\pi r}{L}\left(\frac{k^2}{2}+qk\right)} & for\ L\ even \\ e^{\frac{j2\pi r}{L}\left(\frac{k(k+1)}{2}+qk\right)} & for\ L\ odd \end{cases} \tag{6}$$

Where, $k = 0, 1, 2\ldots L-1$, q is any integer and r is any integer relatively prime to L. From the above equation, q and r comes in the numerator of the exponential power fraction. This means, that their variation can lead to a significant change in the output. This is the key focus of the research reported in this paper. This ZCT sequence can be applied in two forms: pre-coding and post-coding, as discussed below.

2.3 ZCT PRE-CODING BASED OFDM SYSTEM

The block diagram of ZCT pre-coding based OFDM system is shown in Figure 2. In the ZCT pre-coding based OFDM system, the baseband modulated data is passed through S/P convertor which generates a complex vector of size N that can be written as $X = [X_0, X_1, X_2\ldots X_{N-1}]^T$. ZCT pre-coding is then applied to this complex vector which transforms this complex vector into new vector of length N that can be written as $Y=[Y_0, Y_1, Y_2\ldots Y_{N-1}]^T$; where, R is a ZCT based row-wise precoding matrix of size $L=N*N$. By reordering,

$$k = mN + l \tag{7}$$

And the matrix R with row wise reshaping can be written as

$$R = \begin{bmatrix} r_{00} & \cdots & r_{0(N-1)} \\ \vdots & \ddots & \vdots \\ r_{(N-1)0} & \cdots & r_{(N-1)(N-1)} \end{bmatrix} \tag{8}$$

Where, R is a $N*N$, ZCT complex orthogonal matrix with length $L^2 = N*N$. Accordingly, pre-coding X gives rise to Y as follows:

$$Y = RX \tag{9}$$

Or,

$$Y_m = \sum_{l=0}^{N-1} r_{m,l} X_l \qquad m = 0,1,2,\ldots\ldots,N-1 \tag{10}$$

Where, $r_{m,l}$ means the m^{th} row and l^{th} column of pre-coder matrix. Therefore, the complex baseband OFDM signal with N subcarriers with ZCT pre-coding is given by

$$\hat{x}_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \left\{ e^{\left(\frac{j2\pi mn}{N}\right)} \left[e^{j\pi m^2} \sum_{l=0}^{L-1} \left(Y_l . e^{\frac{j\pi l^2}{L^2}} \right) e^{\frac{j2\pi ml}{L}} \right] \right\} \tag{11}$$

The above equation can be expressed as $\hat{x}_n$ the IFFT of constellation data X_l pre-multiplied with ZCT matrix. The BER of this signal is same as before.

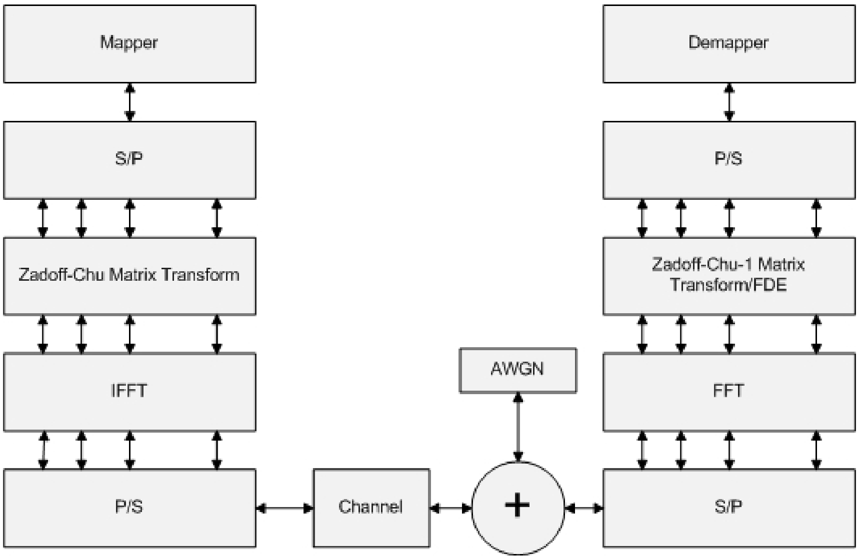

Figure 2: Block diagram of an OFDM system with ZCT pre-coding approach.

2.4 ZCT POST-CODING BASED OFDM SYSTEM

The block diagram of ZCT post-coding based OFDM system is shown in Figure 3. In the ZCT post-coding based OFDM system, the baseband modulated data is passed through S/P convertor which generates a complex vector of size N that can be written as $X = [X_0, X_1, X_2 \ldots X_{N-1}]^T$. Then, IFFT is performed to this complex vector which transforms this complex vector into new vector of length N that can be written as $y = \text{IFFT}\{X\} = [y_0, y_1, y_2 \ldots y_{N-1}]^T$. Similarly, re-ordering as in pre-coding to have:

$$k = m + lN \tag{12}$$

And, ZCT matrix C with column wise reshaping is written as

$$C = \begin{bmatrix} c_{00} & \cdots & c_{0(N-1)} \\ \vdots & \ddots & \vdots \\ c_{(N-1)0} & \cdots & c_{(N-1)(N-1)} \end{bmatrix} \tag{13}$$

In other words, the N^2 point long Zadoff-Chu sequence fills the pre-coding matrix column-wise. The matrix C is $N*N$, ZCT complex orthogonal matrix with length $L^2 = N*N$. Accordingly, postcoding y gives rise to w as follows:

$$w = C.y \tag{14}$$

Where, C is a ZCT column wise post-coding matrix of size $L^2 = N*N$. For $q = 1$ and $r = 1$, the complex baseband ZCT postcoding based OFDM signal with N subcarriers can be written as:

$$w_m = e^{\frac{j\pi m^2}{L^2}} \sum_{l=0}^{L-1} \left[e^{j\pi l^2} . x_l \right] . e^{\frac{j2\pi m l}{L}} ; \quad m = 0,1, \ldots \ldots , N-1 \tag{15}$$

Where, w_m is the IFFT of constellation data X_l premultiplied with quadratic phase and IFFT postcoded, and then alternated with ±1. The BER of this ZCT postcoding based OFDM signal is same as before.

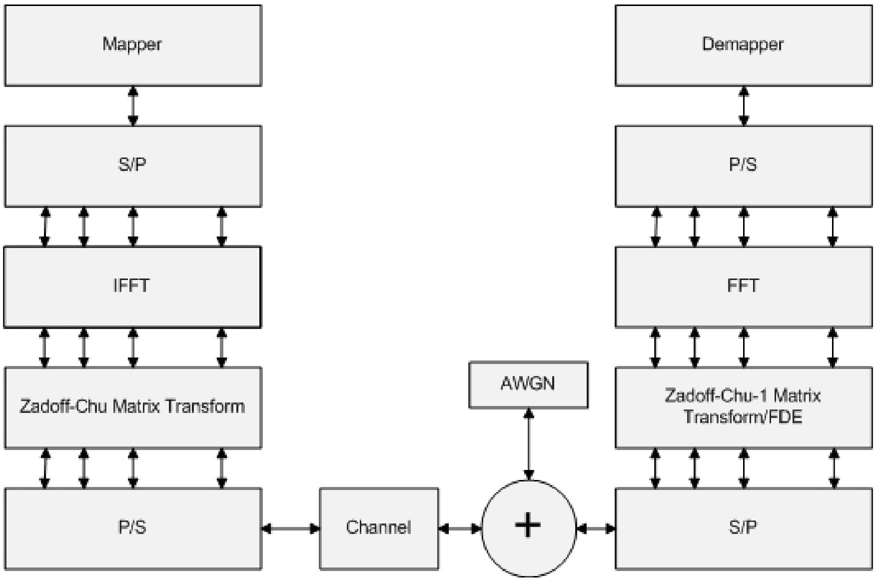

Figure 3: Block diagram of an OFDM system with ZCT post-coding approach.

3. Implementation

To do the BER analysis and to evaluate the performance of ZCT precoding and postcoding based OFDM systems, the steps undertaken are shown in Figure 4 and are detailed below.

1. To do the BER analysis of ZCT precoding and postcoding based OFDM, firstly binary data is generated randomly.
2. The generated binary data is then converted into symbols and is then modulated by M-QAM (where M=4, 16, 64, 256). The ZCT matrix is calculated for the length of the data.
3. The serial data is then converted into parallel data and IFFT is performed. The ZCT matrix (Transform) is applied before and after the IFFT, respectively for precoding and postcoding.
4. The parallel signal is then converted to serial data and is passed through a multipath channel with AWGN noise added.
5. The received signal is again converted from serial to parallel data. ZCT inverse matrix is applied before and after the FFT. The final demodulated signal is parallel data, which is then converted into serial data. The serial data is then demodulated to retrieve back the signal. The BER is calculated by taking the difference of the demodulated data and the input data.

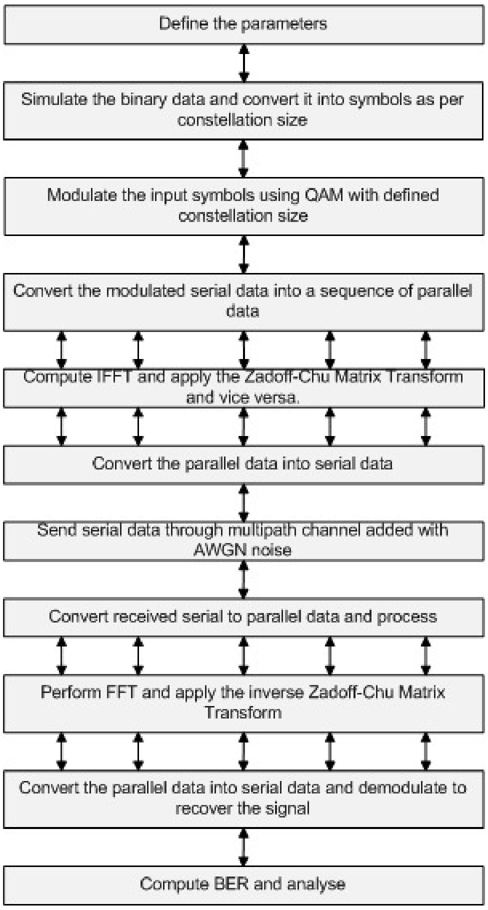

Figure 4: Implementation procedure

4. Results and Discussions

To evaluate the performance of ZCT precoding and postcoding based OFDM systems, extensive simulations have been performed in Matlab. The simulations are performed for different values of *q* and *r*. To show the BER analysis of ZCT precoding and postcoding based OFDM systems, data is generated randomly and then modulated by M-QAM (where M = 4, 16, 64, 256). The order of FFT and IFFT is taken as 64. The results obtained are shown in Figures 5 to 11 and Table 1, for various combinations of *q* and *r*, and EBN0 from 1 to 7. The key conclusions drawn from the results are discussed below:

1. From Figures 5 to 11, it can be interpreted that the results obtained for ZCT precoding and postcoding are different. The BER is more reduced in case of ZCT postcoding for the values of EBN0 (1 to 7), and q=1, 10 and *r*=11 and 111 and M=4, 16, 64 and 256.

2. From Figures 5 to 11, it can be interpreted that the BER decreases with increase in EBN0, for all three cases i.e. no coding, ZCT precoding and postcoding. It is because, with increase in EBN0, the noise will decrease and thus there will be less number of bits in error, which means low BER.

3. From Figures 5 to 11, for ZCT postcoding, the BER is more reduced when value of r is increased from 11 to 111, for any value of q. This can also be seen from data in Table 1. It is because in ZCT postcoding, the matrix is applied column-wise which will lead to higher reduction in PAPR and thus more reduction in BER.

4. From Figures 5 to 11, for ZCT precoding, the BER is also significantly reduced with increase in constellation size for any given SNR.

5. It can also be seen that ZCT postcoding has quick and sharp roll off than ZCT pre-coding. Also, there is less irregularity in ZCT postcoding compared to ZCT precoding.

6. As the constellation size increases, i.e., from M = 4, 16, 64, 256; there is significant reduction in BER; however, not without coding. Therefore, ZCT precoding and postcoding has potential for significant reduction in BER.

7. It can be also seen that for some cases, the BER for ZCT postcoding is not observed. It is because it is very efficient and has quickly reduced the BER to zero.

From the above discussion, the value of q and r significantly changes the BER for any given EBN0 and constellation size M. Also, for same values of q and r, the variation in constellation size also significantly changes the BER. It is also found that ZCT postcoding is more effective than ZCT precoding in reducing the BER. However, ZCT precoding and postcoding and its parameters q and r significantly reduces the PAPR also. Therefore, the selection of value of q, r, M and ZCT precoding/postcoding depends upon the level of PAPR and BER reduction required. Here in this paper, it is preferred to choose a small value of q and large value of r for ZCT post coding and both small values of q and r for ZCT pre-coding. It is preferred to choose ZCT post coding because of its sharp roll-off in reducing BER; however, it also depends upon the desired level of reduction in PAPR.

Constellation Size (M=4)

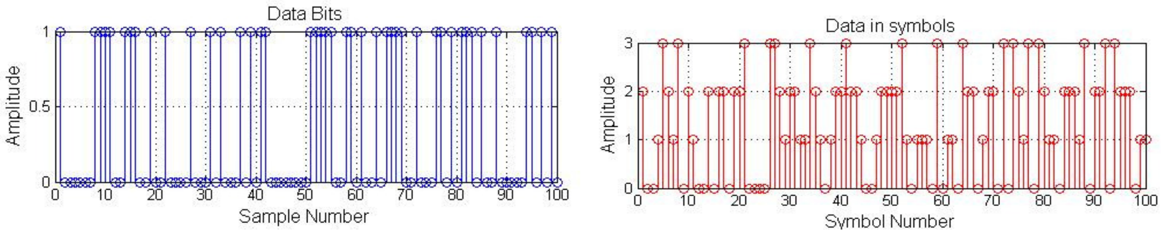

Figure 5: The simulated binary data and its conversation to symbols for constellation size (M=4)

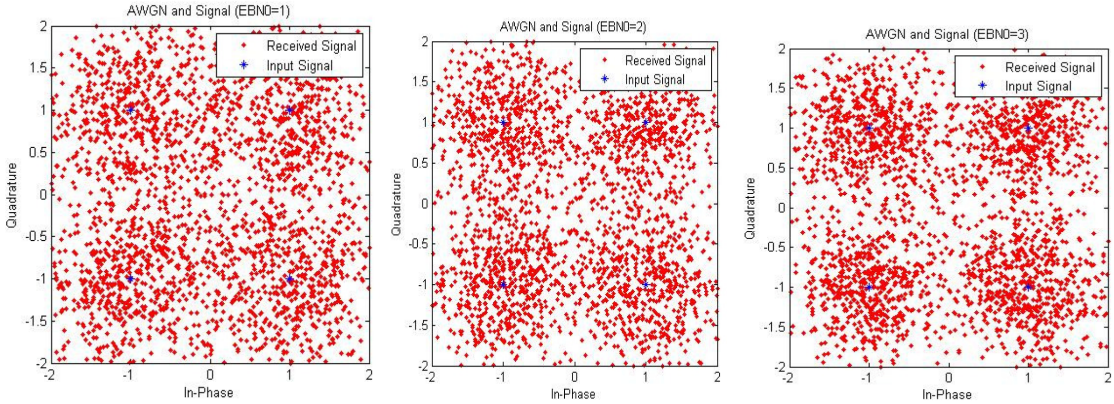

Figure 6: The simulated data modulated using QAM (M=4) and AWGN noise (EBN0=1, 2 and 3).

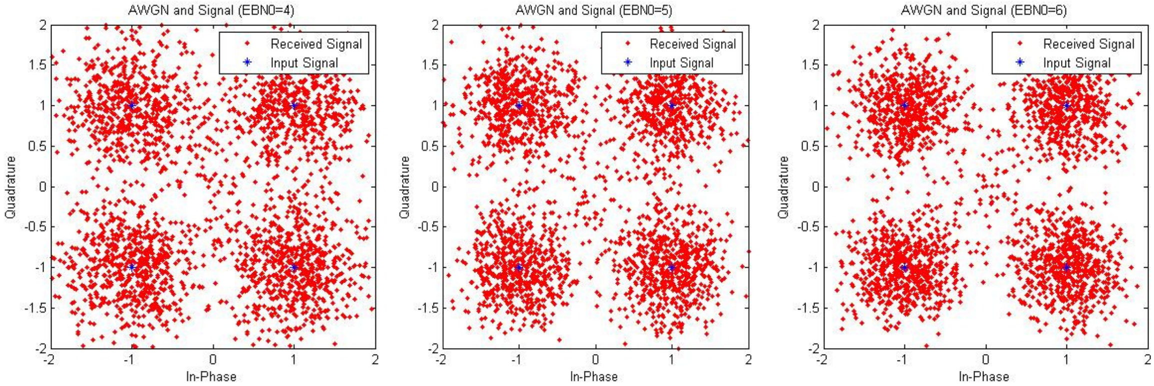

Figure 7: The simulated data modulated using QAM (M=4) and AWGN noise (EBN0=4, 5 and 6)

Constellation Size (M=4)

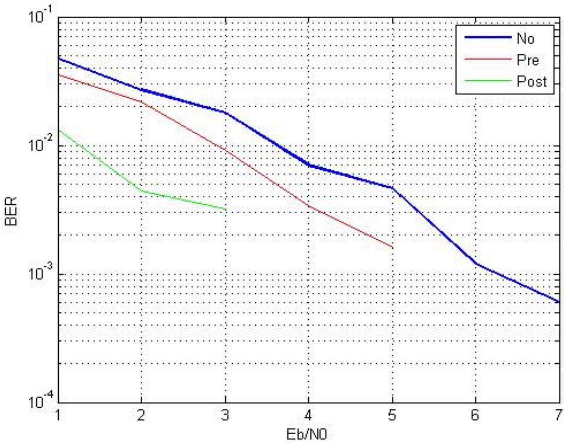

Figure 8: The variation for BER with EBN0 for QAM (M=4) and with ZCT precoding and postcoding and q=1 and r=11.

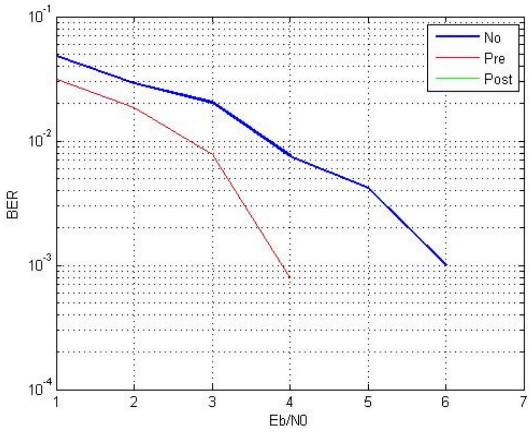

Figure 9: The variation for BER with EBN0 for QAM (M=4) and with ZCT precoding and postcoding and q=10 and r=111.

Constellation Size (M=16)

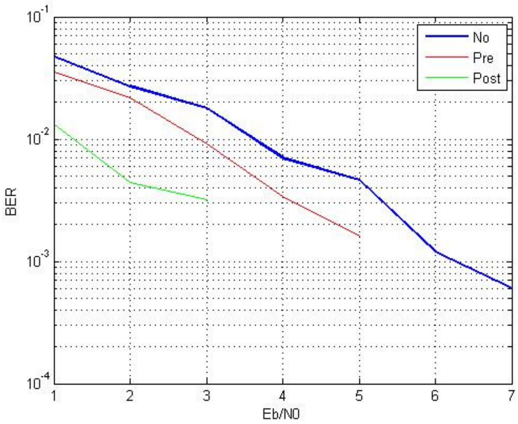

Figure 10: The variation for BER with EN0 for QAM (M=16) and with ZCT precoding and postcoding and q=1 and r=11.

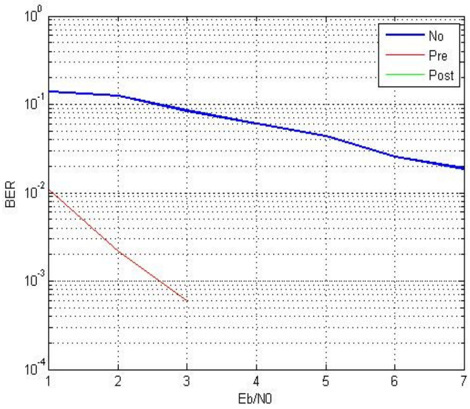

Figure 11: The variation for BER with EN0 for QAM (M=16) and with ZCT precoding and postcoding and q=10 and r=111.

Constellation Size (M=64)

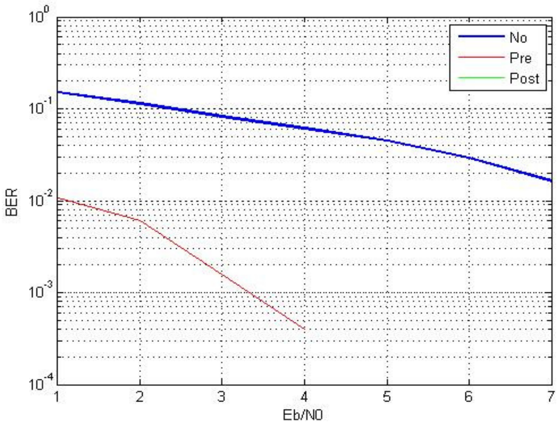

Figure 12: The variation for BER with EN0 for QAM (M=64) and with ZCT precoding and postcoding and q=1 and r=11.

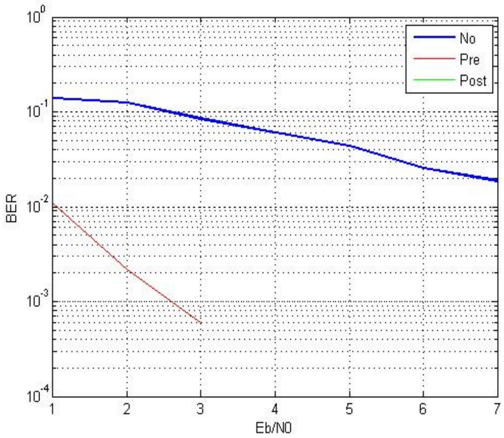

Figure 13: The variation for BER with EN0 for QAM (M=64) and with ZCT precoding and postcoding and q=10 and r=111.

Constellation Size (M=256)

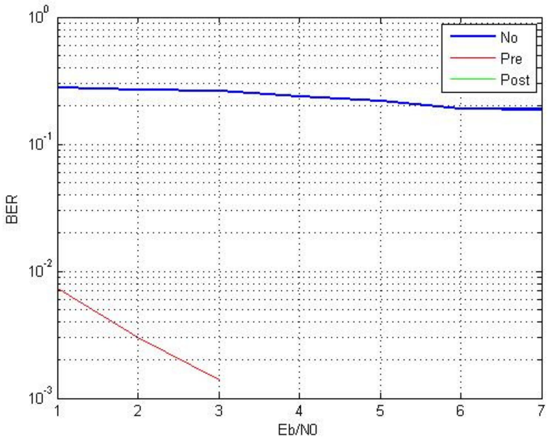

Figure 14: The variation for BER with EBN0 for QAM (M=256) and with ZCT precoding and postcoding and q=1 and r=11.

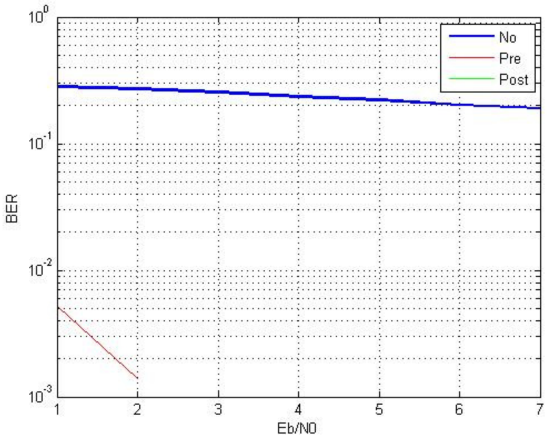

Figure 15: The variation for BER with EBN0 for QAM (M=256) and with ZCT precoding and postcoding and q=10 and r=111.

5. Conclusion

This paper presented the results obtained on applying a ZCT pre-coding and post-coding for reducing BER in OFDM systems with M-QAM modulation, where as M=4, 16, 64, 256 and the parameters q=1, 10 and r=11 and 111. It is observed from the graphs that the response of both ZCT pre-coding and post-coding is different for r and q. The BER is found to significantly vary with the values of M, r and q for a given EBN0. For any particular value of r, q and M, the BER reduction is more for ZCT post-coding than for ZCT pre-coding. From the results, it is preferred to choose ZCT post coding because of its sharp roll-off and quick reduction in BER for a given SNR and M.

Table 1: Variation of Bit Error Rate with EBN0

Constellation Size	**q**	**r**	**EBN0**							
				1	2	3	4	5	6	7
4	1	11	**None**							
			Bits in Error	239	135	89	35	23	6	3
			BER	0.0478	0.0270	0.0178	0.0070	0.0046	0.0012	0.0006
			ZCT Postcoding							
			Bits in Error	66	22	16	0	0	0	0
			BER	0.0132	0.0044	0.0032	0	0	0	0
			ZCT Precoding							
			Bits in Error	177	108	46	17	8	0	0
			BER	0.0354	0.0216	0.0092	0.0034	0.0016	0	0
4	10	111	**None**							
			Bits in Error	240	145	102	38	21	5	0
			BER	0.0480	0.0290	0.0204	0.0076	0.0042	0.0010	0
			ZCT Postcoding							
			Bits in Error	16	0	0	0	0	0	0
			BER	0.0032	0	0	0	0	0	0
			ZCT Precoding							
			Bits in Error	157	92	39	4	0	0	0
			BER	0.0314	0.0184	0.0078	0.0008	0	0	0
256	1	11	**None**							
			Bits in Error	1411	1355	1274	1179	1110	1008	945
			BER	0.2822	0.2710	0.2548	0.2358	0.2220	0.2016	0.1890
			ZCT Postcoding							
			Bits in Error	0	0	0	0	0	0	0
			BER	0	0	0	0	0	0	0
			ZCT Precoding							
			Bits in Error	26	7	0	0	0	0	0
			BER	0.0052	0.0014	0	0	0	0	0
256	10	111	**None**							
			Bits in Error	1388	1340	1307	1190	1090	951	943
			BER	0.2776	0.2680	0.2614	0.2380	0.2180	0.1902	0.1886
			ZCT Postcoding							
			Bits in Error	0	0	0	0	0	0	0
			BER	0	0	0	0	0	0	0
			ZCT Precoding							
			Bits in Error	37	15	7	0	0	0	0
			BER	0.0074	0.0030	0.0014	0	0	0	0

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