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Research Article

ISAR Imaging of Rotating Target with Equal Changing Acceleration Based on the Cubic Phase Function

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When reconstructing the ISAR image of a maneuvering target rotating approximately with the equal changing acceleration, the received signals can be considered as the multicomponent cubic phase signals. If the model of rotating with constant acceleration is used here, the errors of the reconstructed images cannot be neglected, and a lot of pseudoscatterers are produced. Hence, the ISAR imaging of rotating target with equal changing acceleration based on the cubic phase (CP) function is presented in this paper. The CP function is applied to estimate the parameters of the multicomponent cubic phase signals, and the clear instantaneous ISAR images are obtained. The results of simulated data demonstrate the effectiveness of the method proposed.

1. INTRODUCTION

Inverse synthetic aperture radar (ISAR) is a well-established technique for reconstructing the high-resolution radar images of targets [1]. The range-Doppler (RD) algorithm is the classical method used to process the received data. Large bandwidth coded signals are used to get high resolution in the range coordinate, and coherent integration of echoes collected at different viewing angles is used to achieve high cross-range resolution. The RD algorithm is based on the assumption that the Doppler frequency of each scatterer, relative to a reference point taken on the target and called focus point, is constant during the observation time. This hypothesis is usually valid when the target moves smoothly. When the target has a maneuvering flight, the Doppler frequency associated with each scatterer becomes time-varying. In these cases, a novel ISAR technique named range instantaneous Doppler (RID) has been proposed recently. In this technique, the Doppler analysis is replaced by a time frequency transform, such as the Radon-Wigner transform [2], the joint time frequency distribution [3], and the adaptive chirplet decomposition [4]. These algorithms are based on the assumption that the target is rotating with constant acceleration after motion compensation, and the echo of each scatterer is linear frequency modulated (LFM) signal approximately. In fact, in some cases, the target will have a high-maneuvering flight and rotating with equal changing acceleration; the echo of each scatterer is the cubic phase signal [5]. In [6], the product high-order ambiguity function was used to estimate the parameters of multicomponent cubic phase signal, but this method suffered from the computational burden and the error propagation effect. So, in this paper, a new ISAR imaging algorithm of a rotating target with equal changing acceleration based on the cubic phase (CP) function is presented. The cubic phase function is a technique to estimate the instantaneous frequency rate of a quadratic FM signal [7]. In this paper, we extend it to estimate the parameters of the multicomponent cubic phase signals and use it in the ISAR imaging of a rotating target with equal changing acceleration. The results of simulated data demonstrate the validity of the method proposed.

2. THE CHARACTERISTIC OF NONUNIFORM ROTATING SCATTERERS

Assume that the initial velocity, acceleration, and acceleration rate of the rotation are V_0, a, and b, respectively. The instantaneous velocity of a scatterer with cross-range coordinate x_i is

$$V(t) = (V_0 + at + bt^2)x_i, \quad (1)$$

and the corresponding Doppler angular frequency is

$$\omega_i(t) = \frac{4\pi}{\lambda}V(t) = \frac{4\pi}{\lambda}x_i(V_0 + at + bt^2), \quad (2)$$

where λ is the wavelength of the radar. When $a = b = 0$, (2) represents the situation of uniform rotation, and the Doppler frequency is invariant.

Suppose that the complex envelope of each scatterer is A_i $(i = 1, 2, \ldots, p)$, where p is the number of scatterers in a range bin, then the echo within the range bin is

$$s(t) = \sum_{i=1}^{p} A_i \exp\left[j\frac{4\pi}{\lambda}x_i(V_0 t + at^2 + bt^3)\right]. \quad (3)$$

Equation (3) is the expression of the received signals for a target rotating with equal changing acceleration; it is the sum of P-complex cubic phase signals. So, a new algorithm for the detection and parameters estimation of multicomponent cubic phase signals based on the CP function is proposed in this paper, then the algorithm is used in the ISAR imaging of rotating target with equal changing acceleration, and the clear instantaneous ISAR images are obtained.

3. THE CUBIC PHASE FUNCTION

The CP function was introduced in [7] for the purposes of estimating the instantaneous frequency rate law of a quadratic FM signal. In this paper, a new algorithm for the detection and parameters estimation of multicomponent cubic phase signals based on the CP function is developed in the following.

For a monocomponent cubic phase signal:

$$s(t) = a_0 e^{j\phi(t)} = a_0 e^{j(a_1 t + a_2 t^2 + a_3 t^3)}, \quad (4)$$

where $\phi(t)$ is the signal phase, a_0 is amplitude, a_1, a_2, and a_3 are the coefficients of the phase. The CP function is defined as

$$\mathrm{CP}(t,u) = \int_0^{+\infty} s(t+\tau)s(t-\tau)e^{-ju\tau^2}\,d\tau, \quad (5)$$

along with (4), we obtain

$$\mathrm{CP}(t,u) = a_0^2 e^{2j(a_1 t + a_2 t^2 + a_3 t^3)} \int_0^{+\infty} e^{j(2a_2 + 6a_3 t - u)\tau^2}\,d\tau. \quad (6)$$

By using the identity

$$\int_{-\infty}^{+\infty} e^{-jmt^2}\,dt = \sqrt{\frac{\pi}{m}}e^{-j(\pi/4)}, \quad m > 0, \quad (7)$$

we have

$$|\mathrm{CP}(t,u)| = \begin{cases} \infty, & u = 2a_2 + 6a_3 t, \\ \dfrac{a_0^2}{2}\sqrt{\dfrac{\pi}{|2a_2 + 6a_3 t - u|}}, & u \neq 2a_2 + 6a_3 t. \end{cases} \quad (8)$$

It is not hard to see that $\mathrm{CP}(t,u)$ peaks along the curve $u = 2a_2 + 6a_3 t$, which is defined as the instantaneous frequency rate (IFR) of the signal. The slope and intercept indicate the estimation of a_3 and a_2, respectively. So, the estimation of a_3 and a_2 can be implemented as follows.

Estimate two IFRs $u(t_1)$ and $u(t_2)$, at times t_1 and t_2, respectively,

$$\begin{aligned} u(t_1) &= 2a_2 + 6a_3 t_1, \\ u(t_2) &= 2a_2 + 6a_3 t_2. \end{aligned} \quad (9)$$

So, the estimated values $\hat{a}_3$ and $\hat{a}_2$ can be obtained:

$$\begin{aligned} \hat{a}_3 &= \frac{u(t_1) - u(t_2)}{6(t_1 - t_2)}, \\ \hat{a}_2 &= \frac{t_1 u(t_2) - t_2 u(t_1)}{2(t_1 - t_2)}. \end{aligned} \quad (10)$$

Then, the parameters a_1 and a_0 can be estimated by dechirping and finding the Fourier transform peak. In practice, the discrete-time signal is $s(n)$, $-(N-1)/2 \le n \le (N-1)/2$, and the specification of the parameters estimation algorithm is as follows. Compute the CP function of the original signal and estimate $\hat{a}_3$ and $\hat{a}_2$ by finding the peak of it, then find $\hat{a}_1$ and $\hat{a}_0$ by evaluating

$$\begin{aligned} \hat{a}_1 &= \arg\max_{a_1}\left|\sum_{n=-(N-1)/2}^{(N-1)/2} s(n)e^{-j(a_1 n + \hat{a}_2 n^2 + \hat{a}_3 n^3)}\right|, \\ \hat{a}_0 &= \left|\frac{1}{N}\sum_{n=-(N-1)/2}^{(N-1)/2} s(n)e^{-j(\hat{a}_1 n + \hat{a}_2 n^2 + \hat{a}_3 n^3)}\right|. \end{aligned} \quad (11)$$

The CP function, like the ambiguity function, is bilinear. It therefore produces "cross-terms" when multiple components are presented. So, the influence of cross-terms should be studied. The theoretical analysis demonstrates that the sharply peaked "auto-terms" can often be detected against the background of dispersed cross-terms in the CP function which can be verified as follows.

For simplicity, we discuss here the two components case, which are modeled as

$$s(t) = s_1(t) + s_2(t) = a_{01}e^{j(a_{11}t + a_{21}t^2 + a_{31}t^3)} + a_{02}e^{j(a_{12}t + a_{22}t^2 + a_{32}t^3)}. \quad (12)$$

So, we have

$$\begin{aligned} s(t+\tau)s(t-\tau) &= a_{01}^2 e^{2j(a_{11}t + a_{21}t^2 + a_{31}t^3)} e^{j(2a_{21} + 6a_{31}t)\tau^2} \\ &\quad + a_{02}^2 e^{2j(a_{12}t + a_{22}t^2 + a_{32}t^3)} e^{j(2a_{22} + 6a_{32}t)\tau^2} \\ &\quad + a_{01}a_{02}e^{j(a_{11}+a_{12})t + j(a_{21}+a_{22})t^2 + j(a_{31}+a_{32})t^3} e^{jA\tau^3 + jB\tau^2 + jC\tau} \\ &\quad + a_{01}a_{02}e^{j(a_{11}+a_{12})t + j(a_{21}+a_{22})t^2 + j(a_{31}+a_{32})t^3} e^{-jA\tau^3 + jB\tau^2 - jC\tau}, \end{aligned} \quad (13)$$

where

$$A = a_{31} - a_{32},$$
$$B = a_{21} + a_{22} + 3a_{31}t + 3a_{32}t, \quad (14)$$
$$C = (a_{11} - a_{12}) + 2(a_{21} - a_{22})t + 3(a_{31} - a_{32})t^2.$$

The CP function of the "auto-terms" has the form of (8) and peaks along the curves $u = 2a_{21} + 6a_{31}t$ and $u = 2a_{22} + 6a_{32}t$, respectively. Now let us compute the CP function of the "cross-terms":

$$\mathrm{CP}_{\mathrm{cro}}(t,u) = 2a_{01}a_{02}e^{j(a_{11}+a_{12})t+j(a_{21}+a_{22})t^2+j(a_{31}+a_{32})t^3} \times \int_0^{+\infty} e^{j(B-u)\tau^2}\cos(A\tau^3 + C\tau)\,d\tau. \quad (15)$$

If $u = B$, we have

$$|\mathrm{CP}_{\mathrm{cro}}(t,u)| = 2a_{01}a_{02}\left|\int_0^{+\infty}\cos(A\tau^3 + C\tau)\right| < \infty. \quad (16)$$

If $u \neq B$, we have

$$|\mathrm{CP}_{\mathrm{cro}}(t,u)| = 2a_{01}a_{02}\left\{\left(\int_0^{+\infty}\cos[(B-u)\tau^2]\cos(A\tau^3 + C\tau)\,d\tau\right)^2 + \left(\int_0^{+\infty}\sin[(B-u)\tau^2]\sin(A\tau^3+C\tau)\,d\tau\right)^2\right\}^{1/2} < \infty. \quad (17)$$

Equations (16) and (17) come into existence just because the cosine function is bounded and periodic.

We can see from (16) and (17) that the CP function of the "cross-terms" is bounded while the CP function of the "auto-terms" is infinite when $u = 2a_{21} + 6a_{31}t$ or $u = 2a_{22} + 6a_{32}t$. Hence, the existence of the "cross-terms" does not influence the detection of the "auto-terms."

4. ISAR IMAGING OF ROTATING TARGET WITH EQUAL CHANGING ACCELERATION BASED ON THE CP FUNCTION

The algorithm for the detection and parameters estimation of multicomponent cubic phase signal has been presented in Section 3. But in practice, the power of each component is quite different, and the weak component can often be covered up by the strong component. To overcome this problem, the ISAR algorithm based on the CP function is presented along with the "Clean" technique, just as follows.

Step 1. Let $k = 1$, where k is the number of the scatterers, $s(t)$ is the data of each range bin.

Step 2. Compute the CP function of $s(t)$ along with (5), and get its absolute value $|\mathrm{CP}(t,u)|$.

Step 3. Finding the peak of $|\mathrm{CP}(t,u)|$, it is concentrated along the curve $u = 2a_{2k} + 6a_{3k}t$, then the estimated values $\{a_{0k}, a_{1k}, a_{2k}, a_{3k}\}$ of the kth component are obtained along with (10) and (11).

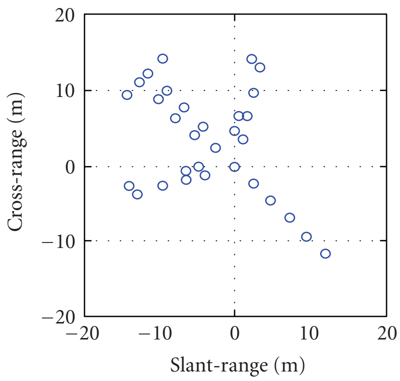


FIGURE 1: The point model of the target.

Step 4. Construct the reference signal $s_{r1}(t) = \exp(-ja_{2k}t^2 - ja_{3k}t^3)$, then multiply it with $s(t)$, we obtain $s_1(t) = s(t)s_{r1}(t)$. Now, the kth component has been compensated to a sinusoidal signal, while the other components are still the cubic phase signals.

Step 5. Design a filter with narrow bandwidth around a_{1k}, then the kth component of $s_1(t)$ is filtered out, and this operation has a little influence on the other components.

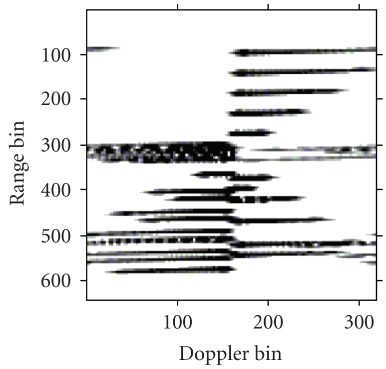
Step 6. Multiply the residual signal with $s_{r2}(t) = \exp(ja_{2k}t^2 + ja_{3k}t^3)$, and the other components can be calibrated to the original form. Hence, the echo signal without the kth component can be obtained.

Step 7. Let $k = k + 1$, repeat Steps 2–6 until the energy of the residual signal is less than a threshold.

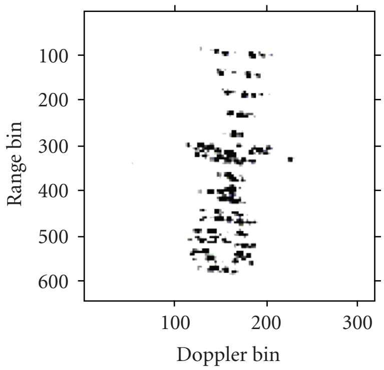
5. THE RESULTS OF ISAR IMAGES

In the simulated experiment, the parameters of the radar system are carrier frequency $f_0 = 5.52$ GHz, bandwidth $B_d = 400$ MHz, pulse width $\tau_w = 25.6$ micro-seconds, sampling frequency $f_s = 5$ MHz, sampling number $N = 128$, pulse repeated frequency PRF = 400 Hz, and the imaging angle $\Delta\theta \approx 4.15°$. Suppose that the target is rotating with equal changing acceleration if the initial velocity is 0.018 rad/s, acceleration is 0.05 rad/s^2, acceleration rate is 6 rad/s^3, the integrated imaging time is 0.32 second, and the number of pulses is 128. The target is a plane model of 25 m × 25 m with 31 points, as shown in Figure 1.

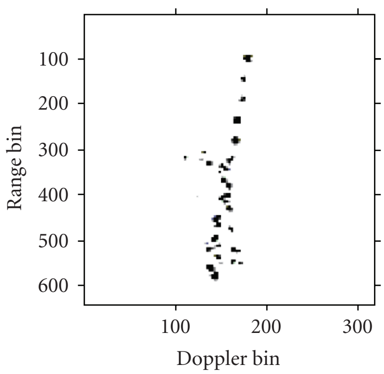
Figure 2(a) is the ISAR image based on the range-Doppler algorithm; we can see that the image is blurred and cannot be recognized. Figure 2(b) is the instantaneous ISAR image based on the assumption that the target is rotating with constant acceleration, the quality of the image improves in a way, but there still exist a lot of pseudoscatterers. Figure 2(c) is the instantaneous ISAR image at the same time as Figure 2(b), but it is based on the assumption that the target is rotating with equal changing acceleration, and the image is obtained by the new algorithm proposed in this paper. We can see that the quality of the image improves greatly.



(a) Range-Doppler algorithm



(b) Instantaneous ISAR image at time $t = 0.08$ second based on the assumption that the target is rotating with constant acceleration



(c) Instantaneous ISAR image at time $t = 0.08$ second based on the assumption that the target is rotating with equal changing acceleration

Figure 2: ISAR imaging of target rotating with equal changing acceleration.

So, the ISAR imaging algorithm based on the CP function is valid.

6. CONCLUSION

A new ISAR imaging algorithm of target rotating with equal changing acceleration based on CP function is presented. This algorithm is based on the assumption that the echo of each scatterer is the cubic phase signal; the results of simulated data demonstrate the validity of the method proposed. At the same time, a new efficient algorithm for detection and parameters estimation of the multicomponent cubic phase signals based on the CP function is presented. Hence, the method proposed in this paper is valuable in practice.

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