# Object detection by $\kappa$-connected seed competition 

Alexandre X. Falcão, Paulo A.V. Miranda, Anderson Rocha and Felipe P.G. Bergo<br>Institute of Computing - State University of Campinas (UNICAMP)<br>CEP 13084-851, Campinas, SP, Brazil<br>\{afalcao,paulo.miranda,anderson.rocha,felipe.bergo\}@ic.unicamp.br


#### Abstract

The notion of "strength of connectedness" between pixels has been successfully used in image segmentation. We present an extension to these works, which can considerably increase the efficiency of object definition tasks. A set of pixels is said a $\kappa$-connected component with respect to a seed pixel when the strength of connectedness of any pixel in that set with respect to the seed is higher than or equal to a threshold. While the previous approaches either assume no competition with a single threshold for all seeds or eliminate the threshold for seed competition, we found that seed competition with different thresholds can reduce the number of seeds and the need for user interaction during segmentation. We also propose automatic and user-friendly interactive methods for determining the thresholds. The improvements are demonstrated through several segmentation experiments involving medical images.


## 1. Introduction

Many image segmentation methods have been proposed, but their success usually depends on user intervention, being automatic segmentation only verified for some specific situations. In view of that, it is important to develop interactive methods, which minimize the user's time and involvement in the segmentation process, such that their automation becomes feasible under certain conditions. For example, we are interested in reducing the user intervention to simple selection of a few pixels in the image.

Fuzzy connectedness/watersheds are image segmentation approaches based on seed pixels, still in progress, and successfully used in many applications $[11,14,15,9]$. These approaches have been used under two region growing paradigms, with and without competition among seeds. In object detection with seed competition [17, 10, 21, 1], the seeds are specified inside and outside the object, each seed defines an influence zone composed by pixels more strongly connected to that seed than to any other, and the object is
defined by the union of the influence zones of its internal seeds. In object definition without seed competition [20], a seed is specified inside the object and the strength of connectedness of each pixel with respect to that seed is computed, such that the object is obtained by thresholding the resulting connectivity image.

We extend these methods using the framework of the image foresting transform (IFT) [5]- a general tool for the design, implementation, and evaluation of image processing operators based on connectivity. In the IFT, the image is interpreted as a graph, whose nodes are the image pixels and whose arcs are defined by an adjacency relation between pixels. The cost of a path in this graph is determined by an application-specific path-cost function, which usually depends on local image properties along the path- such as color, gradient, and pixel position. For suitable path-cost functions and a set of seed pixels, one can obtain an image partition as an optimum-path forest rooted at the seed set. That is, each seed is root of a minimum-cost path tree whose pixels are reached from that seed by a path of minimum cost, as compared to the cost of any other path starting in the seed set. The IFT essentially reduces image operators to a simple local processing of attributes of the forest $[12,4,19,2,3]$.

The strength of connectedness of a pixel with respect to a seed is inversely related to the cost of the optimum path connecting the seed to that pixel in the graph. A set of pixels is said a $\kappa$-connected component with respect to a seed, when they are reached by optimum paths whose costs are less than or equal to $\kappa$. In this sense, when a seed is selected inside an object, its maximal extent is a $\kappa$-connected component composed by only internal pixels. In [20], the object is defined without competition, as the union of all $\kappa$ connected components (minimum-cost path trees) created from each internal seed, separately (which requires one IFT for each seed). Clearly, the initial appeal for seed competition is the possibility to detect multiple objects with a single IFT and without depending on thresholds: external and internal seeds compete among themselves, partitioning the image into an optimum-path forest, and each object is de-
fined by the union of the optimum-path trees rooted at its internal seeds. However, we found that seed competition with a threshold $\kappa_{s}$ for each internal seed $s$ can considerably increase the efficiency of object definition tasks. The method restricts seed competition into regions of pixels that are $\kappa_{s}$-connected to some internal seed $s$, pixels not reached by any seed are considered background, and external seeds are only needed when the extent of a seed is not contained in the object. Of course, this comes with the problem of finding the values $\kappa_{s}$ for each seed $s$, but we provide automatic and user-friendly interactive ways to determining them.

Section 2 describes some definitions related to the IFT, making them more specific for region-based image segmentation. For sake of simplicity, we will describe the methods for gray-scale and two-dimensional images, but they are extensive to multi-parametric and multi-dimensional data sets. The proposed method and its algorithms with automatic and interactive $k_{s}$ detection, respectively, are presented in Sections 3 and 4. Section 5 demonstrates the improvements with respect to the previous approaches and Section 6 states conclusions and discusses future work.

## 2. Background

An image $\hat{I}$ is a pair $\left(D_{I}, I\right)$ consisting of a finite set $D_{I}$ of pixels (points in $\mathbb{Z}^{2}$ ) and a mapping $I$ that assigns to each pixel $p$ in $D_{I}$ a pixel value $I(p)$ in some arbitrary value space.

An adjacency relation $A$ is a binary relation between pixels $p$ and $q$ of $D_{I}$. We use $q \in A(p)$ and $(p, q) \in A$ to indicate that $q$ is adjacent to $p$. Once the adjacency relation $A$ has been fixed, the image $\hat{I}$ can be interpreted as a directed graph $\left(D_{I}, A\right)$ whose nodes are the image pixels in $D_{I}$ and whose arcs are the pixel pairs $(p, q)$ in $A$. We are interested in irreflexive, symmetric, and translationinvariant relations. For example, one can take $A$ to consist of all pairs of pixels $(p, q) \in D_{I} \times D_{I} \backslash\{(0,0)\}$ such that $d(p, q) \leq \rho$, where $d(p, q)$ denotes the Euclidean distance and $\rho$ is a specified constant (i.e. 4-adjacency, when $\rho=1$, and 8 -adjacency, when $\rho=\sqrt{2}$ ).

A path is a sequence $\pi=\left\langle p_{1}, p_{2}, \ldots, p_{n}\right\rangle$ of pixels where $\left(p_{i}, p_{i+1}\right) \in A$, for $1 \leq i \leq n-1$. The path is trivial if $n=1$. Let $\operatorname{org}(\pi)=p_{1}$ and $\operatorname{dst}(\pi)=p_{n}$ be the origin and destination of a path $\pi$. If $\pi$ and $\tau$ are paths such that $d s t(\pi)=\operatorname{org}(\tau)=p$, we denote by $\pi \cdot \tau$ the concatenation of the two paths, with the two joining instances of $p$ merged into one. In particular, $\pi \cdot\langle p, q\rangle$ is a path resulting from the concatenation of its longest prefix $\pi$ and the last $\operatorname{arc}(p, q) \in A$.

A predecessor map is a function $P$ that assigns to each pixel $q \in D_{I}$ either some other pixel in $D_{I}$, or a distinctive marker nil not in $D_{I}$ - in which case $q$ is said to be a root of the map. A spanning forest is a predecessor map

$\operatorname{org}\left(P^{*}(q)\right)$
(a)

(b)

(c)

Figure 1. (a) A spanning forest with two roots. The bold path is $P^{*}(q)$. (b) An image graph with 4 -adjacency. The numbers are the image values $I(p)$ and the bigger dots denote two seeds (One inside the brighter rectangle and one in the darker background around it). The background also has bright pixels. (c) An optimum-path forest for $f_{\max }$, with $\delta(p, q)=|I(q)-I(p)|$. The numbers are the cost values and the rectangle is obtained as a tree rooted at the internal seed.
which contains no cycles - in other words, one which takes every pixel to nil in a finite number of iterations. For any pixel $q \in D_{I}$, a spanning forest $P$ defines a path $P^{*}(q)$ recursively as $\langle q\rangle$, if $P(q)=n i l$, or $P^{*}(p) \cdot\langle p, q\rangle$ if $P(q)=$ $p \neq$ nil (see Figure 1a).

A pixel $q$ is connected to a pixel $p$ if there exists a path in the graph from $p$ to $q$. In this sense, every pixel is connected to itself by its trivial path. Since $A$ is symmetric, we can also say that $p$ is connected to $q$, or simply $p$ and $q$ are connected. Therefore, a connected component is a subset of $D_{I}$ wherein all pairs of pixels are connected.

A path-cost function $f$ assigns to each path $\pi$ a path cost $f(\pi)$, in some totally ordered set $V$ of cost values, whose maximum element is denoted by $+\infty$. A path $\pi$ is optimum if $f(\pi) \leq f(\tau)$ for any other path $\tau$ with $\operatorname{dst}(\tau)=\operatorname{dst}(\pi)$, irrespective of its starting point. The IFT establishes some conditions applied to optimum paths, which are satisfied by only smooth path-cost functions. That is, for any pixel $q \in$ $D_{I}$, there must exist an optimum path $\pi$ ending at $q$ which either is trivial, or has the form $\tau \cdot\langle p, q\rangle$ where
(C1) $f(\tau) \leq f(\pi)$,
(C2) $\tau$ is optimum,
(C3) for any optimum path $\tau^{\prime}$ ending at $p, f\left(\tau^{\prime} \cdot\langle p, q\rangle\right)=$ $f(\pi)$.
The IFT takes an image $\hat{I}$, a smooth path-cost function $f$ and an adjacency relation $A$; and returns an optimum-path forest - a spanning forest $P$ such that $P^{*}(q)$ is optimum for every pixel $q \in D_{I}$. In the forest, there are three important attributes for each pixel: its predecessor in the optimum path, the cost of that path, and the corresponding root (or some label associated with it). The IFT-based image operators result from simple local processing of one or more of these attributes.

For a given seed set $S \subset D_{I}$, the concept of strength of connectedness $[20,16,10]$ of a pixel $q \in D_{I}$ with respect to a seed $s \in S$ can be interpreted as an image property inversely related to the cost of the optimum path from $s$ to $q$ according to the max-arc path-cost function $f_{\text {max }}$ :

$$
\begin{align*}
f_{\max }(\langle q\rangle) & = \begin{cases}0, & \text { if } q \in S \\
+\infty, & \text { otherwise }\end{cases} \\
f_{\max }(\pi \cdot\langle p, q\rangle) & =\max \left\{f_{\max }(\pi), \delta(p, q)\right\} \tag{1}
\end{align*}
$$

where $(p, q) \in A, \pi$ is any path ending at $p$ and starting in $S$, and $\delta(p, q)$ is a non-negative dissimilarity function between $p$ and $q$ which depends on image properties, such as brightness and gradient (see Figures 1b and 1c).

One may think of smoothness as a more general definition for strength of connectedness. In this work, we discuss only $f_{\text {max }}$ because the comparison with previous approaches and our practical experience in region-based segmentation, which shows that $f_{\text {max }}$ often leads to better results than other commonly known smooth cost functions.

## 3. Seed competition with $\kappa$-connectivity

We assume given a seed set $S$ either interactively, by simple mouse clicks, or automatically, based on some $a$ priori knowledge about the approximate location of the object. The adjacency relation $A$ is usually a simple 8 neighborhood, but sometimes it is important to allow farther pixels be adjacent. This may reduce the number of seeds required to detect nearby components of a same object, such as letters of a word in the image of a text. Some examples of $\delta$ functions for $f_{\max }$ are given below:

$$
\begin{align*}
\delta_{1}(p, q) & =K\left(1-e^{\left(\frac{-1}{2 \sigma^{2}}(I(p)-I(q))^{2}\right)}\right)  \tag{2}\\
\delta_{2}(p, q) & =G(q)  \tag{3}\\
\delta_{3}(p, q) & =K\left(1-e^{\left(\frac{-1}{\left.2 \sigma^{2}\left(\frac{I(p)+I(q)}{2}-I(s)\right)^{2}\right)}\right)}\right.  \tag{4}\\
\delta_{4}(p, q) & =\min _{\forall s \in S}\left\{\delta_{3}(p, q)\right\}  \tag{5}\\
\delta_{5}(p, q) & =a \delta_{1}(p, q)+b \delta_{3}(p, q) \tag{6}
\end{align*}
$$

where $K$ is a positive integer (e.g. the maximum image intensity), $\sigma$ is an allowed intensity variation, $G(q)$ is a gradient magnitude computed at $q$, and $I(s)$ is the intensity of a seed $s \in S$, such that $s=\operatorname{org}\left(P^{*}(p)\right)$ in $\delta_{3}$ and $\delta_{4}$ considers all seeds in $S$. The parameters $a$ and $b$ are constants such that $a+b=1$.

Functions $\delta_{1}$ and $\delta_{2}$ assume low inhomogeneity within the object. They represent gradient magnitudes with different image resolutions and lead to smooth functions. In fact, $f_{\text {max }}$ is smooth whenever $\delta(p, q)$ is fixed for any $(p, q) \in A$. The IFT with these functions becomes a watershed transform [12]. Function $\delta_{3}$ exploits the dissimilarity between object and pixel intensities, being the object represented by its seed pixels, but $f_{\text {max }}$ may not be smooth for the general case with multiple seeds [5] (i.e. the IFT results a spanning forest, but it may be non-optimal). This problem was the main motivation for $\delta_{4}$ [17]. However, sometimes $\delta_{3}$ provides better segmentation results than $\delta_{4}$ (see Section 5). Function $\delta_{3}$ may also limit the influence zones of the seeds, when the intensities inside the object vary linearly toward the background. Function $\delta_{5}$ reduces this problem, and $\delta_{3}$ can be replaced by $\delta_{4}$ in Equation 6 to ensure smoothness. Other interesting ideas of dissimilarity functions for $f_{\max }$ are presented in [20, 10, 18, 17].

The basic differences between the formulations proposed in [10] and [17] are that (i) the former assumes $\delta(p, q)=$ $\delta(q, p)$ for all $(p, q) \in A$, and requires smooth path-cost functions, and (ii) the later allows asymmetric dissimilarity relations (e.g. $\delta_{2}$ ), and non-smooth cost functions (e.g. $f_{\max }$ with $\delta_{3}$ and multiple seeds). The strength of connectedness between image pixels in (i) is a symmetric relation, while it may be asymmetric in (ii).

In $[17,10]$, seeds are selected inside and outside the object, and the object is defined by the subset of pixels which are more strongly connected to its internal seeds than to any other. We define the object as the subset of pixels which are more strongly $\kappa$-connected to its internal seeds than to any other. That is, the seeds will compete among themselves for pixels reached from more than one seed by paths whose costs are less than or equal to $\kappa$. In which case, the pixel is conquered by the seed whose path cost is minimum. Note that, even the internal seeds compete among themselves, and a distinct value of $\kappa$ may be required for each seed. When the seed competition fails, these thresholds should limit the influence zones of the seeds avoiding connection between object and background, and the pixels do not conquered by any seed should be considered as belonging to the background.

In general, the use of distinct values of $\kappa$ reduces the number of seeds required to complete segmentation. Figure 2a also illustrates an example where many seeds have to be carefully selected in the background to detect the object. The segmentation fails when some of these seeds are


Figure 2. Segmentation by seed competition of the eye ball in a CT image of the eye orbit. (a) One internal seed and many external seeds are required for segmentation, using $f_{\text {max }}$ with $\delta_{4}$. (b) Segmentation fails when some external seeds are removed. (c) A value of $\kappa$ limits the influence zone of the internal seed where the seed competition fails.
removed (Figure 2b), but it works when we limit the extent of the internal seed to some value of $\kappa$ (Figure 2c).

The algorithms and the problem of determining these thresholds for the internal seeds are addressed next.

## 4. Algorithms

The IFT computes three attributes for each pixel $p \in$ $D_{I}$ [5]: its predecessor $P(p)$ in the optimum path, the cost $C(p)$ of that path, and the corresponding root $R(p)$. In the algorithms presented in this section, we do not need to create the predecessor map $P$. The IFT propagates wavefronts $W_{c s t}$ of same cost $c s t$ around each seed, following the order of the costs $c s t=0,1, \ldots, K$. This process is exploited to compute the values $\kappa_{s}$ of each seed $s \in S$ automatically and interactively.

### 4.1. Automatic computation of $\kappa_{s}$

First consider the wavefronts around a seed $s$ selected inside a given object. All pixels $p$ in the wavefront $W_{c s t}$ around $s$ have optimum cost $C(p)=c s t, 0 \leq c s t \leq K$. If the object is a single $\kappa$-connected component with respect to $s$, then there exists a threshold $\kappa_{s}, 0 \leq \kappa_{s} \leq K$, such that the object can be defined by the union of all wavefronts $W_{c s t}$, for $c s t=0,1, \ldots, \kappa_{s}$. We can specify a fixed $\kappa_{s}$ for this particular application, but this is susceptible to intensity variations. Another alternative is to search for matchings between the shape of the object and the shape of the wavefronts. One drawback is the speed of segmentation, but this may be justified in some applications. A more complex situation occurs when the object definition requires more than one seed pixel. Each seed defines its own maximal extent inside the object and we need to match the shape of the ob-

(a)

(b)

(c)

Figure 3. A CT image of the eye orbit with one seed inside the eye ball. (a) A wavefront of cost $\kappa$ which leads to the maximum extent of this seed inside the eye ball. (b) The wavefront of cost $\kappa+1$ shows a large area growth when it invades the background. (c) The pixel propagation order provides smoother wavefront transitions for interactive selection of $\kappa$.
ject with the shape of the union of their influence zones. The approach presented here is much simpler and yet effective. It stems from the following observation.

The effectiveness of segmentation using $f_{\text {max }}$ depends on assigning lower values of $\delta(p, q)$ to arcs inside (and outside) the objects and higher values to arcs across their boundaries. As consequence, the wavefronts usually present a considerable increase in number of pixels when they cross the object boundaries (Figures 3a and 3b). That is, many pixels in the background are reached by optimum paths whose cost is the value $\delta(p, q)$ of some $\operatorname{arc}(p, q)$ across the boundary.

We substitute the choice of one value $\kappa_{s}$ for each seed $s \in S$ by a threshold $T$ (i.e. a percentage of the total number of pixels divided by the number of internal seeds), which limits the maximum size of their wavefronts. The region growing process of a seed $s$ must stop when the size of its wavefront of cost $c s t$ is greater than $T$, and the value of $\kappa_{s}$ is determined as $\max \{c s t-1,0\}$. The algorithm presented below computes $\kappa_{s}$ for multiple object definition with seed competition.

## Algorithm 1 ObJECT DEFINITION WITH AUTOMATIC $\kappa_{s}$

 DETECTIONInput: $\quad$ Image $\hat{I}=\left(D_{I}, I\right)$, adjacency $A$, size threshold $T$, and a labeled image $\hat{L}=\left(D_{I}, L\right)$, where $L(p)=i$ for seed pixels inside object $0<$ $i \leq k, L(p)=0$ for seeds in the background, $L(p)=-1$ otherwise.
Output: $\quad$ A labeled image $\hat{L}=\left(D_{I}, L\right)$, where $L(p)=i$, $0 \leq i \leq k$.
AUXILIARY: Priority queue $Q$ and maps $C, R, \kappa$, size, and cst defined in $D_{I}$ to store cost and root of each pixel and threshold, wavefront size, and wavefront cost of each seed, respectively.

```
For every pixel \(p \in D_{I}\), do
    \(R(p) \leftarrow p, \operatorname{size}(p) \leftarrow 0, \operatorname{cst}(p) \leftarrow 0, \kappa(p) \leftarrow+\infty\).
        If \(L(p)=-1\), then set \(C(p) \leftarrow+\infty\) and \(L(p) \leftarrow 0\).
        Else, set \(C(p) \leftarrow 0\) and insert \(p\) in \(Q\).
While \(Q \neq \emptyset\), do
        Remove a pixel prom \(Q\) such that \(C(p)\) is minimum.
        If \(\kappa(R(p))=+\infty\) and \(L(R(p)) \neq 0\), then
            If \(C(p) \neq \operatorname{cst}(R(p))\), then
                Set size \((R(p)) \leftarrow 1\) and \(\operatorname{cst}(R(p)) \leftarrow C(p)\).
                L Else, set size \((R(p)) \leftarrow \operatorname{size}(R(p))+1\).
            If size \((R(p))>T\), then
            \(\mathrm{L} \operatorname{Set} \kappa(R(p)) \leftarrow \max \{\operatorname{cst}(R(p))-1,0\}\).
        If \(C(p) \leq \kappa(R(p))\), then
            For every \(q \in A(p)\), such that \(C(q)>C(p)\), do
                Set tmp \(\leftarrow \max \{C(p), \delta(p, q)\}\).
                If \(\operatorname{tmp}<C(q)\), then
                    If \(C(q) \neq+\infty\), then remove \(q\) from \(Q\).
                        Set \(C(q) \leftarrow t m p, R(q) \leftarrow R(p)\).
                    Insert \(q\) in \(Q\).
For every pixel \(p \in D_{I}\), do
    L If \(C(p) \leq \kappa(R(p))\), then set \(L(p) \leftarrow L(R(p))\).
```

The priority queue $Q$ can be implemented as described in $[6,8]$, such that each instance of the IFT will run in time proportional to the number $\left|D_{I}\right|$ of pixels. The root map is used to find in constant time the root of each pixel in $S$. The influence zone of a seed $s \in S$ is limited either when it meets the influence zone of other seed at the same minimum cost or when the value $\kappa_{s}$ of $s$ is found.

### 4.2. Interactive computation of $\kappa_{s}$

A first approach is to initially compute the optimum cost $C(p)$ and root $R(p) \in S$ for each pixel $p \in D_{I}$. Then, the user moves the cursor of the mouse over the image, and for each position $q$ of the cursor, the program displays the influence zone of the corresponding root $s=R(q) \in S$ defined by pixels $p \in D_{I}$, such that $C(p) \leq C(q)$ and $R(p)=R(q)$. This interactive process can be repeated until the user selects a pixel $q$ to confirm the influence zone of $s$ (i.e. $\kappa_{s}=C(q)$ ). The user can repeat this interactive process for each seed $s \in S$.

One drawback of the method above is the abrupt size variations of the wavefronts (Figures 3a and 3b) which makes the selection of pixel $q$ sometimes difficult. We circumvent this problem by exploiting the propagation order $O(p)$ (a number from 1 to $\left|D_{I}\right|$ ) of each pixel $p$ removed from $Q$ during execution of the IFT. Note that, a pixel $p$ propagates before a pixel $q$ (i.e. $O(p)<O(q)$ ) when it is reached by an optimum path from $S$, whose cost $C(p)$ is less than the cost $C(q)$ of the optimum path that reaches $q$. When $C(p)=C(q)$, we assume a first-in-first-out (FIFO) tie-breaking policy for $Q$. That is, among all pixels with the same minimum cost in $Q$, the one first reached by an
optimum path from $S$ is removed for propagation. Therefore, we compute the propagation order $O(p)$ of each pixel $p \in D_{I}$. When the user moves the cursor to a position $q$, the program displays the influence zone of the corresponding root $s=R(q) \in S$ defined by pixels $p \in D_{I}$, such that $O(p) \leq O(q)$ and $R(p)=R(q)$. The rest of the process is the same. Note that, although $\kappa_{s}=C(q)$, only the pixels $p$ in the wavefront $W_{C(q)}$ which have $O(p) \leq O(q)$ are selected as belonging to the influence zone of $s$. This provides smoother transitions between consecutive wavefronts (Figure 3c) as compared to the first idea. The algorithm is presented below.

## Algorithm 2 ObJECT DEFINITION WITH INTERACTIVE $\kappa_{s}$ DETECTION

InPUT: $\quad$ Image $\hat{I}=\left(D_{I}, I\right)$, adjacency $A$, and a labeled image $\hat{L}=\left(D_{I}, L\right)$, where $L(p)=i$ for seed pixels inside object $0<i \leq k, L(p)=0$ for seeds in the background, $L(p)=-1$ otherwise.
Output: A labeled image $\hat{L}=\left(D_{I}, L\right)$, where $L(p)=i$, $0 \leq i \leq k$.
Auxiliary: Priority queue $Q$ and maps $C, R, O$ defined in $D_{I}$ to store cost, root and propagation order of each pixel, respectively.

```
Set ord \(\leftarrow 0\).
For every pixel \(p \in D_{I}\), do
        Set \(R(p) \leftarrow p\).
        If \(L(p)=-1\), then \(\operatorname{set} C(p) \leftarrow+\infty\) and \(L(p) \leftarrow 0\).
        Else, set \(C(p) \leftarrow 0\) and insert \(p\) in \(Q\).
While \(Q \neq \emptyset\), do
        Remove a pixel p from \(Q\) such that \(C(p)\) is minimum.
        Set \(O(p) \leftarrow\) ord +1 and ord \(\leftarrow\) ord +1 .
        For every \(q \in A(p)\), such that \(C(q)>C(p)\), do
            Set \(\operatorname{tmp} \leftarrow \max \{C(p), \delta(p, q)\}\).
            If \(\operatorname{tmp}<C(q)\), then
                If \(C(q) \neq+\infty\), then remove \(q\) from \(Q\).
                Set \(C(q) \leftarrow t m p, R(q) \leftarrow R(p)\).
                Insert \(q\) in \(Q\).
While the user is not satisfied, do
        The user can select a pixel \(q\) on the image.
        For every pixel \(p \in D_{I}\), do
            If \(O(p) \leq O(q)\) and \(R(p)=R(q)\), then
            L Set \(L(p) \leftarrow L(R(p))\).
```

The selection of a pixel $q$ in line 16 is done based on the propagation order as described above.

One advantage of the these algorithms as compared to classical segmentation methods based on seed competition occurs when the object contains several background parts (holes) inside it. In this case, the use of $\kappa_{s}$ usually eliminates the need for at least one background seed at each hole. On the other hand, some small noisy parts of the object may not be conquered by the internal seeds due to the use of $\kappa_{s}$. The labeled image can be post-processed, such that holes

| Object | Description | Modality | \# of Slices |
| :---: | :--- | :--- | :---: |
| O1 | left eye ball | CT-orbit | 15 |
| O2 | left caudate nucleus | MR-brain | 15 |
| O3 | lateral ventricles | MR-brain | 15 |
| O4 | corpus callosum | MR-brain | 10 |
| O5 | patella | CT-knee | 15 |
| O6 | femur | CT-knee | 15 |
| O7 | white matter | MR-brain | 15 |

## Table 1. Description, imaging modality and number of slices for each object used in the experiments.

with area below a threshold are closed [13]. The area closing operator has shown to be a very effective complement for the presented algorithms. In many situations, the objects do not have holes and high area thresholds can be used to reduce the number of internal seeds. These algorithms are compared to the classical segmentation approach based on seed competition in the next section.

## 5. Evaluation

We have selected 100 images from Magnetic Resonance (MR) and Computerized Tomography (CT) data sets of 7 objects for evaluation (see Table 1 and Figure 4). Each object consists of some slices that represent different degrees of challenge for segmentation. The original images have been pre-processed to increase the similarities between pixels inside the objects and the contrast between object and background. Each of four users have performed segmentation over the 100 images using each of two methods:

M1. Object detection with seed competition and automatic $\kappa_{s}$ computation;

M2. Object detection with seed competition and without $\kappa_{s}$ computation.

The experiments aimed to compare these methods with respect to the number of user interactions required to complete segmentation. Although interactive $\kappa_{s}$ detection might reduce the number of external seeds in M1, we decided to avoid it in order to evaluate the combination of seed competition and automatic $\kappa_{s}$ detection with respect to M2.

In order to show the robustness of these approaches, we have chosen the best dissimilarity function for each situation and fixed the parameters of segmentation. We used the 8 -neighborhood as adjacency relation $A$. The size threshold $T$ was set to $1 \%$, except for O 2 where $T=0.2 \%$ in M1. Since objects from O1 to O6 do not have holes, we set the area closing threshold to some arbitrary high value


Figure 4. (a)-(g) Results of slice segmentation of the objects from 1 to 7 , respectively, overlaid with the pre-processed images.
(e.g. 500 pixels). The only exception was O7, whose area threshold could not be higher than 3 pixels due to its holes. Table 2 shows the most suitable dissimilarity function found for each pair of object and method. In function $\delta_{2}$, we used the magnitude of the Sobel's gradient. The value of $\sigma$ was 20 for all cases involving $\delta_{3}$ and $\delta_{4}$. Note that we chose $\delta_{3}$ in some situations, despite $f_{\text {max }}$ not being smooth.

In Medical Image Analysis, it is common to use as ground truth the results of manual segmentation performed by an expert user. This methodology is questionable, because the experts usually make mistakes when they delineate the same object twice. In most cases, the results look like the same but there are small differences along the object boundaries. These small differences, however, seem to

| Object | M1 | M2 |
| :---: | :---: | :---: |
| O1 | $\delta_{2}$ | $\delta_{2}$ |
| O2 | $\delta_{4}$ | $\delta_{2}$ |
| O3 | $\delta_{3}$ | $\delta_{3}$ |
| O4 | $\delta_{4}$ | $\delta_{2}$ |
| O5 | $\delta_{4}$ | $\delta_{4}$ |
| O6 | $\delta_{3}$ | $\delta_{2}$ |
| O7 | $\delta_{3}$ | $\delta_{3}$ |

## Table 2. The dissimilarity functions used for each combination of object and method.

be acceptable in many applications. This similarity measure was defined as follows.

Each object was represented by a set of $l$ binary slices $\hat{L}_{i}=\left(D_{I}, L_{i}\right), i=1,2, \ldots, l$, where $L_{i}(p)=1$ for object pixels and 0 otherwise. Let $\hat{L}_{i}$ and $\hat{L}_{i}^{\prime}$ be the binary images resulting from the segmentation of a same object slice using different methods. The similarity between these results was measured by:

$$
\begin{equation*}
1-\frac{\sum_{i=1}^{i=l} \sum_{\forall p \in D_{I}} L_{i}(p) \oplus L_{i}^{\prime}(p)}{\sum_{i=1}^{i=l} \sum_{\forall p \in D_{I}} L_{i}(p)+\sum_{i=1}^{i=l} \sum_{\forall p \in D_{I}} L_{i}^{\prime}(p)} \tag{7}
\end{equation*}
$$

where $\oplus$ is the "exclusive or" operation. In the case of manual segmentation by an expert user, it has been shown that the similarity values are around 0.96 [7]. Since none of the users is an expert, we required from them acceptable results from the expert's point of view with similarity values around 0.90 between distinct segmentations of a same object, using different methods.

Method M2 represents the classical approach based on relative fuzzy connectedness/watershed transform [17, 12]. The number of user interactions in both methods is the total number of seeds selected inside (NIS - Number of Internal Seeds) and outside (NES - Number of External Seeds) the object. Method M1 is the proposed variant of relative fuzzy connectedness. The number of seeds is expected to be much less with M1 than with M2, due to the automatic $\kappa_{s}$ detection.

Table 3 shows the average number of interactions and similarity values among all users, for both methods. Note that O 3 was detected with a same value of $\kappa$, but the other objects required from $6.8 \%$ to $92.4 \%$ of different $\kappa_{s}$ values. O5 did not count because it was segmented with only one seed per slice. On average, M2 required 2.8 more user interactions than M1.

Table 4 shows in detail the average values of NIS, NES, and AKD for each object and method. Note that the number of automatic $\kappa_{s}$ varied from $59 \%$ to $100 \%$ of NIS ( $88 \%$ on average). This demonstrates the effectiveness of the proposed approach for automatic $\kappa_{s}$ detection and explains the

|  | M1 | M2 | M1,M2 |
| :---: | :---: | :---: | :---: |
| O1 | 29.5 | 77.6 | 0.962 |
| O2 | 29.3 | 38.8 | 0.915 |
| O3 | 31.3 | 61.3 | 0.935 |
| O4 | 27.5 | 46.8 | 0.918 |
| O5 | 15.0 | 61.0 | 0.946 |
| O6 | 26.3 | 37.8 | 0.981 |
| O7 | 46.3 | 284.8 | 0.930 |

Table 3. The average numbers of user interactions for each object and method, and the average similarity values between both methods for a same object.

|  | M1 |  |  | M2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | NIS | NES | AKD | NIS | NES |
| O1 | 18.0 | 11.5 | 13.0 | 26.8 | 50.8 |
| O2 | 25.3 | 4.0 | 24.3 | 18.8 | 20.0 |
| O3 | 30.3 | 1.0 | 30.3 | 30.3 | 31.0 |
| O4 | 22.3 | 5.2 | 19.8 | 22.3 | 24.5 |
| O5 | 15.0 | 0.0 | 15.0 | 44.0 | 17.0 |
| O6 | 26.3 | 0.0 | 15.5 | 22.8 | 15.0 |
| O7 | 46.0 | 0.3 | 46.0 | 66.0 | 218.8 |

Table 4. Average numbers of internal seeds (NIS), external seeds (NES), and automatic $\kappa_{s}$ detections (AKD).
reduction of user interactions and external seeds in M1 with respect to M2. This is an important result for future automation, since seed competition is sensitive to the location of the external seeds due to the heterogeneity of the background.

## 6. Conclusions

We presented two IFT-based algorithms for object detection based on $\kappa$-connected components with seed competition. They differ from the previous approaches in the following aspects: computation of different values of $\kappa$ for each seed, effective automatic $\kappa_{s}$ detection, and userfriendly $\kappa_{s}$ computation, where the user moves the cursor of the mouse to indicate the pixel whose propagation order defines the object. The use of the propagation order rather than the pixel cost is important to create smoother transitions between possible objects, facilitating the user's work. The new methods have considerably reduced the number of user interactions in medical image segmentation with respect to the previous approaches. We believe that these re-
sults are extensive to other image types by suitable choice of pre-processing and dissimilarity function.

We are currently investigating two approaches for 3D segmentation of medical images: (i) automatic segmentation with only internal seeds and automatic $\kappa_{s}$ detection, and (ii) interactive segmentation with automatic $\kappa_{s}$ detection, where the user can add/remove internal and external seeds, and subsequent IFTs are executed in a differential way [2].

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