On Multi-User Gain in MIMO Systems with Rate Constraints

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***Abstract*—In this paper, we study the advantages of multi-user concurrent transmission, measured using the multi-user gain (MUG), in multiple-input multiple-output (MIMO) systems with rate constraints. Our focus is on a maximum eigenmode beamforming (MEB) strategy. We derive a closed-form expression for the transmitted sum power required by the MEB strategy and prove that this strategy is asymptotically optimal. The simple closed-form expression for the MEB sum-power provides many useful insights into the asymptotic behavior of multi-user MIMO systems. Interleave-division multiple-access is adopted as a platform for implementing the proposed MEB scheme. Both numerical and simulation results show that a major part of MUG can be achieved with a quite small number of users.**

Keywords— multiple-input multiple-output (MIMO), multi-user gain (MUG), maximum eigenmode beamforming (MEB)

I. INTRODUCTION

This paper is concerned with the advantages of multi-user concurrent transmission [1], measured using the so-called multi-user gain (MUG) [2][3], in a multiple-input-multiple-output (MIMO) environment. The key problem is sum-power minimization given a fixed rate constraint for every user. This problem is important for delay sensitive services, such as real time video, where scheduling [4][5][6][7] may be difficult due to fairness issues. Several algorithms [8][9] have been proposed to compute the minimum sum power (MSP) solution for this problem. They involve joint optimization on the transmission covariance matrices and decoding order, which becomes very computationally costly even when the number of users, denoted by K below, is only moderately large. The implementation of an optimal multi-user MIMO system is also a challenging issue, involving the feedback of the channel matrix from the receiver(s) and complicated transmitter/receiver design.

In this paper, we focus on a much simpler, sub-optimal strategy, referred to as maximum eigenmode beamforming (MEB), for multi-user systems over a MIMO multiple-access channel (MAC). With MEB, each user transmits information only in the direction of its maximum eigenmode. This strategy reduces system complexity considerably by avoiding joint optimization of covariance matrices and decoding order. The feedback information for each user is also reduced to a vector (instead of a matrix). This strategy has been previously considered in [6][10][11] for throughput maximization in MIMO broadcast channels (BCs).

We show that, although simple, the MEB strategy is asymptotically optimal when K is large. Even for a small K, e.g., 2 or 4, the MEB performance is still quite close to the optimal limit. We derive a closed-form expression for the sum-power of the MEB strategy. The asymptotic optimality and closed-form expression of the MEB sum power provide a fast and reasonably accurate approach to estimate the true MSP for a MIMO system. The MEB strategy can be realized using the interleave-division multiple-access (IDMA) principle [12]. Simulation results show that an impressive amount of MUG is potentially achievable in practice.

Our focus is on MIMO MACs. MIMO BCs will be briefly covered based on the duality principle [13].

II. SYSTEM MODEL

A K-user system over a quasi-static MIMO MAC can be expressed by the following input-output relationship.

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{H}_k$ and $\boldsymbol{x}_k$ are the channel matrix and transmitted signal for user k, respectively, $\boldsymbol{y}$ the received signal at the base station and $\boldsymbol{n}$ a vector of complex additive white Gaussian noise (AWGN) samples with zero mean and unit variance.

For simplicity, we assume that M and N antennas are equipped at the base station and every mobile unit, respectively. $\{\boldsymbol{H}_k\}$ are assumed to be independent and identically distributed (i.i.d.) and perfectly known at the transmitters and the receiver. We further assume that the rate of each user is the same at R/K bits/symbol, where R is referred to as the system *sum-rate*. The corresponding required average (with respect to the distribution of $\{\boldsymbol{H}_k\}$) transmitted MSP is denoted by $P_{N\times M}(K, R)$.

III. MAXIMUM EIGENMODE BEAMFORMING

The sum-power minimization problem for computing $P_{N\times M}(K, R)$ involves joint optimization of the transmission covariance matrices and decoding order for each channel realization [8][9]. The existing methods are highly complex. In this section, we investigate a sub-optimal but very simple MEB strategy for multi-user MIMO MACs. The corresponding MEB sum power provides an upper bound for the true MSP of the MIMO system in (1) and we show that MEB is asymptotically optimal for large K.

In outline, the MEB strategy is as follows:

- Each mobile unit transmits only in its maximum eigenmode direction;
- A simple correlator is used to collect signals from all receive antennas. Successive interference cancellation (SIC) is applied at the receiver. The user with the largest maximum eigenmode is decoded first;
- The power level of each user is optimized based on the above transmitting/receiving operations.

In more detail, for each channel realization we first perform singular value decomposition (SVD) on the channel matrix $\boldsymbol{H}_k$

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of each user k. Denote by $d_{k,\max}$ the maximum singular value of $\boldsymbol{H}_k$. Let $\boldsymbol{u}_{k,\max}$ and $\boldsymbol{v}_{k,\max}$ be the left and right singular vectors corresponding to $d_{k,\max}$, respectively. Let each user k transmit information only in the direction of $\boldsymbol{v}_{k,\max}$, i.e., $\boldsymbol{x}_k = \boldsymbol{v}_{k,\max} \cdot x_k$. Then (1) becomes

$$\boldsymbol{y} = \sum\nolimits_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{v}_{k,\max} x_k + \boldsymbol{n} = \sum\nolimits_{k=1}^{K} d_{k,\max} \boldsymbol{u}_{k,\max} x_k + \boldsymbol{n} . \quad (2)$$

Without loss of generality, we assume that $d_{1,\max} \leq \ldots \leq d_{K,\max}$. Then SIC is applied at the base station with descending decoding order on k.

Clearly, the complexity of MEB is much lower than that of the optimal multi-user MIMO scheme. The latter requires the feedback from the receiver of the channel matrices and involves water-filling and joint power and decoding order optimization. While the MEB strategy only requires the feedback of a vector to steer beamforming at each transmitter.

We now proceed to derive the sum power required by the MEB strategy. When decoding $\boldsymbol{x}_k$, we simply correlate the received signal by $\boldsymbol{u}_{k,\max}$. The signal-to-noise ratio for user k (denoted by SNR_k) at the output of the correlator is

$$SNR_k = \frac{p_k d_{k,\max}^2 \,|\, \boldsymbol{u}_{k,\max}^H \boldsymbol{u}_{k,\max} \,|^2}{1 + \sum_{i=1}^{k-1} p_i d_{i,\max}^2 \,|\, \boldsymbol{u}_{k,\max}^H \boldsymbol{u}_{i,\max} \,|^2} = \frac{p_k d_{k,\max}^2}{1 + \sum_{i=1}^{k-1} p_i d_{i,\max}^2 \phi_{k,i}} \quad (3)$$

where $\phi_{k,i} = |\, \boldsymbol{u}_{k,\max}^H \boldsymbol{u}_{i,\max} \,|^2$ and $p_k = \mathrm{E}[|x_k|^2]$ is the transmitted power of user k. Note that in (3), we assume that the interference from {user i: $i > k$} has been removed by SIC. For each channel realization, $\{p_k\}$ can be computed using (3) recursively according to the channel capacity formula $R/K = \log_2(1+SNR_k)$, $\forall k$.

Denote by $q_k = p_k d_{k,\max}^2$ the received power for user k. Then (3) is rewritten as

$$q_k = \left(2^{R/K} - 1\right)\left(1 + \sum\nolimits_{i=1}^{k-1} q_i \phi_{k,i}\right). \quad (4)$$

The following assumption is the basis for the subsequent discussions. It implies that the the angle distribution of the users is uniform.

Assumption I: $\{\boldsymbol{u}_{k,\max}\}$ are i.i.d., so are $\{d_{k,\max}\}$.

Based on Assumption I, $\phi_{k,i}$ is a random variable with mean $1/M$. Taking averages over $\{\phi_{k,i}\}$ on both sides of (4), we have

$$\mathrm{E}\left(q_k\right) = \left(2^{R/K} - 1\right)\left(1 + \sum\nolimits_{i=1}^{k-1} \mathrm{E}(q_i)/M\right). \quad (5)$$

or in an equivalent non-recursive form,

$$\mathrm{E}\left(q_k\right) = \left(2^{R/K} - 1\right)\left((2^{R/K} - 1)/M + 1\right)^{k-1}. \quad (6)$$

Denote by $f_{N\times M}(\cdot)$ the probability density function (PDF) (and by $F_{N\times M}(\cdot)$ the corresponding cumulative distribution function (CDF)) of $d_{k,\max}^2$ ignoring ordering. From Assumption I, all $\{ d_{k,\max}^2 \}$ have the same PDF $f_{N\times M}(\cdot)$ regardless of k. Now, denote by $f_{N\times M}^{(k:K)}(\cdot)$ the PDF of $d_{k,\max}^2$ (the k-th smallest value) considering ordering. According to order statistics [14], $f_{N\times M}(\cdot)$ and $f_{N\times M}^{(k:K)}(\cdot)$ are related by

$$f_{N\times M}^{(k:K)}(x) = \frac{K!}{(k-1)!(K-k)!} F_{N\times M}^{k-1}(x)(1 - F_{N\times M}(x))^{K-k} f_{N\times M}(x) . \quad (7)$$

The MEB sum power is then calculated as

$$\begin{aligned}
\mathrm{E}\left(\sum\nolimits_{k=1}^{K} p_k\right) &= \sum\nolimits_{k=1}^{K} \mathrm{E}\left(q_k\right) \mathrm{E}\left(1/d_{k,\max}^2\right) \\
&= \sum_{k=1}^{K} \int_0^\infty \left(2^{R/K} - 1\right)\left(\frac{2^{R/K} - 1}{M} + 1\right)^{k-1} \frac{1}{x} f_{N\times M}^{(k:K)}(x) dx \\
&= \sum_{k=1}^{K} \int_0^\infty \left(2^{R/K} - 1\right)\left(\frac{2^{R/K} - 1}{M} + 1\right)^{k-1} \frac{1}{x} f_{N\times M}^{(k:K)}(x) dx \\
&= \sum_{k=1}^{K} \int_0^\infty \left(2^{R/K} - 1\right)\left(\frac{2^{R/K} - 1}{M} + 1\right)^{k-1} \frac{1}{x} \frac{K!}{(k-1)!(K-k)!} \\
&\quad \cdot F_{N\times M}^{k-1}(x)(1 - F_{N\times M}(x))^{K-k} f_{N\times M}(x) dx \\
&= \int_0^\infty \frac{K\left(2^{R/K} - 1\right)}{x} \sum_{k=1}^{K} \frac{(K-1)!}{(k-1)!(K-k)!} (1 - F_{N\times M}(x))^{K-k} \\
&\quad \cdot \left(\left(\left(2^{R/K} - 1\right)/M + 1\right) F_{N\times M}(x)\right)^{k-1} dF_{N\times M}(x) \\
&= \int_0^1 \frac{K\left(2^{R/K} - 1\right)\left(1 + \left(2^{R/K} - 1\right)t/M\right)^{K-1}}{F_{N\times M}^{-1}(t)} dt . \quad (8)
\end{aligned}$$

Since MEB is a particular realization of MIMO systems, (8) is an upper bound for the true MSP of the system in (1).

Theorem 1: Given the target rate of R/K for each user, the sum power required by the MEB strategy is given by

$$\int_0^1 \frac{K(2^{R/K} - 1)\left(1 + (2^{R/K} - 1)t/M\right)^{K-1}}{F_{N\times M}^{-1}(t)} dt , \quad (9a)$$

which serves as an upper bound of the true MSP for the MIMO system in (1), i.e.,

$$P_{N\times M}(K,R) \leq \int_0^1 \frac{K(2^{R/K} - 1)\left(1 + (2^{R/K} - 1)t/M\right)^{K-1}}{F_{N\times M}^{-1}(t)} dt . \quad (9b)$$

In particular, there is only one eigenmode for each user in a single-input single-output (SISO) MAC. In this case, it can be shown that MEB is optimal (this is also true for multiple-input single-output (MISO) MACs), i.e., we can obtain the true MSP for a SISO MAC by setting $M = N = 1$ in (9a).

Corollary 1: The MSP of a SISO MAC is given by

$$P_{1\times 1}(K,R) = \int_0^1 \frac{K(2^{R/K} - 1)\left(1 + (2^{R/K} - 1)t\right)^{K-1}}{F_{1\times 1}^{-1}(t)} dt . \quad (10)$$

IV. MULTI-USER GAIN

It can be shown that, given the target rate of R/K for each user, the MSP of the system in (1) is a monotonically decreasing function of K [3]. Therefore power saving can be achieved by allowing multiple users' to transmit concurrently. This advantage is quantified by the following ratio (referred to as multi-user gain (MUG)):

$$G_{N\times M}(K,R) = \frac{P_{N\times M}(1,R)}{P_{N\times M}(K,R)} . \quad (11)$$

In eqn. (11), the MSP for a single-user system, $P_{N\times M}(1, R)$, can be obtained using SVD and water-filling over all eigenmodes

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[3]. When $K > 1$, we can use the method proposed in [9] to compute $P_{N\times M}(K, R)$.

Fig. 1 shows the MSP of a 2×2 MIMO system where MUG is measured by the difference between the single-user performance and the true MSP. Note that for a multi-user system, the single-user performance can be achieved by TDMA. For comparison, we also show in Fig. 1 the performance achieved by the MEB strategy. The channel condition considered in Fig. 1 is a single-cell environment involving three factors, namely, Rayleigh fading, normalized lognormal fading with $\sigma_s = 8$ and path loss in an edge-length-1 single hexagon cell with uniform user distribution and fourth power path-loss law. We assume independent Rayleigh fading for every transmit-receive antenna link and equal lognormal fading and path loss for all the links seen by a particular user. To avoid deep fading, we allow an outage probability of $P_{out} = \Pr(|h_k|^2 < G_0) = 0.01$, i.e., user k doesn't transmit if its channel gain is below a given threshold G_0.

We can see from Fig. 1 that MUG is significant at high rates. For example, for $R = 8$, allowing 8 users to transmit simultaneously can potentially achieve about 10dB power gain compared with a single-user system. It is also interesting that the difference between the MEB performance and the true MSP is marginal for $R \leq 4$. This indicates that the low-cost MEB approach is nearly optimal if R is not too high.

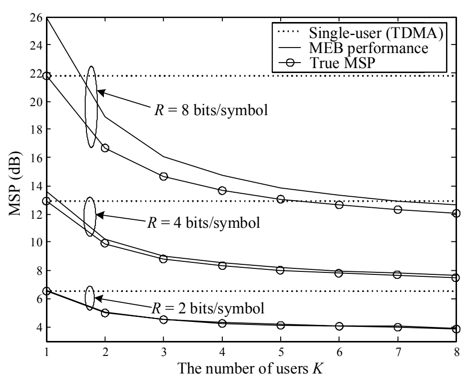

Fig. 1. The MSPs of a 2×2 multiple access system with different K over a single-cell fading channel. $P_{out} = 0.01$.

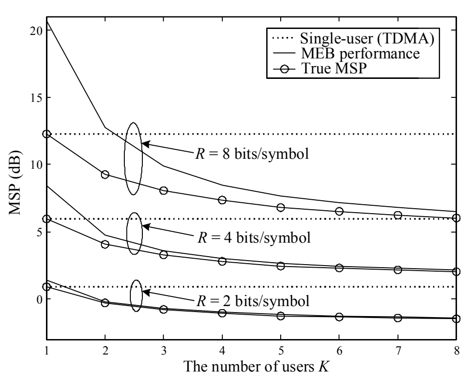

Fig. 2. The MSPs of a 4×4 multiple access system with different K over a single-cell fading channel. $P_{out} = 0.01$.

Fig. 2 shows the MSP and the corresponding MEB performance of a 4×4 MIMO system with different K. The channel condition is the same as that in Fig. 1. Comparing Figs. 1 and 2, we can see that the gap between the MEB performance and the true MSP increases when more antennas are involved and the sum-rate R increases. On the other hand, this gap reduces rapidly as K increases. For example, when $K = 8$, the gap is marginal for both 2×2 and 4×4 schemes.

V. Asymptotic MSP and MUG

It is interesting to examine the limit of such gain when K increases. The existing methods for calculating MSP are highly complicated when K is large. In this section, we derive the limit of the MSP when $K\to\infty$ based on the results of Section III.

A. Asymptotic MSP

We first derive a lower bound for the true MSP of the system in (1). Consider the following multiple access system:

$$\tilde{\boldsymbol{y}} = \sum_{k=1}^{K} d_{k,\max} \boldsymbol{I}_M \cdot \tilde{\boldsymbol{x}}_k + \boldsymbol{n} \tag{12}$$

where $\boldsymbol{I}_M$ is an $M\times M$ identity matrix and $d_{k,\max}$ the maximum singular value of $\boldsymbol{H}_k$. In (12), each user k sees M parallel sub-channels with equal gain, one for each receive antenna. For each channel realization $\{\boldsymbol{H}_k\}$, assume that $\{\boldsymbol{x}_k\}$ achieve the MSP to support the target rate for (1). Construct

$$\tilde{\boldsymbol{x}}_k = \boldsymbol{H}_k / d_{k,\max} \cdot \boldsymbol{x}_k \tag{13}$$

and substitute (13) into (12). Then the same rate profile can be supported in the systems (1) and (12). In this case,

$$\| \tilde{\boldsymbol{x}}_k \|_2 \leq \| \boldsymbol{H}_k \|_2 / d_{k,\max} \cdot \| \boldsymbol{x}_k \|_2 = \| \boldsymbol{x}_k \|_2 \tag{14}$$

where $\|\cdot\|_2$ denotes the 2-norm. Therefore system (12) requires equal or less power to support the same rates as system (1).

System (12) can be viewed as a bank of M identical SISO MACs, each with sum-rate R/M. The corresponding MSP can be achieved with every user transmitting at rate R/MK in every sub-channel. The MSP of each sub-channel can be computed using (10) with R and $F_{1\times1}(t)$ replaced by R/M and $F_{M\times N}(t)$, respectively. (Note: In (10), $F_{1\times1}(t)$ characterizes the distribution of the (only) singular of each user in a SISO channel. While in (12), we artificially set the fading distribution of each user to $F_{M\times N}(t)$ in every sub-channel.) The MSP of the overall system (12) is then M times that of each sub-channel. Hence we obtain a lower bound for the MSP of (1) based on (10) as:

$$P_{N\times M}(K,R) \geq \int_0^1 \frac{MK(2^{R/MK}-1)\left(1+(2^{R/MK}-1)t\right)^{K-1}}{F_{N\times M}^{-1}(t)} dt \,. \tag{15}$$

When $K\to\infty$, (15) becomes,

$$\begin{aligned} P_{N\times M}(\infty,R) &= \lim_{K\to\infty} P_{N\times M}(K,R) \\ &\geq \lim_{K\to\infty} \int_0^1 \frac{MK(2^{R/MK}-1)\left(1+(2^{R/MK}-1)t\right)^{K-1}}{F_{N\times M}^{-1}(t)} dt \\ &= \int_0^1 \frac{R\ln 2 \cdot 2^{Rt/M}}{F_{N\times M}^{-1}(t)} dt \,. \end{aligned} \tag{16}$$

On the other hand, when $K\to\infty$, (9b) becomes

$$P_{N\times M}(\infty,R) = \lim_{K\to\infty} P_{N\times M}(K,R)$$

$$\le \lim_{K\to\infty}\int_0^1 \frac{K(2^{R/K}-1)\left(1+(2^{R/K}-1)t/M\right)^{K-1}}{F_{N\times M}^{-1}(t)}dt$$

$$= \int_0^1 \frac{R\ln 2\cdot 2^{Rt/M}}{F_{N\times M}^{-1}(t)}dt\,. \tag{17}$$

Combining (16) and (17), we have the following.

Theorem 2: When $K\to\infty$, the MSP of the MIMO system in (1) is given by

$$P_{N\times M}(\infty,R)=\lim_{K\to\infty}P_{N\times M}(K,R)=\int_0^1 \frac{R\ln 2\cdot 2^{Rt/M}}{F_{N\times M}^{-1}(t)}dt \tag{18}$$

which is asymptotically achievable by the MEB approach.

B. An Example

Fig. 3 shows the MEB performance with different K and the asymptotic MSPs. We can see from Fig. 3 that the gaps between the MEB performance and the asymptotic limits become marginal at $K = 8$ for both 2×2 and 4×4 systems.

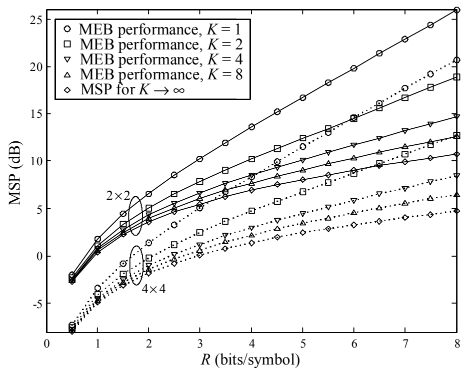

Fig. 3. Comparison between the MEB performance with finite K and the asymptotic MSPs in MIMO systems over a single-cell fading channel. P_{out} = 0.01. The antenna settings $N \times M$ are marked on the curves.

C. Asymptotic Analysis

Some interesting observations can be made from Theorem 2. The first is regarding the asymptotic MSP required for very large R. From (18) we have

$$\lim_{R\to+\infty}\frac{d}{dR}\left(P_{N\times M}(\infty,R)\right)_{\text{dB}}$$

$$=10\log_{10}2\cdot\lim_{R\to+\infty}\frac{d}{dR}\left(\log_2\left(R\ln 2\int_0^1\frac{2^{Rx/M}}{F_{N\times M}^{-1}(x)}dx\right)\right)$$

$$=10\log_{10}2\cdot\lim_{R\to+\infty}\frac{d}{dR}\left(\frac{R}{M}+\log_2\left(\int_0^1\frac{2^{-R(1-x)/M}}{F_{N\times M}^{-1}(x)}dx\right)\right)$$

$$=\frac{10\log_{10}2}{M} \tag{19}$$

where $(A)_{\text{dB}} \equiv 10\log_{10}A$.

Eqn. (19) shows that the asymptotic slope of $(P_{N\times M}(\infty, R))_{\text{dB}}$ depends on M only and is independent of N and the fading distribution. This indicates that the number of antennas at the base station is the most important factor.

Next we examine the impact of M and N. Recall that $F_{N\times M}(\cdot)$ is the CDF of $\{d_{k,\max}^2\}$. Increasing either M or N leads to reduced $F_{N\times M}(x)$ for $\forall x > 0$ (which indicates an increased mean for $d_{k,\max}^2$) and so reduced MSP. Moreover, increasing M has an additional benefit since it also reduces the numerator inside the integral in (18). This can be quantified using (19) as

$$\left(P_{N\times aM}(\infty,aR)\right)_{\text{dB}}\approx\frac{10\log_{10}2}{aM}\cdot aR\approx\left(P_{N\times M}(\infty,R)\right)_{\text{dB}} \tag{20}$$

where a is a positive integer and R is sufficiently large. Eqn. (20) shows that, when R is large, it increases asymptotically linearly with M for a fixed sum power. This observation has interesting implications in cellular systems. Suppose that both the average transmitted sum power and cross-cell interference remain unchanged. Then (20) implies that cellular capacity increases approximately linearly with M. Generally speaking, a cellular system benefits more from increasing the number of antennas at the base station than at each mobile unit.

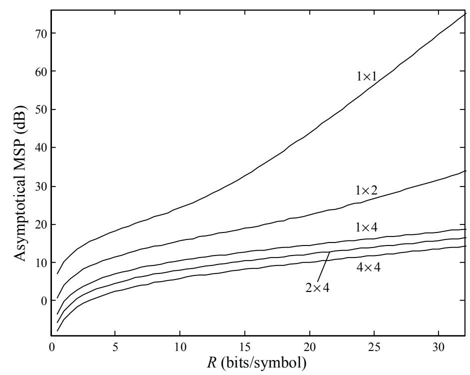

Fig. 4. The asymptotic MSPs of various multiple access systems with different M and N over a single-cell fading channel. P_{out} = 0.01. The antenna settings $N \times M$ are marked on the curves.

The above observations are illustrated using numerical results in Fig. 4. Asymptotic MSPs are plotted for various multiple access systems over a single-cell fading channel. We can clearly see the change of the slope for different M in Fig. 4. Increasing M, i.e., the number of antennas at the base station, has a more significant effect than increasing N.

D. Asymptotic MUG

From (19) above and (20) in [3], we can obtain the asymptotic slope of the MUG in a MIMO system as

$$\lim_{R\to+\infty}\frac{d\left(G_{N\times M}(K,R)\right)_{\text{dB}}}{dR}=\left(\frac{1}{\min(M,N)}-\frac{1}{M}\right)10\log_{10}2\,. \tag{21}$$

We make the following observations from (21):

- When $M \le N$, the asymptotic slope in (21) is zero. When $M > N$ and R is large, the asymptotic MUG increases linearly with R. This again indicates that increasing M has a more significant effect than increasing N.
- The maximum asymptotic slope of the MUG is $10\log_{10}2$ and this is approached by allocating only one antenna at each user side and as many antennas as possible at the base station. This is a single-input-multiple-output (SIMO) situation. This indicates that the advantage of multi-user concurrent transmission is most significant for SIMO channels.

VI. REALIZATION BASED ON IDMA

We demonstrate realization of the MEB strategy using interleave-division multiple-access (IDMA) [12] and illustrate the potential MUG. The channel condition is the same as that used in previous figures. We fix the system sum-rate at 4 bits/symbol and adopt rate-1/2 convolutional coding and length-2 spreading for all users. Multiple coded streams may be assigned to a user based on the principle of superposition coding to achieve a high single-user rate. At the transmitter for user k, the encoded and interleaved signal sequence transmitted over a Gaussian MIMO MAC is multiplied with a proper power control factor and a beamforming vector $\boldsymbol{v}_{k,\max}$. The receiver consists of a MIMO elementary signal estimator (ESE) and a bank of K single-user *a posteriori probability* (APP) decoders (DECs). Extrinsic information for the signals is exchanged between the ESE and DEC parts in a turbo manner. The transmitted power levels for all users are carefully designed using an interior point method [15] for each channel realization. The details regarding detection and power allocation can be found in [15][16].

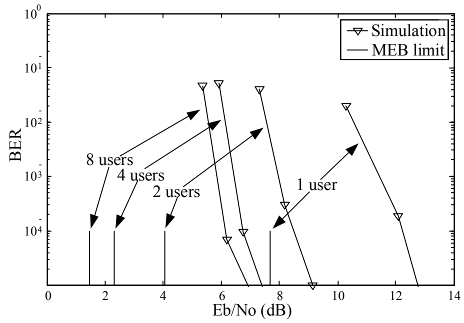

Fig. 5. Simulation results of an MEB-based IDMA system over a 2×2 MIMO MAC. The corresponding MEB limits are plotted for reference.

Fig. 5 shows the simulation results of a convolutionally coded IDMA system over a 2×2 MIMO fading MAC. The theoretical limits achieved by the MEB strategy, cited from Fig. 1, are also plotted for reference. We can see from Fig. 5 that the performance of the IDMA system is only about 5 dB away from MEB limits at BER = 10^{-5}. This performance loss is mainly due to the convolutional codes used. There is about 6 dB MUG between the curves for 1 and 8 users at BER = 10^{-5}, which is in line with the MUG achievable theoretically. It is also interesting to note that a significant portion of the MUG can be achieved with only 2 or 4 users.

VII. MIMO BC

The dual MIMO BC of the system in (1) consists of M antennas at the base station and N antennas at each mobile unit. (In particular, the dual of a SIMO MAC is a MISO BC). The dual MEB strategy for MIMO BCs is similar to that for MIMO MACs, in which the signal for a mobile unit is transmitted in the direction of the maximum eigenmode of the channel between the base station and this mobile unit. Dirty paper encoding and decoding are required at the base station and mobiles respectively. We are currently studying sub-optimal methods that may avoid the use of dirty paper coding.

Based on the duality principle [13], all of the results above, e.g., all the observations related to Theorems 1 and 2 and Figs 1-4, can be directly applied in MIMO BCs. Again, increasing the number of antennas at the base station is a more efficient way to enhance performance than at the mobile units.

VIII. CONCLUSIONS

We have shown that the MEB strategy is asymptotically optimal for large K. It is also nearly optimal even for finite K. Based on this asymptotic optimality and the closed-form expression for the MEB sum power, we have examined the potential MUG in MIMO systems and demonstrated that multi-user concurrent transmission has a significant power advantage over single-user one. IDMA provides an efficient platform to exploit MUG in practice.

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