Wireless MIMO Switching with MMSE Relaying

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***Abstract*— A wireless relay which forms a one-to-one mapping from the inputs (uplinks) to the outputs (downlinks) is called a multiple-input-multiple-output (MIMO) switch. The MIMO switch carries out *precode-and-forward*, where all users send their signals in the uplink and then the MIMO switch precodes the received vector signal for broadcasting in the downlink. Ideally, each user employs a receive filter to recover its desired signal from one other user with no or little interference from other users. We propose a joint design of the precoder and the receive filters to achieve the minimum-mean-square-error (MMSE), assuming full channel state information is available at the relay. Our results indicate that the proposed MMSE relaying scheme outperforms the existing ZF/MMSE schemes.**

***Index Terms*—Beamforming, MIMO switching, MMSE, multi-way relaying network coding, relay.**

I. Introduction

This paper investigates the throughput performance of a wireless MIMO switching network [1], in which a multi-antenna relay helps multiple single-antenna users communicate with one another. We explore two minimum mean square error (MMSE) relaying schemes: MMSE relaying with and without network coding. We show that compared with the existing zero-forcing (ZF)/MMSE relaying schemes, both of our proposed MMSE schemes achieve better MSE and throughput performances.

Two-way relaying has been extensively investigated in recent years (see, e.g., [2]–[4]). By applying physical-layer network coding (PNC) [5], two half-duplex nodes can accomplish bidirectional information exchange in two phases with the help of a half-duplex relay. Much of the current interest is on general multi-way relay serving multiple users. Capacity bounds for the multi-relay channel where the relay has a single antenna has been developed in [6]. So far, the traffic patterns studied include pairwise data exchange [6]–[8], where the users form pairs to exchange data within each pair, and full data exchange, where each user broadcasts to all other users [6], [9]–[11]. In contrast, the traffic pattern studied in this paper is arbitrary unicast, where the mapping from senders to receivers can be an arbitrary permutation, which is more general than pairwise exchanges.

Specifically, we propose and study a precode-and-forward scheme which improves on our previous work [1], [12]. An ordinary zero-forcing relaying with and without network coding was proposed in [12] to maximize the sum rate under a fairness requirement. The ZF scheme eliminates interference at the expense of elevated noise, which degrades the throughput performance, especially for ill-conditioned channels or low SNR communication. The MMSE precode-and-forward scheme strikes a balance between noise and interference. The MMSE precoder and receive filter are studied for single-hop transmission in [13]. The MMSE relaying schemes were investigated in [8], [11], in which MMSE precoders at the relay are used to minimize the sum MSE of all the users. In contrast, in this paper we introduce receive filters at the users and show that joint optimization of the relay precoder and the receive filters can yield significantly better MSE results. In addition, we show how network coding can be introduced into the MMSE system to further improve the MSE and throughput performances. Simulation results show that the network-coded MMSE relaying improves the throughput performance over the existing ZF/MMSE relaying schemes noticeably with moderate complexity increase.

The remainder of the paper is organized as follows: Section II introduces the background of wireless MIMO switching. In Section III, the ordinary and network-coded MMSE relaying schemes are proposed. Section IV presents the simulation results. The paper is concluded by Section V.

II. System Description

Consider K users, numbered $1, \ldots, K$, each with one antenna, as shown in Fig. 1. The users communicate via a relay with N antennas and there is no direct link between any two users. In this paper, we focus on the pure unicast case, in which each user transmits to one other user only. The collection of unicast patterns can be used to realize any general traffic flow pattern (unicast, multicast, broadcast, or a mixture of them) among the users by scheduling a set of different unicast traffic flows.[1]

Each transmission consists of one uplink symbol interval and one downlink symbol interval. In particular, the two symbol intervals are two slots in a time-division system.

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[1]Similarly, in order to accomplish full data exchange, multiple slots are required in [9]–[11].

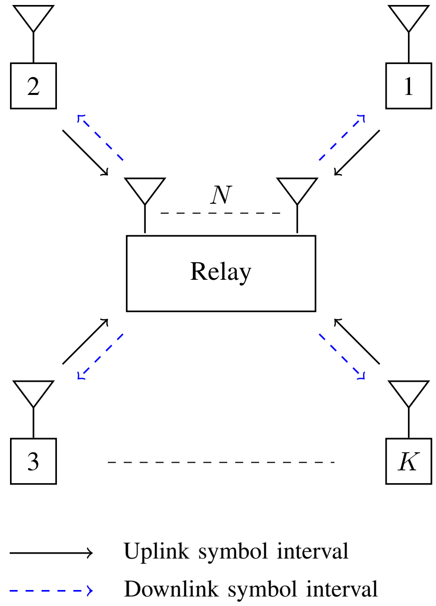

Fig. 1. Wireless MIMO switching.

The uplink symbol interval is for simultaneous uplink transmissions from the users to the relay; the downlink symbol interval is for downlink transmissions from the relay to the users. We assume the two intervals are of equal duration. Each round of uplink and downlink transmission realizes a switching permutation, as described below.

Consider one transmission. Let $\boldsymbol{x} = [x_1, \cdots, x_K]^T$ be the vector representing the signals transmitted by the users. Let $\boldsymbol{y} = [y_1, \cdots, y_N]^T$ be the received signals at the relay, and $\boldsymbol{u} = [u_1, \cdots, u_N]^T$ be the noise vector with independent identically distributed (i.i.d.) noise samples following circularly-symmetric complex Gaussian (CSCG) distribution, i.e., $u_n \sim \mathcal{N}_c(0, \gamma^2)$. Then

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{u}, \tag{1}$$

where $\boldsymbol{H}$ is the uplink channel gain matrix. The relay multiplies $\boldsymbol{y}$ by a precoding matrix $\boldsymbol{G}$ before relaying the signals. In this paper, we assume that the uplink channel and downlink channel are reciprocal, i.e., the downlink channel is $\boldsymbol{H}^T$. Thus, the received signals at the users in vector form are

$$\boldsymbol{r} = \boldsymbol{H}^T\boldsymbol{G}\boldsymbol{y} + \boldsymbol{w} = \boldsymbol{H}^T\boldsymbol{G}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{H}^T\boldsymbol{G}\boldsymbol{u} + \boldsymbol{w}, \tag{2}$$

where $\boldsymbol{w}$ is the noise vector at the receiver, with the i.i.d. noise samples following CSCG distribution, i.e., $w_n \sim \mathcal{N}_c(0, \sigma^2)$. We assume the detector at the users is $\boldsymbol{C}$, where $\boldsymbol{C}$ is diagonal since the users are distributed.

$$\hat{\boldsymbol{r}} = \boldsymbol{C}\boldsymbol{r}. \tag{3}$$

We refer to an $N \times N$ matrix $\boldsymbol{P}$ that has one and only one nonzero element on each row and each column equal to 1 as a *permutation* matrix. Evidently, $\boldsymbol{P}\boldsymbol{x}$ is a column vector consisting of the same elements as $\boldsymbol{x}$ but permuted in a certain order depending on $\boldsymbol{P}$. For example, if

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

then $\boldsymbol{P}\boldsymbol{x} = [x_3, x_1, x_2]^T$. In the case where all diagonal elements of $\boldsymbol{P}$ are zero it is also called a *derangement*. The precoder $\boldsymbol{G}$ realizes a particular permutation represented by the permutation matrix $\boldsymbol{P}$. Note that a symmetric permutation, i.e., $\boldsymbol{P} = \boldsymbol{P}^T$, realizes a pairwise data exchange. By manipulating $\boldsymbol{G}$ and $\boldsymbol{C}$, our purpose is to make $\hat{\boldsymbol{r}}$ close to $\boldsymbol{P}\boldsymbol{x}$ in some sense. The sum rate is written as (4).

$$C = \sum_{i=1}^{N} \frac{1}{2} \log_2 \left(1 + \frac{|\boldsymbol{p}_i^T\boldsymbol{H}^T\boldsymbol{G}\boldsymbol{h}_i|^2}{|\boldsymbol{p}_i^T\boldsymbol{H}^T\boldsymbol{G}\boldsymbol{H}|^2 - |\boldsymbol{p}_i^T\boldsymbol{H}^T\boldsymbol{G}\boldsymbol{h}_i|^2 + \gamma^2|\boldsymbol{p}_i^T\boldsymbol{H}^T\boldsymbol{G}|^2 + \sigma^2}\right). \tag{4}$$

III. MMSE Relaying

The objective of the MMSE relaying scheme is to minimize the sum MSE between the received signals after detection and the transmit signals. In the following, we introduce the design of the detector $\boldsymbol{C}$ and the precoder $\boldsymbol{G}$.

A. Non-PNC MMSE Relaying

For brevity in the context, we let

$$\boldsymbol{M} \triangleq \boldsymbol{H}\boldsymbol{H}^H + \gamma^2\boldsymbol{I}. \tag{5}$$

Given a desired permutation $\boldsymbol{P}$, the optimization problem for determining the optimal precoder and filter is formulated as

$$\min_{\boldsymbol{G},\boldsymbol{C}} \quad \mathbb{E}\|\boldsymbol{P}\boldsymbol{x} - \boldsymbol{C}\boldsymbol{r}\|^2 \tag{6a}$$

$$\text{subject to} \quad \text{Tr}[\boldsymbol{G}\boldsymbol{M}\boldsymbol{G}^H] \le q \tag{6b}$$

$$\boldsymbol{C} \text{ is diagonal}, \tag{6c}$$

where (6b) is the constraint of relaying power consumption. Unfortunately, the problem is nonconvex, making it difficult to solve both $\boldsymbol{G}$ and $\boldsymbol{C}$ directly. However, we notice that for fixed $\boldsymbol{C}$ both the objective and the constraint are quadratic with respect to (w.r.t.) $\boldsymbol{G}$, thus, the problem is a quadratically constrained quadratic program (QCQP). On the other hand, for fixed $\boldsymbol{G}$, the problem can also be rewritten as a QCQP w.r.t. diag$\{\boldsymbol{C}\}$. In this case, we can iteratively optimize $\boldsymbol{G}$ and $\boldsymbol{C}$ until convergence to a local optimum.

The iterative formulas can be determined using the Lagrangian method. First, for fixed $\boldsymbol{C}$, the Lagrangian function is written as

$$J = \mathbb{E}\|\boldsymbol{P}\boldsymbol{x} - \boldsymbol{C}\boldsymbol{r}\|^2 + \lambda\left(\text{Tr}[\boldsymbol{G}\boldsymbol{M}\boldsymbol{G}^H] - q\right), \tag{7}$$

where $\lambda \ge 0$ is the Lagrangian multiplier. Suppose all uplink transmissions are independent and use unit average power, i.e., $\mathbb{E}\{x_i^2\} = 1,\ i = 1, \cdots, K$. Plugging (2) into (7), and then differentiating the cost function w.r.t. $\boldsymbol{G}^*$ to calculate the gradient, we have

$$\frac{\partial J}{\partial \boldsymbol{G}^*} = (\lambda \boldsymbol{I} + \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{C} \boldsymbol{H}^T) \boldsymbol{G} \boldsymbol{M} - \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{P} \boldsymbol{H}^H = 0. \quad (8)$$

Thus, the optimal beamformer can be calculated as

$$\tilde{\boldsymbol{G}}_N = (\lambda \boldsymbol{I} + \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{C} \boldsymbol{H}^T)^{-1} \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{P} \boldsymbol{H}^H \boldsymbol{M}^{-1}. \quad (9)$$

Plugging $\tilde{\boldsymbol{G}}_N$ into (7) and setting the derivative w.r.t. λ to 0, the Lagrangian multiplier can be obtained as $\lambda = \frac{N\sigma^2}{q}$. Thus, the MMSE precoder can be written as

$$\boldsymbol{G}_N = \alpha \left(\frac{N\sigma^2}{q} \boldsymbol{I} + \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{C} \boldsymbol{H}^T \right)^{-1} \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{P} \boldsymbol{H}^H \boldsymbol{M}^{-1}, \quad (10)$$

where α is a scaling factor to guarantee the relay transmits the received signal with its maximal transmit power, i.e.,

$$\alpha = \sqrt{\frac{q}{\mathrm{Tr}\left[\tilde{\boldsymbol{G}}_N \boldsymbol{M} \tilde{\boldsymbol{G}}_N^H \right]}}. \quad (11)$$

Then we consider the solution of $\boldsymbol{C}$ given $\boldsymbol{G}$. Let the column vector $\boldsymbol{c} = \mathrm{diag}\{\boldsymbol{C}\} \in \mathcal{C}^{N\times 1}$. The cost function can be rewritten as

$$J = N - \boldsymbol{c}^H \boldsymbol{s} - \boldsymbol{s}^H \boldsymbol{c} + \boldsymbol{c}^H \boldsymbol{D} \boldsymbol{c} + \lambda \left(\mathrm{Tr}[\boldsymbol{G} \boldsymbol{M} \boldsymbol{G}^H] - q \right), \quad (12)$$

where

$$\boldsymbol{s} = \mathrm{diag}\{\boldsymbol{P} \boldsymbol{H}^H \boldsymbol{G}^H \boldsymbol{H}^*\} \in \mathcal{C}^{N\times 1}, \quad (13)$$

$$\boldsymbol{D} = \boldsymbol{I} \odot (\boldsymbol{H}^T \boldsymbol{G} \boldsymbol{M} \boldsymbol{G}^H \boldsymbol{H}^* + \sigma^2 \boldsymbol{I}) \in \mathcal{C}^{N\times N}, \quad (14)$$

where $\odot$ denotes element-by-element multiplication, i.e., the Hadamard product. We calculate the gradient w.r.t. $\boldsymbol{c}$, and the optimal solution is then

$$\boldsymbol{C} = \mathrm{diag}\{\boldsymbol{D}^{-1} \boldsymbol{s}\} \in \mathcal{C}^{N\times N}. \quad (15)$$

The details are as outlined by the following pseudo-code.

```
1: init: C = C_ZF, G = G_ZF;
2: while the MSE can be improved by more than ε do
3:    Calculate G according to (10);
4:    Calculate C according to (15);
5: end while
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Remark 1: Since the iterative algorithm can not guarantee the global optimum, the trick is using the ZF result [1], [12] to initialize $\boldsymbol{G}$. Thus, the MMSE scheme could achieve better MSE and throughput performances than the ZF scheme.[2]

B. Network-coded MMSE Relaying

For the network-coded MMSE relaying, we permit the self-interference to exist at the side of receivers. Then the self-interference can be canceled from the denominator of the SNR in the sum rate (4). As we proposed earlier in [12], we define a diagonal matrix $\boldsymbol{B}$ to denote the weights of self-interference. We will show that by proper design of $\boldsymbol{B}$, the throughput performance outperforms that of ordinary MMSE relaying in which $\boldsymbol{B} = 0$.

With network coding, we could cancel the self information from $\boldsymbol{C}\boldsymbol{r}$ (i.e., $\boldsymbol{C}\boldsymbol{r} - \boldsymbol{B}\boldsymbol{x}$) at the receivers. The square error is then $\|\boldsymbol{P}\boldsymbol{x} - (\boldsymbol{C}\boldsymbol{r} - \boldsymbol{B}\boldsymbol{x})\|^2$. The optimization problem can be written as

$$\min_{\boldsymbol{G},\boldsymbol{B},\boldsymbol{C}} \quad \mathbb{E}\|(\boldsymbol{P}+\boldsymbol{B})\boldsymbol{x} - \boldsymbol{C}\boldsymbol{r}\|^2 \quad (16a)$$

$$\text{subject to} \quad \mathrm{Tr}[\boldsymbol{G}\boldsymbol{M}\boldsymbol{G}^H] \le q \quad (16b)$$

$$\boldsymbol{B} \text{ and } \boldsymbol{C} \text{ are diagonal.} \quad (16c)$$

The problem is nonconvex. However, for fixed $\boldsymbol{B}$ and $\boldsymbol{C}$, it is a QCQP w.r.t. $\boldsymbol{G}$; for fixed $\boldsymbol{G}$, it is a QCQP w.r.t. $(\boldsymbol{B}, \boldsymbol{C})$. Thus, we can solve this problem in the same iterative algorithm for solving (6).

For fixed $\boldsymbol{B}$ and $\boldsymbol{C}$, the Lagrangian function is written as

$$J = \mathbb{E}\|(\boldsymbol{P}+\boldsymbol{B})\boldsymbol{x} - \boldsymbol{C}\boldsymbol{r}\|^2 + \lambda \left(\mathrm{Tr}[\boldsymbol{G}\boldsymbol{M}\boldsymbol{G}^H] - q \right). \quad (17)$$

In order to calculate the gradient, we differentiate the cost function w.r.t. $\boldsymbol{G}^*$, and the optimal beamformer can be calculated as

$$\tilde{\boldsymbol{G}}_P = (\lambda \boldsymbol{I} + \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{C} \boldsymbol{H}^T)^{-1} \boldsymbol{H}^* \boldsymbol{C}^H (\boldsymbol{P}+\boldsymbol{B}) \boldsymbol{H}^H \boldsymbol{M}^{-1}. \quad (18)$$

The result is similar to the MMSE precoder in (9). Hence, given $\boldsymbol{B}$ and $\boldsymbol{C}$ we can find the solution of the Lagrangian multiplier $\lambda = \frac{N\sigma^2}{q}$ in the same way. Thus, the MMSE-PNC precoder is written as

$$\begin{aligned} \boldsymbol{G}_P = &\beta \left(\frac{N\sigma^2}{q} \boldsymbol{I} + \boldsymbol{H}^* \boldsymbol{C}^H \boldsymbol{C} \boldsymbol{H}^T \right)^{-1} \\ &\boldsymbol{H}^* \boldsymbol{C}^H (\boldsymbol{P}+\boldsymbol{B}) \boldsymbol{H}^H \boldsymbol{M}^{-1}, \end{aligned} \quad (19)$$

where β is a scaling factor to guarantee the relay transmits the received signal with its maximal transmit power, i.e.,

$$\beta = \sqrt{\frac{q}{\mathrm{Tr}\left[\tilde{\boldsymbol{G}}_P \boldsymbol{M} \tilde{\boldsymbol{G}}_P^H \right]}}. \quad (20)$$

For fixed $\boldsymbol{G}$, let the column vectors $\boldsymbol{b} = \mathrm{diag}\{\boldsymbol{B}\} \in \mathcal{C}^{N\times 1}$, $\boldsymbol{c} = \mathrm{diag}\{\boldsymbol{C}\} \in \mathcal{C}^{N\times 1}$. The cost function can be rewritten as

$$\begin{aligned} J =& N + \boldsymbol{b}^H \boldsymbol{b} - \boldsymbol{c}^H \boldsymbol{s} - \boldsymbol{c}^H \boldsymbol{D}_1 \boldsymbol{b} - \boldsymbol{s}^H \boldsymbol{c} - \boldsymbol{b}^H \boldsymbol{D}_1^H \boldsymbol{c} \\ &+ \boldsymbol{c}^H \boldsymbol{D}_2 \boldsymbol{c} + \lambda \left(\mathrm{Tr}[\boldsymbol{G}\boldsymbol{M}\boldsymbol{G}^H] - q \right) \\ =& (\boldsymbol{D}_3 \boldsymbol{c} - \boldsymbol{D}_3^{-1} \boldsymbol{s})^H (\boldsymbol{D}_3 \boldsymbol{c} - \boldsymbol{D}_3^{-1} \boldsymbol{s}) - \boldsymbol{s}^H \boldsymbol{D}_3^{-2} \boldsymbol{s} + N \\ &+ (\boldsymbol{b} - \boldsymbol{D}_1^H \boldsymbol{c})^H (\boldsymbol{b} - \boldsymbol{D}_1^H \boldsymbol{c}) + \lambda \left(\mathrm{Tr}[\boldsymbol{G}\boldsymbol{M}\boldsymbol{G}^H] - q \right). \end{aligned} \quad (21)$$

where

$$\boldsymbol{s} = \mathrm{diag}\{\boldsymbol{P} \boldsymbol{H}^H \boldsymbol{G}^H \boldsymbol{H}^*\} \in \mathcal{C}^{N\times 1}, \quad (22)$$

$$\boldsymbol{D}_1 = \boldsymbol{I} \odot (\boldsymbol{H}^H \boldsymbol{G}^H \boldsymbol{H}^*) \in \mathcal{C}^{N\times N}, \quad (23)$$

$$\boldsymbol{D}_2 = \boldsymbol{I} \odot (\boldsymbol{H}^T \boldsymbol{G} \boldsymbol{M} \boldsymbol{G}^H \boldsymbol{H}^* + \sigma^2 \boldsymbol{I}) \in \mathcal{C}^{N\times N}, \quad (24)$$

$$\boldsymbol{D}_3 = (\boldsymbol{D}_2 - \boldsymbol{D}_1 \boldsymbol{D}_1^H)^{\frac{1}{2}} \in \mathcal{C}^{N\times N}. \quad (25)$$

According to (21), given $\boldsymbol{G}$ the MSE is minimized by

$$\boldsymbol{C} = \mathrm{diag}\{\boldsymbol{D}_3^{-2} \boldsymbol{s}\}, \boldsymbol{B} = \mathrm{diag}\{\boldsymbol{D}_1^H \boldsymbol{c}\} = \mathrm{diag}\{\boldsymbol{D}_1^H \boldsymbol{D}_3^{-2} \boldsymbol{s}\}. \quad (26)$$

[2]As mentioned in [1], [12], our earlier proposed ZF scheme also outperforms the traditional ZF scheme since we optimize the power allocation over the relay antennas instead of using equal power at each antenna [11].

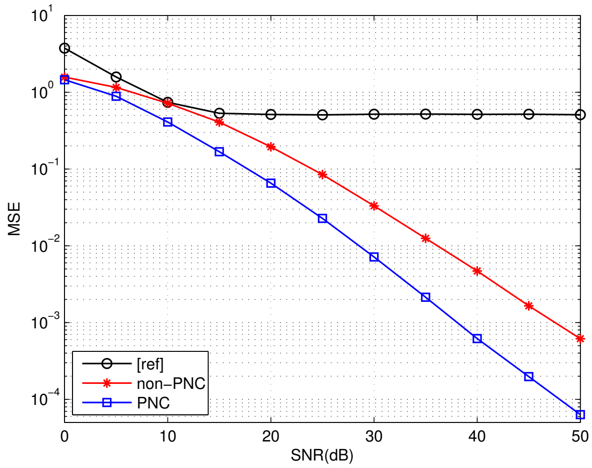

Fig. 2. MSE of different MMSE relaying schemes when $K = N = 2$.

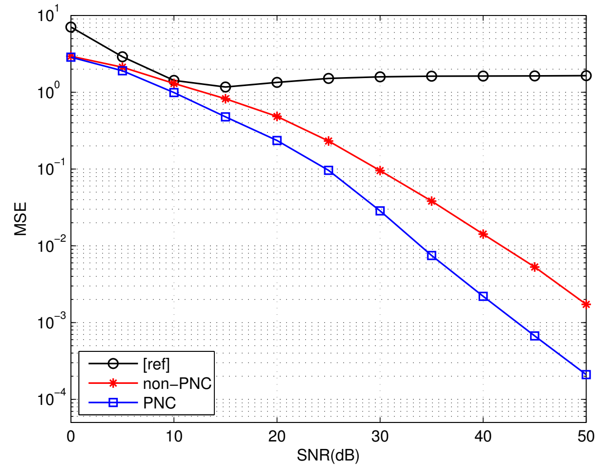

Fig. 4. MSE of different MMSE relaying schemes when $K = N = 4$.

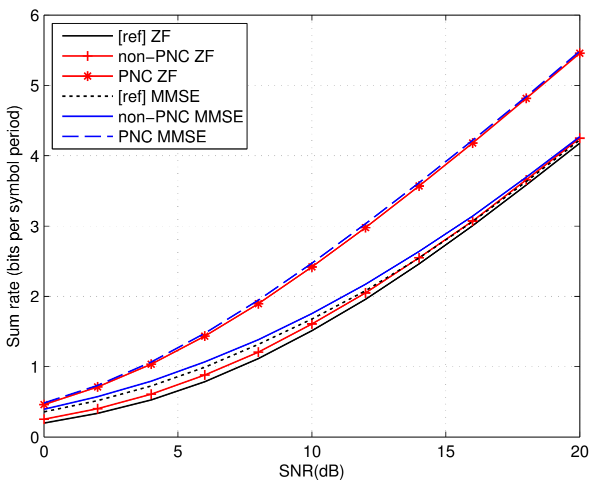

Fig. 3. Throughput comparison of different ZF and MMSE relaying schemes when $K = N = 2$.

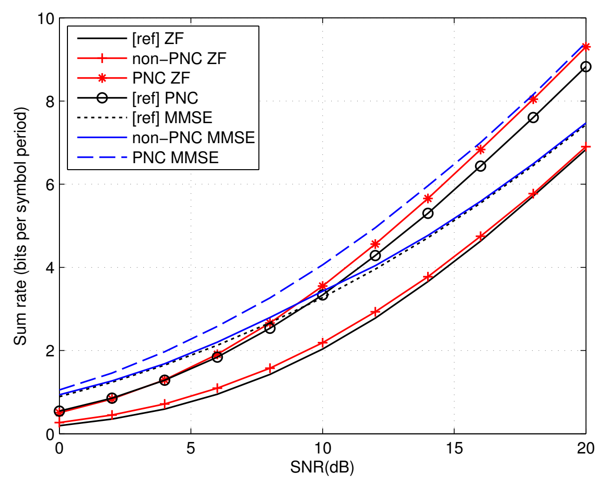

Fig. 5. Throughput comparison of different ZF and MMSE relaying schemes when $K = N = 4$.

The details are as outlined by the following pseudo-code.

Init: $\boldsymbol{C} = \boldsymbol{C}_{\text{ZF-PNC}}$, $\boldsymbol{G} = \boldsymbol{G}_{\text{ZF-PNC}}$;
while the MSE can be reduced by more than ϵ **do**
  Calculate $\boldsymbol{G}$ according to (19);
  Calculate $\boldsymbol{C}$ and $\boldsymbol{B}$ according to (26);
end while

Lemma 1: The MSE of both the non-PNC and the PNC schemes converge as the number of iterations increases.

Proof: For the non-PNC scheme, given $\boldsymbol{C}$ the MSE function is convex w.r.t. $\boldsymbol{G}$. If we use the Lagrangian method, we can minimize MSE w.r.t. $\boldsymbol{G}$. Furthermore, given $\boldsymbol{G}$ the MSE function is convex w.r.t. $\boldsymbol{C}$. We can also use the Lagrangian method to minimize MSE w.r.t. $\boldsymbol{C}$. As a results, the MSE keeps decreasing until some local optimum. The convergence of the PNC scheme can be proved similarly. ■

IV. Numerical Results

In this section, we evaluate the MSE and the throughputs of the proposed MMSE relaying schemes. We assume the maximum transmit power of the relay is the same as that of each user, i.e., $q = 1$, and the noise levels at the relay and the users are the same. Our simulations yield essentially the same general results for different symmetric (resp. asymmetric) permutations, i.e., pairwise (resp. non-pairwise) information exchange. Thus, we just pick one symmetric permutation $\boldsymbol{P} = [\boldsymbol{e}_2, \boldsymbol{e}_1, \boldsymbol{e}_4, \boldsymbol{e}_3]$ and present three results here:

Observation 1: The proposed MMSE relaying schemes have better MSE performance than the MMSE schemes in [8], [11].

The MSE performance is evaluated in Fig. 2 and Fig. 4. The sum MSE of the MMSE scheme [8], [11] saturates at high SNR, since they optimize the precoders at the relay only. In contrast, we jointly optimize both the precoder at the relay and the detectors at the users, thus our MSE keeps decreasing. Importantly, the MSE can be further decreased by network coding. Comparing the MSE of different number of users, we see that the sum MSE increases as the number of users grows.

Observation 2: The proposed network-coded MMSE relaying scheme generally has better throughput performance than the relaying schemes in [1], [8], [11], [12].

We compare our proposed relaying schemes with the ZF schemes in [1], [8], [11], [12], the MMSE scheme in [8], [11] and the network-coded schemes in [11], [12]. The throughput performance is evaluated in Fig. 3 and Fig. 5. For non-PNC schemes, the proposed non-PNC MMSE scheme has similar throughput performance as the MMSE scheme in [8], [11], and they both outperform the non-PNC ZF schemes in [1], [11]. The effect of the receive filter $\boldsymbol{C}$ in throughput performance is not obvious. By joint optimization with $\boldsymbol{C}$, the optimal $\boldsymbol{B}$ and $\boldsymbol{G}$ are different from that in the case without $\boldsymbol{C}$. The gain is shown in Fig. 3 and Fig. 5.

The throughput performance can be further improved by network coding. In particular, our proposed network-coded MMSE scheme outperforms our prior network-coded ZF relaying scheme as well as the network-coded scheme in [11]. Summing up the gains induced by MMSE and network coding, the network-coded MMSE relaying scheme can achieve around 5 dB gain over the traditional ZF/MMSE schemes. In addition, when the number of users grows, the sum rate increases; however, the average rate decreases. Note that the gap between network-coded ZF and MMSE schemes is trivial when $N = 2$. The reason is that the loss due to zero-forcing disappears after adding $\boldsymbol{B}$ when $N = 2$, i.e., there is no zero element in $\boldsymbol{P}+\boldsymbol{B}$. However, when $N > 2$ the MMSE scheme achieves larger gain over the ZF scheme since the latter forces some elements in $\boldsymbol{P}+\boldsymbol{B}$ to be zero.

Observation 3: The complexities of the network-coded schemes are close to the non-PNC schemes. The MMSE schemes have higher complexity than the ZF schemes.

As mentioned in Remark 1, we initialize the precoders with the ZF results proposed in [1], [12]. The iteration stops when the throughput improvement is smaller than 10^{-3}. To evaluate the complexity of our proposed schemes, we first evaluate the numbers of iterations required for our proposed MMSE schemes to achieve convergence. Fig. 6 indicates that with the ZF result initialization, the number of iterations decreases as SNR increases, or as the number of users decreases. The network-coded schemes have faster convergence, but little additional complexity in terms of the number of multiplications needed, as shown in Fig. 6. The comparison results of the numbers of multiplications and additions are similar, thus we show the figure of multiplication only.

We remark that for implementation, the optimization computation itself can be performed solely at the relay, with the computed detectors' gains $\boldsymbol{C}$ and the self-interference weights $\boldsymbol{B}$ conveyed to the users. Since both $\boldsymbol{C}$ and $\boldsymbol{B}$ are diagonal, the amounts of data to be conveyed are not large.

V. Conclusion

In this paper, we have proposed two MMSE relaying schemes in the wireless MIMO switching, i.e., a general multi-way relay channel. The schemes, especially the network-coded scheme has substantially better MSE and throughput performances than the existing ZF/MMSE relaying schemes.

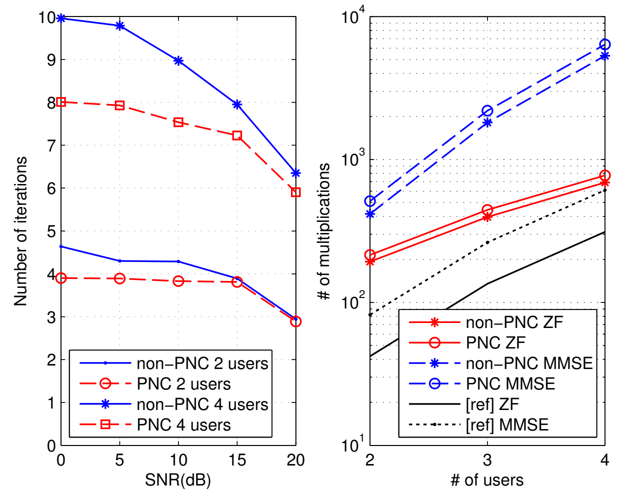

Fig. 6. Computational complexity comparison with different numbers of users.

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