RANDOMIZED SPACE-TIME BLOCK CODING FOR DISTRIBUTED AMPLIFY-AND-FORWARD COOPERATIVE RELAYS

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ABSTRACT

Cooperation diversity schemes employing space-time block coding (STBC) techniques have been proposed for wireless networks to increase network capacity and coverage even when each node is equipped with a single antenna. Such schemes allow several relay stations distributed in space to assist the transmission between a given source-destination pair. A key design problem in cooperative networks is to take advantage from spatial diversity by reducing the amount of signaling and processing overhead as far as possible. In this paper, capitalizing on randomized STBC (RSTBC), a coding method which has been recently developed for decode-and-forward (D&F) relay nodes, a totally decentralized cooperative communication scheme is proposed for amplify-and-forward (A&F) relays, where each relay is unaware of both the effective STBC being employed by the other nodes and the number of cooperating stations. Numerical results are provided to highlight the effectiveness of the proposed scheme in comparison to its D&F counterpart.

Index Terms— Amplify-and-forward relaying, cooperative wireless networks, relay design, space-time randomized coding.

1. INTRODUCTION

Multiantenna techniques offer significant improvements in link reliability through the use of multiple antennas at the transmitter and/or receiver side, without involving system losses in terms of delay and bandwidth efficiency. However, due to space and power constraints, the use of multiple antennas might not be feasible at mobile stations in cellular systems, as well as in *ad hoc* or multihop wireless networks. To overcome such practical limitations and not to renounce the performance enhancement introduced by multiantenna approaches, a viable strategy consists of exploiting cooperative diversity arising from the presence of multiple terminals distributed in space [1], which may serve as relay stations. *Decode-and-forward (D&F)* and *Amplify-and-forward (A&F)* relaying are popular cooperation protocols: in the former one, the relay node forwards the source symbols if it has correctly decoded; in the latter one, the relay node amplifies the received signal and retransmits it to the destination. One possible approach for involving more than one cooperative relay without a significant loss in spectral efficiency is to use *space-time block coding (STBC)* among the relays [1].

The use of conventional STBC rules in a distributed fashion is a challenging design problem. The main impediment stems from the fact that, to provide diversity and coding gains, coordination among the source, the destination and the relays is strictly needed before using a specific STBC, which is typically designed for a fixed number of transmit antennas. Unfortunately, due to the distributed and *ad hoc* nature of cooperative links, the number of virtual antennas (i.e., cooperating nodes) is generally unknown and time-varying. With reference to D&F relaying, *randomized STBC (RSTBC)* was proposed in [2], which does not require a preliminary coordination among the participants in the cooperative communication. Such a strategy allows achieving diversity order equal to the minimum between the number of cooperating nodes N and the number of antennas L assumed in the space-time code structure, irrespective of the number of relays; moreover, under the maximum likelihood (ML) detection rule, it approaches the performance of centralized STBC D&F-based schemes [1], both in diversity and coding gain.

In general, D&F cooperation protocols provides performance gains under the assumption that each relay node can decide whether it has correctly decoded or not. However, such an assumption imposes practical limitations on the D&F cooperative systems. Moreover, if the assumption of correct decision at the relay nodes is not met exactly, because of the errors at the relay nodes, error propagation from the relays to the destination [1] would highly degrade system bit-error-rate (BER) performance. In this paper, we focus on A&F relaying. Since this operation mode requires no decoding at relay nodes, it involves a less processing burden with respect to D&F relaying and, thus, it is well-suited to systems with simple relay units such as wireless sensor networks and practical *ad hoc* or multihop wireless networks. In [3], A&F cooperation protocols using full-rate linear dispersion space-time codes were analyzed and it is shown that, for very large values of the total transmitted power, the system approximatively achieves diversity order equal to the minimum between N and the block length K and, additionally, at low and high signal-to-noise ratio (SNR), maximum coding gain. However, the work of [3] did not account for code design criteria of the space-time codes in a decentralized fashion. In this paper, relying on the idea of randomized coding, we propose a distributed A&F cooperation protocol which is fully decentralized and admits single-symbol ML decoding. Numerical simulation results show that the proposed diversity scheme performs comparably to its centralized counterpart and outperforms the D&F-based RSTBC scheme of [2], even in the high SNR region where errors at the relay nodes are rare.

2. SYSTEM MODEL

The considered wireless network is composed of N randomly and independently placed relay nodes, one source station (S) and one destination terminal (D), each one employing a single transmit/receive antenna. The channel between each node pair is assumed frequency non-selective and quasi-stationary, i.e., it is characterized by a single fading coefficient that remains constant within one frame

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of data symbols but may vary from frame to frame. Specifically, for $i \in \{1, 2, \ldots, N\}$, let f_i and η_i denote the channel gain and the distance, respectively, between S and the ith relay, whereas g_i and μ_i denote the channel gain and the distance, respectively, between the ith relay and D; the coherence time of f_i is larger than K symbol intervals, whereas the coherence time of g_i is larger than P symbol intervals.[1] The channel vectors $\mathbf{f} \triangleq [f_1, f_2, \ldots, f_N]^T \in \mathbb{C}^N$ and $\mathbf{g} \triangleq [g_1, g_2, \ldots, g_N]^T \in \mathbb{C}^N$ are statistically independent of each other and are modeled as follows: $\mathbf{f} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{\Sigma_f})$, with $\mathbf{\Sigma_f} \triangleq \mathrm{diag}(\sigma^2_{f_1}, \sigma^2_{f_2}, \ldots, \sigma^2_{f_N}) \in \mathbb{R}^{N\times N}$, and $\mathbf{g} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{\Sigma_g})$, with $\mathbf{\Sigma_g} \triangleq \mathrm{diag}(\sigma^2_{g_1}, \sigma^2_{g_2}, \ldots, \sigma^2_{g_N}) \in \mathbb{R}^{N\times N}$, whereas we define $\sigma^2_{f_i} \triangleq \eta_i^{-\rho}$ and $\sigma^2_{g_i} \triangleq \mu_i^{-\rho}$, where ρ is the path-loss exponent. Following the related literature, e.g., [1, 2], perfect synchronization is assumed at the symbol level among all the terminals.

Let S send the block $\mathbf{a} \triangleq [a_1, a_2, \ldots, a_K]^T \in \mathbb{C}^K$ composed of independent and identically distributed (i.i.d.) zero-mean symbols having variance σ_a^2, with the aid of the N relays. The random vector $\mathbf{a}$ assumes equiprobable values in $\mathcal{A} \triangleq \{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \ldots, \boldsymbol{\alpha}_Q\}$. The cooperative transmission takes place in two phases. In Phase I, which spans a time interval of K consecutive symbol periods, S broadcasts to all the potential relays the vector $\mathbf{a}$ and, consequently, a block of K consecutive samples of the discrete-time baseband equivalent received signal at the ith relay can be expressed as $\mathbf{y}_i = f_i\,\mathbf{a} + \mathbf{n}_i$, for $i \in \{1, 2, \ldots, N\}$, where $\mathbf{n}_i \sim \mathcal{CN}(\mathbf{0}_K, \sigma_i^2\,\mathbf{I}_K)$ denotes additive white Gaussian noise (AWGN), which is statistically independent of $\mathbf{a}$, $\mathbf{f}$ and $\mathbf{g}$, with $\mathbf{n}_{i_1}$ and $\mathbf{n}_{i_2}$ statistically independent of each other for $i_1 \neq i_2$. Observe that in this framework the power constraint of the source transmission is $P_\mathrm{s} \triangleq \mathrm{E}(\|\mathbf{a}\|^2) = K\,\sigma_a^2$. In Phase II, all the N relays simultaneously transmit in the same frequency band a block of data containing the information of $\mathbf{a}$. Let $\mathbf{x}_i \in \mathbb{C}^P$ denote the block of $P \geq K$ data transmitted by relay i, the baseband equivalent discrete-time signal received at D assumes the form[2] $\mathbf{y}_\mathrm{d} = \mathbf{X}\,\mathbf{g} + \mathbf{n}_\mathrm{d}$, where $\mathbf{X} \triangleq [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N] \in \mathbb{C}^{P\times N}$ and $\mathbf{n}_\mathrm{d} \sim \mathcal{CN}(\mathbf{0}_P, \sigma_\mathrm{d}^2\,\mathbf{I}_P)$ denotes AWGN, which is statistically independent of $\mathbf{a}, \mathbf{f}, \mathbf{g}$ and $\{\mathbf{n}_i\}_{i=1}^N$. Note that the code rate turns out to be K/P and the transmission time for Phase II is equal to P symbols intervals. Finally, assuming coherent detection at the destination, the block $\mathbf{y}_\mathrm{d}$ received in Phase II is used by D to produce the optimal (in the minimum SEP sense) ML estimate $\widehat{\mathbf{a}}_\mathrm{opt}$ of $\mathbf{a}$.

3. PROPOSED COOPERATIVE DIVERSITY SCHEME

In the sequel, we assume that the ith relay has perfect knowledge of the fading coefficient f_i previously obtained via training. Herein, we describe our proposed randomized coding rule, by explicating the structure of the aforementioned matrix $\mathbf{X}$. Let us focus on the processing carried out at the ith relay, with $i \in \{1, 2, \ldots, N\}$. The relay undertakes three actions. As a first step, the received signal $\mathbf{y}_i$ is scaled by the factor $\lambda_i > 0$ and multiplied by f_i^{-1}, thus obtaining the new data block

$$\mathbf{z}_i = \lambda_i\, f_i^{-1}\,\mathbf{y}_i = \lambda_i\,\mathbf{a} + \lambda_i\,\overbrace{\mathbf{n}_i\, f_i^{-1}}^{\widetilde{\mathbf{n}}_i \in \mathbb{C}^K} = \lambda_i\,\mathbf{a} + \lambda_i\,\widetilde{\mathbf{n}}_i\,, \qquad (3.1)$$

where $\widetilde{\mathbf{n}}_i \sim \mathcal{CN}(\mathbf{0}_K, |f_i^{-1}|^2\,\sigma_i^2\,\mathbf{I}_K)$. Since we have assumed that there is complete CSI at the relays, an appropriate constraint [1] is to ensure that a given transmitted power is maintained, that is,

$$\lambda_i = \sqrt{\frac{P_i}{\sigma_a^2 + |f_i^{-1}|^2\,\sigma_i^2}}\,, \qquad (3.2)$$

which keeps the power constraint $\mathrm{E}(\|\mathbf{z}_i\|^2) = K\,P_i > 0$. In the second step, as done in standard space-time coding [4], the vector $\mathbf{z}_i$ is mapped onto a space-time code matrix $\boldsymbol{\mathcal{C}}(\mathbf{z}_i) \in \mathbb{C}^{P\times L}$, where P is the block length and L denotes the number of antennas in the underlying space-time code. The impact of L on the system performance will become clear in Section 4; for the moment, we underline only that there is *no* relationship between L and the number of cooperating relays N. The STBC is distributed among the relays such that each node virtually acts as a single antenna in a multiple antennas transmitter, by transmitting a random linear combination of the columns of $\boldsymbol{\mathcal{C}}(\mathbf{z}_i)$. Therefore, as a last step, let $\mathbf{r}_i \in \mathbb{C}^L$ be a random vector containing the linear combination coefficients for the ith node, with $\mathrm{E}(\|\mathbf{r}_i\|^2) = 1$, the code $\mathbf{x}_i \in \mathbb{C}^P$ transmitted by the ith relay is given by $\mathbf{x}_i = \boldsymbol{\mathcal{C}}(\mathbf{z}_i)\,\mathbf{r}_i$. For the time being, we do not make any specific assumption on the statistical model of the *randomization matrix* $\boldsymbol{\mathcal{R}} \triangleq [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N] \in \mathbb{C}^{L\times N}$, which collects the randomization vectors used by all the relays; we highlight only that the proposed coding rule is completely decentralized since the ith relay chooses $\mathbf{r}_i$ locally from a given distribution, which does not depend on the node index i. It is noteworthy that, apart from $\boldsymbol{\mathcal{R}}$, the matrix $\mathbf{X} = [\boldsymbol{\mathcal{C}}(\mathbf{z}_1)\,\mathbf{r}_1, \boldsymbol{\mathcal{C}}(\mathbf{z}_2)\,\mathbf{r}_2, \ldots, \boldsymbol{\mathcal{C}}(\mathbf{z}_N)\,\mathbf{r}_N]$ also depends on the source block $\mathbf{a}$, the channel vector $\mathbf{f}$ and the noise vectors $\{\mathbf{n}_i\}_{i=1}^N$ at all the relays. Hence, to explicitly write the block received at D by separating the source signal from the effective noise contribution, we have to focus on a specific code structure.

In this paper, we consider *complex* orthogonal space-time block codes (COSTBCs) [4], which are able to achieve full diversity at a low symbol-by-symbol ML decoding complexity. In complex OSTBCs, the code matrix $\boldsymbol{\mathcal{C}}(\mathbf{z}_i)$ is a widely linear (WL) transformation of $\mathbf{z}_i \triangleq [z_{i,1}, z_{i,2}, \ldots, z_{i,K}]^T$ given by (3.1), i.e., the entries of $\boldsymbol{\mathcal{C}}(\mathbf{z}_i)$ are complex linear combinations of the complex variables $z_{i,1}, z_{i,2}, \ldots, z_{i,K}$ and their conjugates $z^*_{i,1}, z^*_{i,2}, \ldots, z^*_{i,K}$. Following [4], we assume for the space-time coded matrix $\boldsymbol{\mathcal{C}}(\mathbf{z}_i)$ the following orthogonal structure:

$$\boldsymbol{\mathcal{C}}(\mathbf{z}_i) = \sum_{k=1}^{K} z_{i,k}\,\mathbf{\Phi}_k + \sum_{k=1}^{K} z^*_{i,k}\,\mathbf{\Psi}_k\,, \qquad (3.3)$$

where $\mathbf{\Phi}_1, \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_K, \mathbf{\Psi}_1, \mathbf{\Psi}_2, \ldots, \mathbf{\Psi}_K$ are constant *weight* matrices in $\mathbb{C}^{P\times L}$ designed such that the columns of $\boldsymbol{\mathcal{C}}(\mathbf{z}_i)$ are orthogonal vectors, i.e., $\boldsymbol{\mathcal{C}}(\mathbf{z}_i)^H\,\boldsymbol{\mathcal{C}}(\mathbf{z}_i) = \|\mathbf{z}_i\|^2\,\mathbf{I}_L, \forall \mathbf{z}_i \in \mathbb{C}^K - \{\mathbf{0}_K\}$, which imposes that $P \geq L$ necessarily. In this case, accounting for (3.1), it is readily seen that $\boldsymbol{\mathcal{C}}(\mathbf{z}_i) = \lambda_i\,[\boldsymbol{\mathcal{C}}(\mathbf{a}) + \boldsymbol{\mathcal{C}}(\widetilde{\mathbf{n}}_i)]$. Consequently, the signal received by D can be decomposed as

$$\mathbf{y}_\mathrm{d} = \sum_{i=1}^{N} \lambda_i\, g_i\,\boldsymbol{\mathcal{C}}(\mathbf{a})\,\mathbf{r}_i + \mathbf{w}_\mathrm{d} = \boldsymbol{\mathcal{C}}(\mathbf{a})\,\mathbf{h} + \mathbf{w}_\mathrm{d}\,, \qquad (3.4)$$

[1] Boldface upper [lower] case letters (e.g., $\mathbf{A}$ or $\mathbf{a}$) are matrices [vectors]; $\mathbb{C}^{m\times n}$ [$\mathbb{R}^{m\times n}$] is the field of $m \times n$ complex [real] matrices; $\mathbb{C}^m$ [$\mathbb{R}^m$] is a shorthand for $\mathbb{C}^{m\times 1}$ [$\mathbb{R}^{m\times 1}$]; $\{\mathbf{a}\}_i$ is the ith element of $\mathbf{a}$; $|a|$ denotes the magnitude of $a \in \mathbb{C}$; $*, T, H, -1$ denote the conjugate, the transpose, the Hermitian and the inverse of a matrix; let $\mathbf{a} = [a_1, a_2, \ldots, a_n]^T \in \mathbb{C}^n$ and $\mathcal{I} \subseteq \{1, 2, \ldots, n\}$, the ith entry $\{\mathbf{a}^{[*]\mathcal{I}}\}_i$ of the vector $\mathbf{a}^{[*]\mathcal{I}}$ is $\{\mathbf{a}^*\}_i$ if $i \in \mathcal{I}$, otherwise $\{\mathbf{a}^{[*]\mathcal{I}}\}_i = \{\mathbf{a}\}_i$; $\mathbf{0}_m \in \mathbb{R}^m$, is the null vector and $\mathbf{O}_{m\times n} \in \mathbb{R}^{m\times n}$ and $\mathbf{I}_m \in \mathbb{R}^{m\times m}$ are the null and the identity matrices; $\|\mathbf{a}\|$ is the Euclidean norm of $\mathbf{a}$; $\mathbf{A} = \mathrm{diag}(a_1, a_2, \ldots, a_n) \in \mathbb{C}^{n\times n}$ is a diagonal matrix whose (i,i)th entry is a_i; $\mathrm{E}[\cdot]$ and $j \triangleq \sqrt{-1}$ denote ensemble averaging and imaginary unit; a circular symmetric complex Gaussian random vector $\mathbf{x} \in \mathbb{C}^n$ with mean $\boldsymbol{\mu} \in \mathbb{C}^n$ and covariance matrix $\mathbf{K} \in \mathbb{C}^{n\times n}$ is denoted as $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{K})$.

[2] For a fair comparison with the results of [2], it is assumed that there is no direct link between S and D. For instance, this is the case when the distance between S and D is large enough such that the direct link strength is negligible. However, our framework can be straightforwardly modified by allowing S to re-transmit the block $\mathbf{a}$ in Phase II together with the N relays.

where $\mathbf{h} \triangleq [h_1, h_2, \ldots, h_L]^T = \boldsymbol{\mathcal{R}}\, \boldsymbol{\Lambda}\, \mathbf{g} \in \mathbb{C}^L$ is the "composite" channel seen by D, with $\boldsymbol{\Lambda} \triangleq \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N) \in \mathbb{R}^{N \times N}$, and

$$\mathbf{w}_{\mathrm{d}} \triangleq \sum_{i=1}^{N} \lambda_i\, g_i\, \boldsymbol{\mathcal{C}}(\widetilde{\mathbf{n}}_i)\, \mathbf{r}_i + \mathbf{n}_{\mathrm{d}} \tag{3.5}$$

denotes the effective noise at the receiver. It should be observed that the receiver can estimate from (3.4) the vector $\mathbf{h}$ by completely ignoring what $\mathbf{f}$, $\mathbf{g}$ and $\boldsymbol{\mathcal{R}}$ are. Specifically, estimation of $\mathbf{h}$ can be performed by resorting to standard training-based identification methods. In this case, each data transmission is preceded by a training period, wherein all the cooperating relays transmit a symbol sequence known to D, by using certain randomization vectors $\{\mathbf{r}_i\}_{i=1}^N$ that will be maintained until a new training phase is initiated.

A nice property of COSTBC rules is that $\boldsymbol{\mathcal{C}}(\cdot)$ can be designed so as to obtain a *linearized* signal model at the destination. This can be accomplished if there exists a subset $\mathcal{I} \subseteq \{1, 2, \ldots, P\}$ such that

$$[\boldsymbol{\mathcal{C}}(\boldsymbol{\beta})\, \boldsymbol{\gamma}]^{[*]\mathcal{I}} = \boldsymbol{\mathcal{D}}(\boldsymbol{\gamma})\, \boldsymbol{\beta}\,, \quad \text{for any } \boldsymbol{\beta} \in \mathbb{C}^K \text{ and } \boldsymbol{\gamma} \in \mathbb{C}^L\,, \tag{3.6}$$

where $\boldsymbol{\mathcal{D}}(\boldsymbol{\gamma}) \in \mathbb{C}^{P \times K}$ is a complex orthogonal WL transformation of $\boldsymbol{\gamma}$, i.e., $\boldsymbol{\mathcal{D}}^H(\boldsymbol{\gamma})\, \boldsymbol{\mathcal{D}}(\boldsymbol{\gamma}) = \|\boldsymbol{\gamma}\|^2\, \mathbf{I}_K$, $\forall \boldsymbol{\gamma} \in \mathbb{C}^L - \{\mathbf{0}_L\}$, whose structure is similar to that of $\boldsymbol{\mathcal{C}}(\boldsymbol{\beta})$ and whose weight matrices are uniquely determined from $\boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2, \ldots, \boldsymbol{\Phi}_K, \boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \ldots, \boldsymbol{\Psi}_K$, which are *known* at D. It can be shown [5] that (3.6) holds if and only if the pth row of $\boldsymbol{\mathcal{C}}(\cdot)$, $\forall p \in \mathcal{I}$, has all its non-zero entries conjugated (*conjugate row*), whereas the pth row of $\boldsymbol{\mathcal{C}}(\cdot)$, $\forall p \in \{1, 2, \ldots, P\} - \mathcal{I}$, has all its non-zero entries non-conjugated (*non-conjugate row*). For instance, the well-known 2×2 COSTBC design of proposed by Alamouti, for which $P = K = L = 2$, fulfills condition (3.6) with $\mathcal{I} = \{2\}$. By virtue of (3.6), from (3.4) and (3.5), one has the linearized (L) model

$$\mathbf{y}_{\mathrm{d,L}} \triangleq \frac{1}{\|\mathbf{h}\|^2}\, \boldsymbol{\mathcal{D}}^H(\mathbf{h})\, \mathbf{y}_{\mathrm{d}}^{[*]\mathcal{I}} = \mathbf{a} + \underbrace{\frac{1}{\|\mathbf{h}\|^2}\, \boldsymbol{\mathcal{D}}^H(\mathbf{h})\, \mathbf{w}_{\mathrm{d}}^{[*]\mathcal{I}}}_{\mathbf{w}_{\mathrm{d,L}} \in \mathbb{C}^K}\,, \tag{3.7}$$

with

$$\mathbf{w}_{\mathrm{d}}^{[*]\mathcal{I}} = \sum_{i=1}^{N} \lambda_i\, \boldsymbol{\mathcal{D}}(g_i\, \mathbf{r}_i)\, \widetilde{\mathbf{n}}_i + \mathbf{n}_{\mathrm{d}}^{[*]\mathcal{I}}\,. \tag{3.8}$$

Although at first sight the signal model (3.4) might appear similar to that reported in [2, Eq. 3], there is a key difference. The effective noise $\mathbf{w}_{\mathrm{d}} \in \mathbb{C}^P$ at the receiver depends on the source-to-relay channel vector $\mathbf{f}$ and on the relay-to-destination channel vector $\mathbf{g}$, as well as on the randomization matrix $\boldsymbol{\mathcal{R}}$. This implies that, given both $\mathbf{g}$ and $\boldsymbol{\mathcal{R}}$, the vector $\mathbf{w}_{\mathrm{d}}$ may be additive correlated Gaussian noise (ACGN); instead, assuming correct decision at the relays, the noise term at the receiver is AWGN in [2]. To this respect, capitalizing on the linearized models (3.7) and (3.8), it is readily seen that $\mathbf{w}_{\mathrm{d,L}} \sim \mathcal{CN}(\mathbf{0}_K, \mathbf{K}_{\mathrm{d,L}})$ conditioned on $\boldsymbol{\mathcal{R}}$, $\mathbf{f}$ and $\mathbf{g}$, where $\mathbf{K}_{\mathrm{d,L}} \triangleq \mathrm{E}[\mathbf{w}_{\mathrm{d,L}}\, \mathbf{w}_{\mathrm{d,L}}^H \,|\, \boldsymbol{\mathcal{R}}, \mathbf{f}, \mathbf{g}] = \frac{1}{\|\mathbf{h}\|^4}\, \boldsymbol{\mathcal{D}}^H(\mathbf{h})\, \mathbf{K}_{\mathrm{d}}\, \boldsymbol{\mathcal{D}}(\mathbf{h})$, with

$$\begin{aligned}\mathbf{K}_{\mathrm{d}} &\triangleq \mathrm{E}[\mathbf{w}_{\mathrm{d}}^{[*]\mathcal{I}}\, (\mathbf{w}_{\mathrm{d}}^{[*]\mathcal{I}})^H \,|\, \boldsymbol{\mathcal{R}}, \mathbf{f}, \mathbf{g}] \\ &= \sum_{i=1}^{N} \lambda_i^2\, |f_i^{-1}|^2\, \sigma_i^2\, \boldsymbol{\mathcal{D}}(g_i\, \mathbf{r}_i)\, \boldsymbol{\mathcal{D}}^H(g_i\, \mathbf{r}_i) + \sigma_{\mathrm{d}}^2\, \mathbf{I}_P\,. \end{aligned} \tag{3.9}$$

Interestingly, if the STBC used by the relays has full rate 1, i.e., $P = K$, one has $\boldsymbol{\mathcal{D}}(g_i\, \mathbf{r}_i)\, \boldsymbol{\mathcal{D}}^H(g_i\, \mathbf{r}_i) = \boldsymbol{\mathcal{D}}^H(g_i\, \mathbf{r}_i)\, \boldsymbol{\mathcal{D}}(g_i\, \mathbf{r}_i) = |g_i|^2\, \|\mathbf{r}_i\|^2\, \mathbf{I}_K$, which implies that the vector $\mathbf{w}_{\mathrm{d,L}}$ turns out to be AWGN, whose autocorrelation matrix assumes the simplified form

$$\mathbf{K}_{\mathrm{d,L}} = \frac{1}{\|\mathbf{h}\|^2} \left(\sum_{i=1}^{N} \lambda_i^2\, |f_i^{-1}|^2\, \sigma_i^2\, |g_i|^2\, \|\mathbf{r}_i\|^2 + \sigma_{\mathrm{d}}^2 \right) \mathbf{I}_K\,. \tag{3.10}$$

In this case, optimum ML decoding corresponds to the decision rule

$$\widehat{\mathbf{a}} = \arg\min_{\mathbf{a} \in \mathcal{A}} \|\mathbf{y}_{\mathrm{d,L}} - \mathbf{a}\|^2\,. \tag{3.11}$$

For instance, this is the case of the Alamouti's code for $L = 2$. However, in the case of COSTBC design, it was proven in [6] that the code rate cannot be greater than $3/4$ for $L \geq 3$. In such a case, the noise is correlated at the destination, thus, under the assumption that realizations of $\mathbf{h}$ and $\mathbf{K}_{\mathrm{d,L}}$ are perfectly known at the receiver, one-shot optimum ML decoding can be obtained by preventively whitening $\mathbf{w}_{\mathrm{d,L}}$. Although the matrix $\mathbf{K}_{\mathrm{d,L}}$ could be estimated at D,[3] a simpler receiving strategy consists of resorting to the suboptimal detector which is designed as if the effective noise vector $\mathbf{w}_{\mathrm{d}}$ were AWGN, i.e., it implements the rule (3.11). This detector turns out to be optimum for COSTBC exhibiting full rate 1, which happens when $L = 2$ as previously mentioned. However, for $L \geq 3$, we will show that the suboptimum detector achieves satisfactory performance.

4. NUMERICAL PERFORMANCE ANALYSIS

In this section, to perform a comparative performance study of the proposed A&F randomized approach and its D&F counterpart developed in [2], we resort to Monte Carlo computer simulations. We remember that both schemes are decentralized by construction: the code order L is independent of the actual number of cooperative nodes, and this allows to decentralize the relay selection procedures. We also report the performance of the A&F centralized approach. In a *centralized* scheme [1], if $N \geq L$, the nodes are divided into L equal groups and each group emulates a *pre-assigned* single antenna in a L-dimensional virtual multiple-antenna transmitter.[4] If $N < L$, the nodes emulate N out of the L preselected virtual antennas. For simplicity, in the subsequent examples, we consider the case when N is a multiple of L, i.e., $N = M\, L$, with $M \in \mathbb{Z}$. In this case, the centralized A&F schemes can be still described by (3.4) and (3.5), with $\boldsymbol{\mathcal{R}}$ being a deterministic matrix given by

$$\boldsymbol{\mathcal{R}} = \Big[\underbrace{\mathbf{e}_1, \ldots, \mathbf{e}_1}_{M} \underbrace{\mathbf{e}_2, \ldots, \mathbf{e}_2}_{M} \ldots \underbrace{\mathbf{e}_L, \ldots, \mathbf{e}_L}_{M} \Big]\,, \tag{4.1}$$

where $\mathbf{e}_\ell \triangleq [\mathbf{0}_{\ell-1}^T, 1, \mathbf{0}_{L-\ell}^T]^T \in \mathbb{R}^L$.

In all the experiments, the following simulation setting is adopted. All the nodes are uniformly and independently distributed in a circle of radius 10 meters centered around D. The position of D is kept fixed, while the position of S changes randomly from run to run, with the distance between S and D set to 10 meters. The source uses QPSK signaling, i.e., $a_k \in \{\pm 1/\sqrt{2} \pm j 1/\sqrt{2}\}$, for $k \in \{1, 2, \ldots, K\}$, thus $\sigma_a^2 = 1$. We set the path-loss exponent $\rho = 2$; the transmitted power from the ith relay is equal to $P_i = 1$; the noise power at the relay is assumed equal to the noise power at the destination, i.e., $\sigma_{\mathrm{d}}^2 = \sigma_i^2 = \sigma^2$ and, consequently, the SNR is defined as $\gamma \triangleq 1/\sigma^2$. For the considered decentralized approaches, the entries of $\boldsymbol{\mathcal{R}}$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance $1/L$. Finally, as performance measure, we resorted to the *average* BER (ABER) at the receiver output as a function of the SNR ranging from 0 to 30 dB: for each of the 10^4 Monte Carlo run carried out (wherein, besides the network configuration, all the channel coefficients, independent

[3] It is easily seen that $\mathbf{K}_{\mathrm{d,L}} = \mathrm{E}[\mathbf{y}_{\mathrm{d,L}}\, \mathbf{y}_{\mathrm{d,L}}^H \,|\, \boldsymbol{\mathcal{R}}, \mathbf{f}, \mathbf{g}] - \sigma_a^2\, \mathbf{I}_K$, where $\mathrm{E}[\mathbf{y}_{\mathrm{d,L}}\, \mathbf{y}_{\mathrm{d,L}}^H \,|\, \boldsymbol{\mathcal{R}}, \mathbf{f}, \mathbf{g}]$ can be consistently estimated from the received data.

[4] If N is not a multiple of L, then at the remaining nodes, the power is equally distributed among the L antennas.

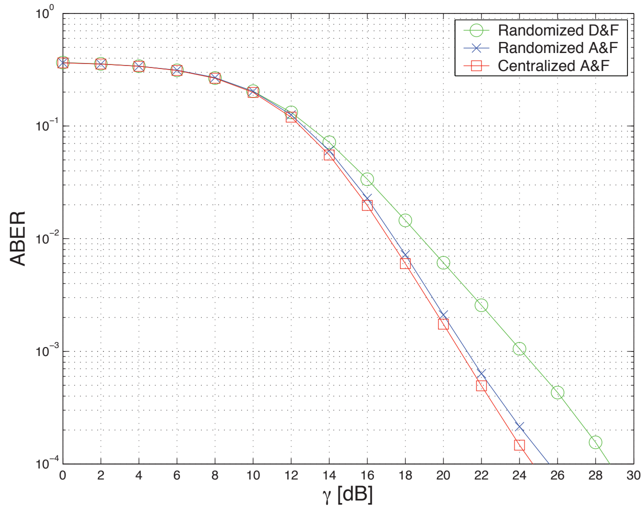

Fig. 1. ABER versus γ (Example 1, $N = 4$).

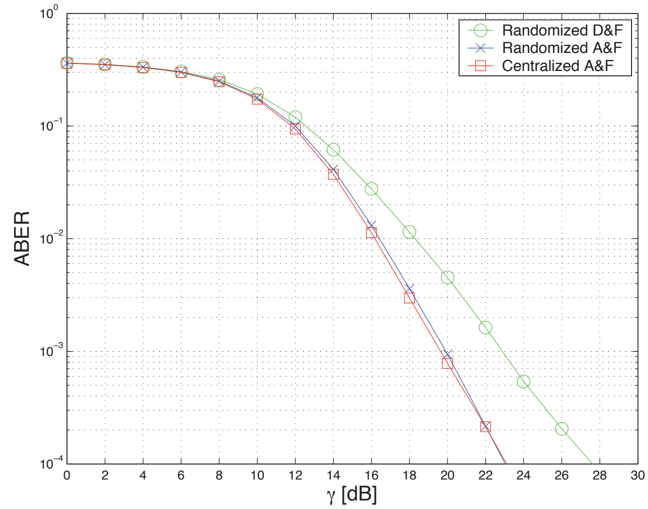

Fig. 2. ABER versus γ (Example 2, $N = 6$).

sets of noise, data sequences and randomization coefficients are randomly generated), an independent record of 10^2 symbols is considered to evaluate the ABER. Note that, for all the methods under comparison, transmission and reception processes are simulated in both Phase I and II. Thus, for the D&F scheme, the obtained results account for incorrect decisions at the relays.

Example 1: Alamouti space-time code of order $L = 2$

In this example, we consider as space-time coding rule the full-rate Alamouti code of order $L = K = P = 2$ given by

$$\boldsymbol{\mathcal{C}}(\mathbf{a}) = \begin{bmatrix} a_1 & a_2 \\ -a_2^* & a_1^* \end{bmatrix} \Longrightarrow \boldsymbol{\mathcal{D}}(\mathbf{h}) = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} . \tag{4.2}$$

We remember that, in this case, the decision rule (3.11) is optimum, i.e., $\widehat{\mathbf{a}} = \widehat{\mathbf{a}}_{\text{opt}}$. In this example, the number of relay nodes is equal to $N = 4$. It is apparent from Fig. 1 that the proposed randomized A&F scheme achieves the same diversity order of its centralized counterpart, by paying only a negligible performance penalty in the high SNR regime in terms of coding gain. Remarkably, the randomized A&F cooperative protocol outperforms the randomized D&F coding rule proposed in [2] for all the considered values of the SNR. Specifically, for an ABER of 10^{-4}, the SNR required for the randomized D&F scheme is about 4 dB more than that for the proposed randomized A&F approach. This performance gap is essentially due to the error propagation from the D&F relays to the destination, because of the errors at the relay nodes. Numerical results, not reported here due to the lack of space, show that the ABER at the output of the ML detector regarding the "worse" relay node, i.e., the one that is farthest from the source, varies from 10^{-2} to 10^{-3}, for $18 \leq \gamma[\text{dB}] \leq 23$, whereas it varies from 10^{-3} to 10^{-4}, for $24 \leq \gamma[\text{dB}] \leq 27$, and, finally, it is below 10^{-4}, for $\gamma \geq 28$ dB. Henceforth, we can state that, if the BERs at the relay nodes are not really negligible, noise propagation from the proposed A&F relay nodes to the source is less harmful than error propagation from the D&F relay nodes of [2].

Example 2: Alamouti space-time code of order $L = 3$

In this example, we consider as space-time coding rule the Alamouti code of order $L = K = 3$ given by

$$\boldsymbol{\mathcal{C}}(\mathbf{a}) = \begin{bmatrix} a_1 & a_2 & a_3 \\ -a_2^* & a_1^* & 0 \\ -a_3^* & 0 & a_1^* \\ 0 & -a_3^* & a_2^* \end{bmatrix} \Longrightarrow \boldsymbol{\mathcal{D}}(\mathbf{h}) = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_2^* & -h_1^* & 0 \\ h_3^* & 0 & -h_1^* \\ 0 & h_3^* & -h_2^* \end{bmatrix} , \tag{4.3}$$

whose rate is $K/P = 3/4$. We remember that, in this case, the decision rule (3.11) is suboptimum. In this example, the number of relay nodes is equal to $N = 6$. Results of Fig. 2, besides confirming the superiority of the randomized A&F coding strategy with respect to its D&F counterpart for all the considered SNR values, show that the A&F-based receivers based on the suboptimum decision rule (3.11), which ignore the fact that the noise correlation matrix (3.9) is not diagonal, exhibit very satisfactory performances. In particular, the proposed randomized A&F scheme essentially achieves the same performance of the centralized space-time code in terms of both diversity order and coding gain.

5. REFERENCES

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