A UNIFIED FRAMEWORK FOR GLRT-BASED SPECTRUM SENSING OF SIGNALS WITH COVARIANCE MATRICES WITH KNOWN EIGENVALUE MULTIPLICITIES

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ABSTRACT

In this paper, we create a unified framework for spectrum sensing of signals which have covariance matrices with known eigenvalue multiplicities. We derive the generalized likelihood-ratio test (GLRT) for this problem, with arbitrary eigenvalue multiplicities under both hypotheses. We also show a number of applications to spectrum sensing for cognitive radio and show that the GLRT for these applications, of which some are already known, are special cases of the general result.

1. INTRODUCTION

One of the most essential parts of cognitive radio is spectrum sensing. An erroneous decision results in either increased interference for the primary users (missed detection), or underutilized spectrum (false alarm). Therefore, it is important to design good detectors, that exploit most of the available knowledge about the signal to be detected. All man-made signals have some structure, which is intentionally introduced for example by the channel coding, the modulation and by the use of space-time codes. Usually, some of these properties of the signal are known from standards.

In this work, we consider a discrete-time model, and the structure of the signal is then inherent in the covariance matrix of the signal if the signal is stationary. Such structures incurs that some of the eigenvalues of the signal covariance matrix are larger than others, even though the exact eigenvalues or their multiplicities may not be known. Detection of correlated signals, exploiting features with unknown parameters is often referred to as blind detection. Blind detectors based on functions of eigenvalues of the sample covariance matrix were proposed and analyzed e.g. in [1, 2]. These detectors are blind in the sense that they do not exploit any knowledge of the eigenvalues nor their multiplicities. In this work, however, we consider the eigenvalues of the signal covariance matrix to have known multiplicities. This can occur, for example, when a single signal is received by multiple antennas (SIMO) [2, 3, 4, 5], when the signal is encoded with an orthogonal space-time block code (OSTBC) [6], or if the signal is an OFDM signal [7].

A related problem was considered in [8], also dealing with covariance matrices with known eigenvalue multiplicities. The problem of [8] was not only to detect the presence or absence of a signal, but rather to detect the number of signal sources embedded in noise. The paper [8] assumed that each signal source gives rise to a distinct eigenvalue, and that the remaining eigenvalues are equal to the noise power. This is a special case of the problem we consider in this paper, which allows arbitrary eigenvalue multiplicities. The work of [8] made use of the results on principal component analysis in [9]. In this work, we also use the results of [9], in particular for the maximum-likelihood estimation of the covariance matrices.

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Contributions: We derive the generalized likelihood-ratio test (GLRT) when the covariance matrices have arbitrary and known eigenvalue multiplicities under both hypotheses. We show that this is a unifying framework for some applications of spectrum sensing, which are special cases of the general result. In particular, we show that the GLRT of [2, 3, 4, 5], for detection of a single signal using multiple antennas, is a special case of the problem. Furthermore, we show that two of the detectors proposed in [6], for signals encoded with an OSTBC, are equivalent to the GLRT. We also derive the eigenvalues and their multiplicities of a synchronized OFDM signal in an AWGN channel, using the model in [7]. From this, we derive the GLRT for detection of an OFDM signal in AWGN.

2. MODEL AND PROBLEM FORMULATION

Let $\mathbf{y}_k$, $k = 1, \ldots, K$, be the observed N-length column vectors. We wish to discriminate between the two hypotheses

$$\begin{aligned} H_0 &: \mathbf{y}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_0), \text{ i.i.d. } k = 1, \ldots, K \\ H_1 &: \mathbf{y}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_1), \text{ i.i.d. } k = 1, \ldots, K, \end{aligned} \tag{1}$$

where $\mathbf{Q}_i$ has r_i distinct eigenvalues $\lambda_{1,i} > \lambda_{2,i} > \ldots > \lambda_{r_i,i}$, with known multiplicities $q_{1,i}, \ldots, q_{r_i,i}$ respectively, and $\mathbf{y}_k \in \mathbb{R}^{N\times 1}$. Then, $\sum_{j=1}^{r_i} q_{j,i} = N$. Note that the model is real-valued. This is not a restriction in most cases, since a complex valued model can be split into its real and imaginary parts.

Let $\mathbf{Y} \triangleq [\mathbf{y}_1\ \mathbf{y}_2\ \ldots\ \mathbf{y}_K] \in \mathbb{R}^{N\times K}$, and denote by $\widehat{\mathbf{R}}$ the sample covariance matrix

$$\widehat{\mathbf{R}} \triangleq \frac{1}{K}\sum_{k=1}^{K} \mathbf{y}_k\mathbf{y}_k^T = \frac{1}{K}\mathbf{Y}\mathbf{Y}^T. \tag{2}$$

Then the likelihood functions of $\mathbf{Y}$, under the two hypotheses can be written

$$p(\mathbf{Y}|\mathbf{Q}_i) = \frac{1}{(2\pi)^{NK/2}\det(\mathbf{Q}_i)^{K/2}} \exp\left(-\frac{K}{2}\mathrm{tr}\left(\mathbf{Q}_i^{-1}\widehat{\mathbf{R}}\right)\right). \tag{3}$$

3. DETECTION

In general, the covariance matrices $\mathbf{Q}_i$ are unknown. A standard technique to deal with unknown parameters, that usually performs well, is the generalized likelihood-ratio test (GLRT):

$$\frac{p\left(\mathbf{Y}|H_1, \widehat{\mathbf{Q}_1}\right)}{p\left(\mathbf{Y}|H_0, \widehat{\mathbf{Q}_0}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{4}$$

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where $\widehat{\mathbf{Q}_i}$ is the maximum-likelihood (ML) estimate of $\mathbf{Q}_i$, and the multiplicities $q_{1,i}, \ldots, q_{r_i,i}$ of the eigenvalues of $\mathbf{Q}_i$ are assumed to be known. As already mentioned, this can be the case for example in a SIMO transmission [2, 3, 4, 5], if the signal is encoded with an orthogonal space-time block code [6], or if the signal is OFDM modulated as we will show in Section 4.4. This also includes the special case of [10] when the structure of the transmitted signal is assumed to be completely unknown, so that all eigenvalues of the covariance matrix are assumed to have multiplicity one.

3.1. ML-Estimation of the Covariance Matrices

In this subsection, we will show the maximum-likelihood estimates that are required for the GLRT. The main work in deriving the ML estimates of $\mathbf{Q}_i$ was done in [9]. We will then use the result of [9] to derive the likelihood functions and the GLRT in (4).

Let $\mathbf{u}_{1,i}, \ldots, \mathbf{u}_{N,i}$ denote the eigenvectors of $\mathbf{Q}_i$, normalized so that $\|\mathbf{u}_{j,i}\| = 1,\ \forall j, i$. Define the set of indices

$$\mathcal{S}_{k,i} \triangleq \left(\sum_{j=1}^{k-1} q_{j,i}\right) + 1, \ldots, \sum_{l=1}^{k} q_{l,i} \tag{5}$$

($\Rightarrow \bigcup_{k=1}^{r_i} \mathcal{S}_{k,i} = 1, \ldots, N$). For example, if there are two distinct eigenvalues with multiplicities $q_{1,1}$ and $q_{2,1}$ $(= N - q_{1,1})$ respectively under hypothesis H_1, then $\mathcal{S}_{1,1} = 1, \ldots, q_{1,1}$ and $\mathcal{S}_{2,1} = q_{1,1} + 1, \ldots, N$. The covariance matrix $\mathbf{Q}_i$ is completely defined by its eigenvalues and eigenvectors, and can be written

$$\mathbf{Q}_i = \sum_{k=1}^{r_i} \sum_{j \in \mathcal{S}_{k,i}} \lambda_{k,i} \mathbf{u}_{j,i} \mathbf{u}_{j,i}^T.$$

Denote by d_k and $\mathbf{v}_k$, $k = 1, \ldots, N$, the eigenvalues sorted in descending order, and the corresponding normalized eigenvectors respectively of the sample covariance matrix $\widehat{\mathbf{R}}$. Following [9], the ML estimates of the eigenvalues and eigenvectors are

$$\begin{aligned} \widehat{\lambda}_{k,i} &= \frac{1}{q_{k,i}} \sum_{j \in \mathcal{S}_{k,i}} d_j, \quad k = 1, \ldots, r_i, \\ \widehat{\mathbf{u}}_{k,i} &= \mathbf{v}_k, \quad k = 1, \ldots, N. \end{aligned} \tag{6}$$

3.2. Generalized Likelihood-Ratio Test

Inserting the ML estimates (6) into the likelihood function (3) yields (7). Now, consider the likelihood functions of the two hypotheses. Then, inserting (7) for both hypotheses into (4) yields

$$\frac{p\left(\mathbf{Y}|H_1, \widehat{\mathbf{Q}_1}\right)}{p\left(\mathbf{Y}|H_0, \widehat{\mathbf{Q}_0}\right)} = \left(\frac{\prod_{k=1}^{r_0} \left(\frac{1}{q_{k,0}} \sum_{l \in \mathcal{S}_{k,0}} d_l\right)^{q_{k,0}}}{\prod_{j=1}^{r_1} \left(\frac{1}{q_{j,1}} \sum_{i \in \mathcal{S}_{j,1}} d_i\right)^{q_{j,1}}}\right)^{K/2}$$

We state the result in a theorem.

Theorem 1 *The GLRT of (1), where $\mathbf{Q}_i$ has distinct eigenvalues $\lambda_{1,i} > \lambda_{2,i} > \ldots > \lambda_{r_i,i}$, with known multiplicities $q_{1,i}, \ldots, q_{r_i,i}$ respectively, is*

$$\frac{\prod_{k=1}^{r_0} \left(\widehat{\lambda}_{k,0}\right)^{q_{k,0}}}{\prod_{j=1}^{r_1} \left(\widehat{\lambda}_{j,1}\right)^{q_{j,1}}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{8}$$

where

$$\widehat{\lambda}_{k,i} = \frac{1}{q_{k,i}} \sum_{j \in \mathcal{S}_{k,i}} d_j,$$

the sets $\mathcal{S}_{k,i}$ are given by (5), and d_j are the eigenvalues of the sample covariance matrix given by (2) sorted in descending order.

4. SPECTRUM SENSING APPLICATIONS

In spectrum sensing for cognitive radio, the problem is to discriminate between noise only and a signal embedded in noise. In the following, we will show a number of spectrum sensing applications, that are special cases of our general result in Theorem 1. A standard assumption is that the noise is zero-mean white, so that

$$\mathbf{y}_k | H_0 \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}).$$

That is, under H_0 there is only one eigenvalue with multiplicity N. This assumptions yields that the numerator in (8) is

$$\left(\frac{1}{N} \sum_{i=1}^{N} d_i\right)^N = \left(\frac{1}{N} \mathrm{tr}(\widehat{\mathbf{R}})\right)^N.$$

We will use this assumption in the sequel of this section.

4.1. Multiple Receive Antennas (SIMO)

The first special case we consider is when the detector have multiple antennas, which was analyzed in [2, 3, 4, 5]. Assume that there are $n_r = N > 1$ receive antennas at the detector. Then, under H_1, the received signal can be written

$$\mathbf{y}_k = \mathbf{h} x_k + \mathbf{w}_k,\ k = 1, \ldots, K, \tag{9}$$

where $\mathbf{h}$ is the channel vector, x_k is the transmitted signal sample, and $\mathbf{w}_k$ is the noise vector. The signal is assumed to be zero-mean Gaussian, i.e. $x_k \sim \mathcal{N}(0, \gamma^2)$. The noise is the same as under H_0, i.e. $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$. Then, the covariance matrix under H_1 is $\mathbf{Q}_1 = \gamma^2 \mathbf{h}\mathbf{h}^T + \sigma^2 \mathbf{I}$.

Now we have the following eigenvalues with their corresponding multiplicities under the two hypotheses

$$\begin{aligned} H_0 &: \ \lambda_{1,0} = \sigma^2,\ q_{1,0} = N, \\ H_1 &: \ \begin{cases} \lambda_{1,1} = \gamma^2 \|\mathbf{h}\|^2 + \sigma^2,\ q_{1,1} = 1, \\ \lambda_{2,1} = \sigma^2,\ q_{2,1} = N - 1. \end{cases} \end{aligned} \tag{10}$$

Inserting these into (8) yields the GLRT

$$\frac{\left(\frac{1}{N} \mathrm{tr}(\widehat{\mathbf{R}})\right)^N}{d_1 \left(\frac{1}{N-1} \sum_{i=2}^{N} d_i\right)^{N-1}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

This test is equivalent to [4, eq. (35)], showing that the GLRT of [2, 3, 4, 5] is a special case of Theorem 1.

4.2. Orthogonal Space-Time Block Codes

Now consider a slightly more general case, when the transmitted signal is encoded with an orthogonal space-time block code (OSTBC). This problem was considered in [6], and we use the same model here.

Assume that there are n_r receive antennas and n_t transmit antennas. The OSTBC code matrix $\mathbf{X} \in \mathbb{C}^{n_t \times T}$ is a linear function of n_s symbols $s_1, \ldots, s_{n_s}$ and their complex conjugates. The coded symbols (columns of $\mathbf{X}$) are transmitted over T time intervals. Let $\mathbf{Y} \in \mathbb{C}^{n_r \times T}$ be the received matrix that consists of the space-time coded signal plus noise, i.e.

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}, \tag{11}$$

$$p(\mathbf{Y}|\widehat{\mathbf{Q}}_i) = \frac{\exp\left(-\frac{K}{2}\mathrm{tr}\left(\widehat{\mathbf{Q}}_i^{-1}\widehat{\mathbf{R}}\right)\right)}{(2\pi)^{NK/2}\det\left(\widehat{\mathbf{Q}}_i\right)^{K/2}} = \frac{\exp\left(-\frac{K}{2}\mathrm{tr}\left(\sum_{k=1}^{r_i}\sum_{m\in\mathcal{S}_{k,i}}\frac{1}{\widehat{\lambda}_{k,i}}\mathbf{v}_m\mathbf{v}_m^T\sum_{j=1}^{N}d_j\mathbf{v}_j\mathbf{v}_j^T\right)\right)}{(2\pi)^{NK/2}\det\left(\widehat{\mathbf{Q}}_i\right)^{K/2}} = \frac{\exp\left(-\frac{K}{2}\sum_{k=1}^{r_i}\sum_{m\in\mathcal{S}_{k,i}}\frac{d_m}{\widehat{\lambda}_{k,i}}\right)}{(2\pi)^{NK/2}\left(\prod_{k=1}^{r_i}\widehat{\lambda}_{k,i}^{q_{k,i}}\right)^{K/2}}$$

$$= \frac{\exp\left(-\frac{K}{2}\sum_{k=1}^{r_i}\frac{\sum_{m\in\mathcal{S}_{k,i}}d_m}{\frac{1}{q_{k,i}}\sum_{l\in\mathcal{S}_{k,i}}d_l}\right)}{(2\pi)^{NK/2}\left(\prod_{k=1}^{r_i}\left(\frac{1}{q_{k,i}}\sum_{j\in\mathcal{S}_{k,i}}d_j\right)^{q_{k,i}}\right)^{K/2}} = \frac{1}{\left(\prod_{k=1}^{r_i}\left(\frac{1}{q_{k,i}}\sum_{j\in\mathcal{S}_{k,i}}d_j\right)^{q_{k,i}}\right)^{K/2}}\frac{\exp\left(-\frac{KN}{2}\right)}{(2\pi)^{NK/2}} \quad (7)$$

where $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix, and $\mathbf{W} \in \mathbb{C}^{n_r \times T}$ is a matrix of noise. Following [6], we have assumed perfect time and frequency synchronization. Then, following [6], we can for an OSTBC equivalently write the model as

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{w},$$

where $\mathbf{G}$ is a real-valued $2n_rT \times 2n_s$-matrix ($n_s < n_rT$) with the property $\mathbf{G}^T\mathbf{G} = \|\mathbf{H}\|^2\mathbf{I}$, and

$$\mathbf{x} = \left[\mathrm{Re}(s_1), \ldots, \mathrm{Re}(s_{n_s}), \mathrm{Im}(s_1), \ldots, \mathrm{Im}(s_{n_s})\right]^T \in \mathbb{R}^{2n_s \times 1}.$$

Now consider K space-time blocks $\mathbf{Y}_k$, or equivalently K vectors $\mathbf{y}_k$, received in a sequence. Moreover, we assume that the channel is slow fading, such that the generator matrix $\mathbf{G}$ remains the same during the whole time of reception. Then, under H_1, we have the model

$$\mathbf{y}_k = \mathbf{G}\mathbf{x}_k + \mathbf{w}_k,\ k = 1\ldots, K. \quad (12)$$

We assume that the elements of $\mathbf{x}_k$ are i.i.d. $\mathcal{N}(0, \gamma^2)$. In this case, the covariance matrix under H_1 is $\mathbf{Q}_1 = \gamma^2\mathbf{G}\mathbf{G}^T + \sigma^2\mathbf{I}$. Therefore, we have the following eigenvalue properties under H_1

$$\begin{cases}\lambda_{1,1} = \gamma^2\|\mathbf{H}\|^2 + \sigma^2,\ q_{1,1} = 2n_s,\\ \lambda_{2,1} = \sigma^2,\ q_{2,1} = 2n_rT - 2n_s,\end{cases}$$

and under H_0 the same as in (10). Using these particular eigenvalue multiplicities in Theorem 1, we obtain the GLR

$$\frac{\left(\frac{1}{2n_rT}\mathrm{tr}(\widehat{\mathbf{R}})\right)^{2n_rT}}{\left(\widehat{\lambda}_{1,1}\right)^{2n_s}\left(\widehat{\lambda}_{2,1}\right)^{2n_rT-2n_s}}$$

$$= \frac{1}{(2n_rT)^{2n_rT}}\frac{\left(2n_s\widehat{\lambda}_{1,1} + (2n_rT - 2n_s)\widehat{\lambda}_{2,1}\right)^{2n_rT}}{\left(\widehat{\lambda}_{1,1}\right)^{2n_s}\left(\widehat{\lambda}_{2,1}\right)^{2n_rT-2n_s}}$$

$$= \frac{1}{(2n_rT)^{2n_rT}}\left(\frac{\widehat{\lambda}_{2,1}}{\widehat{\lambda}_{1,1}}\right)^{2n_s}\left(\frac{2n_s\widehat{\lambda}_{1,1} + (2n_rT - 2n_s)\widehat{\lambda}_{2,1}}{\widehat{\lambda}_{2,1}}\right)^{2n_rT}$$

$$= \frac{1}{(2n_rT)^{2n_rT}}\left(\frac{\widehat{\lambda}_{2,1}}{\widehat{\lambda}_{1,1}}\right)^{2n_s}\left(2n_s\frac{\widehat{\lambda}_{1,1}}{\widehat{\lambda}_{2,1}} + 2n_rT - 2n_s\right)^{2n_rT}$$

By taking the derivative of this GLR, with respect to $\widehat{\lambda}_{1,1}/\widehat{\lambda}_{2,1}$, one can show that the GLR is a monotonously increasing function of $\widehat{\lambda}_{1,1}/\widehat{\lambda}_{2,1}$. Therefore, the GLRT can be equivalently written

$$\frac{\widehat{\lambda}_{1,1}}{\widehat{\lambda}_{2,1}} \underset{H_0}{\overset{H_1}{\gtrless}} \overline{\eta}. \quad (13)$$

Now, we have shown that the estimate of the covariance matrix that was proposed in [6], and referred to as "near-ML", is actually the true ML-estimate. Moreover, the ad-hoc detector proposed in [6] is identical to (13). Thus, we have shown that the ad-hoc detector of [6] is also equivalent to the GLRT. This explains why the numerical performances of these detectors were identical in [6].

4.3. Signal with Unknown Correlation Structure

Now consider the case when the signal correlation is unknown, so that all eigenvalues of the covariance matrix are assumed to have multiplicity one. That is, under H_1

$$q_{k,1} = 1,\ k = 1, \ldots, N.$$

Again, we assume that the noise is white Gaussian, so that there is only one distinct eigenvalue with multiplicity N under H_0, as in (10). Using these assumptions in Theorem 1, we obtain the GLRT

$$\frac{\left(\frac{1}{N}\mathrm{tr}(\widehat{\mathbf{R}})\right)^N}{\prod_{j=1}^N d_j} \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

This is of course equivalent to the GLRT obtained in [10] for this special case of the problem, and also to the sphericity test of [11].

4.4. OFDM

In this section we consider an OFDM signal with a cyclic prefix (CP). We will use the vector-matrix model of [7], and show the eigenvalue properties of the received OFDM signal in an AWGN channel. Again, we assume perfect synchronization.

Now, let $\mathbf{x}_k$ be the N_d-vector of data associated with the kth OFDM symbol. This data vector is the output of the IFFT operation, used to create the OFDM data. An OFDM symbol is then created by repeating the last N_c elements of $\mathbf{x}_k$ at the beginning of the symbol. Following [7], the received OFDM symbol can be modelled by (12), where

$$\mathbf{G} = \begin{bmatrix}\mathbf{0}_{N_c\times N_d-N_c} & \mathbf{I}_{N_c}\\ \multicolumn{2}{c}{\mathbf{I}_{N_d}}\end{bmatrix} \in \mathbb{R}^{(N_c+N_d)\times N_d}.$$

Here $\mathbf{0}_{n\times m}$ denotes the $n \times m$ all-zero matrix, and $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. Then, G has the property

$$\mathbf{G}^T\mathbf{G} = \mathrm{diag}(\underbrace{1, \ldots, 1}_{N_d-N_c}, \underbrace{2, \ldots, 2}_{N_c}) \in \mathbb{R}^{N_d\times N_d}. \quad (14)$$

Since the matrices $\mathbf{G}^T\mathbf{G}$ and $\mathbf{G}\mathbf{G}^T$ have the same non-zero eigenvalues, this means that the matrix $\mathbf{G}\mathbf{G}^T$ have eigenvalues $2, 1$ and 0 with multiplicities $N_c, N_d - N_c$ and N_c respectively. Again, the covariance matrix is $\mathbf{Q}_1 = \gamma^2\mathbf{G}\mathbf{G}^T + \sigma^2\mathbf{I}$ under H_1, and we get the following eigenvalue properties

$$\begin{cases}\lambda_{1,1} = 2\gamma^2 + \sigma^2,\ q_{1,1} = N_c,\\ \lambda_{2,1} = \gamma^2 + \sigma^2,\ q_{2,1} = N_d - N_c,\\ \lambda_{3,1} = \sigma^2,\ q_{3,1} = N_c.\end{cases}$$

With these eigenvalue multiplicities, the GLR in Theorem 1 becomes

$$\frac{\left(\frac{1}{N}\text{tr}(\widehat{\mathbf{R}})\right)^N}{\left(\widehat{\lambda}_{1,1}\right)^{N_c}\left(\widehat{\lambda}_{2,1}\right)^{N_d-N_c}\left(\widehat{\lambda}_{3,1}\right)^{N_c}}.$$

Here, we assumed real-valued OFDM samples, but in reality they are complex-valued. This is not a restriction. Since the generator matrix $\mathbf{G}$ is real valued, we can split the received vectors into real and imaginary parts and deal with them separately. The only consequence of this is that the dimension of the received vector and the multiplicities of the eigenvalues will increase with a factor of two.

5. MONTE-CARLO SIMULATIONS

In the following, we will show some numerical results of the proposed GLRT exemplified by a signal encoded with the Alamouti code. We will compare the proposed GLRT, that exploits the known signal structure, with a few eigenvalue-based blind detectors.

5.1. Benchmarks

There were two blind detectors proposed in [1], based on functions of the eigenvalues of the sample covariance matrix. The detectors of [1] use the tests

$$\frac{d_1}{d_N} \underset{H_0}{\overset{H_1}{\gtrless}} \eta', \quad \frac{\text{tr}(\widehat{\mathbf{R}})}{d_N} \underset{H_0}{\overset{H_1}{\gtrless}} \eta''. \tag{15}$$

A similar test is

$$\frac{d_1}{\text{tr}(\widehat{\mathbf{R}})} \underset{H_0}{\overset{H_1}{\gtrless}} \tilde{\eta}. \tag{16}$$

The detector (16) works as a blind detector for any kind of correlated signal, although it was also shown in [2, 3, 4] to be equivalent to the GLRT for the SIMO scenario of Section 4.1.

5.2. Numerical Results

The Alamouti code is an OSTBC, so the GLRT in this case is given by (13). As a comparison, we show the detection performance of the detectors presented in Section 5.1. Note in passing that none of the detectors requires knowledge of the noise variance. Each detector received $K = 100$ code blocks, using $n_r = 8$ receive antennas. The SNR in dB is defined as $10\log_{10}(\gamma^2/\sigma^2)$. Performance is given as the probability of missed detection, P_{MD}, as a function of SNR. The channel coefficients were drawn from a complex circularly symmetric $\mathcal{N}(0, 1)$ distribution. The probability of false alarm was chosen to $P_{\text{FA}} = 0.05$. The optimal decision thresholds were computed empirically from a set of noise-only realizations, to achieve the chosen P_{FA}. The results are shown in Figure 1. It is clear that the GLRT, exploiting the known eigenvalue structure, performs better than the blind detectors.

6. CONCLUDING REMARKS

We generalized and unified numerous recent problems in spectrum sensing. It should be noted that the general result also includes the problem of discriminating two signals, of different kind, from one another. This problem may be of large interest in the context of cognitive radio, when one wishes to distinguish between primary and secondary user's signals.

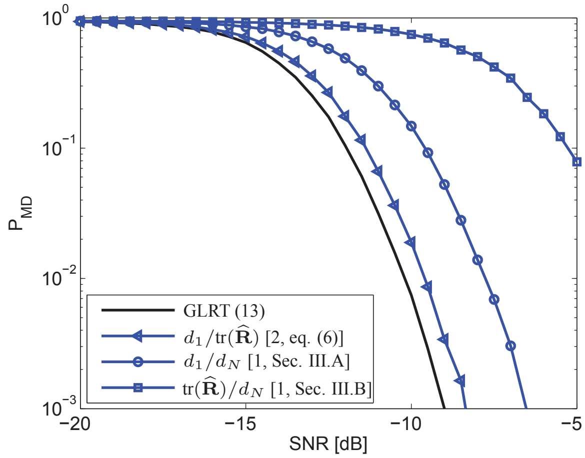

Fig. 1. Probability of missed detection versus SNR for detection of an Alamouti coded signal. $P_{\text{FA}} = 0.05$, $K = 100$, $n_r = 8$.

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