A PARTICLE FILTERING ALGORITHM FOR COOPERATIVE BLIND EQUALIZATION USING VB PARAMETRIC APPROXIMATIONS

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ABSTRACT

We introduce in this paper a new distributed sequential Monte Carlo (SMC) algorithm for blind equalization of frequency-selective broadcast channels. In the considered setup, multiple receiving nodes sense independently distorted versions of the same broadcast signal and cooperate to recover it. The proposed approach innovates by using parametric approximations based on the Variational Bayes (VB) method that allow the inter-node communication burden to be greatly reduced compared to previous communication-intensive distributed SMC algorithms. We verify via numerical simulations that the proposed method yields better performance than alternative methods that employ ad hoc parametric approximations, while preserving roughly the same computational cost.

***Index Terms*—** Distributed Algorithms, Particle Filters, Variational Bayes Methods, Blind Equalization.

1. INTRODUCTION

We consider in this paper a scenario where a single transmitter broadcasts a sequence of discrete-valued symbols to multiple receivers located at different nodes in a network. Rather than forwarding the local observations at each node to a centralized data fusion center, our goal is to derive instead a decentralized algorithm where the different receivers process their own local measurements independently, but also cooperate with each other, via some form of message passing over the network, to come up with the best global estimate of the transmitted sequence given the observations at *all* nodes.

Previous work, e.g. [1],[2], introduced distributed algorithms that approximate linear minimum-mean-square error (LMMSE) estimates of a hidden parameter vector or signal observed by multiple receivers located at remote nodes in a sensor network. However, the algorithms in [1], [2] are not ideally suited for distributed equalization of digital broadcast channels, as, due to the non-Gaussian distribution of the transmitted signals, the optimal minimum probability of error estimate of the transmitted data stream may differ significantly from the LMMSE estimates approximated by conventional adaptive or Kalman filters.

In [3], we filled the gap in the literature by introducing novel maximum-a-posteriori (MAP) particle-filter-based distributed equalization algorithms. To reduce the heavy inter-node communication burden required by the (asymptotically) optimal distributed particle filter equalizer, [3] also proposes the use of suboptimal parametric approximations to certain particle-dependent quantities, at the expense of a modest degradation in performance. Previous work [4], [5] also considered distributed particle filtering algorithms, but in the context of target tracking rather than digital communications. Furthermore, [4],[5] assumed perfect knowledge of the observation model parameters at each network node, whereas, in [3], we assumed unknown, random channel parameters.

In this paper, we innovate by proposing a new parametric approximation scheme based on the Variational Bayes (VB) [6] method. The VB method allows joint probability density functions (p.d.f's) to be approximated by the product of *separable* marginal p.d.f's in a way that the Kullback-Leibler (KL) divergence between the joint p.d.f. and its separable approximation is minimized. Using the VB method, moment matching operations needed for the determination of parametric density approximations can be performed while avoiding ad hoc schemes that were previously verified to perform poorly at low-noise levels [3].

The remainder of the paper is organized as follows: in Sec. 2 we describe the signal model, briefly introducing in Sec. 3 the particle filter approach for blind equalization. In Section 4, we present the new variational Bayes approximate distributed particle filter (VB-ADPF) scheme, whose performance is evaluated in Sec. 5. Finally, we draw our conclusions in Sec. 6.

2. SIGNAL MODEL

Denote by $\{b_n\}$ an independent, identically distributed (i.i.d.) binary bit sequence and by $\{x_n\}$, $x_n \in \{\pm 1\}$, the corresponding differentially encoded symbols. We assume that the observations $y_{r,0:n} \triangleq \{y_{r,0}, \ldots, y_{r,n}\}$ at the $r-$th node of a network of R receivers are obtained as the output of the additive noise frequency-selective FIR channel

$$y_{r,n} = \mathbf{h}_r^H \mathbf{x}_n + v_{r,n} \ , \quad (1)$$

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where $\mathbf{h}_r \in \mathbb{C}^{L\times 1}$ is a vector with the (time-invariant) channel impulse response terms, $\mathbf{x}_n \triangleq [x_n \dots x_{n-L+1}]^T$, and $v_{r,n}$ represents an i.i.d zero-mean complex Gaussian random process of variance σ_r^2.

The *unknown, random* parameters $\mathbf{h}_r$ and σ_r^2, $1 \le r \le R$, are assumed to be independent for $r \neq s$, and distributed a priori as $\sigma_r^2 \sim \mathcal{IG}(\sigma_r^2|\alpha;\beta)$ and $\mathbf{h}_r \mid \sigma_r^2 \sim \mathcal{N}_L(\mathbf{h}_r|0; I\sigma_r^2/\epsilon^2)$, where $\mathcal{N}_L$ and $\mathcal{IG}$ denote respectively an $L-$variate Gaussian and an inverse Gamma p.d.f., and $\{\alpha,\beta,\epsilon\}$ are the model's hyperparameters.

Under these hypotheses, we aim at developing a recursive method for obtaining smoothed MAP estimates $\hat{b}_{n-d} = \arg\max_{b_{n-d}} p(b_{n-d}|y_{1:R,0:n})$, where $d \ge 0$ and $y_{1:R,0:n} \triangleq \{y_{1,0:n} \cdots y_{R,0:n}\}$.

3. BLIND EQUALIZATION VIA PARTICLE FILTERS

The posterior probability mass function (p.m.f) of the transmitted bits can be approximated via particle filters as

$$p(b_{n-d}|y_{1:R,0:n}) \approx \sum_{q=1}^{Q} w_n^{(q)} \mathcal{I}\left\{b_{n-d} = b_{n-d}^{(q)}\right\}, \quad (2)$$

where $\mathcal{I}\{\cdot\}$ stands for the indicator function; Q is the number of particles $b_n^{(q)}$, sampled from the importance function $\pi(\cdot)$, and $w_n^{(q)}$ are the importance weights. Exploiting the fact that each distinct bit sequence $b_{-L:n-1}^{(q)}$ uniquely defines a corresponding state sequence $\mathbf{x}_{0:n}^{(q)}$, the *optimal importance function* [7] can be expressed as $\pi(b_n|b_{-L:n-1}^{(q)}, y_{1:R,0:n}) = p(\mathbf{x}_n \mid \mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n})$, which can be evaluated as

$$p(\mathbf{x}_n \mid \mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n}) = \frac{p(\mathbf{x}_n, \mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n})}{\sum_{\mathbf{x}_n} p(\mathbf{x}_n, \mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n})}. \quad (3)$$

The *importance weights* can be propagated in turn by the recursion [7]

$$w_n^{(q)} \quad \propto \quad w_{n-1}^{(q)} \sum_{\mathbf{x}_n} \frac{p(\mathbf{x}_n, \mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n})}{p(\mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n-1})}. \quad (4)$$

From the a priori independence of the unknown parameters for each receiver's channel, one deduces that, see [3],

$$p(\mathbf{x}_{0:n}^{(q)}, y_{1:R,0:n}) \propto \prod_{r=1}^{R} p(\mathbf{x}_{0:n}^{(q)}, y_{r,0:n}). \quad (5)$$

Using the channel parameter priors defined in Section 2, it can be also shown after tedious algebraic manipulations [7] that

$$\begin{aligned} p(\mathbf{x}_{0:n}, y_{r,0:n}) &= \int_{\mathbb{R}^+}\int_{\mathbb{C}^L} p(\mathbf{x}_{0:n}, y_{r,0:n}, \mathbf{h}_r, \sigma_r^2)\, d\mathbf{h}_r\, d\sigma_r^2 \\ &\propto |\mathbf{\Sigma}_n|\,[\beta_{r,n}]^{-\alpha_n} \end{aligned} \quad (6)$$

where Σ_n, $\beta_{r,n}$, and α_n can be recursively computed via

$$\alpha_n = \alpha_{n-1} + 1, \quad (7)$$

$$\beta_{r,n} = \beta_{r,n-1} + \gamma_n^{-1}\,\|e_{r,n}\|^2, \quad (8)$$

$$\bar{\mathbf{h}}_{r,n} = \bar{\mathbf{h}}_{r,n-1} + \gamma_n^{-1}\mathbf{\Sigma}_{n-1}\mathbf{x}_n e_{r,n}^*, \quad (9)$$

$$\mathbf{\Sigma}_n = \mathbf{\Sigma}_{n-1} - \gamma_n^{-1}\mathbf{\Sigma}_{n-1}\mathbf{x}_n\mathbf{x}_n^H\mathbf{\Sigma}_{n-1}, \quad (10)$$

with $e_{r,n} \triangleq y_{r,n} - \bar{\mathbf{h}}_{r,n-1}^H \mathbf{x}_n$, $\gamma_n^{-1} \triangleq 1 + \mathbf{x}_n^H \mathbf{\Sigma}_{n-1}\mathbf{x}_n$, $\alpha_{-1} = \alpha$, $\beta_{r,-1} = \beta$, $\bar{\mathbf{h}}_{r,-1} = 0$, and $\mathbf{\Sigma}_{-1} = \mathbf{I}\epsilon^{-2}$.

4. COOPERATIVE BLIND EQUALIZATION

Substituting (5) into (3), the expression of the optimal importance function can be rewritten as

$$p(\mathbf{x}_n|\mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n}) = \frac{\prod_{r=1}^R \lambda_{r,n}^{(q)}(\mathbf{x}_n)}{\sum_{\mathbf{x}_n}\prod_{r'=1}^R \lambda_{r',n}^{(q)}(\mathbf{x}_n)} \quad (11)$$

where $\lambda_{r,n}^{(q)}(\mathbf{x}_n) \triangleq p(\mathbf{x}_n, \mathbf{x}_{0:n-1}^{(q)}, y_{r,0:n})$. Likewise, the weight update rule can be obtained by plugging in (5) into (4), which yields

$$w_n^{(q)} \propto w_{n-1}^{(q)} \sum_{\mathbf{x}_n}\prod_{r=1}^{R} \frac{\lambda_{r,n}^{(q)}(\mathbf{x}_n)}{\lambda_{r,n-1}^{(q)}(\mathbf{x}_{n-1}^{(q)})}. \quad (12)$$

The DcPF-II algorithm in [3] is an exact decentralized implementation of (11)-(12). Despite its asymptotic optimality, that algorithm has heavy inter-node communication requirements ($2Q$ real numbers per node per bit in a BPSK system [3]), which preclude its use in practical applications. To mitigate this, we propose here a method to eliminate the need to broadcast the Q distinct coefficients $\lambda_{r,n}^{(q)}$ by replacing those quantities in the $s-$th node ($s \neq r$) with the approximation $\tilde{\lambda}_{r,n}(\mathbf{x}_n)$, defined so as to depend on the $r-$th receiver particles only via a predefined set of parameters that are independent of the particle label q.

To this aim, we first observe that, from the a priori independence of the unknown parameters, it can be verified that $p(\mathbf{h}_r, \sigma_r^2|\mathbf{x}_{0:n}, y_{1:R,0:n-1}) = p(\mathbf{h}_r, \sigma_r^2|\mathbf{x}_{0:n-1}, y_{1:R,0:n-1})$ and

$$p(\mathbf{h}_r, \sigma_r^2|\mathbf{x}_{0:n-1}, y_{1:R,0:n-1}) = p(\mathbf{h}_r, \sigma_r^2|\mathbf{x}_{0:n-1}, y_{r,0:n-1}). \quad (13)$$

Assuming now the prior pdf's in Section 2, it can be shown that [7] $p(\mathbf{h}_r|\sigma_r^2, \mathbf{x}_{0:n-1}^{(q)}, y_{r,0:n-1}) = \mathcal{N}(\mathbf{h}_r|\bar{\mathbf{h}}_{r,n-1}^{(q)}, \sigma_r^2\mathbf{\Sigma}_{n-1}^{(q)})$ and $p(\sigma_r^2|\mathbf{x}_{0:n-1}^{(q)}, y_{r,0:n-1}) = \mathcal{IG}(\sigma_r^2|\alpha_{n-1}, \beta_{r,n-1}^{(q)})$ where $\Sigma_n^{(q)}$, $\bar{\mathbf{h}}_{r,n}^{(q)}$, $\alpha_n^{(q)}$, and $\beta_{r,n}^{(q)}$ are given by the recursions in (7)-(10). Given then a Monte Carlo representation of $p(\mathbf{x}_{0:n-1} \mid y_{r,0:n-1})$ by the properly weighted set $\{\mathbf{x}_{0:n-1}^{(q)}, w_{n-1}^{(q)}\}$ and using (13), we can make the approximation

$$p(\mathbf{h}_r,\sigma_r^2|y_{r,0:n-1}) \approx \sum_{q=1}^{Q} w_{n-1}^{(q)} p(\mathbf{h}_r,\sigma_r^2|\mathbf{x}_{0:n-1}^{(q)}, y_{r,0:n-1})$$
$$= \sum_{q=1}^{Q} w_{n-1}^{(q)} \mathcal{IG}(\sigma_r^2|\alpha_{n-1},\beta_{r,n-1}^{(q)}) \mathcal{N}(\mathbf{h}_r|\bar{\mathbf{h}}_{r,n-1}^{(q)}, \sigma_r^2 \mathbf{\Sigma}_{n-1}^{(q)}). \quad (14)$$

4.1. VB-ADPF Algorithm

To drop the need to transmit particle-specific quantities, we propose to replace (14) with the *single-term* approximation

$$\tilde{p}(\mathbf{h}_r,\sigma_r^2|\mathbf{x}_{0:n},y_{r,0:n-1}) = \mathcal{N}(\mathbf{h}_r|\tilde{\mathbf{h}}_{r,n-1},\sigma_r^2\tilde{\mathbf{\Sigma}}_{r,n-1})\mathcal{IG}(\sigma_r^2|\tilde{\alpha}_{r,n-1},\tilde{\beta}_{r,n-1}), \quad (15)$$

where the parameters $\widetilde{(\cdot)}$, determined in the sequel, are *invariant* with $\mathbf{x}_{0:n-1}$. The joint density in (15) was *designed* to allow the analytic derivation [7] of the approximate *likelihood*

$$\tilde{p}(y_{r,n}|\mathbf{x}_{0:n},y_{r,0:n-1}) \triangleq \iint p(y_{r,n}|\mathbf{h}_r,\sigma_r^2,\mathbf{x}_{0:n},y_{r,0:n-1}) \times \tilde{p}(\mathbf{h}_r,\sigma_r^2|\mathbf{x}_{0:n},y_{r,0:n-1})\, d\mathbf{h}_r d\sigma_r^2 \quad (16)$$
$$= \frac{\tilde{\gamma}_n^{-1}\ \tilde{\beta}_{r,n-1}^{\tilde{\alpha}_{r,n-1}}}{\left(\tilde{\beta}_{r,n-1}+\tilde{\gamma}_n^{-1}\|y_{r,n}-\tilde{\mathbf{h}}_{r,n-1}^H\mathbf{x}_n\|^2\right)^{(\tilde{\alpha}_{r,n-1}+1)}}, \quad (17)$$

where $\tilde{\gamma}_n \triangleq 1+\mathbf{x}_n^H\tilde{\mathbf{\Sigma}}_{r,n-1}\mathbf{x}_n$, from which we finally obtain that

$$\tilde{\lambda}_{r,n}(\mathbf{x}_n) \triangleq \tilde{p}(\mathbf{x}_{0:n},y_{r,0:n}) \propto \prod_{i=0}^{n}\tilde{p}(y_{r,i}|\mathbf{x}_{0:i},y_{r,0:i-1}) \quad (18)$$

from the iterative application of Bayes' law.

4.1.1. Parameter Determination

Our aim is to determine the parameters $\widetilde{(\cdot)}$ so that the first and second moments of (15) match, as close as possible, those of (14). Unfortunately, the exact solution of the aforementioned moment matching problem is intractable, which forces us to resort instead to the application of the Variational Bayes method [6]. First, we make the approximation

$$p(\mathbf{h}_r,\sigma_r^2|\mathbf{x}_{0:n-1}^{(q)},y_{1:R,0:n-1}) \approx f^{(q)}(\mathbf{h}_r)f^{(q)}(\sigma_r^2), \quad (19)$$

where $f^{(q)}(\cdot)$ are VB marginals [6], defined as the set of *separable* densities that minimize the KL divergence between the p.d.f's on the left and right-hand sides of (19). The derivation of VB marginals is a somewhat cumbersome inductive process (see [6, Ch. 3]) and is omitted here for lack of space. After long algebraic manipulations, it can be shown that

$$f^{(q)}(\mathbf{h}_r) = \mathcal{N}\left(\mathbf{h}_r|\bar{\mathbf{h}}_{r,n-1}^{(q)},\frac{\beta_{r,n-1}^{(q)}}{\alpha_{n-1}}\Sigma_{n-1}^{(q)}\right),$$
$$f^{(q)}(\sigma_r^2) = \mathcal{IG}\left(\sigma_r^2|\alpha_{n-1}{+}L,\frac{\alpha_{n-1}+L}{\alpha_{n-1}}\beta_{r,n-1}^{(q)}\right). \quad (20)$$

Replacing (20) into (14) and integrating out σ_r^2 allows us to analytically determine the *approximate marginal*

$$\hat{p}(\mathbf{h}_r|y_{1:R,0:n-1}) \approx \sum_{q=1}^{Q} w_{n-1}^{(q)} f^{(q)}(\mathbf{h}_r), \quad (21)$$

whose first and second moments $\widehat{(\cdot)}$ can be computed as

$$\hat{\mathbf{h}}_{r,n-1} = \sum_{q=1}^{Q} w_{n-1}^{(q)}\bar{\mathbf{h}}_{r,n-1}^{(q)}, \quad (22)$$
$$\hat{\Sigma}_{r,n-1} = \sum_{q=1}^{Q}\left[w_{n-1}^{(q)}\left(\bar{\mathbf{h}}_{r,n-1}^{(q)}\bar{\mathbf{h}}_{r,n-1}^{(q)H} + \Sigma_{r,n-1}^{(q)}\right)\right] - \hat{\mathbf{h}}_{r,n-1}\hat{\mathbf{h}}_{r,n-1}^H, \quad (23)$$

since the r.h.s. of (21) is a sum of Gaussian p.d.f's. Likewise, replacing $\mathbf{h}_r$ with σ_r^2 on both sides of (21) yields an approximation to $p(\sigma_r^2|y_{1:R,0:n-1})$. Exploiting properties of the inverse Gamma distribution, namely that if $\sigma^2 \sim \mathcal{IG}(\sigma^2|\alpha,\beta)$, then $\mathrm{E}[\sigma^2]=\beta/(\alpha-1)$ and $\mathrm{VAR}[\sigma^2]=\mathrm{E}^2[\sigma^2]/(\alpha-2)$, we obtain that

$$\hat{\mathrm{E}}_{n-1}[\sigma_r^2] = \frac{(\alpha_{n-1}{+}L)\sum_{q=1}^{Q} w_{n-1}^{(q)}\beta_{r,n-1}^{(q)}}{\alpha_{n-1}(\alpha_{n-1}{+}L{-}1)}, \quad (24)$$
$$\widehat{\mathrm{VAR}}_{n-1}[\sigma_r^2] = \frac{(\alpha_{n-1}{+}L)^2}{\alpha_{n-1}^2(\alpha_{n-1}{+}L{-}1)(\alpha_{n-1}{+}L{-}2)} \times \sum_{q=1}^{Q}\left[w_{n-1}^{(q)}(\beta_{r,n-1}^{(q)})^2\right] - \hat{\mathrm{E}}_{n-1}^2[\sigma_r^2]. \quad (25)$$

We now wish to determine the parameters of the *separable approximation*

$$\mathcal{N}\left(\mathbf{h}_r|\hat{\mathbf{h}}_{r,n-1},\hat{\Sigma}_{n-1}\right)\mathcal{IG}\left(\sigma_r^2|\hat{\alpha}_{n-1},\hat{\beta}_{r,n-1}\right), \quad (26)$$

such that the moments of (26) match (22)-(25). The parameters for the normal distribution in (26) are directly given by (22) and (23). Solving for the parameters of the inverse Gamma distribution, we obtain that $\hat{\alpha}_{r,n-1} = 2+\widehat{\mathrm{E}}_{n-1}^2[\sigma_r^2]/\widehat{\mathrm{VAR}}_{n-1}[\sigma_r^2]$, and $\hat{\beta}_{r,n-1} = (\hat{\alpha}_{r,n-1}-1)\widehat{\mathrm{E}}_{n-1}[\sigma_r^2]$.

Finally, to analytically evaluate (16), we need to determine the parameters of the *nonseparable* approximation (15) that best matches (26). To this aim, we employ the VB approximation (20) for a second time, which yields

$$\tilde{\alpha}_{n-1} = \hat{\alpha}_{n-1}-L, \quad (27)$$
$$\tilde{\beta}_{r,n-1} = \frac{\tilde{\alpha}_{n-1}}{\tilde{\alpha}_{n-1}+L}\hat{\beta}_{r,n-1}, \quad (28)$$
$$\tilde{\mathbf{h}}_{r,n-1} = \hat{\mathbf{h}}_{r,n-1}, \quad (29)$$
$$\tilde{\mathbf{\Sigma}}_{r,n-1} = \hat{\mathbf{\Sigma}}_{r,n-1}\frac{\tilde{\alpha}_{n-1}}{\tilde{\beta}_{r,n-1}}. \quad (30)$$

The proposed VB-ADPF equalizer runs then as follows: at instant n, the $r-$th receiver evaluates (18) for all 2^L possible values[1] of $\mathbf{x}_n$ and broadcasts those quantities. Upon

[1] Note that the communication cost can be reduced to $\mathcal{O}(L^2)$ without further approximations by broadcasting $\tilde{\mathbf{h}}_{r,n-1}$, $\tilde{\mathbf{\Sigma}}_{r,n-1}$, $\tilde{\alpha}_{n-1}$, $\tilde{\beta}_{r,n-1}$ and $y_{r,n}$, which allow (18) to be evaluated at the remote receiver.

receiving the data broadcast by the remaining receivers, the $r-$th receiver evaluates the approximate optimal importance function and the corresponding importance weights via modified versions, respectively of Equation (11) and (12), where, for $s = 1, \ldots, R$, $\lambda_{s,n}^{(q)}(\mathbf{x}_n)$ is computed via (6) if $r = s$ and replaced, if $s \neq r$, with $\tilde{\lambda}_{s,n}(\mathbf{x}_n)$ in (18). In the actual implementation of (22) and (23), a *pivoting* technique [7] is used to mitigate phase ambiguity in $\bar{\mathbf{h}}_{r,n-1}^{(q)}$.

5. SIMULATION RESULTS

The performance of the proposed algorithm was evaluated via simulations consisting of 200 independent Monte Carlo runs. In each realization, we computed the mean bit error rate (BER) as a function of E_B/N_0, transmitting a random sequence of 300 i.i.d bits, with the first 150 bits discarded to allow for convergence. For comparison, we ran with the same setup the previous ADPF and DcPF-II algorithms from ref. [3]. The simulated communication system has $R = 2$ receivers and the filters employed $Q = 300$ particles. All algorithms perform (synchronized) residual resampling [4] at all iterations. The transmission channels $\mathbf{h}_r$ have $L = 3$ coefficients, and were obtained by sampling independently in each realization and for each receiver from a complex Gaussian pdf $\mathcal{N}(\mathbf{0}; \Lambda)$, $\Lambda = \text{diag}(2, 1, 0.5)$, and normalized so that $\|\mathbf{h}_r\|^2 = 1$. The noise variances were determined as $\sigma^2 = \|\mathbf{h}_r\|^2 N_0/E_B$. The model hyperparameters were set to $\alpha = 3$, $\beta = 0.1$ and $\epsilon = 1$.

The results are displayed in Fig. 1. For comparison, we also show in Fig. 1 the mean BER when the receivers operate independently and do not cooperate to improve their local signal estimate. The mean BER for the centralized Forward-Backward algorithm with perfect knowledge of the channel parameters is shown as a lower bound to performance. As one may observe, the cooperative algorithms (DcPF-II, ADPF and VB-ADPF) outperform the isolated receivers. The VB-ADPF performance gap relative to DcPF-II at low noise levels is less than that observed for the previous ADPF algorithm from [3], indicating the better quality of the VB parametric approximations compared to the ad hoc approach employed in [3].

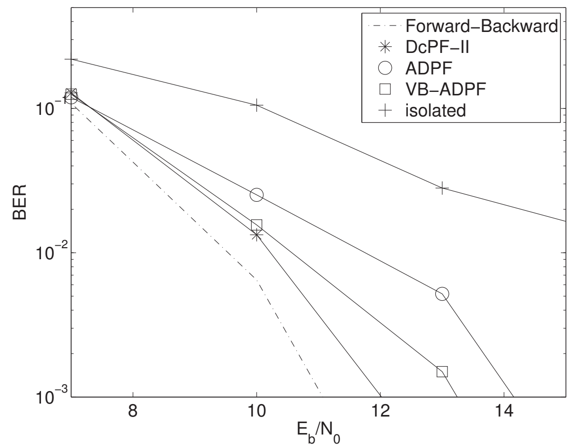

Fig. 1. Mean bit error rate (BER) estimated in 200 independent runs.

6. CONCLUSIONS

We introduced in this paper a novel cooperative particle filter (PF) algorithm for *blind* equalization of frequency-selective broadcast channels. The proposed algorithm relies on parametric approximations to reduce the inter-node communication burden associated with traditional distributed PF methods. Specifically, in the simulated example shown in the paper, data broadcast requirements were reduced from $2Q = 600$ real numbers per node per bit (DcPF-II algorithm [3]) to $2^L = 8$ real numbers per node per bit, while maintaining roughly the same computational cost as the communication-intensive DcPF-II scheme. By replacing the ad hoc parametric approximations in [3] with the more principled VB approach, we were able in our numerical simulations to reduce the performance gap relative to the theoretically optimal DcPF-II algorithm by a margin of roughly 1 dB in low-to-medium SNR's. However, as future work, the asymptotic convergence of the proposed VB-ADPF algorithm still needs to be proven analytically.

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