On the Modeling and Optimization of Discontinuous Network Congestion Control Systems

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Abstract—**AIMD is a widely-used network congestion control scheme. Despite its discontinuous control behavior, the majority of contemporary literature employed a statistically-averaged continuous model to approximate AIMD without considering its discontinuity. The design of a discontinuous control system must be based on rules that are entirely different from that of continuous control systems. Ignoring discontinuity issues results in great discrepancy between analytical models and the practice. In this paper we use the sliding mode control (SMC) theory to investigate congestion control without ignoring its discontinuity. Based on the SMC theory, the design of discontinuous (congestion) control systems must consider the relative degree and zero dynamics of the system, in order to guarantee asymptotic stability. This framework can precisely reflect the behavior of the control rules and the controlled objective of a congestion control system.**

We show that the relative degree of the control system of rate-based, AIMD flow-control algorithms is two. That is, to apply sound control principles to the design of AIMD algorithms, one should use both the queue length error and its first order time derivative to construct the switching function of the control model of an active queue management scheme. Based on the SMC model, one can quantify the tradeoffs among the convergence speed, the amount of throttling adjustments, and the degree of oscillations. We show quantitatively that one can guarantee stability conditions, drastically reduce oscillation of AIMD without significant loss of fairness and stability, and quantitative understanding of the tradeoffs among oscillation, delay and fairness.

Keywords— congestion control; AIMD; discontinuous control; relative degree; asymptotical stability

I. Introduction

Congestion control of persistent transport sessions is critical to the overall Internet performance. In this paper, we study the system dynamics of the rate-based *additive-increase-multiplicative-decrease* (AIMD) scheme. From the analysis we generated a design space to make a performance tradeoff without compromising the system stability. Although our study focuses on the rate-based congestion control systems, our method is applicable to other congestion control schemes that satisfy the basic assumptions of the discontinuous control models. Using well-established control theories, our analysis makes quantitative design tradeoffs between complex system factors based on the dynamics of the network resources and control algorithms. Our results show that the proposed approach makes substantial improvement of the system performance.

A key issue in AIMD congestion control systems is the discontinuity of control. The control switches with some measured performance indicator. For instance, when the congestion bit (e.g., ECN) is set, AIMD halves the sending rate; otherwise, it increases the sending rate linearly. When the ECN bit is used in *active queue management* (AQM) schemes to mark the traffic conditions, i.e., *congestion* or *not*, *packet loss* or *not*, etc., such decisions are discrete and discontinuous. When an *error function* (also called the *switching function* in the rest of the paper) based on some measured performance indicator, is negative (positive), then a positive (negative) control action is engaged.

Three critical issues related to the system dynamics of congestion control schemes are (1) timing of setting the ECN bit, (2) construction of the switching functions, and (3) the amount of control adjustments, i.e., the increase or decrease of the transmission rate. Many AQM schemes investigated the issue of when and how to set the ECN bit, especially on how to optimize the congestion window sizes of TCP sessions. For instance, RED [1][6], BLUE [4], proportional-integral (PI) controller [5], proportional-differential (PD) controller [34] [35], AVQ [10], R-SMVS (a *sliding mode variable structure control* scheme proposed by Ren *et al.* in [13]), several versions of RED [14]-[18], and the "binary feedback" scheme proposed by Ramakrishnan and Jain (called RJBF in remaining discussion) [19]. They are designed to work with the AIMD scheme [20] of TCP Tahoe/Reno.

The second of the three issues, construction of switching functions, is the focus of congestion avoidance schemes, such as AIMD and binomial control [12]. Existing literatures just simply define the switching function as one measurement minus its desired value. Based on the SMC theory, construction of switching functions must consider the relative degree and zero dynamics of the system, in order to guarantee asymptotic stability. Zhang and Shin [26] proposed an α-control scheme with the goal of controlling the maximum queue length to a chosen range without buffer overflow, by adjustments to the increase rate of AIMD. Lee *et al.* proposed AIMD/H (AIMD with history) [27], which updates the decrease ratio of AIMD according to history information to

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smooth rate/window variation. Our study shows that designing algorithms without considering the relative degree of the control system would only have limited effects.

Most AQM and congestion avoidance schemes are closely related to each other. Those AQM schemes not designed to work with AIMD, e.g. REM [3] and GKVQ (Gibbens-Kelly virtual queue) [8][9], are to provide users link pricing information based on marking probability [22][23]. They are not a BDC system and not within the scope of this paper.

AIMD can be modeled as a *binary decision control* (BDC) system. Analysis and optimization of the asymptotic stability and transient behavior of a BDC control system is generally considered a very difficult issue. Due to discontinuity of the control actions, the state trajectory of the control system makes instantaneous changes when the control action switches according to the + and - sign of the switching function. When, and how much, to make the changes need to be carefully designed, so that a transmission session can effectively adapt to current conditions.

Despite its profound importance, the discontinuity in a BDC system is largely overlooked in contemporary AQM schemes that adopted the AIMD for congestion control [1][5][6][10][13]-[18][34][35]. Instead, most of them adopted a nonlinear, continuous fluid dynamics model, such as that of window-based AIMD scheme [7] in the TCP Tahoe/Reno

$$\frac{dW(t)}{dt}=\frac{1}{R(t)}-\frac{W(t)W(t-R(t))}{2R(t)}\rho(t-R(t)), \qquad (1)$$

or a slightly changed version of (1) (e.g. in [34]), where ρ is the packet drop/mark probability, W is the congestion window size and R is the *round trip time* (RTT). Then they linearized (1) within a small neighbor of the desired equilibrium point. Based on the linear model, they used frequency domain (e.g. [1], [5], [6] and [10]) or time domain (e.g. [13] and [34]) method to design AQM controllers.

The control mechanism of a congestion control system based on (1) can be depicted in Fig. 1 (a). Given a control objective q_d, a control scheme such as RED or PI uses q_d and a measured performance indicator $q(t)$ as its inputs to determine the necessary the packet drop/mark probability $\rho(t)$. This takes into account the effects of the dynamics of the congestion control mechanisms (such as AIMD) and the queuing dynamics of the target buffer. In this model, only the statistically-averaged behavior of the system dynamics is considered, and the discontinuity of AIMD is omitted from the design of the control algorithms. Control algorithms that are based on (1), e.g. P (*proportional*), PI, PD or even PID (*proportional-integral-differential*) control schemes, etc., are optimized for the statistical average of the controlled objective, with little or no consideration of the short term behavior of AIMD. As a result, one cannot guarantee the asymptotic system stability. Parameter settings would be subject to recurrent traffic oscillations, and significant packet delays or packet losses in short time windows, even when only a small number of connections share a common path.

In this paper, we propose a modeling approach that incorporates the discontinuity of control actions and the queue dynamics of the congested link without using statistical approximation of the congestion control system. This simpler model is depicted in Fig. 1 (b) which gives accurate account of the transient system behavior.

To illustrate our method's application in the design of congestion control algorithms, we use it to model a rate-based, AIMD congestion control scheme as follows. Letting the control objective be the queue length of the congested link, one can use the output y to represent the difference between the actual queue length and its desired value, the control variable u to represent the changing rate of the source sending rate, and the state variable vector x to represent the queue length and the data sending rate, where x will be defined shortly. Many congestion control algorithms used only a single bit to indicate the sign of the switching function for congestion control. As a result, we will focus on the stability analysis of a single-bit AIMD congestion control scheme. We will then show the applicability of our model to the cases that use more indication bits for congestion control. The modeling technique is also applicable to window-based schemes.

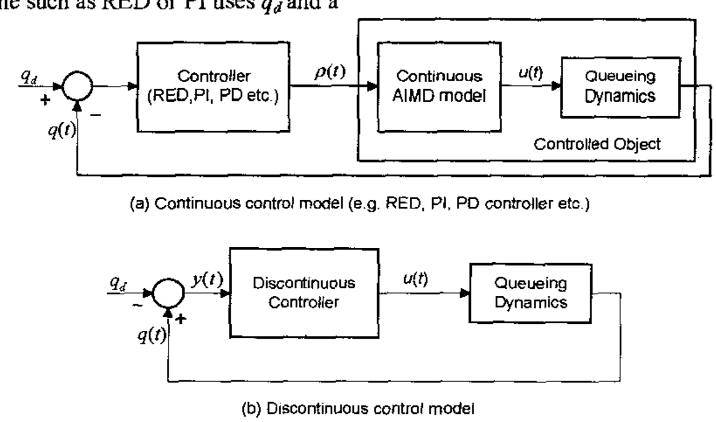

Fig. 1. Structural models of congestion control schemes. $\rho(t)$ is the mark/drop probability, $q(t)$ is the actual queue length and q_d is the desired queue length, $u(t)$ is variant rate of data sending rate or congestion window size.

The rest of this paper is organized as follows. Section II first discusses the conditions under which the design principle of the congestion control schemes can be formalized, based on the sliding mode control rules. Section III presents our main results on rate-based congestion control. Section V provides a theoretical proof of our results. We discuss the transient behavior and delay effects in section VI. Conclusions are made in Section VII.

II. Problem Formulation

In the design of a rate-based AIMD scheme, our objective is to guarantee asymptotic stability, and to optimize the transient dynamics of network traffic flow. In addition to the issue of discontinuity that departs our work from the existing approaches [1][5][6][10][13]-[18][34], our model directly takes into account the non-linearity of the queuing dynamics without linearization approximation.

A major advantage of using BDC to model AIMD congestion control schemes is that it provides a well-defined region of stable operations under different parameter settings while treating the controlled object, i.e., the queuing dynamics of the congested path, as a black box [31]. Therefore, it places little restrictions on the types of AQM management schemes that can employ this modeling technique for performance analysis, even in the presence of uncontrolled routers.

In the control theory literature, BDC can be considered a *variable structure control* (VSC) system, because its control rule switches with the (+ and -) sign of the error (switching) function, *i.e.*, $y(t)$ in Fig. 1. The objective of a BDC system is to drive the error function to zero, i.e., the control objective is at its optimal value, via switching of the control rules. By using the output-based *sliding mode control* (SMC) [23] theory as the basis of the congestion control rules, we can cope with parameter and model uncertainties, which are called *structured* and *unstructured uncertainties*, respectively, in control theory literature. The robustness analysis is simple, and accurate characterization of the discontinuity property has led to significant improvement on the performance of the rate-based congestion control schemes in our study.

The basic idea of the SMC theory is to drive the system state to a subset of the state space, i.e., the manifold (called *switching manifold* or *sliding mode*) defined by $S = 0$, where S is the switching function, in finite time. The system's state trajectory then "slides" along the dynamics defined by the switching mode to the desired equilibrium state, provided that the sliding mode dynamics is asymptotically stable.

It is known that within the switching manifold, the control system dynamics is reduced to the sliding mode dynamics [24]. The SMC theory allows one to simplify the control system design into a "zero-keeping" problem of the switching function [25]. (That is, trying to move the value of the switching function to zero.) We will take advantage of this property to analyze the rate-based AIMD algorithm.

The necessary and sufficient condition [25] for stability of SMC is that the sliding mode is asymptotically stable, and

$$S\dot{S} \le -\eta|S|, \tag{2}$$

where $\eta>0$, and S is the switching function.

Eq. (2) is called the *sliding condition* [25] and guarantees existence of the sliding mode. The physical meaning of (2) is that when $\frac{d}{dt}S^2 < 0$, the distance to the switching manifold monotonically decreases, in order to bring the control objective closer to its target value. Furthermore, the state trajectory will reach the switching manifold within finite time $\frac{|S_0|}{\eta}$, where S_0 is the initial value of S. η determines the convergence rate to the switching manifold [25].

Given these two conditions, a dynamic system defined by

$$\begin{cases} \dot{x} = f(x,u,t) \\ y = h(x,t) \end{cases}, \tag{3}$$

can be stabilized by a properly-designed BDC controller [30], where t is the time variable, $x \in R^n, u, y \in R$, f and h are some unknown smooth function. Referring to Fig. 1 (b), x stands for the state variable of the queuing dynamics, y the difference between $q(t)$ and q_d.

The overall control rules of the BDC controller are summarized as follows,

$$u = \begin{cases} u^-(x,t), \text{ if } S < 0 \\ u^+(x,t), \text{ if } S > 0 \end{cases}, \tag{4}$$

where

$$S = y + \sum_{i=1}^{r-1} c_i \frac{d^i y}{dt^i}, \tag{5}$$

$u^-(x, t)$ and $u^+(x, t)$ are the control rules of the BDC controller when the sign of S is $-$ and $+$, respectively. To guarantee overall system asymptotical stability, i.e., the controlled object (3) together with the controller (4), we must meet the following conditions [30]:

I) The system's relative degree with respect to y is r;
II) The *zero dynamics* of the system defined by (3) with output y is asymptotically stable, i.e., it is minimum phase when the output is y;
III) $c_{r-1}p^{r-1} + c_{r-2}p^{r-2} + \ldots + c_1 p + 1$ is a Hurwitz polynomial.

Informally, the *relative degree* of the system can be derived by taking derivatives of y until the control variable u appears in the right-hand side of the equation, without being encapsulated in any other function.[2] For a nonlinear system like ours, its zero dynamics is equivalent to the role of *zeros* for a linear system.

[2] The reader is referred to contemporary nonlinear control literature for its formal mathematical definition.

Condition I) is equivalent to the condition that the relative degree of the system defined in (3) is one, with respect to S defined in (5). When condition III) holds, condition II) is equivalent to the condition that the zero dynamics with output S defined in (5) is asymptotically stable, or, it is *minimum phase* with output S. In fact, the zero dynamics with output S defined in (5), which is exactly the dynamics of the sliding mode, is the combination of two elements [24][30]: the *zero dynamics* with output y, and

$$S = y + \sum_{i=1}^{r-1} c_i \frac{d^i y}{dt^i} = 0 \cdot \quad (6)$$

Condition III) guarantees the differential equation (6) is asymptotically stable. We further note that c_n, c_{n-1}, ... c_1 are manually determined. $u^-(x, t)$ and $u^+(x, t)$ determine the range of the attractive zone of the sliding mode, so that within the attractive zone, the control rules can bring the state trajectory back to the switching manifold. Their values are proportional to the speed of state trajectory changes. When the system has non-negligible delays, measurement errors, or finite control frequency, large control values will aggravate the degree of oscillation that are caused by these factors. In the worst case, the control stability may become divergent. The quantitative tradeoff between these factors is essential to the optimal design of the congestion control algorithms.

III. MAIN RESULTS

On the basis of the sliding-mode control theory, and a BDC based control behavior model, we have developed the analytical modeling of the rate-based AIMD scheme, and have verified the results via extensive ns-2 simulation. We summarize the main results in this section, and will discuss their analytical details in next section.

A. Relative Degree of Rate-Based Congestion Control

Our first assessment is that the relative degree of the rate-based congestion control system is two (see Theorem 1 for details) when the control objective is queue length. Both theoretical and empirical results show that correct incorporation of the relative degree of the control model has predominant impact on the performance of AIMD. Other factors, such as increment and decrement values, and delay have relatively minor performance effects.

The result in the relative degree issue is derived from the law of physics of the rate-based control systems, and thus, all rate-based AIMD congestion control algorithms should take into account this factor, regardless of details of their design. To single out the effects of relative degree on the system performance, we focus on the relationship between relative degree and the asymptotic stability, without considering the transient on-set behavior of a new session. In the experiments, we tested the effects of relative degree by using two different switching functions,

SF1: $$S = y, \quad (7)$$

and

SF2: $$S = y + c_1 \dot{y}, \quad (8)$$

where $y = q - q_d$. q is the actual queue length and q_d is the target queue length. Eqs. (7) and (8) represent feedback control schemes that have relative degrees of one and zero, respectively. The sender keeps sending data at the current rate until it receives a single-bit ECN feedback from the router via an ACK packet. It corresponds to the + and - signs of its switching function, and the sender adjusts its sending rate according to the congestion bit value in the ACK packet. The source linearly increases its sending rate with rate $\alpha(>0)$ if the congestion bit is zero; otherwise, the sending rate is decreased exponentially with time constant $\beta(>0)$. This particular AIMD setting satisfies the constraints of the controller defined by (4) and (5), and thus, their properties are applicable to the analysis of our system. In our simulation, the source does not retransmit lost packets, and it adjusts the transmission rates only according to the value of the ECN bit. Retransmission has no effects on the relative degree properties.

In this ns-2 simulation, we assume that a single link of capacity 10Mbps is shared by fifteen connections with the roundtrip time ranging from 40 ms to 200 ms. The fifteen connections started randomly within the first 0.1 second. We also assume that packets have an average size of 1000 bytes, the buffer size being 100 packets. We set q_d =50 packets as our control goal. Because we are mainly interested in the recurrent behavior of AIMD, we simply initialized the sending rate of each sources as 650 Kbps and set α=10 pkt/sec^2, β=8.3 sec, without considering the on-set phase of a session. We always set c_1=1 unless stated otherwise explicitly. This configuration satisfies the condition that $\alpha\beta=C/N$, where C is the link capacity. This means that, around the equilibrium point, the increase rate is equal to the decrease rate [29].

The queue lengths of the target buffer simulated by ns-2 by using SF1 and SF2 are plotted in Fig. 2. The degree of oscillation of the queue length for SF2 is much smaller than that of SF1, clearly indicating the significance of the relative degree. We note that statistical average of the queue lengths is a very poor performance indicator, because in these two examples, the average queue lengths of the two scenarios are very close to each other. Nevertheless, SF2 can provide much better delay quality than SF1. Repeated and consistent results from numerical and ns-2 simulations showed that oscillation was drastically reduced when the relative degree was taken into account.

To gain insight into why SF2 performs much better than SF1, we take a closer look at their system state trajectories. As mentioned earlier, a robust switching function should be able to bring the state trajectory as close to the switching manifold as possible. Referring to Fig. 3, the switching manifold is the straight dotted line connecting coordinates (0, 50) and (50, 0). The state trajectory of SF2 did chatter around the switching manifold, but stayed at its close proximity. Chattering was caused by delay, non-zero control period and measurement error, etc. because these aspects result in that control switching does not precisely occur on the switching manifold.

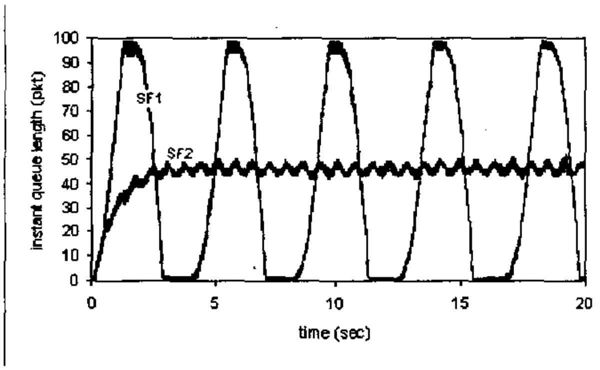

Fig. 2. Instant queue length traces based on switching functions SF1 and SF2.

For SF1, as depicted in Fig. 4, where the switching manifold of SF1 is the vertical dotted line passing the coordinate (50, 0), the state trajectory is a relatively much large circle that is not even close to the switching manifold, making SF1 vulnerable to oscillations. From the design viewpoint, the results suggest that without taking into account the relative degree, it will be very difficult to contain the control behavior of the system to a certain range of the target utility function, not to mention any notion of performance guarantee.

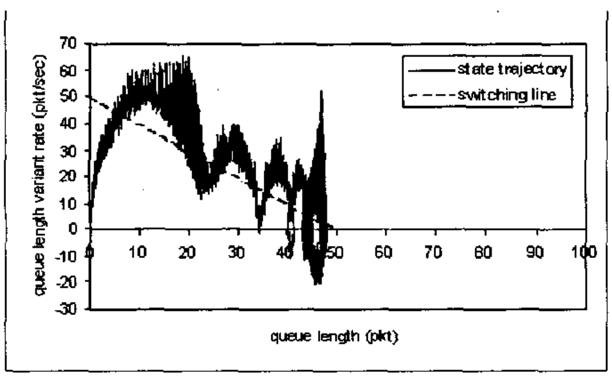

Fig. 3. State trajectory for SF2 with c_1=1.

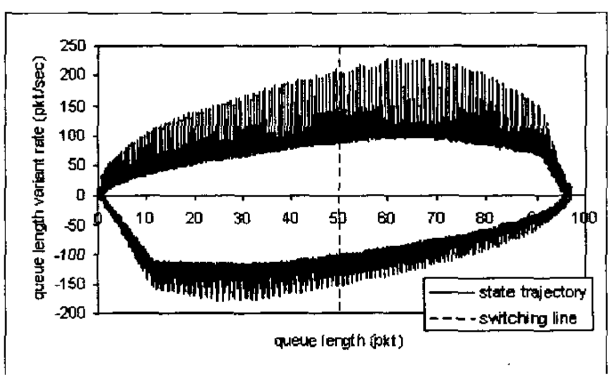

Fig. 4. State trajectory for SF1

Next, we compared the performance results of using different α and β values, with and without incorporating the relative degree in the switching function. The round trip transmission time is set to be less than 10^{-6} sec to rule out the effects of delays. Simulation results with different α and β values in SF1 are plotted in Fig. 5. It is clear that for SF1 the degree of oscillation in using different parameter configurations is very close to each other, despite the significant differences in their phases.

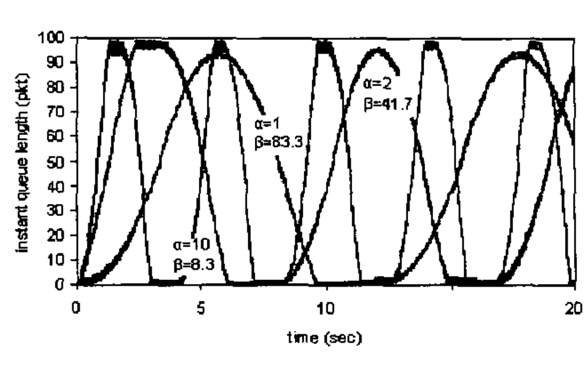

Fig. 5. Traces of queue lengths for SF1 with different α and β values.

We must caution that in addition to correct consideration of the relative degree in the switching function, both α and β values need to be properly assigned to prevent oscillation. For the SF2 example in Fig. 6, one can see that if α is too large and β too small, the system will still be subject to oscillation.

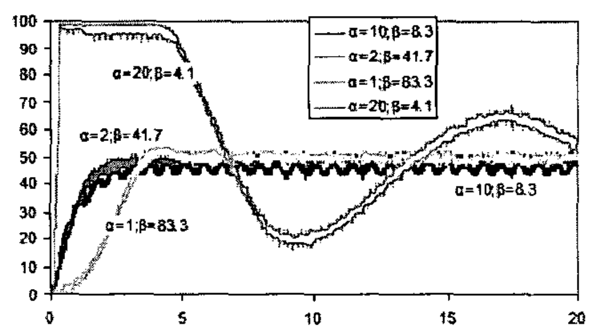

Fig. 6. Traces of queue lengths for SF2 with different α and β values.

IV. Minimum Phase of the Queuing Dynamics

An output-based SMC system is asymptotically stable only if the original system's zero dynamics is asymptotically stable, i.e., the original system is minimum-phase. The following simulation results give a simple and quick checking of the minimum phase property. Let S denote the system output, referring to (8) the output becomes unstable when c_1 is negative, implying that the system becomes non-minimum phase. The outputs remain stable when the value of c_1 remains positive.

AIMD is an *asymmetric controller*, because the amounts of control (linear rate) for the + value of the switching function is smaller than that (exponential rate) of the - value of the switching function. This average behavior is clearly depicted in an asymptotically stable system. Referring to Fig. 7, one can see that even when we set the target value at 50, and when the system was stable, the queue length could never reach the target value.

On the other hand, when we changed the AIMD rule to *additive increase* and *additive decrease* (AIAD), with identical increase/decrease amplitudes, the system reached the target value, see Fig. 8. The steady-state errors caused by asymmetry of the increment and decrement amounts only occur to the cases with non-ideal control switching. Here by non-ideal we mean the control switching takes place not exactly on the switching manifold. This may be caused by delay, measurement error, etc.

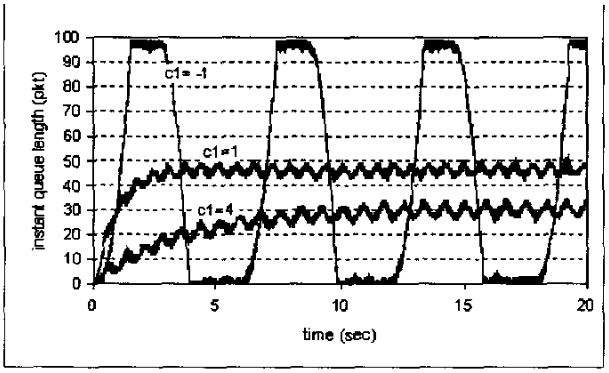

Fig. 7. Instant queue length traces for SF2 with different c_1.

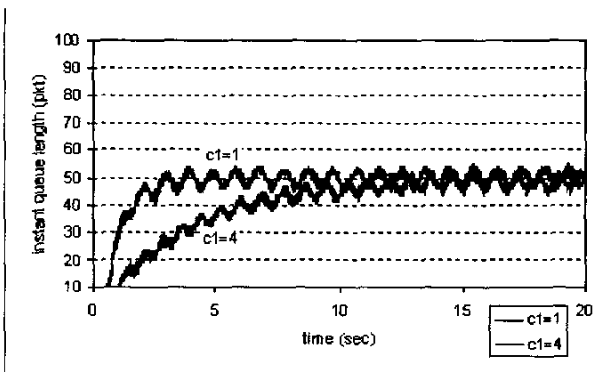

Fig. 8. The steady state of the AIAD controller

V. DYNAMICS ANANYSIS

In this section, we analyze the AIMD systems based on the SMC control theory, to prove their asymptotic stability conditions, and other related issues.

A. Network Model

We assume that the bottleneck link is shared by N sources. Let q denote the queue length of the link, C link capacity, λ_i the sending rate of source i, ζ the total input rate to the link, $\zeta(t)=\sum_{i=1}^{N}\lambda_i(t-\tau_{Fi})$, τ_{Fi} the (outbound) delay from source i to the link. We get

$$\dot{q}(t)=-CG(q(t),\zeta(t))+\zeta(t)+\hat{C}(t), \tag{9}$$

where $\hat{C}(t)$ represents non-responsive traffic, and $G(\cdot,\cdot)$ the instantaneous link utilization. In a general sense, we assume that $G(\cdot,\cdot)$ is a functional of q and ζ, so that it can represent any bandwidth allocation schemes on the router that make adaptive allocation decisions based on current queue length and input rate.

Obviously, $G(q,\zeta)\in(0,1)$ for $q>0$ and $\zeta>0$. For mathematical correctness, we further assume that $G(q,\zeta)$ are twice differentiable with respect to q and ζ. In a real system, any change in ζ should lead to consistent changes in $\dot{q}$. That is, $\frac{\partial\dot{q}}{\partial\zeta}>0$ should hold for real systems. By plugging this condition into (9), it is easy to get:

$$\frac{\partial G}{\partial\zeta}<\frac{1}{C}. \tag{10}$$

The control system design aims to drive $y=q-q_d$ to zero through adjustment of λ_i. To reduce chattering of λ_i, we adjust $\dot{\lambda}_i$ in the same way as the existing AIMD scheme. We introduce new control variables $u_i\underline{\underline{\Delta}}\dot{\lambda}_i$ and get

$$\dot{\zeta}(t)=\sum_{i=1}^{N}u_i(t-\tau_{Fi}). \tag{11}$$

On the basis of the network analytical model defined above, we present the proof of the system properties discussed so far in the following subsection.

B. System Properties

The first property that we want to address is the relative degree of rate-based congestion control schemes.

Theorem 1: The relative degree of the system defined by Eqs. (9) and (11) is two, when the output is y and control variable is u_i, respectively.

Proof: It is easy to verify that $\frac{\partial}{\partial u_i}\dot{y}=0$ and $\frac{\partial}{\partial u_i}\ddot{y}>0$. This means that control input u_i explicitly appears in $\ddot{y}$ but not in $\dot{y}$. We just proved the theorem based on the definition of the relative degree. ∎

Recall that we have drawn the same conclusion in the simulation that the relative degree of rate-based AIMD is two. According to conditions I) and III) in Section II, we now choose $S = y + c_1\dot{y}$, $c_1>0$, as the switching function for the rate-based congestion control system.

Next, we consider the stability of the sliding mode. The dynamics of the sliding mode is just the zero dynamics when the output is the switching function [24][30].

Theorem 2: The zero dynamics of the system defined by Eqs. (9) and (11) is asymptotically stable when the output is $S = y + c_1\dot{y}$, $c_1>0$.

Proof: We define a transform

$$\begin{cases} z_1 = S \\ z_2 = y \end{cases}. \tag{12}$$

Its Jacobi matrix

$$\begin{bmatrix} \frac{\partial z_1}{\partial q} & \frac{\partial z_1}{\partial \zeta} \\ \frac{\partial z_2}{\partial q} & \frac{\partial z_2}{\partial \zeta} \end{bmatrix} = \begin{bmatrix} 1 - c_1 C\frac{\partial G}{\partial q} & c_1 - c_1 C\frac{\partial G}{\partial \zeta} \\ 1 & 0 \end{bmatrix} \tag{13}$$

is nonsingular because $\frac{\partial G}{\partial \zeta} \neq \frac{1}{C}$ for arbitrary q and ζ. Thus, transform (12) is a global *diffeomorphism*. Applying (12) to (9) and (11), we get

$$\dot{S} = \dot{z}_1 = (1 - c_1 C\frac{\partial G}{\partial q})\dot{q} + c_1(1 - C\frac{\partial G}{\partial \zeta})\zeta + c_1\dot{\hat{C}}, \tag{14}$$

$$\dot{z}_2 = \frac{z_1 - z_2}{c_1}. \tag{15}$$

Substituting $z_1=0$ into (15), we conclude that, when the output is $S = y + c_1\dot{y}$, the zero dynamics of the system defined by (9) and (11) is expressed as

$$\dot{z}_2 = \frac{-z_2}{c_1}. \tag{16}$$

Obviously (16) is asymptotically stable for $c_1>0$. ∎

Theorem 2 is equivalent to the condition that, the zero dynamics of the system defined by (9) and (11) is asymptotically stable when the output is y.

C. AIMD Parameter Setting

There are three parameters to configure an AIMD controller with the switching function SF2: c_1, α and β. First, we consider c_1, which determines the dynamics of the sliding mode. Generally, we set $c_1=1$ empirically. If this value is too large, there will be a significant damping effect on the sliding mode. If too small, it will be necessary to increase (decrease) sharply α (β), in order to guarantee stability, i.e., see (21) and (22). Both alternatives will run into implementation problems.

Next, we discuss how to determine α and β values in AIMD. By definition, the AIMD adjustment rules can be expressed as

$$u_i(t) = \begin{cases} \alpha, \text{ if } S(t - \tau_{Bi}) < 0 \\ -\frac{\lambda_i(t)}{\beta}, \text{ else} \end{cases}, \tag{17}$$

where τ_{Bi} is the backward delay from the link to source i. In selecting the α and β values, one needs to ensure that the system (including the AIMD controller) reaches the asymptotical stability in the shortest time.

For practicality, our goal is to derive a simple design rule for selection of α and β without overly complicating the analysis. First, we consider the case of homogeneous sources. That is,

$$\zeta \equiv N\lambda_i. \tag{18}$$

It will become clear from simulation results that the impact of delay is relatively small, and thus we omit the delay terms in the subsequent discussion. To satisfy the sliding condition (2), we discuss the following cases:

1) When $S < 0$, $\dot{S} > \eta$. From (14), (10), (17) and (18), we have

$$\alpha > \frac{\eta - (1 - c_1 C\frac{\partial G}{\partial q})\dot{q} - c_1\dot{\hat{C}}}{(1 - C\frac{\partial G}{\partial \zeta})c_1 N}. \tag{19}$$

2) Similarly, when $S > 0$, $\dot{S} < -\eta$. We have

$$\frac{1}{\beta} > \frac{\eta + (1 - c_1 C\frac{\partial G}{\partial q})\dot{q} + c_1\dot{\hat{C}}}{(1 - C\frac{\partial G}{\partial \zeta})c_1\zeta}. \tag{20}$$

where $\eta>0$. (19) and (20) together guarantee the existence of the sliding mode. That is, the state trajectory will reach $S = y + c_1\dot{y} = 0$ within time $\frac{|S_0|}{\eta}$, where S_0 is the initial value of S. This condition gives a clear guideline on how long one needs to wait before the system can reach its sliding mode, given the initial state. Based on Theorem 2, the asymptotical stability of the sliding mode is guaranteed. Given the above, we directly arrive at the following theorem.

Theorem 3: Under the AIMD control law (17) with a switching function $S = y + c_1\dot{y}$, $c_1>0$, the queuing dynamic system defined by (9) and (11) is asymptotical-ly stable if (19) and (20) holds.

Note that the stability conditions (19) and (20) are inequalities. They define the attractive zone of the switching manifold $S = y + c_1\dot{y} = 0$. Within the attractive zone, $|S|$ monotonically decreases until reaching the switching manifold. Increasing α and $1/\beta$ values will increase the attractive zone, and also increase the rate of convergence to the sliding mode (see (14)). However, due to delay, measurement error, etc., the switching between increase and decrease cannot take place exactly on the switching manifold. The level of chattering is proportional to the values of α and $1/\beta$ for the case with non-ideal increase-decrease switching.

Theorem 4: Given that a desired attractive zone is Z_A and disturbance $\hat{C}$ is bound within D, we should choose the following α and $1/\beta$ values to guarantee that the switching manifold is attractive within Z_A:

$$\alpha = \sup_{(q,\zeta)\in Z_A}\left[\frac{\eta-(1-c_1C\frac{\partial G}{\partial q})\dot{q}+c_1\dot{\hat{C}}}{(1-C\frac{\partial G}{\partial \zeta})c_1N}\right], \quad (21)$$

$$\frac{1}{\beta} = \sup_{(q,\zeta)\in Z_A}\left[\frac{\eta+(1-c_1C\frac{\partial G}{\partial q})\dot{q}+c_1\dot{\hat{C}}}{(1-C\frac{\partial G}{\partial \zeta})c_1\zeta}\right]. \quad (22)$$

By (19) and (20), it is trivial to prove that these conditions hold. For heavily loaded cases, $G \cong 1$, (21) and (22) can be simplified into

$$\alpha = \sup\left[\frac{\eta-\zeta+C-\hat{C}-c_1\dot{\hat{C}}}{c_1N}\right], \quad (23)$$

$$\frac{1}{\beta} = \sup\left[\frac{\eta+\zeta-C+\hat{C}+c_1\dot{\hat{C}}}{c_1\zeta}\right]. \quad (24)$$

When the disturbance is slowly-changing ($\dot{\hat{C}} \cong 0$), and the required attractive zone of the switching manifold is defined by $\zeta \in [k_1C, k_2C]$, where $0<k_1<1<k_2$, and $\hat{C} \in [0, k_3C]$, $0<k_3<1$, we could ignore $\dot{\hat{C}}$ and simplify (23), and (24), into the following forms:

$$\alpha \cong \frac{\eta+(1-k_1)C}{c_1N}, \quad (25)$$

$$\beta \cong c_1\left[1+\frac{k_3}{k_1}-\frac{1}{k_2}+\frac{1}{k_1}\frac{\eta}{C}\right]^{-1}. \quad (26)$$

At the equilibrium point, when the system remains stable, it would be useful to keep the increase rate the same as the decrease rate [29]. In that case, we can set

$$\alpha\beta \cong \frac{C}{N}. \quad (27)$$

If this constraint is adopted, then one of the two equations (25) and (26) can be omitted, yet the conditions guaranteed by the omitted term will become void.

Given the theoretical bounds derived above, one still needs to exercise caution in physical design, to make sure that the degree of oscillation within the guaranteed zone is acceptable. For instance, the α and $1/\beta$ values used in the example of Fig. 6 are smaller than the bounds defined in (23) and (24). Even though the system eventually converges, the level of chattering is quite visible.

VI. Transient Behavior and Delay Effects

η determines the rate of convergence toward the switching manifold. On the switching manifold, the system dynamics is dominated by $S = y + c_1\dot{y} = 0$. To reduce the system response time, we prefer a larger η, or equivalently, larger α and β^{-1}. On the other hand, in order to reduce the chattering caused by delay, we need to decrease the values of α and β^{-1}. Clearly, one needs to choose values for α and β^{-1} to balance the two competing factors. A tradeoff technique is to use the *boundary layer* method [24][32][33]. It has been proven that this method can contain the chattering in a bounded range. The basic idea of the boundary layer method is to suppress chattering by smoothing the discontinuity of the control within a neighborhood of the switching manifold. It is based on an observation that the control variable should be reduced with S approaching to zero. As such, we adopt the following boundary layer control law as follows:

$$u_i = \begin{cases} \alpha, S<-\sigma \\ -\alpha\frac{S}{\sigma}, -\sigma \le S < 0 \\ -\frac{\lambda_i S}{\beta\sigma}, 0 \le S \le \sigma \\ -\frac{\lambda_i}{\beta}, S>\sigma \end{cases}, \quad (28)$$

where $\sigma(>0)$ is the boundary layer width. By experience, we choose σ to be equal to one quarter of the buffer size. If it is too small, the performance effect is not obvious. But if too large, it tends to reduce the sensitivity of the control. α and β are determined based on (25) and (26). To implement (28), it must use explicit feedback messages that carry the numerical values of S to the sources for execution. Although this method may improve the performance, its cost is significant. To reduce the implementation cost, one can reduce the amount of feedback information, but using more than one bit to indicate the congestion states. For example, if the system can use only two bits for signaling of congestion conditions, (28) can be simplified as:

$$u_i = \begin{cases} \alpha, S<-\sigma \\ 0, 0 \le S \le \sigma \\ -\frac{\lambda_i}{\beta}, S>\sigma \end{cases}. \quad (29)$$

An alternative solution to this problem is to make the two coefficients α and β adaptive to system states. In an ideal system, (19) and (20) can be reduced to $\alpha > \frac{\eta - \zeta + C}{c_1 N}$ and $\frac{1}{\beta} > \frac{\eta + \zeta - C}{c_1 \zeta}$. Within these two bounds for them, one can create adaptive adjustments to α and β. For instance, one can take the difference between the inbound traffic and outbound traffic to determine the amounts of adjustments to the two values, based on selected optimization criteria.

VII. CONCLUSION

In this paper, we proved that the relative degree of the rate-based AIMD congestion control system is two and its zero dynamics is asymptotically stable. It is noted that without considering these two factors, heuristic attempts to optimize AIMD or similar rate-based congestion control schemes have only marginal effects.

Using the sliding mode control theory, we developed a systematic design principle, in which the complex interplay between different system parameters can be governed by a few simple equations and conditions. Our method permits one to make tradeoffs between the amount of feedback information, delay, and the expected performance outcomes, under explicitly defined conditions. There is little restriction on the system model, making this framework broadly applicable to a wide range of congestion control algorithms.

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REFERENCES

[1] S. Floyd, V. Jacobson, "Random early detection gateways for congestion avoidance," *IEEE/ACM Transactions on Networking*, August 1993.

[2] S. Floyd, "TCP and explicit congestion notification," *ACM Computer Communication Review*, vol. 24, pp. 10-23, October 1994.

[3] S. Athuraliya, D.E. Lapsley, S.H. Low, "Random early marking for Internet congestion control," *IEEE/ACM Transactions on Networking*, vol. 15, no. 3, pp. 48-53, May/June 2001.

[4] W. Feng, D. Kandlur, D. Saha, K. Shin, "Blue: A new class of active queue management algorithms," *IEEE/ACM Transactions on Networking*, August 2002.

[5] C.V. Hollot, V. Misra, D. Towsley, W. Gong, "On designing improved controllers for AQM routers supporting TCP flows," *IEEE INFOCOM 2001*, Alaska, Anchorage, April 2001.

[6] C.V. Hollot, V. Misra, D. Towsley, W. Gong, "A control theoretic analysis of RED," *IEEE INFOCOM 2001*, Alaska, Anchorage, April 2001.

[7] V. Misra, W.B. Gong, D. Towsley, "Fluid-based Analysis of a Network of AQM Routers Supporting TCP Flows with an Application on RED," *ACM/SIGCOMM 2000*.

[8] R.J. Gibbens, F.P. Kelly, "Distributed connection acceptance control for a connectionless network," *Proceeding of the 16th International Teletraffic Congress*, Edinburgh, Scotland, June 1999.

[9] R.J. Gibbens, F.P. Kelly, "Resource pricing and the evolution of congestion control," *Automatica* 35 (1999), pp. 1969-1985.

[10] S. Kunniyur, R. Srikant, "Analysis and design of an adaptive virtual queue (AVQ) algorithm for active queue management," *ACM SIGCOMM 2001*.

[11] D. Chiu, R. Jain, "Analysis of the Increase/Decrease Algorithms for Congestion Avoidance in Computer Networks," *Journal of Computer Networks and ISDN*, vol. 17, no. 1, pp. 1-14, June 1989.

[12] D. Bansal, H. Balakrishnan, "Binomial Congestion Control Algorithms," *IEEE INFOCOM 2001*.

[13] F. Ren, C. Lin, X. Ying, X. Shan, F. Wang, "A robust Active queue management algorithm based on sliding mode variable structure control," *IEEE INFOCOM 2002*.

[14] T.J. Ott, T.V Lakshman, L.H. Wong, "SRED: Stabilized RED," *IEEE INFOCOM 1999*.

[15] D. Lin, R. Morris, "Dynamics of Random Early Detection," *ACM SIGCOMM 1997*.

[16] W. Feng, D. Kandlur, D. Saha, K. Shin, "A Self-Configuring RED Gateway", *IEEE INFOCOM 1999*, March 1999.

[17] M. May, T. Bonald, T. Bolot, "Analytic Evaluation of RED Performance," *IEEE INFOCOM 2000*, March 2000.

[18] M. Christiansen, K. Jeffay, D. Ott, F.D. Smith, "Tuning RED for Web Traffic," *ACM SIGCOMM 2000*, August 2000.

[19] K.K. Ramakrishnan, Raj Jain, "A Binary Feedback Scheme for Congestion Avoidance in Computer Networks with a Connectionless Network Layer," *ACM SIGCOMM 1988*.

[20] V. Jacobson, "Congestion avoidance and control," in *ACM SIGCOMM*, 1988.

[21] L.S. Brakmo, L.L. Peterson, "TCP Vegas: End to end congestion avoidance on a global Internet," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1465–1480, Oct. 1995.

[22] F.P. Kelly, A. Maulloo, and D. Tan, "Rate control for communication networks: Shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, no. 49, pp. 237-252, 1998.

[23] Steven H. Low, David E. Lapsley, "Optimization Flow Control, I: Basic Algorithm and Convergence," *IEEE/ACM Transactions on Networking*, 1999.

[24] V.I. Utkin, "Sliding Modes in Control Optimization," Springer-Verlag Berlin, Heidelberg 1992.

[25] J. E. Slotine, W. Li, *Applied Nonlinear Control*, Prentice Hall, 1991.

[26] X. Zhang, K.G. Shin, "Second-Order Rate-Control Based Transport Protocols", *IEEE ICNP 2001*, Riverside, California, November, 2001.

[27] K.W. Lee, T. Kim, V. Bharghavan, "A Comparison of End-to-End Congestion Control Algorithms: The Case of AIMD and AIPD," *IEEE INFOCOM 2001*.

[28] D. Tan, A. Zakhor, "Real-time Internet video using error resilient scalable compression and TCP-friendly transport protocol," *IEEE/ACM Transactions on Multimedia*, May 1999.

[29] J. Bolot and A. Shankar, "Dynamical behavior of rate-based flow control mechanism," *ACM SIGCOMM Computer Communication Review*, vol. 20, no. 4, pp. 35-49, April 1990.

[30] L. Fridman, A. Levant, "Sliding Modes of Higher Order as a Natural Phenomenon in Control Theory," *Robust Control Via Variable Structure & Lyapunov Techniques* (*Lecture Notes in Control and Information Science*, 217), F. Garafalo, L. Glielmo eds., Springer-Verlag, London, 1996.

[31] A. Levant, "Higher Order Sliding: Differentiation and Black-Box Control", in *Proc. of the IEEE Conference on Decision and Control*, December 2000.

[32] J.A. Burton, A.S.I. Zinober, "Continuous approximation of variable structure control," *Int. J. System Sci.*, vol. 17, no. 6, 1986.

[33] I.E. Slotine, J.A. Coetsee, "Adaptive sliding controller synthesis for nonlinear systems," *Int. J. Control*, vol. 43, no. 6, 1986.

[34] K.B. Kim, Ao Tang, Steven H. Low, "Design of AQM in supporting TCP based on well-known AIMD Model," *IEEE Globecom 2003*.

[35] J.S. Sun, G.R. Chen, K.T. Ko, S. Chan, M. Zukerman, "PD-controller: A new active queue management scheme," *IEEE Globecom 2003*.