Computers and Electrical Engineering 39 (2013) 1587–1600

Contents lists available at SciVerse ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

Efficient feedback scheme based on compressed sensing in MIMO wireless networks ☆

Wei Lu *, Yingzhuang Liu, Desheng Wang

Department of Electronics and Information Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
Division of Optical Network and Intelligent Communication, Wuhan National Laboratory for Optoelectronics, Wuhan 430074, China

A R T I C L E I N F O

Article history:
Available online 7 March 2012

A B S T R A C T

In multiple-input–multiple-output (MIMO) broadcast channel the throughput can be enhanced by channel state information (CSI) feedback, but it is resources and feedback expensive. We propose a compressed sensing (CS) feedback scheme for zero-forcing beamforming (ZFBF) in MIMO broadcast channel, which can reduce the feedback load and resource consumption. The feedback channels are shared and opportunistically accessed by users who are self pre-selected based on their channel vectors' norm and correlation. Multiple measurement vectors (MMV) CS is used for the CSI feedback. Orthogonal matching pursuit and reduced MMV and boost algorithms are applied for CS recovery to get the CSI. Semi-orthogonal user selection and ZFBF are implemented at the base station to achieve spatial multiplexing gain. Both the analog and digital CS feedback schemes are analyzed. Simulations show that the proposed CS feedback has good performances compared with traditional feedback schemes in points of feedback load and throughput due to user self pre-selection algorithm and CS feedback.

1. Introduction

Multi-user multiple-input–multiple-output (MU-MIMO) communication with partial channel state information (CSI) can substantially increase channel capacity while it requires low feedback overhead and is practical in engineering [1–4]. However, it requires that each user reports its CSI, such as channel quality indicator (CQI) and channel direction information (CDI), which is resources expensive. On the other hand, there is a throughput loss because of the feedback inaccuracy induced by the feedback quantization. In order to improve the feedback performance and save the resources, we need to adopt more efficient feedback scheme for the MIMO broadcast system.

Compressed sensing (CS) is a promising technology in signal processing and communications and has attracted lots of researchers' attention. Recently some authors have applied CS technology to the wireless broadcast channel, multiple users access channel and sensor networks to reduce the resource consumption and enhance the accuracy of the transmitted information. A high-precision feedback technique by using sparse approximation and compression called compressive feedback has been proposed in [5], which quantifies the channel vector by the linear combination of several unit vectors in the defined codebook, then the base station can obtain more accurate channel information, since the linear combination can be recovered by CS. In [6] the authors propose a CS based opportunistic feedback protocol for feedback resource reduction in MIMO broadcast channel with random beamforming (RBF), and the users in each beam with better channel quality, which means that user's CQI is above a threshold, will feed back their CQI and will be the candidates in the user set for transmission. Signal

☆ Reviews processed and approved for publication by Editor-in-Chief Dr. Manu Malek.

* Corresponding author at: Department of Electronics and Information Engineering, Huazhong University of Science and Technology, Wuhan 430074, China.

E-mail addresses: lvweiwhut@yahoo.com.cn (W. Lu), liuyz@hust.edu.cn (Y. Liu), dswang@hust.edu.cn (D. Wang).

0045-7906/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compeleceng.2012.02.006

to interference plus noise ratio (SINR) based user self-selection algorithm is proposed in [7], and the channel estimation of selected users are acquired via CS by exploiting the fact that only partial users are selected which is also a kind of sparcity. It should be noticed that analog feedback of the channel gain vector is used in [5–7].

A simple on-off random multiple access scheme is introduced in [8], in which CS can be applied to the multiple user detection. In [9] the authors propose a novel CQI feedback schemes in a wireless Orthogonal Frequency Division Multiplexing (OFDM) system. The CS is used for compressing and recovering the CQI. The orthogonal matching pursuit algorithm and subspace CS algorithm are used for the CQI recovery, and the subspace CS outperform the cosine transform based CQI feedback.

Meanwhile CS has been applied to the sensor networks and can greatly reduce the resource consumption. In [10] the sparse nature of monitored sensor network is exploited and a random access CS scheme is proposed in which the sensors transmit at random to a fusion center. Meanwhile the author has applied the random access CS scheme to the underwater sensor networks [11]. In [12] CS is applied to sensor data gathering for large-scale wireless sensor networks, which can reduce global scale communication cost without introducing intensive computation or complicated transmission control. What's more, the proposed scheme can cope with abnormal sensor readings by using overcomplete hybrid dictionary.

It can be seen that CS can make the wireless communications more efficient in transmission or resource consumption, however, most of the above researches are based on single measurement vector (SMV) CS and recovery algorithms, which have been introduced in [13] and other related literatures. In the MIMO communication, the user selection is based on SNR or SINR which are scalars, and the analysis are with RBF [6,7]. Since multiple-user and multiple-antenna are the essential natures of MIMO broadcast channel, the channel vector of each user is a vector and only partial users are selected and active which means that the channel vectors for all the selected users composite a sparse multiple dimensional vectors. Hence it is straightforward to apply multiple measurement vectors (MMV) CS to the MIMO broadcast channel. MMV CS has been discussed in [14–18]. In these literature and references therein, some conditions for MMV CS and recovery algorithm such as matching pursuit (MP), orthogonal matching pursuit (OMP), reduced MMV and boost algorithm (ReMBo) and convex relaxation, are thoroughly discussed.

In [4] the zero-forcing beamforming (ZFBF) and random vector quantization for MU-MIMO are introduced. Semi-orthogonal user selection (SUS) and greedy selection are illustrated, and the performance bounds are analyzed. In the paper, we propose a CS feedback scheme in ZFBF MU-MIMO broadcast channel with CS MMV algorithm which is different from the previous research with CS SMV and RBF, and the proposed feedback scheme can reduce the feedback resource consumption. A self pre-selection algorithm is based on the channel vector's norm and correlation, which is different from the previous researches based on SNR and SINR. What's more, both the analog and digital CS feedback are analyzed and compared with the quantization feedback of all users. The MMV OMP and ReMBo algorithms are used for the feedback information recovery.

The main contributions of our work are as following:

- A self pre-selection algorithm based on the channel vector's norm and correlation is introduced which can reduce the feedback load and obtain better orthogonality among selected users at high probability.
- The CS MMV is firstly applied to the MIMO channel state information feedback. Both the analog and digital CS MMV feedback are considered, and the recovery rate and accuracy are analyzed.
- The feedback recovery performances of MMV OMP algorithm and ReMBo algorithm with noise are analyzed and simulated. MMV OMP outperforms the ReMBo algorithm.
- The impacts of CS digital/analog feedback on ZFBF are given, and the noise and quantization error on the sum throughput are presented.

The remaining of the paper is organized as follows. Section 2 presents broadcast channel model and feedback channel model. CS MMV background and CS feedback strategy are explained in Section 3. Section 4 evaluates the performances of sum throughput, feedback reduction, CS MMV recovery performance and algorithm complexity. Simulations and conclusion are given in Sections 5 and 6.

2. System model

2.1. MIMO broadcast channel model

In this paper, we focus on the MIMO broadcast channel. The base station is equipped with N_t antennas, and each user has a single antenna. There are $U > N_t$ users in the system. We assume that users are in flat Rayleigh fading, and the channel is assumed to be constant during each transmission period. For the MIMO broadcast channel, we assume that there are N_t users scheduled simultaneously. The received signal at user k is given by

$$y_k = \sqrt{\rho}\mathbf{h}_k\mathbf{x} + z_k \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times N_t}$ is the channel vector with zero mean unit variance i.i.d complex Gaussian entries, $\mathbf{x}$ is the transmitted symbol vector and $E(\mathbf{x}^H\mathbf{x}) = 1$, z_k is the additive noise with zero mean and unit variance. It should be noticed that although Rayleigh fading channel is considered here, the proposed scheme can be directly extended to other channel models such as Rician channel since the proposed scheme does not depend on the channel model. We assume that all the scheduled users are equal power allocated and have the same SNR ρ. The transmitted symbol vector $\mathbf{x}$ is given by

$$\mathbf{x} = \sum_{i \in T} \mathbf{w}_i s_i \tag{2}$$

where s_i and $\mathbf{w}_i$ are the transmitted symbol and beamforming vector for user i, T is the scheduled user set. Then the received symbol for user k is

$$y_k = \sqrt{\rho}\mathbf{h}_k \mathbf{w}_k s_k + \sum_{j \neq k, j \in T} \sqrt{\rho}\mathbf{h}_k \mathbf{w}_j s_j + z_k \tag{3}$$

For the choice of beamforming vectors, it is well know that linear zero-forcing precoding is optimal and simple. The base station will compute the beamforming vectors provided the channel direction information of the scheduled users, which are mutually near-orthogonal. After user set T is selected according to the semi-orthogonal user selection algorithm, define $\hat{\mathbf{H}} = (\hat{\mathbf{h}}_{\pi(1)}, \ldots, \hat{\mathbf{h}}_{\pi(|T|)})$, $\hat{\mathbf{h}}_{\pi(i)}$ is the channel gain vector obtained by the base station for user $\pi(i)$. Then the beamforming vectors are

$$\hat{\mathbf{W}} = (\hat{\mathbf{H}}^H \hat{\mathbf{H}})^{-1} \hat{\mathbf{H}}^H \tag{4}$$

The beamforming vector is obtained by normalizing the ith column of $\hat{\mathbf{W}}$, and the scheduled users will receive signal from the base station.

2.2. Feedback channel model

In the wireless communication system, the feedback information of different users are distinguished by frequency, time slot or orthogonal code. Taking long term evolution (LTE) system for example, the feedback information is transmitted on allocated resource blocks (RB), which are time-frequency resources. The frequency, time slot or orthogonal code resources used for feedback are regarded as the general feedback channel in the paper.

We present a general feedback channel model for the multiuser feedback with M feedback channel among U users, in which users report their CSI/CDI to the base station. We assume that the feedback channels are constant and Rayleigh fading during the feedback period, the received signal by base station is given by

$$\mathbf{Y} = \sqrt{\rho}\mathbf{A}\mathbf{V} + \mathbf{Z} \tag{5}$$

where $\mathbf{Y} \in \mathbb{C}^{M \times N}$ is the received signal matrix, $\mathbf{A} \in \mathbb{C}^{M \times U}$ is the channel gain matrix of feedback channels, $\mathbf{V} \in \mathbb{C}^{U \times N}$ is the feedback information from U users, and $\mathbf{Z} \in \mathbb{C}^{M \times N}$ is complex Gaussian noise matrix with i.i.d zero mean unit variance entries. The ith row vector in $\mathbf{V}$ is feedback information from user i. When N is 1, our feedback model is the same as the model in [6]. It should be noticed that in this feedback channel model the feedback channels are general channel and can be different resources, such as frequency band, time-frequency element and others.

When M is equal to U, it means that each user has its dedicated feedback channel, which is popular case in the existing communication systems. When M is less than U, it means that U users share M feedback channels together, which can be an resource-efficient way to feed back information to the base station. However, how can we retrieve the feedback information since the feedback channels are shared among the users? Fortunately the emerging compressed sensing technology offers a solution. If only parts of the users feed back information on all the shared channels to the base station, which means that $\mathbf{V}$ is row sparse, we can identify and recover the feedback information via compressed sensing.

In order to implement zero-forcing beamforming, the users report the CSI to the base station through feedback channels. In [6] this channel model is used for feeding back the channel quality information (CQI) which can be represented by one scalar or one bit. In our feedback channel model, the CSI information, such as channel direction information (CDI) or channel quantization indexes are transmitted through feedback channels.

3. Compressed sensing feedback

Before discussing the proposed compressed sensing feedback algorithm, we present some important results for MMV CS used in our work.

3.1. Compressed sensing background

CS is a technology that can capture and represent compressible signals by employing nonadaptive linear projection. CS can be used for reconstructing single measurement vector (SMV) and multiple measurement vector (MMV) [19,20]. In this paper, we focus on applying the MMV CS to the multiuser MIMO feedback scheme. MMV CS can be considered as how to recover multiple SMV with the same support simultaneously. Give a multiple measurement vectors $\mathbf{B}$ and a redundant dictionary $\mathbf{A}$, the MMV problem in a noiseless scenario can be represented as

$$\mathbf{B} = \mathbf{A}\mathbf{X} \tag{6}$$

where $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathbf{X} \in \mathbb{R}^{n\times l}$ and $\mathbf{B} \in \mathbb{R}^{m\times l}$. If there is only one column in $\mathbf{X}$ and $l = 1$, MMV problem is degraded to a SMV problem. The column vectors in $\mathbf{X}$ are sparse and with the same support. A noiseless MMV CS is an optimization problem as

$$(P_0) : \min R_0 = \left\| \left(\sum_{j=1}^{l} |x_{ij}| \right)_{i\in\Omega} \right\|_0, \quad \Omega \in 1, 2, \ldots, n, \quad s.t.\ \mathbf{B} = \mathbf{AX} \tag{7}$$

where R_0 is sparsity and also the number of nonzero row of $\mathbf{X}$. The optimization problem for problem (P_0) requires searching all the subset of $\{1, 2, \ldots, n\}$ and is NP-hard problem, and the complexity grows exponentially with the size n.

Because of the difficulty of problem (P_0), it can be relaxed as

$$(P_1) : \min R_1 = \left\| \left(\sum_{j=1}^{l} |x_{ij}| \right)_{i\in\Omega} \right\|_1, \quad \Omega \in 1, 2, \ldots, n, \quad s.t.\ \mathbf{B} = \mathbf{AX} \tag{8}$$

It has been proved that problem (P_0) and problem (P_1) have identical and unique solution when the sparse level R_0 and the matrix mutual coherence M_c satisfy the following restriction [14]

$$R_0 < \left(1 + \frac{1}{M_c}\right) \Big/ 2 \tag{9}$$

$$M_c(\mathbf{A}) = \max_{1\leqslant i,j\leqslant n, i\neq j} |\mathbf{G}(i,j)| \tag{10}$$

where $\mathbf{G}(i,j)$ is the (i,j)th entry of the Gram matrix $\mathbf{G}$: $\mathbf{G} = \mathbf{A}^H\mathbf{A}$. This restriction for MMV is similar to the Theorem 12 in [17]. It is possible to show that the coherence of a matrix is always in the range of $\left[\sqrt{(n-m)/m(n-1)}, 1\right]$ [21]. Then (9) is given by

$$R_0 < \left(1 + \sqrt{\frac{m(n-1)}{n-m}}\right) \Big/ 2 \tag{11}$$

Problem (P_1) can be solved by greedy algorithm, such as matching pursuit (MP) and orthogonal matching pursuit (OMP) [14,20]. Since problem (P_1) is a convex optimization problem, it can also be solved by convex optimization algorithm [16,20]. Problem (P_0) and problem (P_1) are noiseless model, for noisy scenario the MMV problem can be represented as

$$\mathbf{B} = \mathbf{AX} + \mathbf{N} \tag{12}$$

where $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathbf{X} \in \mathbb{R}^{n\times l}$, $\mathbf{B} \in \mathbb{R}^{m\times l}$ and $\mathbf{N} \in \mathbb{R}^{m\times l}$. The optimization problem is given by

$$(P_2) : \min R_1 = \left\| \left(\sum_{j=1}^{l} |x_{ij}| \right)_{i\in\Omega} \right\|_1, \quad \Omega \in 1, 2, \ldots, n, \quad s.t.\ \|\mathbf{B} - \mathbf{AX}\|_F \leqslant \varepsilon \tag{13}$$

where ε is related to the noise $\mathbf{N}$ and the recovery accuracy requirement. Problem (P_2) can be solved by greedy algorithm and convex relaxation algorithm [15,16]. It should be noticed that above discussion for MMV is in $\mathbb{R}$, however, the results can be easily extended to the situation in $\mathbb{C}$, such as the CS applications in [5,6].

3.2. Algorithm of compressed sensing feedback

We apply the emerging compressed sensing technology to the feedback algorithm in MU-MIMO system, which can reduce the feedback resource consumption. The system model is illustrate in Fig. 1. Unlike the zero-forcing beamforming scheme in [1–3], in our system the mobile users are firstly self pre-selected according to the user pre-selection algorithm, then the pre-selected users feed back the channel state information via compressed sensing feedback scheme on all the feedback channels. The pre-selected users set is $S = \{\pi(1), \ldots, \pi(|S|)\}$. The CSI information can be analog or digital according to the feedback strategies which will be further explained in the following sections. At the base station, the compressed CSI will be recovered and the pre-selected users will be identified via compressed sensing recovery algorithm, such as MP, OMP or convex optimization. The base station will select N_t users among the pre-selected users, then zero-forcing beamforming will be carried out for the selected users. The selected user set is $T = \{\pi(1), \ldots, \pi(|T|)\}$. The details of the process are illustrated as following.

3.2.1. User self pre-selection algorithm

The self pre-selection algorithm is to select partial users to report their channel information to the base station, which is a kind of sparsity that is vital for compressed sensing. Firstly we give the cumulative distribution function (CDF) of the squared absolute inner product between two uniformly distributed m-dimensional unit complex vectors, which will be used in the self pre-selection algorithm.

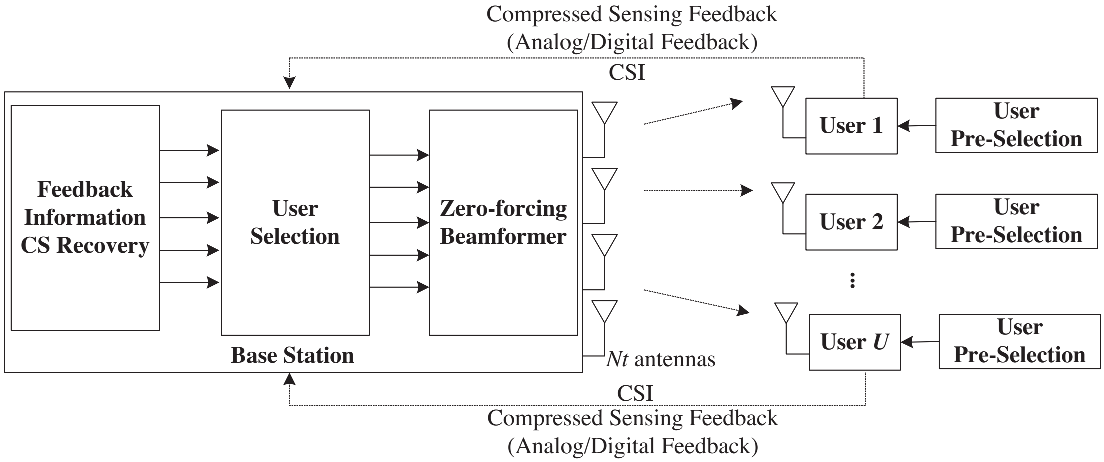

Fig. 1. MIMO broadcast model with compressed sensing feedback and zero-forcing beamforming.

Lemma 1. *The CDF of the squared absolute inner product x between two uniformly distributed m-dimensional unit complex vectors is give by*

$$1-(1-x)^{m-1}, \quad \textit{for } x \in [0,1]$$

Proof. The uniformly distributed m-dimensional unit complex vector can be seen as the point on the surface of m-dimensional unit complex hypersphere in Fig. 2. As the two vectors are random distributed, it is reasonable to assume that one vector, such as $\mathbf{c}_1$, is fixed and another vector, such as $\mathbf{c}_i$, is random on the surface of unit complex hypersphere as shown in Fig. 2. The squared absolute inner product for the vectors are shown. It can be seen that the points (such as $\mathbf{c}_2$ and $\mathbf{c}_3$) on the spherical cap which is intersected by the hyperplane $|d|^2 \geqslant \delta$ filled with the diagonal line have squared absolute inner product with $\mathbf{c}_i$ larger than δ. Therefore, the probability of $|\mathbf{c}_i\mathbf{c}_j|^2 \geqslant \delta$ is the ratio between the surface area of the spherical cap and the surface area of the hypersphere.

The surface area of a m-dimensional complex hypersphere of radius r is [3]

$$A(r) = \frac{2\pi^m r^{2m-1}}{(m-1)!} \tag{14}$$

The surface area of the spherical cap formed by the intersection of the subspace $|d|^2 \geqslant \delta$ and the m-dimensional complex hypersphere of radius r is

$$A(S(r)) = \frac{2\pi^m r(r^2-\delta)^{m-1}}{(m-1)!} \tag{15}$$

Then the ratio between the surface area of the spherical cap and the surface area of the hypersphere is given by

$$P' = \left(1-\frac{\delta}{r^2}\right)^{m-1} \tag{16}$$

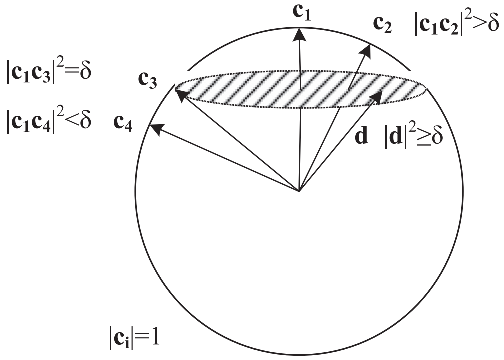

Fig. 2. Complex unit vectors and their square of inner product in the unit complex hypersphere.

Hence the CDF of the squared absolute inner product $\times$ between two uniformly distributed m-dimensional unit complex vectors is give by $1 - P'$ with $r = 1$. $\square$

The channel vector $\mathbf{h}_k$ is a N_t dimensional complex vector where each entry is distributed as complex Gaussian with zero mean and unit variance. $\gamma = \sum_{i=1}^{N_t} |h_{k,i}|^2$ is the sum of the squares of $2N_t$ Gaussian random variables with zero mean and 1/2 variance, and has a central chi-square distribution with $2N_t$ degree of freedom. The probability density function and cumulative distribution function of γ are given by [22]

$$P_\gamma(\gamma) = \frac{\gamma^{N_t-1} e^{-\gamma}}{(N_t - 1)!} \tag{17}$$

$$F_\gamma(\gamma) = 1 - e^{-\gamma} \sum_{k=0}^{N_t-1} \frac{\gamma^k}{k!} \tag{18}$$

Then we can have that

$$P(\gamma > \zeta) = 1 - P(\gamma \leqslant \zeta) = 1 - F_\gamma(\zeta) = e^{-\zeta} \sum_{k=0}^{N_t-1} \frac{\zeta^k}{k!} \tag{19}$$

In the self pre-selection algorithm, each user generates an N_t-dimensional unit complex random vector $\mathbf{g} \in \mathbb{C}^{1 \times N_t}$. The normalized channel gain vector for mobile user k is $\widetilde{\mathbf{h}}_k = \mathbf{h}_k / \|\mathbf{h}_k\|$, and $\gamma_k = \|\mathbf{h}_k\|^2$. Each user will compute $|\widetilde{\mathbf{h}}_k \mathbf{g}|^2$, and compare it with the threshold δ. The self pre-selection user set is given by

$$S = (k : |\widetilde{\mathbf{h}}_k \mathbf{g}|^2 < \delta, \gamma_k > \zeta, \quad k \in (1, \ldots, U)) \tag{20}$$

We need to set a proper value of δ and ζ which will determine the number of mobile users who will report their CSI and occupy feedback resources. The algorithm to determine δ and ζ is given by Lemma 2.

Lemma 2. *If k_p users out of U users are chosen to feed back CSI during the self pre-selection strategy, then δ and ζ are given by*

$$\delta = 1 - \left(1 - \frac{k_p}{U \cdot P(\gamma > \zeta)}\right)^{1/(N_t-1)}$$

Proof. We want the probability of the selection of k_p users is maximized, and it requires that

$$F(\delta, \zeta) = \arg\max_{\delta \in [0,1]} \binom{U}{k_p} P(\delta, \zeta)^{k_p} (1 - P(\delta, \zeta))^{U-k_p} \tag{21}$$

$$P(\delta, \zeta) = P(|\widetilde{\mathbf{h}}_k \mathbf{g}_k|^2 < \delta, \gamma_k > \zeta) \tag{22}$$

By deriving $F(\delta, \zeta)$ respect to $P(\delta, \zeta)$, we can get

$$\frac{\partial F(\delta, \zeta)}{\partial P(\delta, \zeta)} = \binom{U}{k_p} \left\{ k_p P(\delta, \zeta)^{k_p-1} (1 - P(\delta, \zeta))^{U-k_p} - (U - k_p) P(\delta, \zeta)^{k_p} (1 - P(\delta, \zeta))^{U-k_p-1} \right\} \tag{23}$$

We can get the maximum $F(\delta, \zeta)$ by

$$\frac{\partial F(\delta, \zeta)}{\partial P(\delta, \zeta)} = 0 \Rightarrow P(\delta, \zeta) = \frac{k_p}{U} \tag{24}$$

According to the Lemma 1, the CDF of $|\widetilde{\mathbf{h}}_k \mathbf{g}|^2$ is given by

$$P(|\widetilde{\mathbf{h}}_k \mathbf{g}_k|^2 < \delta) = 1 - (1 - \delta)^{(N_t-1)} \tag{25}$$

where $|\widetilde{\mathbf{h}}_k \mathbf{g}_k|^2$ and γ_k are independent to each other. In the self pre-selection algorithm, we choose those users that their signals are strong by setting proper value of ζ, then we can get the threshold of δ by

$$P(|\widetilde{\mathbf{h}}_k \mathbf{g}_k|^2 < \delta) = \frac{k_p}{U \cdot P(\gamma > \zeta)} \tag{26}$$

Then substituting (25) and (26), and we can get Lemma 2. $\square$

By self pre-selection algorithm only part of users, which have better channel qualities and have better orthogonality to each other with high probability, will feed back their CSI to the base station. So there is sparsity among the users, and compressed sensing can make use of the sparsity to recover the feedback information and identify which users have reported the CSI. In the following section we will discuss the compressed sensing feedback and recovery.

3.2.2. Compressed sensing feedback and recovery

The feedback information can be analogy feedback or digital feedback which occupy different amount of feedback resources and have different robustness to the noise. In the CS feedback, the k_p self pre-selected users will feed back their analog channel vectors or the digital index of quantified channel vector to the base station. According to the feedback channel model in Section 2.2, we can get

$$\mathbf{Y}_a = \sqrt{\rho}\mathbf{A}\mathbf{V}_a + \mathbf{Z} \tag{27}$$

$$\mathbf{Y}_d = \sqrt{\rho}\mathbf{A}\mathbf{V}_d + \mathbf{Z} \tag{28}$$

where the row vectors in $\mathbf{V}_a$ are the analog channel vectors of users, the row vectors in $\mathbf{V}_d$ are the digital indexes of the quantified channel vectors. It should be noticed that $\mathbf{V}_a$ and $\mathbf{V}_d$ are row sparse since the non-selected users will not feed back their CSI and the corresponding row vectors are 0. As assumed in Section 2.2, there are M feedback channel among U users. It is obviously that the feedback model can be regarded as a CS MMV problem. As discussed in Section 3.1, feedback channel M, user number U and self pre-selected user number k_p should satisfy (10) and (11). Otherwise, the CS MMV recovery algorithm cannot guarantee a high recovery probability. The impacts of feedback channel M and noise $\mathbf{Z}$ on the recovery probability and accuracy will be discussed in the simulations. It should be noticed that in order to compare the performance of CS analog and digital feedback, we let the column numbers of $\mathbf{V}_a$ and $\mathbf{V}_d$ are identical in the simulation, which means that both of them occupy the same feedback resources. For example if there are 4 transmitting antenna at the base station, the channel gain vector is 4 dimensional, and the column number of $\mathbf{V}_a$ is 4 which means the selected users transmit the feedback information in 4 times. Then in CS MMV digital feedback the quantization bits is 8 for QPSK modulation which needs 4 times to transmit the 4 symbols.

After receiving the feedback information at the base station, CS MMV recovery algorithm is applied to recovery the CSI information. There are typically 2 classes of recovery algorithms. one class of algorithm is to recover the row-sparse matrix itself by greedy algorithm or convex relaxation [14–17]. Another class of algorithm is to recover the support of the row-sparse matrix firstly and then to recover the matrix. In this paper, we will apply both of the 2 classes of algorithms. For the first class of algorithm, we will apply MMV OMP algorithm to the CSI recovery. For the second class of algorithm, the ReMBo algorithm is used [18]. The feedback CS models are given as (27) and (28). The MMV OMP, outlined in Algorithm 1, is directly extended form the SMV OMP.

Algorithm 1. Orthogonal matching pursuit for MMV.

(1) Initialization: residual $\mathbf{R}_0 = \mathbf{Y}$.
(2) At the tth iteration (or iteration ending condition):
(a) Choose the column vector $\mathbf{a}_{k_t}$, which satisfies $\mathbf{a}_{k_t} = \arg\max_{a_k} \|\mathbf{Y}_k\|^2$, where $\mathbf{Y}_k = \mathbf{R}_{t-1}^H \mathbf{a}_k$.
(b) Let $\mathbf{A}_t = [\mathbf{A}_{t-1}, \mathbf{a}_{kt}]$, and $\mathbf{V}^* = \arg\min \|\mathbf{A}_t\mathbf{V} - \mathbf{Y}\|_F^2$, $\mathbf{Y}_t = \mathbf{A}_t\mathbf{V}^*$.
(c) Set $\mathbf{R}_t = \mathbf{Y} - \mathbf{Y}_t$.

The ReMBo algorithm proceeds by taking a random vector $\mathbf{d}$ and combining the individual observations in $\mathbf{Y}$ into a single weighted observation $\mathbf{y}' := \mathbf{Y}\mathbf{d}$. The random combination yields an SMV with the same nonzero location set as the MMV. Then, an SMV problem is solved in order to find the support of and SMV and MMV. Finally, the MMV can be recovered by the sub-matrix of $\mathbf{A}$ which is composed of the column vectors indicated by the support.

Algorithm 2. ReMBo algorithm for MMV.

(1) Initialization: give $\mathbf{Y}$ and $\mathbf{A}$ and set iteration to 0.
(2) At the tth iteration (or iteration ending condition):
(a) Draw a random vector $\mathbf{d}$, and $= \mathbf{Y}'\mathbf{d}$.
(b) Solve $\mathbf{y}' = \mathbf{A}\mathbf{v}'$ using SMV recovery algorithm to recover the support $\widehat{S}$ and solution $\hat{\mathbf{v}}'$.
(c) if $|\widehat{S}| \leqslant sparsitylevel$ and $\|\mathbf{y}' - \mathbf{A}\mathbf{v}'\|_2 \leqslant \epsilon$, then go to step 3), else go to step 2).
(3) Based on the support $\widehat{S}$ and the corresponding submatrix $\mathbf{A}_s$ indicating by the support $\widehat{S}$, get the nonzero submatrix of $\mathbf{V}_s = (\mathbf{A}_s) \dagger \mathbf{Y}$,$\mathbf{A}_s \dagger$ is the pseudo-inverse of $(\mathbf{A}_s)$.
(4) Recover $\mathbf{V}$ by $\mathbf{V}_s$ and setting row vector of $\mathbf{V}$: $\mathbf{v}_i = 0, i \notin \widehat{S}$.

3.2.3. User selection and zero-forcing beamforming

The base station performs user selection algorithm to select N_t out of k_p users at one time based on the feedback CSI information. Although the user number for base station to select has been reduced from U to k_p, finding the optimal user set that maximizes the throughput requires a large search. In the paper, we use a user selection algorithm based on the semi-orthogonal user selection (SUS) procedure [1]. The SUS algorithm is as follows.

Algorithm 3. Semi-orthogonal user selection (SUS) algorithm.

(1) Initialization: Candidate user set $S = \{1, 2, \dots, k_p\}$, and the selected user set is $T_0 = \{\pi(1)\}$, where $\pi(1) = \arg\max \|\mathbf{h}_k\|, k \in S$.
(2) At the ith iteration ($i \leqslant N_t$):
(a) Find the user set $I_i = \{1 \leqslant k \leqslant k_p : |\mathbf{h}_k \mathbf{h}^H_{\pi(j)}| \leqslant \delta', 1 \leqslant j \leqslant i$.
(b) Find the $(i+1)$th user that $\pi(i+1) = \arg\max \|\mathbf{h}_k\|, k \in I_i$.
(c) Then the selected user set is $T_i = \{T_{i-1}, \pi(i+1)\}$.

In the semi-orthogonal user selection algorithm, δ' is related to the allowed correlation threshold. The CSI information used by the SUS algorithm can either be the analog feedback information or the digital feedback information according to the CS MMV feedback strategy.

4. Performance evaluation

4.1. Throughput of CS feedback in ZFBF

Firstly we will discuss the throughput of CS digital feedback in ZFBF. According to the broadcast channel model given in Section 2.1, the signal to interference plus noise ratio (SINR) of user k is as

$$\mathrm{SINR}_k = \frac{\rho|\mathbf{h}_k\mathbf{w}_k|^2}{1+\rho\sum_{j\neq k}|\mathbf{h}_k\mathbf{w}_j|^2} = \frac{\rho\|\mathbf{h}_k\|^2|\tilde{\mathbf{h}}_k\mathbf{w}_k|^2}{1+\rho|\mathbf{h}_k\|^2\sum_{j\neq k}|\tilde{\mathbf{h}}_k\mathbf{w}_j|^2}, \quad 1 \leqslant k \tag{29}$$

where $\tilde{\mathbf{h}}_k$ is the normalized vector of $\mathbf{h}_k$, $\mathbf{w}_k$ is the beamforming vector.

When feedback channel number M and self pre-selected users k_p are chosen properly, the MMV recovery algorithm can perfectly recover the feedback information with high probability and accuracy. If the feedback information is digital CSI and modulated by BPSK and QPSK, it is more robust to the noise effect, and the feedback impacts on the throughput is mainly due to the quantization error of channel vector. Denote θ_k as the angle between $\tilde{\mathbf{h}}_k$ and $\hat{\mathbf{h}}_k$, then we have [1]

$$\tilde{\mathbf{h}}_k = \cos\theta_k\hat{\mathbf{h}}_k + \sin\theta_k\mathbf{g}_k \tag{30}$$

where $\cos\theta_k = |\tilde{\mathbf{h}}_k\hat{\mathbf{h}}_k^*|$, $\hat{\mathbf{h}}_k$ and $\mathbf{g}_k$ are the quantified channel vector and the error vector, and they are orthogonal to each other. Then the SINR can be presented as

$$\mathrm{SINR}_k = \frac{\rho|\mathbf{h}_k\mathbf{w}_k|^2}{1+\rho\sum_{j\neq k}|\mathbf{h}_k\mathbf{w}_j|^2} = \frac{\rho\|\mathbf{h}_k\|^2|\tilde{\mathbf{h}}_k\mathbf{w}_k|^2}{1+\rho\|\mathbf{h}_k\|^2\sin^2\theta_k\sum_{j\neq k}|\mathbf{g}_k\mathbf{w}_j|^2} \tag{31}$$

In the CS digital feedback strategy, the self pre-selected users feed back its quantified CDI $\hat{\mathbf{h}}_k$ and the channel magnitude $\|\mathbf{h}_k\|$. In high SNR regime, the throughput upper bound is given by [1]

$$E(R_d) = E\left\{\sum_{i\in T}\log_2(1+\mathrm{SINR}_i)\right\} \leqslant \frac{N_t}{N_t-1}(B+\log_2 k_p) \tag{32}$$

In the CS analog feedback strategy, the feedback information are the channel vectors. When there is no noise and feedback channel number M and self pre-selected users k_p are chosen properly, CS MMV recovery algorithm can almost recovery the vector perfectly. We assume that the actual channel gain vector is impacted by the noise according to different SNR ratios. Directly analyzing the impact of noise on the throughput is complicate, and the mathematic derivation is tricky. For analysis simplicity, we can regard the impact of noise as the quantization error for digital feedback which has been analyzed in many literatures and reference therein [1–3].

As shown in Fig. 2 and Refs. [1–3], in the quantization channel feedback the complex hypersphere is divided into many spherical caps, and each cap is represented as a codeword in the codebook. When the channel gain vector is projected into the spherical cap, the corresponding codeword is used to present the quantified channel gain vector. The quantization error is represented by the angle between the actual channel vector and quantified channel vector.

The noise has the same impact on the channel vector as the quantization error. When the SNR is high, the received channel gain vector will fluctuate around the actual channel gain vector in a small area, which is similar to a small spherical cap in quantization feedback, and vice versa. Hence we need to map the effect of noise to the number of quantization bits. The relationship of noise impact and the equivalent quantization error impact is given in Lemma 3.

Lemma 3. *The noise impact on channel vector with SNR (SNR > 0) is equivalent to the quantization error impact on the channel gain vector by quantization bits B, and they satisfy*

$$B \approx (N_t - 1)\log_2(1+\mathrm{SNR}) \tag{33}$$

Proof. As shown in (30), the SNR can be approximated by

$$\mathrm{SNR}_k = \frac{\|\cos\theta_k \hat{\mathbf{h}}_k\|^2}{\|\sin\theta_k \hat{\mathbf{g}}_k\|^2} = \frac{\cos^2\theta_k}{\sin^2\theta_k} \tag{34}$$

According to the quantization error analysis in [4], the upper bound of $\sin^2\theta_k$ is given by

$$E(\sin^2\theta_k) < 2^{-\frac{B}{N_t-1}} \tag{35}$$

In the worst situation that the quantization error is maximal, we have

$$E(\sin^2\theta_k) \approx 2^{-\frac{B}{N_t-1}} \tag{36}$$

Substituting (36) to (34), we can get

$$B \approx (N_t - 1)\log_2(1 + \mathrm{SNR}) \qquad \square \tag{37}$$

Then in high SNR regime, the throughput upper bound of CS analog feedback is given by

$$E(R_a) \approx E\left\{\sum_{i\in T}\log_2(1+\mathrm{SINR}_i)\right\} \leqslant \frac{N_t}{N_t-1}\left((N_t-1)\log_2(1+SNR) + \log_2 k_p\right) \tag{38}$$

4.2. Feedback resource reduction

The number of feedback channels required for CS feedback strategy is greatly reduced as only $M = O(k_p log(U/k_p))$ feedback channels are needed compared to U feedback channels for ZFBF without CS in which each users need to feed back their CSI. The comparisons of resource consumption for CS analog/digital feedback and ZFBF feedback without CS are shown in Table 1.

In our simulations, M is set to $U/2$ to achieve a steady recovery probability and accurate CSI. QPSK is used to modulate the digital feedback information which is widely used in LTE Release 8/9 to ensure the robustness of the feedback, then $B/2$ symbols are needed to transmit B bits. Consequently, when $N_t = B/2$ the feedback resource reduction can be up to 50% for CS digital feedback compared to ZFBF feedback without CS. For CS analog/digital feedback, the total feedback resource consumption is linear with MN_t.

4.3. CS feedback recovery performance

CS recovery performance is vital for the CSI information recovery and has important impacts on the performance of the ZFBF. The feedback channel model is illustrated in Section 2.2. The recovery performance for CS digital feedback is better than the CS analog feedback, and it is more robust to the noise. The simulation in next section will show that the robustness of CS MMV digital feedback. Hence, we only analyze the recovery performance for CS analog feedback. As discussed in Section 3.2.2, we will simulate the performance of MMV OMP(M-OMP) algorithm and ReMBo algorithm. We adopt 2 rules for measuring the performance of the 2 algorithms [20]:

- Recovery rate: the percentage of identifying the non-zero rows correctly.
- The relative mean squared error (MSE): the relative mean squared error between the true and the estimated solution, and is given by

$$\mathrm{MSE} = E\left(\frac{\|\hat{\mathbf{V}} - \mathbf{V}\|_F^2}{\|\mathbf{V}\|_F^2}\right) \tag{39}$$

Fig. 3 shows the recovery rate and MSE for different SNR. The simulations are with 100 users, 16 feedback users and 50 feedback channels for different SNR. Fig. 4 shows the recovery rate and MSE for different number of feedback channel. The simulations are with 100 users, 16 feedback users and 10dB SNR for different feedback channels. It can seen that with the increase of SNR and channel number, the recovery rates for both M-OMP and ReMBo increase dramatically and the MSEs reduce quickly. Meanwhile the M-OMP algorithm is superior to the ReMBo algorithm in the sense of recovery rate and MSE. Fig. 5 shows the recovery rate and MSE for different sparsity level. As the increase of sparsity level, the recovery rates for M-OMP and ReMBo reduce quickly while there is a sparsity threshold for M-OMP algorithm. When the sparsity level is below the sparsity threshold the recovery rate for M-OMP is almost to 1.

From the analysis above, it can be seen that the M-OMP algorithm is superior to the ReMBo algorithm. The CS feedback is suitable for moderate and high SNR regime, and the feedback channel number is good to set to the half of the users. The impacts of SNR and feedback channel number are equivalent, which means that in low SNR we can obtain high recovery rate

Table 1
Feedback resource comparisons for different feedback strategies.

	CS analog feedback	CS digital feedback	Feedback without CS
Number of feedback channels	M	M	U
Feedback resource per user	N_t symbol	B bits	B bits
Total feedback resource	MN_t symbol	$MB/2$ symbol	$UB/2$ symbol

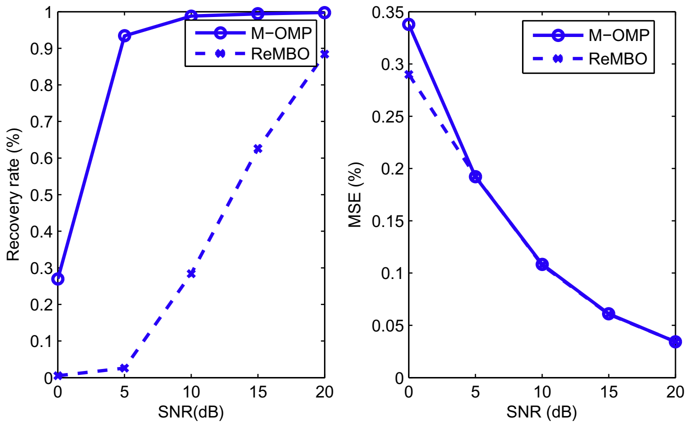

Fig. 3. Recovery rate and MSE for different SNR.

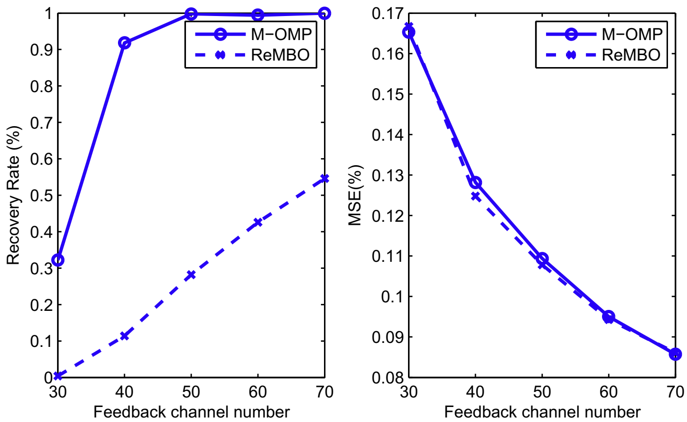

Fig. 4. Recovery rate and MSE for different feedback channel number.

and accuracy by more feedback channels and vice verse. In the low SNR regime, we need to increase the feedback channel number to insure the high recovery rate. The impact of SNR and feedback channel number on the throughput will be further discussed in next section.

Next we discuss the complexity of the proposed feedback scheme. The computation in this scheme is mainly implemented on the base station side. In practical the processor of the base station is powerful and can handle the complexity. However, there are different CS recovery algorithm which have different complexity. It is important to apply the efficient algorithm to make the feedback information up-to-date. In [20] the complexity of different CS recovery algorithms are presented. Generally the M-OMP and MMV order recursive matching pursuit (M-ORMP) are with about less than 1/4 complexity of the MMV FOcal underdetermined system solver (M-FOCUSS) and regularized M-FOCUSS in the point of computational complexity, however, the MSE of M-OMP and M-ORMP is slightly worse than M-FOCUSS and regularized M-FOCUSS. In [23] the author compares the linear programming and greedy algorithms for the CS recovery, and show that greedy algorithms such as M-OMP and its evolutions are with less complexity. Hence greedy algorithm can be regarded as a practical algorithm for the implementation of the proposed scheme in the engineering applications.

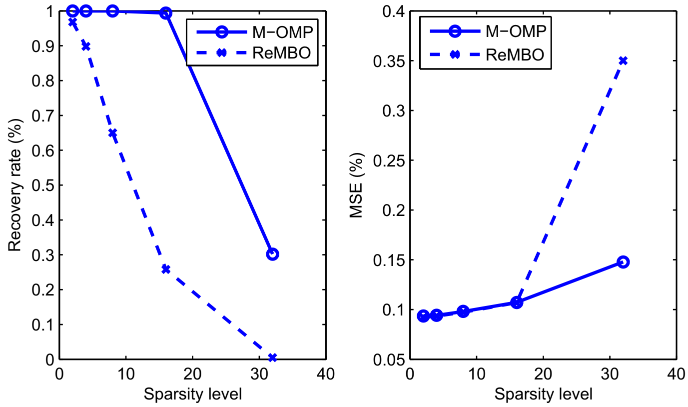

Fig. 5. Recovery rate and MSE for different sparsity level.

5. Simulations and analysis

5.1. Simulation environment

In our simulations, we set the number of transmit antennas at the base station to $N_t = 4$, the total user number is 100. The semi-orthogonal user selection algorithm for zero-forcing beamforming is applied. Since the M-OMP algorithm has better performance than the ReMBo algorithm, the M-OMP is used for the CSI information recovery in our simulations. We simulate the CS analog and digital feedback algorithms, and compare their performance with ZFBF with perfect CSI and quantization CSI with all users. Meanwhile we compare the proposed CS feedback to ZFBF with the same users as CS feedback.

5.2. Results

Fig. 6 compares the sum throughput for CS digital feedback and analog feedback with different feedback users by self pre-selection algorithm. The SNR is 10dB, and the feedback channel number is 50. The sum throughput results for perfect CSI and quantified CSI for all users and numbered feedback users are also shown. Numbered feedback users are the same as the users in the CS feedback. The results show that both CS digital feedback and analog feedback have capacity gain as the increase of the feedback users, while the capacity gain with CS analog feedback increases more obviously. CS analog feedback outperforms CS digital feedback, and even better than the ZFBF with quantified CSI of all users. The quantization bits for digital feedback and quantified CSI is 8 bits with QPSK for the fair comparison with CS analog feedback in the sense of occupying the same feedback resources. The proposed CS feedback algorithm can save 50% feedback resource compared with the ZFBF algorithms in [1–4]. On the other hand, the CS analog feedback outperforms the prefect CSI for numbered feedback users, and the CS digital feedback outperforms the quantified CSI for numbered feedback users. The reason is that CS feedback can pre-select the better users among all the users, and there is a user-selection gain. For the analog CSI of all users with 8 bits feedback for each user which is the same as the scalar quantization in [24], the throughput has no obvious increase due to the inaccuracy of feedback is the main aspect that impacts the throughput. Since the feedback channel number is fixed as 50, with the increase of self pre-selected users the CS recovery accuracy will degrade, which will impact the sum throughput. This is the reason why the sum throughput for CS MMV digital/analog feedback increase slowly when pre-selection users is more than 12. However the feedback channel number can be chosen adaptively according to the number of self pre-selection users.

Fig. 7 shows the sum-throughput versus SNR for different feedback strategies. The self pre-selected users are 16, and the feedback channels are 50. It shows that with the increase of SNR, the sum-throughput for CS analog feedback increases as the same rate of the sum-throughput for ZFBF with perfect CSI of all users, and the sum-throughput for CS digital feedback increases as the same rate of the sum-throughput for ZFBF with quantified CSI of all users. Meanwhile, the sum-throughput for CS analog feedback outperforms the ZFBF with quantified CSI of all users, while the sum-throughput for CS digital feedback is slightly lower than it. However, CS feedback with the simulation parameters can save 50% feedback resources. When the CSI feedback user number is the same as the CS feedback, the throughput of perfect CSI for numbered feedback users is slight lower than CS MMV analog feedback, and the throughput of quantified CSI for numbered feedback users is slight lower than CS MMV digital feedback. For the analog CSI of all users with 8 bits feedback for each user [24], the throughput increase with the SNR but reach a plateau with SNR larger than 10 dB. This is because the feedback accuracy has larger impacts on the throughput than the SNR.

Fig. 8 shows the sum-throughput versus feedback channel number. The SNR is 10 dB, and the self pre-selected users are 16. It shows that with the increase of feedback channel number, the throughput for CS analog feedback increase dramatically, and

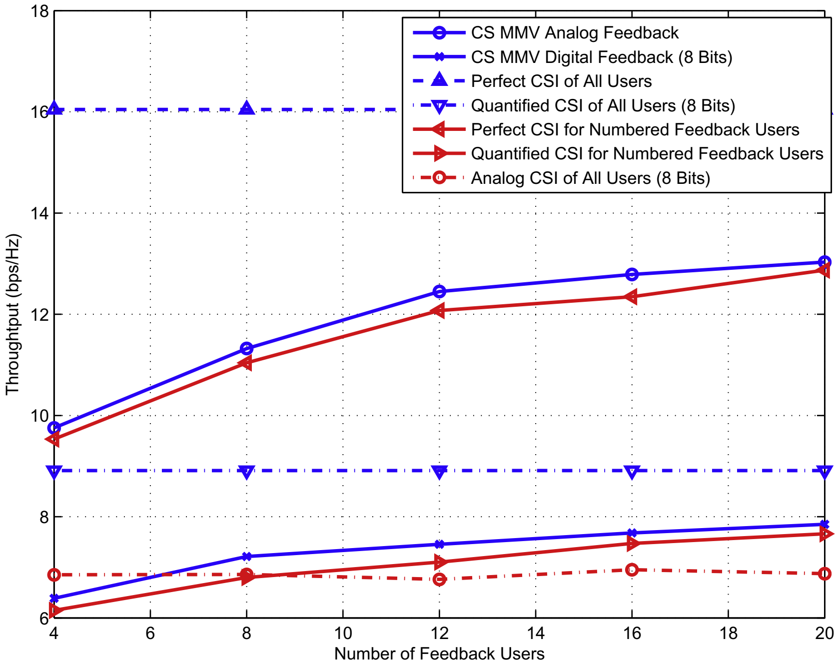

Fig. 6. Sum-throughput versus number of feedback users.

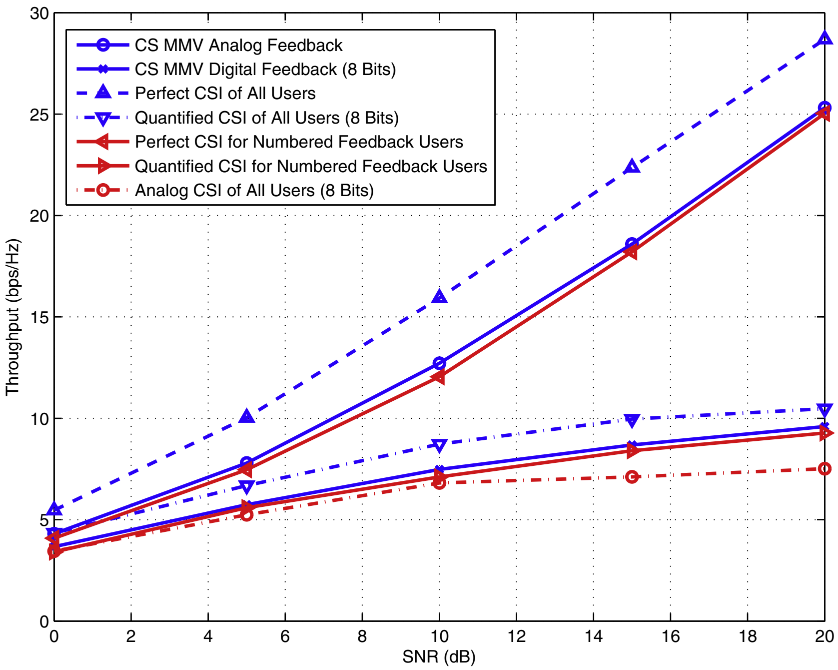

Fig. 7. Sum-throughput versus signal-to-noise ratio.

then reaches a plateau because that the CSI is accurate enough as the feedback channel reach a threshold, such as 50 for this scenario. The throughput for CS digital feedback increases slowly as the increase of feedback channel, which is because that the CSI is quantified and robust to the impact of noise and recovery accuracy. Even the recovery accuracy is low with less feedback channels, the base station can recovered it to quantization bits successfully. Hence, the CSI quantization error is the key factor that affects the throughput for CS digital feedback, which means that we can further reduce the feedback resource for CS digital feedback.

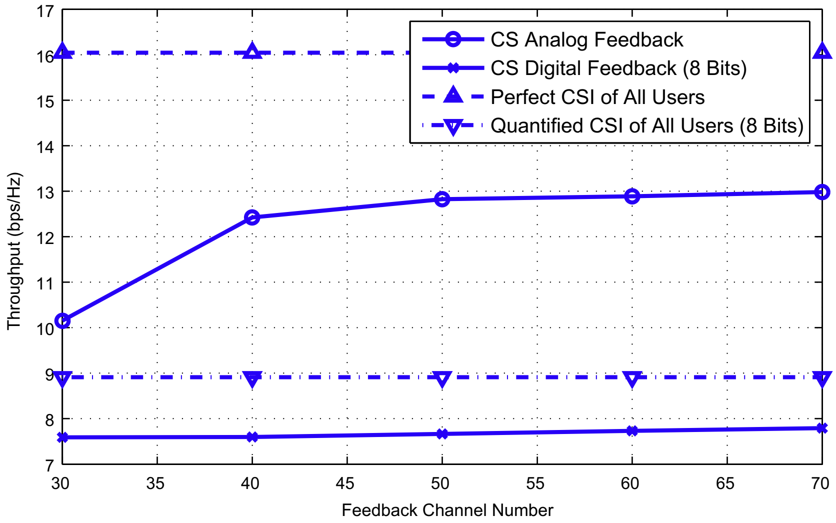

Fig. 8. Sum-throughput versus feedback channel number.

6. Conclusion

We have proposed a resource-efficient feedback scheme based on compressed sensing for multiple users ZFBF MIMO systems. Due to the self pre-selection algorithm, the proposed CS feedback can greatly save the feedback resources. Compared with the ZFBF with quantified CSI of all users and numbered users, simulations and analysis show that the proposed CS analog feedback has better performance than the ZFBF with quantified feedback of all users, and the proposed CS digital feedback has a close performance with the ZFBF with quantified feedback of all users while occupying less feedback resources than the ZFBF with quantified feedback of all users. Meanwhile, the simulations show that M-OMP algorithm has better recovery rate and MSE performance than ReMBo algorithm for CS recovery with noise.

Acknowledgements

This work is supported by National Natural Science Foundation of China 60972015, China-EU International Scientific and Technological Cooperation Program (0902), China National Science and Technology Major Project 2009ZX03003-007-03, 2009ZX03003-003-02 and 2010ZX03003-001-02. Thanks for the reviewers' valuable comments.

References

[1] Yoo T, Jindal N, Goldsmith A. Multi-antenna downlink channels with limited feedback and user selection. IEEE J Select Areas Commun 2007;25(7):1478–91.
[2] Lu P, Yang HC. Sum-rate analysis of multiuser MIMO system with zero-forcing transmit beamforming. IEEE Trans Commun 2009;57(9):2585–9.
[3] Mukkavilli KK, Sabharwal A, Erkip E, Aazhang B. On beamforming with finite rate feedback in multiple-antenna systems. IEEE Trans Inform Theory 2003;49(10):2562–79.
[4] Jindal N. MIMO broadcast channels with finite-rate feedback. IEEE Trans Inform Theory 2006;52(11):5045–60.
[5] Song H, Seo W, Hong D. Compressive feedback based on sparse approximation for multiuser MIMO systems. IEEE Trans Vehicular Technol 2010;59(2):1017–23.
[6] Qaseem ST, Al-Naffouri TY. Compressive sensing for reducing feedback in MIMO broadcast channels. In: 2010 IEEE international conference on communications; 2010. p. 1–5.
[7] Davis LM, Hanly SV, Tune P, Bhaskaran SR, IEEE. Multi-antenna downlink broadcast using compressed-sensed medium access. In: 2010 IEEE international conference on communications; 2010. p. 1–5.
[8] Flecher AK, Rangan S, Goyal VK. On-off random access channels: a compressed sensing framework. URL: <http://arxiv.org/abs/0903.1022>.
[9] Li YJ, Song RF. Novel schemes of CQI feedback compression based on compressive sensing for adaptive OFDM transmission. KSII Trans Internet Inform Syst 2011;5(4):703–19.
[10] Fazel F, Fazel M, Stojanovic M. Design of a random access network for compressed sensing. In: Proc. of information theory and applications workshop (ITA); 2011. p. 1–9.
[11] Fazel F, Fazel M, Stojanovic M. Random access compressed sensing in underwater acoustic networks. In: Proc. of Allerton conference on communications, control, and computing; 2010. p. 1–7.
[12] Luo C, Wu F, Sun J, Wenchen C. Compressive data gathering for large-scale wireless sensor networks. In: Proc. of international conference on mobile computing and networking (MobiCom); 2009. p. 145–56.
[13] Candes EJ, Tao T. Decoding by linear programming. IEEE Trans Inform Theory 2005;51(12):4203–15.
[14] Chen J, Huo XM. Theoretical results on sparse representations of multiple-measurement vectors. IEEE Trans Signal Process 2006;54(12):4634–43.
[15] Tropp JA, Gilbert AC, Strauss MJ. Algorithms for simultaneous sparse approximation. Part I: greedy pursuit. Signal Process 2006;86(3):572–88.
[16] Tropp JA. Algorithms for simultaneous sparse approximation. Part II: convex relaxation. Signal Process 2006;86(3):589–602.
[17] Donoho D, Elad M. Optimally sparse representation in general (nonorthogonal) dictionaries via l1 minization. In: Proceedings of the national academy of sciences; 2003. p. 2197–202.

[18] Mishali M, Eldar YC. Reduce and boost: recovery arbitrary sets of jointly sparse vectors. IEEE Trans Signal Process 2008;56(10):4692–702.
[19] Lei J. Generalized reconstruction algorithm for compressed sensing. Comput Electrical Eng 2011;37(4):570–88.
[20] Cotter SF, Rao BD, Engan K, Kreutz-Delgado K. Sparse solutions to linear inverse problems with multiple measurement vectors. IEEE Trans Signal Process 2005;53(7):2477–88.
[21] Rosemfeld M. The mathematics of Paul Erdös II. In praise of the Gram matrix. Berlin: Springer; 1996.
[22] Simon MK. Probability distributions involving gaussian random variables. New York: Springer; 2002.
[23] Rakotomamonjy A. Surveying and comparing simultaneous sparse approximation (or group-lasso) algorithms. Signal Process 2011;91(7):1505–26.
[24] Ravindran N, Jindal N. Multi-user diversity vs. accurate channel feedback for MIMO broadcast channel. In: 2008 IEEE international conference on communications; 2008. p. 1–5.

Wei Lu received the M.S. degree from the Department of Information Engineering at Wuhan University of Technology, China, in 2007. He was the system engineer of CDMA system in Nortel R&D Lab in Guangzhou, China, from 2007 to 2008. He is currently pursuing his Ph.D. degree at Huazhong University of Science and Technology and Wuhan National Laboratory for Optoelectronics, China. His main research interests include MIMO and compressed sensing.

Yingzhuang Liu received his Ph.D. degree in communications and information system in 2000 from Huazhong University of Science and Technology. He was the post doctor at University of Paris-Sud 11, France, from 2000 to 2001. Now he is the Professor at Huazhong University of Science and Technology and Wuhan National Laboratory for Optoelectronics in China.

Desheng Wang received his Ph.D. degree in communications and information system in 2006 from Huazhong University of Science and Technology. Now he is the Associate Professor in Department of Electronics and Information Engineering at Huazhong University of Science and Technology.