simulated data is shown in Figs. 4 and 5. At 4 GHz, the measured insertion loss was 0.5 dB for the capacitive path and 0.4 dB for the inductive path, with a differential phase shift of 91°. The measured broadside axial ratio was 1.5 dB with 0.9 dB insertion loss at the centre frequency. Axial ratio <3 dB is maintained over a 9% bandwidth.

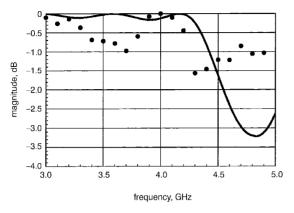


Fig. 5 Four cascaded grids in the capacitive orientation; experimental data compared with full-wave simulation

experimental data
 full-wave simulation

Conclusion: We have presented the design and experimental results of a linear-to-circular polarization converter consisting of four cascaded grids with a metal pattern, never before demonstrated. This method of polarisation conversion provides relatively low insertion loss, good axial ratio, and the design can be adapted to operate at other frequencies.

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## Composite QAM sequences with low PMEPR

H.R. Sadjadpour and S. Yoon

An upper bound for the peak-to-mean envelope power ratio (PMEPR) of composite quadrature amplitude modulation (QAM) sequences in orthogonal frequency division multiplexing (OFDM) systems is derived. The composite QAM sequence is constructed by linear weighted and shifted sum of quadrature phase-shift keying (QPSK) sequences.

Introduction: While most studies for low peak-to-mean envelope power ratio (PMEPR) in OFDM systems utilising Reed-Muller (RM) codes consider PSK signal constellation [1], there are many other applications that require M-QAM constellation. It has been

shown in [2] that M-QAM sequences can be constructed by linear weighted sum of QPSK sequences. In this Letter, the idea in [2] is generalised to construct composite QAM sequences and the upper bound for the PMEPR is derived. The kth symbol of an M-QAM sequence with square constellation can be described, as [2],

$$a_k[M - QAM] = \sum_{i=0}^{m/2-1} 2^i \left(\frac{\sqrt{2}}{2}\right) \exp\left(\frac{\pi j}{4}\right) a_k^i [QPSK]$$
 (1)

where the signal constellation size  $M = 2^m$  and  $a'_k[QPSK]$  is the kth symbol of the ith QPSK sequence. A QPSK symbol can be realised as  $j^{x_k}$  with  $x'_k \in Z_4 = \{0, 1, 2, 3\}$ . The transmitted signal in an OFDM system is then represented as:

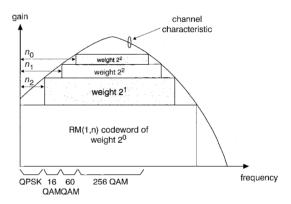
$$S_{\mathbf{a}}(t) = \sum_{k=0}^{N-1} a_k [M - QAM] \exp[2\pi j (f_o + kf_s)t]$$
 (2)

where  $f_o$  is the carrier frequency and  $f_s$  is the bandwidth of each subchannel. Combining (1) and (2) results in

$$S_{\mathbf{a}}(t) = \sum_{i=0}^{m/2-1} 2^{i} \left(\frac{\sqrt{2}}{2}\right) S_{\mathbf{a}^{(i)}[QPSK]}(t)$$
 (3)

where

$$S_{\mathbf{a}^{(i)}[QPSK]}(t) = \sum_{k=0}^{N-1} j^{\gamma_k^i} \exp \left[ 2\pi j (f_o + k f_s) t + \frac{j\pi}{4} \right]$$



**Fig. 1** Construction of composite QAM sequence by weighted sum of elementary QPSK sequences

Composite QAM construction: Golay sequences occur as co-sets of the first-order RM code within the second-order RM code [1]. Construction of Golay (or Golay-like) sequences with low PMEPR based on RM codes is introduced for codewords of length 32 or less. Tables of trade-offs between PMEPR, code rate, and error correction capability are given in [1]. If the size of FFT is greater than 32, then a combination of these QPSK Golay (-like) sequences can be used to construct M-QAM sequences of larger size. For example, a 64-QAM sequence of length 128 is constructed by combining 12 QPSK sequences of length 32. In some other applications such as discrete multitone (DMT), different signal constellation size is assigned to each subchannel. A composite QAM sequence can be used for these applications as well [3] (see Fig. 1). A composite QAM sequence can be expressed as the sum of weighted and shifted coded QPSK sequences. In these examples, a weighted and shifted sum of Golay (-like) sequences is utilised, while in [2] only a weighted sum of Golay sequences is used. Its kth symbol is given by:

$$a_{k}[\text{composite } QAM] = \sum_{j} w_{j} \exp\left(\frac{j\pi}{4}\right) a_{k-n_{j}}^{(j)}[QPSK]$$

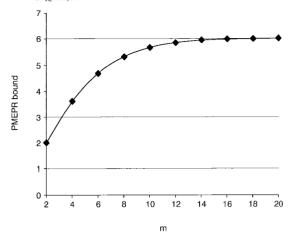
$$\text{for } 0 < k - n_{i} < N$$
(4)

where  $w_j$  and  $n_j$  are the weight and the shift for the QPSK sequence of length  $N_j \le N$ . The value of  $w_j$  depends on the construction of composite QAM sequence considering (2). The corresponding trans-

mitted signal for composite QAM sequence is given by:

$$S_{\mathbf{a}[\text{composite }QAM]}(t) = \sum_{i} w_{j} S_{\mathbf{a}^{(i)}[QPSK]}(t)$$
 (5)

where  $S_{\mathbf{a}^{(i)}[OPSK]}(t)$  is defined similar to (3) and thereafter.



**Fig. 2** PMEPR bound for  $2^m$ -OAM sequence (with m even) constructed by m/2 OPSK Golay sequences

PMEPR bound of composite QAM sequences:

Observation 1: The jth QPSK sequence  $\mathbf{a}^{(j)}[QPSK]$  of length  $N_j$  has an

average power of  $E[S_{\mathbf{a}^{(j)}}(t)]^2 = N_j$  [1]. Observation 2:  $|S_{\mathbf{a}^{(j)}}(t)| = C_j$ , where  $C_j = \sqrt{(2N_j)}$  if the codeword is a Golay sequence. In general, these QPSK codewords can be chosen using any Golay-like sequence with relatively small PMEPR [1].

Lemma 1: If the QPSK codewords are generated from independent input information sequence, then  $E[S_{\mathbf{a}^{(i)}}(t)S_{\mathbf{a}^{(i)}}^*(t)] = 0$  for  $i \neq j$ , where \* denotes complex conjugate.

Proof:

$$\begin{split} &E[S_{\mathbf{a}^{(j)}}(t)S_{\mathbf{a}^{(j)}}^*(t)] \\ &= E\left[\sum_{k_1=0}^{N_t-1}\sum_{k_2=0}^{N_j-1}a_{k_1}^i(a_{k_2}^j)^*\exp(2\pi j(k_1-k_2)f_st)\right] \\ &= \sum_{k_1=0}^{N_j-1}\sum_{k_2=0}^{N_j-1}E(a_{k_1}^i(a_{k_2}^j)^*)\exp(2\pi j(k_1-k_2)f_st) = 0 \end{split} \tag{6}$$

We assume these codewords are statistically independent and each QPSK symbol is uniformly distributed resulting in  $E(a_k^i) = 0$ , therefore  $E(a_{k_1}^i \ (a_{k_2}^j)^* = 0.$ 

Theorem 1: The peak envelope power is upper bounded as  $(\sum_i |w_i|C_i)^2$ .

$$P(t) = \left| \sum_{j} w_{j} S_{\mathbf{a}^{(j)}[QPSK]}(t) \right|^{2}$$

$$\leq \left( \sum_{j} |w_{j}| |S_{\mathbf{a}^{(j)}[QPSK]}(t)| \right)^{2}$$

$$\leq \left( \sum_{j} |w_{j}| C_{j} \right)^{2}$$
(7)

We used observation 2 and triangular inequality to derive this upper bound.

Theorem 2: The average power is  $\sum_{i} |w_{i}|^{2} N_{i}$ . Proof:

$$E(P(t)) = E \left| \sum_{j} w_{j} S_{\mathbf{a}^{(j)}[QPSK]}(t) \right|^{2}$$

$$= \sum_{j} |w_{j}|^{2} E |S_{\mathbf{a}^{(j)}[QPSK]}(t)|^{2}$$

$$= \sum_{i} |w_{j}|^{2} N_{j}$$
(8)

Observation 1 and lemma 1 are used for computation of the mean envelope power.

Theorem 3: The PMEPR is upper bounded as  $(\sum_i |w_i|C_i)^2 / \sum_i |w_i|^2 N_i$ *Proof:* The proof follows directly from theorems 1 and 2.

If only Golay sequences are used, then the PMEPR is upper bounded as  $2(\sum_i |w_i|)^2 / \sum_i |w_i|^2$ . Fig. 2 shows the PMEPR bound for an M-QAM sequence constructed by m/2 QPSK Golay sequences of the same length for different even values of m.

Conclusions: A new construction of composite M-QAM sequence with low PMEPR is derived. The PMEPR upper bound for these types of sequences is computed.

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## Design of dividable interleaver for parallel decoding in turbo codes

Jaevoung Kwak and Kwyro Lee

A dividable interleaving method is proposed for turbo codes with parallel architecture to achieve high-throughput. This method not only solves the memory conflict problem in extrinsic information memory, but reduces the required memory for the interleaver. Many kinds of interleaver type are applicable to this method, the BER performance is similar to that achieved by other interleavers.

Introduction: Turbo codes [1] show an impressive BER performance close to the Shannon limit. However one of the drawbacks of turbo codes is the significant amount of decoding delay which is induced by iterative decoding. Two kinds of structure using several processors have been proposed to reduce the decoding delay. One is the pipelining method; the other is block partitioning. Several papers have discussed the pipelining method [1, 2]. Each processor calculates the entire block for I/W iterations and passes extrinsic information (Le) to the next processor (here, I is iteration number, W is the number of processors). Though decoding delay is reduced to O(N/W) from O(N), the register space for the received data and Le is increased to O(NW) (here, N is block size). The parallel decoding method using block partitioning was suggested in [3]. Each information block is divided into partially overlapped W SISO processors, and these processors decode in parallel. This scheme not only reduces decoding delay to O(N/W), but maintains required register space to O(N). However, the memory bandwidth for Le, which is handled at the same time, is increased by W, and it needs to be interleaved at the same time. Consequently, there can be a data conflict at the read/write operation between W processors and the Le memory block. Previously, a collision-free interleaver using cycle shift was proposed to solve this problem [4], but this method can be applicable to block or multi-stage interleavers. In this Letter, we propose a more general interleaving method, the dividable interleaver, to solve the problem of