Instantaneous Radar Polarimetry with Multiple Dually-polarized Antennas

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***Abstract*— Fully polarimetric radar systems are capable of simultaneously transmitting and receiving in two orthogonal polarizations. Instantaneous radar polarimetry exploits both polarization modes of a dually-polarized radar transmitter and receiver on a pulse by pulse basis, and can improve the radar detection performance and suppress range sidelobes . In this paper, we extend the use of instantaneous radar polarimetry for radar systems with multiple dually-polarized transmit and receive antennas. Alamouti signal processing is used to coordinate transmission of Golay pairs of phase codes waveforms across polarizations and multiple antennas. The integration of multi-antenna signal processing with instantaneous radar polarimetry can further improve the detection performance, at a computational cost comparable to single channel matched filtering.**

I. Introduction

There are many proposals for distributed aperture radar, emanating from the conventional single aperture radar system where transmitter and receiver are colocated. Colocation makes it easy for transmitter and receiver to share a common stable clock (local oscillator), which is required for both range and Doppler measurements. Signal processing for distributed aperture radars (see [1] and the references therein) with widely dispersed antenna elements is currently a very active research area, in part because of significant advances in hardware capabilities. Distributed aperture radar enables multiple views of the scene, and a (wide angle) tomographic approach to the recovery of the scene from the data, and hence results in substantial improvement in target detection. When system elements are widely dispersed, the coherent implementation of distributed aperture radar is rendered difficult by the problem of clock synchronization. Another challenge is the degree of computation necessary to recover the scene, or detect a target, by integrating multiple views.

Target scattering profiles depend significantly both on aspect angle and illumination and receive polarizations (see [2], Section 2.7). In [3], Howard *et al.* proposed a new approach to multi-channel radar that uses polarization to provide essentially independent channels for viewing the target. The introduction of multiple polarizations increases the degrees of freedom in the waveform design space, and complements the effort on waveform design that is specific to distributed aperture radar [4]–[9]. Polarization diversity enables detection of smaller radar cross section (RCS) targets, and avoids the physical, mathematical, and engineering challenges of time-of-arrival coherent combining. The advantage of polarization diversity over spatial diversity is that diversity gains are possible with colocated antennas.

Howard *et al.* [3] employ Golay pairs of phase-coded waveforms [10],[11] to provide synchronization and enable use of the Alamouti space-time code [12] to coordinate transmission of Golay pairs across polarizations. The use of Golay pairs and Alamouti signal processing enables radar ambiguity polarimetry on a *pulse by pulse basis* and allows for estimating the full polarimetric properties of the target, at computational cost comparable to single channel matched filtering. Unlike conventional radar polarimetry, where polarized waveforms are transmitted sequentially and processed independently (non-coherently), the approach in [3] allows for *instantaneous radar polarimetry*, where polarization modes are combined coherently on a pulse by pulse basis. Compared to a radar system with a singly-polarized transmitter and a singly polarized receiver the instantaneous radar polarimetry approach in [3] can achieve the same detection performance (same false alarm and detection probabilities) with a substantially smaller transmit energy, or alternatively it can detect at substantially greater ranges for a given transmit energy.

In this paper, we extend the result of [3] to enable the use of instantaneous radar polarimetry for multiple dually-polarized transmit and receive antennas. We use Alamouti block coding to coordinate the transmission of Golay pairs across polarizations and antennas during four time slots. As we show, the integration of multi-antenna signal processing and instantaneous radar polarimetry further improves the detection performance, while the signal processing complexity at the receiver remains comparable to that for single channel matched filtering. In addition, simultaneous processing of multiple Golay pairs (4 pairs in this paper) results in significant suppression of range sidelobes, making the radar ambiguity function closer to a spike. Finally, we note that although in this paper we assume that the transmitters (receivers) are colocated, the mathematical machinery we develop is also applicable to the case where the transmitters (receivers) are distributed.

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II. Fully Polarimetric Radar Systems

Fully polarimetric radar systems are able to simultaneously transmit and receive on two orthogonal polarizations. The combined signal then has an electric field vector that is modulated both in direction and amplitude by the waveforms on the two polarization channels, and the receiver measures both components of the reflected waveform.

The radar cross section of an extended target such as an aircraft or a ship is highly sensitive to the angle of incidence and angle of view of the sensor (see [2], Sections 2.78). In general the reflection properties that apply to each polarization component are also different; indeed reflection can change the direction of polarization. Polarimetric radars are able to measure the scattering tensor of a target, i.e.,

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_{VV} & \sigma_{VH} \\ \sigma_{HV} & \sigma_{HH} \end{pmatrix}, \tag{1}$$

where σ_{VH} denotes the target scattering coefficient into the vertical channel due to a horizontally polarized incident field. In fact what is measured is the combination of three matrices

$$\mathbf{H} = \begin{pmatrix} h_{VV} & h_{VH} \\ h_{HV} & h_{HH} \end{pmatrix} = \mathbf{C}_{R_x} \mathbf{\Sigma} \mathbf{C}_{T_x} \tag{2}$$

where $\mathbf{C}_{Rx}$ and $\mathbf{C}_{Tx}$ characterize the polarization coupling properties of the transmit and receive antennas, whereas $\mathbf{\Sigma}$ characterizes the target. In most radar systems the transmit and receive antennas are common and $\mathbf{C}_{Rx} = \mathbf{C}_{Tx}^H$. The cross-coupling terms in the antenna polarization matrices are frequency and antenna geometry dependent but for the linearly polarized case this value is typically no greater than about -20dB.

III. Instantaneous Radar Polarimetry for Radar Detection

In this section, we describe the instantaneous radar polarimetry technique proposed in [3]. Much of the language and terminology is hence drawn from [3].

In instantaneous radar polarimetry, we employ both polarization modes of a dually polarized radar transmitter and receiver, as shown in Fig. 1. We use Alamouti code to coordinate the transmission of four waveforms w_H^1, w_V^1, w_H^2, and w_V^2 over the V and H channels. Here the superscripts ^{1} and ^{2} denote two consecutive time slots for a pair of pulses separated by one Pulse Repetition Interval (PRI). We have

$$w_H^2 = \widetilde{w}_V^1 \tag{3}$$

$$w_V^2 = -\widetilde{w}_H^1, \tag{4}$$

where $\widetilde{\cdot}$ denotes complex conjugate time reversal; namely,

$$\widetilde{w}(D) = D^N \overline{w}(D^{-1}), \tag{5}$$

with $N = \deg w$ the degree of the polynomial $w(D)$ in the delay operator D, and $^{-}$ complex conjugation. This operation plays the same role for these polynomials as complex conjugation does in coding.

Let

$$\mathbf{R} = \begin{pmatrix} r_V^1(D) & r_V^2(D) \\ r_H^1(D) & r_V^2(D) \end{pmatrix}, \tag{6}$$

where $r_V^j(D)$ ($r_H^j(D)$), $j = 1, 2$ is the signal measured at time slot j on the vertical (horizontal) polarization channel at the receiver. The returns $r_V^j(D)$ and $r_H^j(D)$ are viewed as polynomials in the delay operator D. Then, the radar measurement equation is given by

$$\mathbf{R} = \mathbf{H}\mathbf{W} + \mathbf{Z}, \tag{7}$$

where

$$\mathbf{W} = \begin{pmatrix} w_V^1(D) & -\widetilde{w}_H^1(D) \\ w_H^1(D) & \widetilde{w}_V^1(D) \end{pmatrix}, \tag{8}$$

and

$$\mathbf{H} = \begin{pmatrix} h_{VV} & h_{VH} \\ h_{HV} & h_{HH} \end{pmatrix}. \tag{9}$$

The phase coded waveforms $w_V^1(D)$ and $w_H^1(D)$ are also viewed as polynomials in the delay operator D and their coefficients are fourth roots of unity since these are QPSK waveforms. The waveform matrix $\mathbf{W}$ has the form of an Alamouti space-time code matrix used in MIMO communications [12]. The entries of $\mathbf{H}$ are taken to be constant since they corresponds to a fixed time (range). The elements of $\mathbf{Z}$ are four independent realizations of a zero-mean complex Gaussian random process with variance N_0 per complex dimension.

The simplicity of Alamouti signal processing follows from the unitary character of the matrix $\mathbf{W}$, specifically

$$w_V^1(D)\widetilde{w}_V^1(D) + w_H^1(D)\widetilde{w}_H^1(D) = 2(N+1)w_V^1 D^N. \tag{10}$$

Polynomials with coefficients that are fourth roots of unity and that satisfy (10) are complex Golay complementary pairs [11]. These include the classical Golay pairs whose coefficients are ± 1. It might appear that the correlation side-lobes vanish only at delays that are multiples of the chip length, but in fact 10 is a property of Golay pairs that holds for all possible non-zero delays. It is this property that enables detection based on energy thresholds that is independent of polarization cross-coupling of the antenna. Correlation properties of Golay pairs have been widely studied in the radar and communications literature. In particular, Golay pairs have been constructed with degrees $N = 2^n - 1$ for all positive integers n [13].

We analyze the detection performance of our fully polarimetric scheme using a slowly fluctuating point target model.

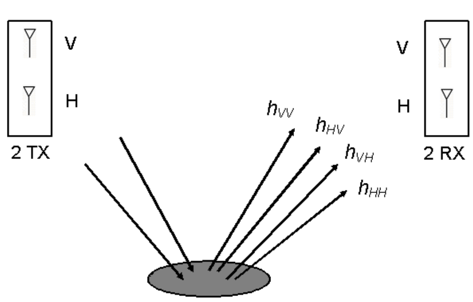

Fig. 1. Scattering model for a fully polarimetric radar, with a dually-polarized transmit antenna and a dually-polarized receive antenna.

That is, we take the matrix $\mathbf{H}$ (considered as a four component vector $\mathbf{h}$) as zero-mean Gaussian distributed with covariance matrix $E[\mathbf{h}\mathbf{h}^H] = \mathbf{\Lambda}$. For simplicity, we also make the assumption that the components of $\mathbf{H}$ are independent and identically distributed, so that $\mathbf{\Lambda} = 2\sigma^2\mathbf{I}$. We also assume that the noise on each receive channel is additive zero-mean white Gaussian noise with power N_o per complex dimension.

Under these assumptions a sufficient statistic for detecting a stationary point target (zero Doppler) at delay t_d is given by $||\mathbf{q}(t_d)||^2$, where

$$\mathbf{q} = \begin{pmatrix} [\mathbf{R}\widetilde{\mathbf{W}}]_{11} \\ [\mathbf{R}\widetilde{\mathbf{W}}]_{12} \\ [\mathbf{R}\widetilde{\mathbf{W}}]_{21} \\ [\mathbf{R}\widetilde{\mathbf{W}}]_{22} \end{pmatrix} \tag{11}$$

and $[\mathbf{R}\widetilde{\mathbf{W}}]_{ij}$ is the ijth element of $\mathbf{R}\widetilde{\mathbf{W}}$.

Given that the pulses w_V and w_H have unit energy and that the total transmit energy across the two polarization channels is E_t, the detection problem may be posed as

$$\mathbf{q} = \begin{cases} \mathbf{n} \sim CN[\mathbf{0}, 2N_o\mathbf{I}] & : \mathrm{H}_0 \\ 2\sqrt{E_t/4}\mathbf{h} + \mathbf{n} \sim CN[\mathbf{0}, (2E_t\sigma^2 + 2N_o)\mathbf{I}] & : \mathrm{H}_1 \end{cases} \tag{12}$$

where H_1 denotes the hypothesis that the target is present. The properties of the waveforms w_V and w_H imply that $E(\mathbf{n}\mathbf{n}^H) = 2N_o\mathbf{I}$. Since $E(\mathbf{h}\mathbf{h}^H) = \mathbf{\Lambda} = 2\sigma^2\mathbf{I}$. The likelihood ratio detector for (12) is simply an energy detector and is equivalent to the test

$$||\mathbf{q}||^2 > \gamma, \tag{13}$$

for some threshold $\gamma > 0$. The probability of false alarm for this detector is

$$\begin{aligned} P_F &= \Pr(||\mathbf{q}||^2 > \gamma \,|H_0) \\ &= \frac{1}{48N_o^4}\int_{\sqrt{\gamma}}^{\infty} z^7 \exp(-\frac{z^2}{2N_o})\, dz = \Phi\left(\frac{\gamma}{2N_o}\right), \end{aligned} \tag{14}$$

where

$$\Phi(x) = (1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3)e^{-x}. \tag{15}$$

Similarly, the probability of detection is

$$P_D = \Pr(||\mathbf{q}||^2 > \gamma \,|H_1) = \Phi\left(\frac{\gamma}{2\sigma^2 E_t + 2N_o}\right). \tag{16}$$

IV. Instantaneous Radar Polarimetry with Multiple Antennas

We now extend the use of instantaneous radar polarimetry for a radar system with multiple dually-polarized transmit and receive antennas. We consider two colocated dually-polarized transmit and two colocated dually-polarized receive antennas, as shown in Fig. 2. We use Alamouti signal processing to coordinate the transmission of Golay pairs w_V and w_H across two polarizations and two antennas over four PRIs. The waveform matrix we transmit is

$$\mathbf{W}_{4\times4} = \begin{pmatrix} \mathbf{W} & -\widetilde{\mathbf{W}} \\ \mathbf{W} & \widetilde{\mathbf{W}} \end{pmatrix}, \tag{17}$$

where $\mathbf{W}$ is the 2 by 2 Alamouti waveform matrix in (8).

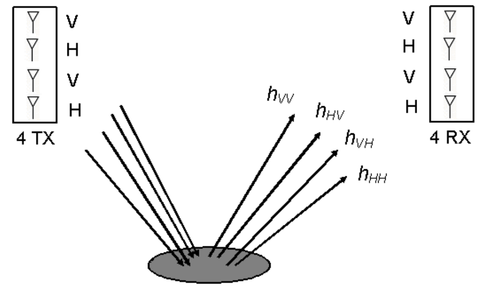

Fig. 2. Scattering model for a fully polarimetric radar with two dually-polarized transmit antennas and two dually-polarized receive antennas.

The radar measurement equation is given by

$$\mathbf{R}_{4\times4} = \mathbf{H}_{4\times4}\mathbf{W}_{4\times4} + \mathbf{Z}_{4\times4}, \tag{18}$$

where

$$\mathbf{H}_{4\times4} = \begin{pmatrix} \mathbf{H}_{11} = \mathbf{H} & \mathbf{H}_{12} = \mathbf{H} \\ \mathbf{H}_{21} = \mathbf{H} & \mathbf{H}_{22} = \mathbf{H} \end{pmatrix} \tag{19}$$

and $\mathbf{H}_{ij}$ denotes the polarization scattering matrix between transmitter j and receiver i. Since we have assumed that in the radar system in Fig. 2 the transmitters (receivers) are colocated it is natural to assume that all $\mathbf{H}_{ij}$ are equal to the polarization scattering matrix $\mathbf{H}$ in (9).

The signal processing complexity required for range detection will remain comparable to that for a single channel matched filtering if the Golay pairs w_V and w_H are chosen to make $\mathbf{W}_{4\times4}$ unitary, i.e.,

$$\mathbf{W}_{4\times4}\widetilde{\mathbf{W}}_{4\times4} = \mathbf{I}_{4\times4}. \tag{20}$$

This unitary condition is equivalent to

$$w_V^2 + \widetilde{w}_V^2 = w_H^2 + \widetilde{w}_H^2. \tag{21}$$

One possible choice is to select $w_H = \widetilde{w}_V$. In this case, given that the total transmit energy across polarizations, antennas, and time slots is E_t, a sufficient statistic for detecting a stationary point target (zero Doppler) at delay t_d is given by $||\mathbf{q}'(t_d)||^2$, where

$$\mathbf{q}' = \begin{pmatrix} [\mathbf{R}\widetilde{\mathbf{W}}]_{11} + [\mathbf{R}\widetilde{\mathbf{W}}]_{13} + [\mathbf{R}\widetilde{\mathbf{W}}]_{31} + [\mathbf{R}\widetilde{\mathbf{W}}]_{33} \\ [\mathbf{R}\widetilde{\mathbf{W}}]_{12} + [\mathbf{R}\widetilde{\mathbf{W}}]_{14} + [\mathbf{R}\widetilde{\mathbf{W}}]_{32} + [\mathbf{R}\widetilde{\mathbf{W}}]_{34} \\ [\mathbf{R}\widetilde{\mathbf{W}}]_{21} + [\mathbf{R}\widetilde{\mathbf{W}}]_{23} + [\mathbf{R}\widetilde{\mathbf{W}}]_{41} + [\mathbf{R}\widetilde{\mathbf{W}}]_{43} \\ [\mathbf{R}\widetilde{\mathbf{W}}]_{22} + [\mathbf{R}\widetilde{\mathbf{W}}]_{24} + [\mathbf{R}\widetilde{\mathbf{W}}]_{42} + [\mathbf{R}\widetilde{\mathbf{W}}]_{44} \end{pmatrix}, \tag{22}$$

where the subscript 4×4 in $\mathbf{R}_{4\times4}$ and $\widetilde{\mathbf{W}}_{4\times4}$ is dropped for notational convenience.

The detection problem may be posed as

$$\mathbf{q}' = \begin{cases} \mathbf{n} \sim CN[\mathbf{0}, 8N_o\mathbf{I}] & : \mathrm{H}_0 \\ 4\sqrt{E_t/16}\mathbf{h} + \mathbf{n} \sim CN[\mathbf{0}, (32E_t\sigma^2 + 8N_o)\mathbf{I}] & : \mathrm{H}_1 \end{cases} \tag{23}$$

where H_1 denotes the hypothesis that the target is present. The energy detector is equivalent to the test

$$||\mathbf{q}'||^2 > \gamma, \tag{24}$$

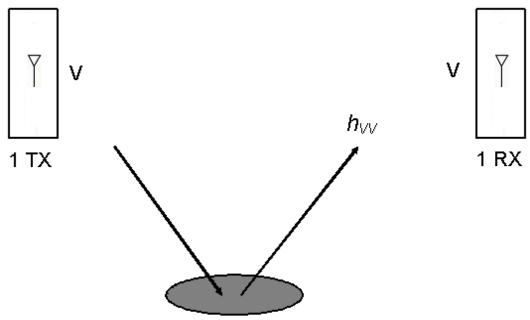

Fig. 3. Scattering model for the baseline system with a singly-polarized transmitter and a singly-polarized receiver.

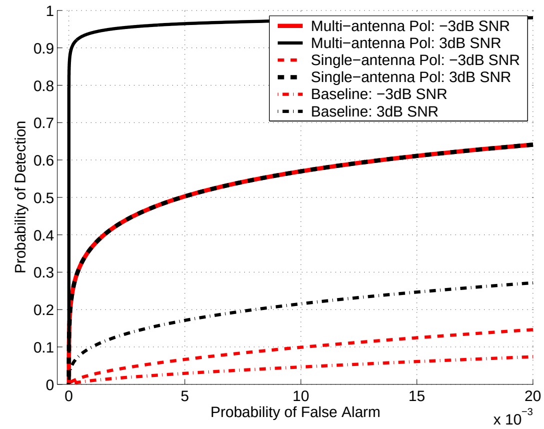

Fig. 4. ROC plots for the fully polarimetric systems and the baseline system at different target SNRs.

for some threshold $\gamma > 0$, and the probabilities of false alarm and detection are given by

$$\begin{aligned} P_F &= \Pr(||\mathbf{q}||^2 > \gamma \,|H_0) &= \Phi\left(\tfrac{\gamma}{8N_o}\right) \\ P_D &= \Pr(||\mathbf{q}||^2 > \gamma \,|H_1) &= \Phi\left(\tfrac{\gamma}{32E_t\sigma^2+8N_o}\right) \end{aligned} \quad (25)$$

V. Performance Improvement Due to Instantaneous Radar Polarimetry

We now compare the performance of the fully polarimetric schemes (shown in Figs. 1 and 2) with the baseline radar system shown in Fig. 3, which employs a single polarization with one transmit and one receive antenna. The baseline system has the same total transmit energy E_t as the fully polarimetric systems.

The probabilities of false alarm and detection, and the receiver operating characteristic (ROC) curve for the single channel radar are given by [14, Chap. 9]

$$\begin{aligned} P_F &= \exp(-\tfrac{\gamma}{2N_o}) \\ P_D &= \exp(-\tfrac{\gamma}{2\sigma^2E_t+2N_o}) \\ P_F &= P_D^{(S+1)} \end{aligned} \quad (26)$$

where $S = \frac{2\sigma^2E_t}{2N_0}$ is the signal-to-noise ratio at the receiver (or target SNR) in the baseline system.

Figure 4 compares the ROC plots for the two fully polarimetric schemes and the baseline scheme, with equal total transmit energy at target SNR $S = -3$dB and $S = 3$dB. We notice that that for a given target SNR the *single antenna* fully polarimetric scheme (Fig. 1) offers substantially better detection performance compared to the baseline system, and that using instantaneous radar polarimetry with multiple antennas leads to even greater improvement in detection performance. We also notice that the ROC curve for the single antenna fully polarimetric scheme at $3dB$ target SNR is the same as the ROC curve for the *multi-antenna* fully polarimetric scheme at $-3dB$ target SNR. This is due to the fact that in the multi-antenna scheme we receive 4 identical copies of the polarization scattering vector $\mathbf{h}$, resulting in 6dB SNR gain compared to the fully polarimetric system in Fig. 1.

We also consider the extra SNR S_1 that the baseline system needs to ensure the same P_D and P_F as the fully polarimetric schemes. Using (26) with P_F and P_D given by (14) and (16), we have

$$\frac{S_1}{S} = \frac{\log\left[\Phi\left(\frac{\gamma}{2N_o}\right)\Big/\Phi\left(\frac{\gamma}{2N_o(S+1)}\right)\right]}{S\log\Phi\left(\frac{\gamma}{2N_o(S+1)}\right)}. \quad (27)$$

This is the extra SNR required by the baseline system to match the probabilities of detection and false alarm of the dually polarized scheme in Fig. 1, for a given target SNR $S = 2\sigma^2E_t/2N_o$.

Similarly, the extra SNR required by the baseline system to match the probabilities of detection and false alarm of the multi-antenna fully polarimetric scheme in Fig. 2 is given by

$$\frac{S_2}{S} = \frac{\log\left[\Phi\left(\frac{\gamma}{8N_o}\right)\Big/\Phi\left(\frac{\gamma}{8N_o(4S+1)}\right)\right]}{S\log\Phi\left(\frac{\gamma}{8N_o(4S+1)}\right)}. \quad (28)$$

Figure 5 shows the extra SNRs S_1/S and S_2/S in dB versus probability of detection for target SNRs $S = -3$dB and $S = 3$dB. As can be seen for any probability of detection greater than 0.5 the fully polarimetric schemes give equivalent performance to the baseline system for substantially smaller transmit energy, or alternatively they allow for detection at substantially greater ranges for a given transmit energy. For instance the single-antenna fully polarimetric scheme requires 5dB less transmit energy than the baseline system to achieve $P_D = 0.7$, when the baseline system operates at $S = -3$dB target SNR, while the multi-antenna scheme requires 12.2dB less transmit energy to do the same. We notice that here the difference between the extra SNRs required by the baseline system to produce the same P_D and P_F as the two fully polarimetric schemes is 7.2dB, which is 1.2dB higher than the extra SNR required by the single-antenna fully polarimetric scheme to produce the same P_D and P_F as the multi-antenna scheme. This additional SNR gain is due to the difference in distribution of the test statistics for the fully polarimetric systems and the baseline system. For both fully polarimetric systems the distribution of the test statistic is χ^2_4, and the only difference between the two distribution is due to SNR.

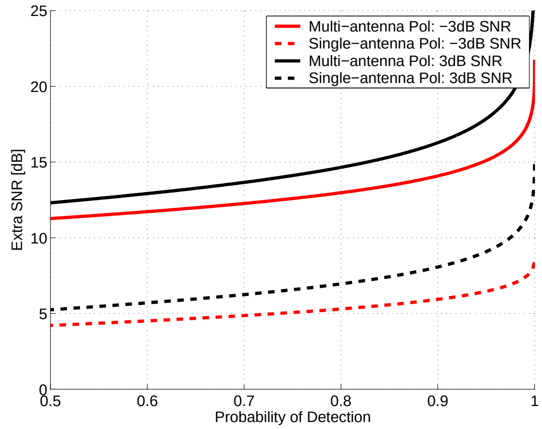

Fig. 5. Extra SNR required for the baseline to get the same P_F and P_D as the instantaneous fully polarimetric schemes.

However, for the baseline system the distribution of the test statistics is χ^2.

Finally, we note that the radar cross-sections for the HV and VH channels may be substantially smaller in magnitude than the VV and HH channels. However, the gains for the fully polarimetric systems persists due to the presence of off-diagonal elements in the polarization scattering matrix $\mathbf{H}$, although these gains are diminished.

VI. Conclusions

In this paper, we extended the result of [3] to enable the use of instantaneous radar polarimetry for multiple dually-polarized transmit and receive antennas. Alamouti signal processing was used to coordinate transmission of Golay pairs of phase codes waveforms across polarizations and multiple antennas. The interplay between Alamouti signal processing, Golay pairs, and polarization diversity allows for estimating the full polarimetric properties of the target on a pulse by pulse basis, at computational cost comparable to single channel matched filtering. We compared our multi-antenna fully polarimetric scheme to a conventional radar system with a singly-polarized transmitter ad a singly-polarized receiver, and with the fully polarimetric system in [3]. Simulation results show that instantaneous radar polarimetry can significantly improve the detection performance compared to the baseline system, specially when it used with multiple transmit and receive antennas.

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