Fixed Effort MIMO Decoders for Wireless Indoor Channels: Theory and Practical Field Trials.

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***Abstract*—We propose a fixed-effort MIMO decoder for frequency selective indoor channels that are characterized by strong Line-of-Sight (LOS) components. Contrarily to the maximum likelihood (ML) approach, where all possible hypotheses are investigated by the metrics calculation, the proposed MIMO detector performs the search over a reduced set of candidates. This search set contains a reduced but representative set of hypotheses around the linear solution obtained at the first detection stage. The candidates are selected according to precomputed search probabilities. A decision feedback equalizer (DFE) is applied in order to remove the effect of the inter symbol interferences (ISI) caused by the channel dispersion. The method provides a near ML performance using a fixed computational effort determined by the hardware resources. The proposed detector also shows a significant complexity reduction compared to popular MIMO detectors such as the V-BLAST and the sphere decoder. Moreover, the proposed detector provides a soft output information for each transmitted bit, using the preselected candidates from the reduced search set which presents a promising aspect for the coded transmission.**

***Index Terms*—MIMO detection, line-of-sight, frequency selective channel, RS and IRS detectors, soft output information**

I. Introduction

Multiple input multiple output (MIMO) transmission systems promise higher data rates and better reliability for a given signal to noise ratio and a fixed bandwidth [1], [2]. The ML detector is the optimum detector in terms of minimizing the bit error rate (BER) for MIMO systems, but stays unfeasible in practice because of its extremely high computational effort. The sphere decoder (SD) has been proposed as an approach to provide ML performance with reduced complexity. The SD derived in [3]–[5] is based on a successive layer detection using decreasing search radii which results in a reduced number of candidates. However, the SD presents a variable complexity depending on the noise level and the channel conditions. The SD complexity converges to an exponential complexity for a bad conditioned channel matrix and a low signal-to-noise ratio (SNR). This makes its implementation unfeasible in practice. The reduced search (RS) algorithm has been proposed in [6] to reduce the number of candidates per level. The RS approach is based on a linear detection providing the starting solution followed by a RS ML. The starting solution is an initial set that will be expanded by the RS algorithm depending on pre-computed search probabilities at each level. The method provides a near ML performance at a fixed computational effort and shows a strongly reduced complexity compared to the SD. However, contrarily to the RS, where a simple set expansion strategy is separately used in the real and imaginary dimensions of the constellation, the improved reduced search algorithm (IRS) offers a joint set expansion in the complex constellation according to precomputed search probabilities, resulting in a further complexity reduction. Moreover, Both RS detectors provide a soft output information for each transmitted bit applying simple probability calculations. This strategy presents a promising aspect for coded transmission.

Hence, the rest of the paper is structured as follows: In section II, the MIMO system model is presented. In section III, the proposed reduced search detectors (RS and IRS) are described. Section IV is dedicated to the soft output information delivered by the RS decoders. In section V, we compare the performance of the RS algorithms to some other MIMO detectors by applying them to the commonly used simulated flat Rayleigh channel, as most of the results of other detectors are available only for this type of channel. In section VI, measurements are performed in a real indoor frequency selective channel using a real 4×4 MIMO communication test bed so that we are closer to the practical circumstances. The performance of the RS algorithms will again be analyzed and compared to the performance of other, common detectors. Finally, our work will be concluded in section VII.

II. System model

We consider a symmetric MIMO system consisting of M transmit (Tx) and M receive (Rx) antennas and a frequency selective channel which will be described in discrete-time representation. The complex channel coefficients include the channel phase information. Assuming the channel to be time invariant over a burst, the channel impulse response (CIR) is of the form $\boldsymbol{H}[l] = \sum_{k=0}^{K-1} \boldsymbol{H}_k \ \delta[l-k]$, where the sum is evaluated over the K different transmission paths consisting of $(K-1)$ reflected signal parts, and where the LOS signal component is incorporated by the index $k = 0$. The complex matrix $\boldsymbol{H}_k \in \mathbb{C}^{M\times M}$ denotes the k-th MIMO transmission path. We define $\boldsymbol{s}[l] = [s_1[l], ..., s_M[l]]^{\mathrm{T}} \in \mathcal{A}^M$ to be the transmitted vector consisting of M data symbols arbitrarily chosen from the quadrature amplitude modulation (QAM) constellation $\mathcal{A}$ with the same probability of occurrence for each symbol.

$\boldsymbol{R_s} = \mathrm{E}[\boldsymbol{s}[l]\boldsymbol{s}^{\mathrm{H}}[l]] = \mathrm{diag}(\sigma_{s_1}^2, .., \sigma_{s_M}^2) \in \mathbb{R}^{M \times M}$ denotes the covariance matrix of the transmitted vector, where $\sigma_{s_m}^2$ denotes the Tx power of the m-th Tx antenna. The received signal vector $\boldsymbol{x}[l]$ can be expressed as:

$$\boldsymbol{x}[l] = \sum_{k=0}^{K-1} \boldsymbol{H}_k \ \boldsymbol{s}[l-k] + \boldsymbol{n}[l] \quad \in \mathbb{C}^M, \tag{1}$$

where $\boldsymbol{n}[l] \in \mathbb{C}^M$ denotes the additive white noise. We assume the noise to be zero-mean complex Gaussian with covariance matrix $\boldsymbol{R_n} = \mathrm{E}[\boldsymbol{n}[l]\boldsymbol{n}^{\mathrm{H}}[l]] = \mathrm{diag}(\sigma_{n_1}^2, .., \sigma_{n_M}^2) \in \mathbb{R}^{M \times M}$, where $\sigma_{n_m}^2$ denotes the noise power at the m-th Rx antenna.

III. Reduced Search Algorithms

We commence with the case of flat fading channels. In this case the CIR $\boldsymbol{H}[l]$ has only one transmission path. For ease of notation, time index $[l]$ is dropped in the following. The reduced search detectors are based on a first linear detection providing the starting solution followed by a RS ML. The search sets are adjusted by the RS algorithms depending on pre-computed search probabilities at each level. The method provides a near ML performance while it uses a fixed computational effort. The zero-forcing (ZF) equalizer $\boldsymbol{G} = \boldsymbol{H}^{-1}$ as well as the minimum mean square error (MMSE) equalizer $\boldsymbol{G} = \sigma_s^2 \boldsymbol{H}^{\mathrm{H}}(\sigma_s^2 \boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_M)^{-1}$ could be applied for the determination of the starting search set. Both equalizers offer a relatively simple filter design. The resulting noise at the equalizer output $\boldsymbol{\eta} = \boldsymbol{G}\boldsymbol{n} \in \mathbb{C}^M$ has the following noise power at level i

$$\sigma_{\eta_i}^2 = \sum_{m=1}^{M} |g_{im}|^2 \sigma_{n_m}^2. \tag{2}$$

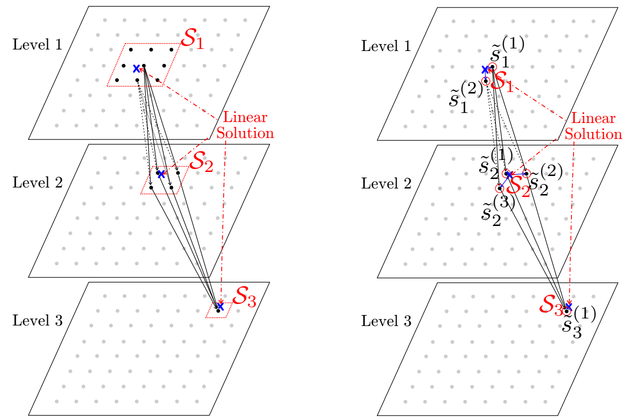
The calculation of the noise power at the equalizer output offers the possibility to determine the reduced search sets. Consequently, a larger search set will be needed for the symbols belonging to the least reliable data streams and a smaller search set for the symbols belonging to the most reliable data streams is expected.

A. Reduced Search Detector (RS)

The RS solution, already derived in [6], is obtained according to

$$\hat{\boldsymbol{s}}_{\mathrm{RS}} = \arg \min_{\hat{\boldsymbol{s}} \in \mathcal{S}} \|\boldsymbol{x} - \boldsymbol{H}\hat{\boldsymbol{s}}\|^2, \tag{3}$$

where $\mathcal{S} = \mathcal{S}_1 \times .. \times \mathcal{S}_M$ is a subset of $\mathcal{A}^M$ denoting the set of calculated hypotheses over all data streams whose cardinality equals the total number of candidates. Contrarily to the SD, where candidates in a hypersphere with a radius R around the received signal are investigated, the RS algorithm considers quadratic search sets $\mathcal{S}_i$ at each complex level i since the powers of real and imaginary noise parts belonging to the i-th data stream are equal[1]. Fig. 1 (a) shows an example of determined candidates located in the frame. The distribution scheme was $\boldsymbol{n}_{\mathcal{S}} = [n_1, n_2, n_3]^{\mathrm{T}} = [9, 4, 1]^{\mathrm{T}}$, where n_i denotes the number of candidates at the i-th level. The different subsets are derived around the linear solution which is marked with a cross at each level.



(a) RS (b) IRS

Fig. 1. Example of derived search subsets for the different decoders. The solution obtained by the linear equalizer is marked by a cross at each level.

B. Improved Reduced Search Detector (IRS)

Similar to the RS, the IRS algorithm is also based on pre-computed search probabilities determining the search set. However, contrarily to the RS algorithm, where a relatively simple set expansion strategy is applied resulting in a quadratic search set, the proposed IRS exclusively selects all candidates of a search set in the complex constellation according to the search probabilities. Applying the ZF equalizer to the received signal $\boldsymbol{x}$, we get the signal y_i corresponding to the i-th level

$$y_i = s_i + \eta_i, \tag{4}$$

where $\eta_i = \sum_{m=1}^{M} g_{im} n_m$ denotes the correlated noise at the equalizer output with the power $\sigma_{\eta_i}^2$, already computed in (2). In the following, we normalize equation (4) with $1/\sigma_{\eta_i}$, so that the normalized noise has power 1 at all levels.

The total search set $\mathcal{S}$ is expanded with the next hypothetical symbol $\tilde{s}'_i$ showing the next highest value of the conditional probability density function at level i

$$\begin{aligned} f\left(y'_i|\tilde{s}'_i\right) &= \frac{1}{\pi} \exp\left(-|y'_i - \tilde{s}'_i|^2\right) \\ &= \frac{1}{\pi} \exp\left(-\frac{|y_i - \tilde{s}_i|^2}{\sigma_{\eta_i}^2}\right), \end{aligned} \tag{5}$$

where $y'_i = \frac{1}{\sigma_{\eta_i}} y_i$ and $\tilde{s}'_i = \frac{1}{\sigma_{\eta_i}} \tilde{s}_i$ corresponds to the symbol chosen from the normalized constellation. Since the exponential function is a monotonically increasing function, it will be sufficient to investigate the ratios $\delta_i = \frac{d_i^{(q)}}{\sigma_{\eta_i}}$, where $d_i^{(q)} = |y_i - \tilde{s}_i^{(q)}|$ denotes the distance between the linear solution and the q-th hypothetical symbol at level i. We assume that the candidates $\tilde{s}_i^{(1)}, .., \tilde{s}_i^{(Q)}$ are already enumerated in an increasing order according to their respective distances $d_i^{(q)}$ at

[1] We assume that the constellation border is not reached.

each level i. The algorithm proceeds in this way until a stop criterion, when the total number of candidates $N_{\mathcal{S}} = \prod_{i=1}^{M} n_i$ supported by the hardware resources is achieved. Table I illustrates the IRS algorithm. Fig. 1 (b) shows an example of

Initialization: $c = 0$, $\mathcal{S} = \{\tilde{s}_1^{(1)}, .., \tilde{s}_M^{(1)}\}$
for $i = 1, .., M$
    $k_i = 1$
    select next candidate $\tilde{s}_i^{(1+k_i)}$ at level i
    $\delta_i = \frac{d_i^{(1+k_i)}}{\sigma_{\eta_i}}$
end
while $c = 0$
    $\mu = \arg\min_i [\delta_1, .., \delta_M]$
    $N = \prod_{\substack{i=1 \\ i \neq \mu}}^{M} k_i \cdot (k_\mu + 1)$
    if $N < N_{\mathcal{S}}$ and $k_\mu < Q$
        Expansion of $\mathcal{S}$ with $\tilde{s}_\mu^{(1+k_\mu)}$
        $k_\mu = k_\mu + 1$
        if $k_\mu < Q$
            select next candidate $\tilde{s}_\mu^{(1+k_\mu)}$ at level μ
            $\delta_\mu = \frac{d_\mu^{(1+k_\mu)}}{\sigma_{\eta_\mu}}$
        else
            $\delta_\mu = \infty$
        end
    else
        $c = 1$
    end
end

TABLE I
IRS ALGORITHM: DETERMINATION OF THE CANDIDATES AT THE DIFFERENT LEVELS.

determined candidates located in the circles at each level with a total distribution scheme $\boldsymbol{n}_{\mathcal{S}} = [2, 3, 1]^{\mathrm{T}}$. Some distances are also marked.

C. Frequency selective channel

The reduced search algorithms are now extended to the case of a frequency selective indoor channel. In this paper, we focus on measured indoor channels utilizing a real MIMO transmission test bed. Figure 2 exemplarily shows a typical, measured discrete-time MIMO channel represented by its 4×4 channel impulse responses in a bandwidth of 4 MHz.

The performed measurements show that the number of the transmission paths K is strongly limited in such a bandwidth and can, thus, be restricted to 2. Consequently, there are the LOS MIMO transmission path with strong power and one MIMO reflected signal part. The remaining samples of the CIRs are at the noise level.

The extended RS algorithms for the measured channel model are illustrated in Figure 3. For the first detection step we again refer to a linear equalizer which takes into account only the effect of the dominant LOS signal parts. This results in a simple design of the linear equalizer which has the same complexity as the linear equalizer used for the flat fading channel. In the second step, the RS algorithm will be applied for the data detection. In the third step, we use the DFE

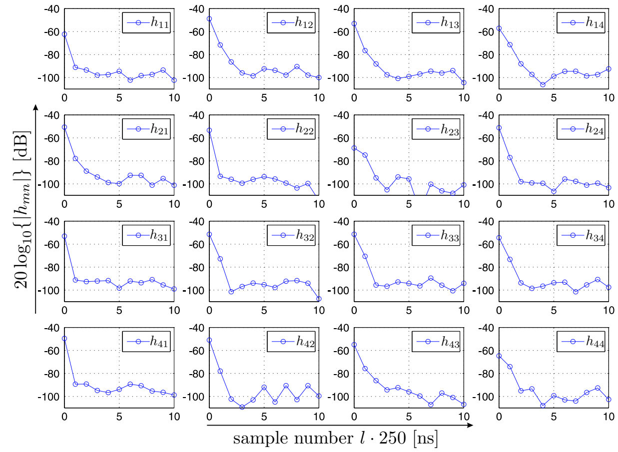


Fig. 2. Example of measured 4×4 MIMO channel impulse responses.

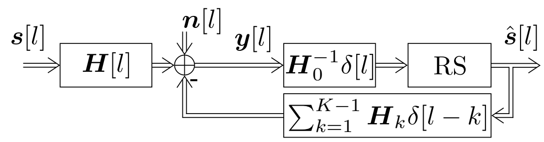


Fig. 3. Extended detection scheme using reduced search ML for frequency selective channels composed of K transmission paths.

in order to remove the ISI effect caused by the channel dispersion:

$$\boldsymbol{y}[l] = \sum_{k=0}^{K-1} \boldsymbol{H}_k \boldsymbol{s}[l-k] + \boldsymbol{n}[l] - \sum_{k=1}^{K-1} \boldsymbol{H}_k \hat{\boldsymbol{s}}[l-k], \qquad (6)$$

where $\hat{\boldsymbol{s}}[l]$ denotes the detected symbols delivered by the RS detector. As the measured MIMO indoor channel has two taps, equation (6) can be written as

$$\boldsymbol{y}[l] = \boldsymbol{H}_0 \boldsymbol{s}[l] + \boldsymbol{H}_1 \boldsymbol{s}[l-1] + \boldsymbol{n}[l] - \boldsymbol{H}_1 \hat{\boldsymbol{s}}[l-1]. \qquad (7)$$

An efficient removal of the ISI effect would be possible for a very low BER at the RS output. In this case, we get a flat channel model ($\boldsymbol{y}[l] = \boldsymbol{H}_0 \boldsymbol{s}[l] + \boldsymbol{n}[l]$) and the RS algorithms work in similar way as it has already been indicated in the previous subsections. Otherwise, an incorrect detection at the RS output could result in an error propagation for the next symbol detection. But this effect is limited due to the fact that the most channel power resulting from the strong LOS components is incorporated in the matrix $\boldsymbol{H}_0$. Therefore, a correct detection at the RS output stays also possible, even if the previous symbols are not detected correctly. Consequently, the extended detection scheme presents a promising approach for strong LOS frequency selective indoor channels without a significant increase of the computational effort compared to the flat fading case.

IV. SOFT OUTPUT INFORMATION

For coded transmission, the used detector has to provide soft output information, i.e. a posteriori probability about each transmitted bit. However, the estimation of the soft output information can be of high complexity when using a sphere detector, since the a posteriori probability of each detected bit depends on the detected bits in the current as well as in the previous levels according to the SD principle. So, some approaches have been proposed in order to simplify the soft output calculation, and therefore, to reduce its complexity [7].

Furthermore, the RS algorithms present a simple calculation of the soft output information using the already determined reduced search set. The log-likelihood ratio (LLR) corresponding to the k-th transmitted bit b_{i_k} at the i-th level is calculated according by

$$L(b_{i_k}|y_i) = \ln \frac{P(b_{i_k}=1|y_i)}{P(b_{i_k}=0|y_i)}, \tag{8}$$

where y_i denotes the unquantized linear solution at level i as defined in the previous section. Considering only the pre-selected symbols in the reduced search set $\mathcal{S}_i$, the LLR is approximated to

$$L(b_{i_k}|y_i) \approx \ln \frac{\sum_{\tilde{s}_i \in \mathcal{S}_i} f(y_i|\tilde{s}_i(b_{i_k}=1))}{\sum_{\tilde{s}_i \in \mathcal{S}_i} f(y_i|\tilde{s}_i(b_{i_k}=0))}, \tag{9}$$

where $f(y_i|\tilde{s}_i(b_{i_k}))$ is the one-dimensional conditional probability density function, defined as follows:

$$f(y_i|\tilde{s}_i(b_{i_k})) = \frac{1}{\sqrt{\pi}\sigma_{\eta_i}} \exp(-\frac{\mathcal{D}\{y_i - \tilde{s}_i(b_{i_k})\}^2}{\sigma_{\eta_i}^2}). \tag{10}$$

$\mathcal{D}$ denotes a real or imaginary operator depending on the considered k-th bit. We note that the distances $y_i - \tilde{s}_i(b_{i_k})$ have already been computed during the RS detection. Therefore, the effort required by the LLR estimation stays very low. Assuming the case that all candidates in the set $\mathcal{S}_i$ have the same value for a certain bit b_{i_k}, the set $\mathcal{S}_i$ will be extended with the next close candidates till we obtain a new candidate showing a different value for the bit b_{i_k}.

V. Performance analysis based on simulations

For comparison purposes, the following simulation assumptions will be considered. We consider a flat Rayleigh fading channel, resulting in a channel matrix $\boldsymbol{H} \in \mathbb{C}^{M \times M}$, containing uncorrelated complex Gaussian fading gains with unit variance. The channel matrix is additionally scaled by $1/\sqrt{M}$ as usual. A perfect channel knowledge at the Rx is assumed. Fig. 4 illustrates some simulation results of the IRS detector compared to the results of the SD, RS and V-BLAST. We refer to a 4×4 system using 64 QAM. The computational effort $\mathcal{F}$ is given for each detector. The MMSE IRS ($\mathcal{F} = 4 \times 10^3$) shows a loss of about 1 dB at BER 10^{-3} compared to the SD. With enhanced effort the MMSE IRS shows closer performance to the SD. The MMSE IRS ($\mathcal{F} = 2 \times 10^4$) shows about 0.3 dB loss at BER 10^{-3} compared to the SD with a tenth of the effort. We additionally observe that the MMSE IRS offers better performance than the ZF IRS showing the same complexity. Although the ZF provides an unbiased data detection and delivers, therefore, precise probability density functions as described in III-B, it suffers from an enhanced

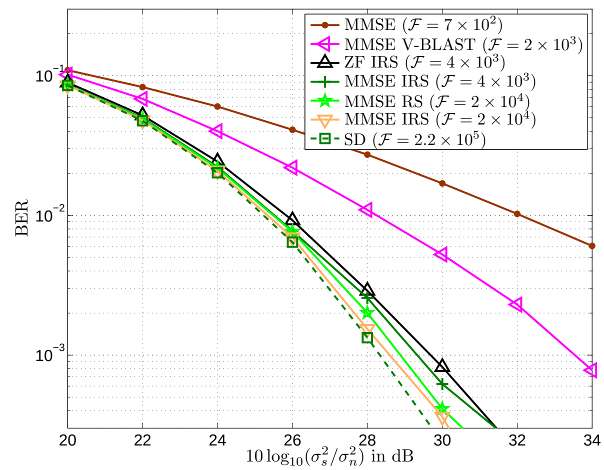


Fig. 4. BER performance and computational effort $\mathcal{F}$ for a 4×4 system using 64 QAM.

noise amplification compared to the MMSE. Thus, the use of MMSE equalizers at the first detection stage should be preferred. Otherwise, the ZF IRS achieves a gain of about 4 dB at BER 10^{-3} with a double complexity compared to the MMSE V-BLAST. Furthermore, the performances of the MMSE IRS decoders compared to the performance of the MMSE equalizer show the large gain which could be achieved using the reduced search strategy.

VI. Performance analysis based on measurements

In this section, we show the performance of the extended RS detectors in a real frequency selective indoor channel, so that we are close to the real circumstances. The equipment is explained in more details in [8]. Furthermore, the different antenna arrangements achieving maximum and minimum LOS capacities have been marked in Figure 5 by ”LOS orthogonal channel” and ”LOS keyhole channels”, respectively. The LOS orthogonal channel is obtained by a broadside antenna arrangement of Tx and Rx where the antenna spacings have exactly been optimized for a 4×4 MIMO system according to [9].

Figure 5 shows two calculated capacity CDFs based on measurements for the orthogonal and keyhole channels. A capacity gap between the optimum LOS configuration and an arrangement that leads to a pure LOS channel of almost keyhole channel transfer matrix (CTM) condition is observed. In comparison to the optimum the 90% outage capacity decreases about 10%. Nevertheless, the capacity, which is obtained in the keyhole LOS setup, is far from the real keyhole capacity. This is the result of the impinging reflections that increase the CTM condition number, and therefore, avoid a pure keyhole channel.

Besides, the extended RS algorithms as well as some commonly used MIMO detectors MMSE and MMSE V-BLAST have been applied for every measurement resulting in respective mutual information CDFs for the different antenna

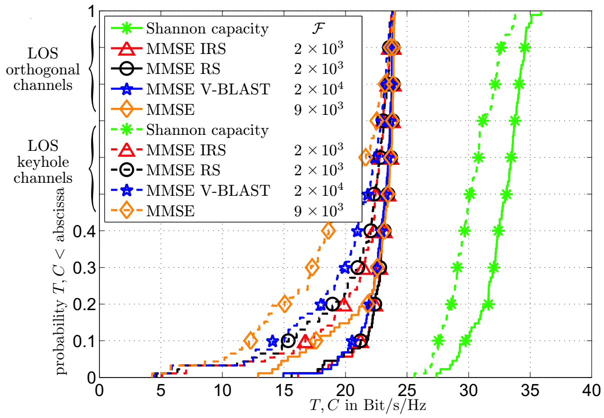


Fig. 5. 4×4 MIMO channel capacity (C) and mutual information (T) for different antenna setups and different MIMO detectors using 64-QAM. CIRs are composed of 2 taps. Computational effort $\mathcal{F}$ is given for each detector.

arrangements. We observe a mutual information degradation of MMSE and MMSE V-BLAST compared to the RS detectors, especially for keyhole channels. Thus, the 90% outage IRS mutual information (about 16.7 Bit/s/Hz) shows a gain of about 34% compared to MMSE (12.5 Bit/s/Hz) and 11% compared to MMSE V-BLAST (15 Bit/s/Hz). One reason for the mutual information degradation is the fact that MMSE and MMSE V-BLAST equalizers for some channel situations do not properly exhibit the noise correlation produced by the equalizer at the different data streams. This drawback is also a main reason for the deterioration of many MIMO equalizers in low-rank MIMO channels compared to their high-rank counterparts, where we observe only a slight decrease of the mutual information. We additionally note that the V-BLAST detector suffers from a possible error propagation through its successive interference cancellation. Otherwise, contrarily to the MMSE V-BLAST, where the information about the noise correlation will be provided in an unidirectional way from the already detected data streams to detect the next data streams, the RS algorithms present an approach, where the noise correlation will be exploited in an efficient way between all different data streams.

In the legend of Figure 5, the computational effort $\mathcal{F}$ is given for the corresponding detector. We observe that the RS detectors show a superior performance compared to the MMSE V-BLAST with a tenth of the effort. We again note that the used RS equalizers do not use the common Toeplitz matrix for frequency selective channels with its large dimension. Consequently, we get a remarkable complexity reduction for the RS algorithms with a relatively high performance. Since the RS algorithms offer a flexible trade-off between complexity and performance, the performances of the RS detectors in Figure 5 can be improved with an enhanced computational effort according to the results presented in Figure 4.

VII. Conclusion

A novel MIMO detector for frequency selective indoor channels was presented. The detector is capable of combining a near ML detection performance with a very low computational effort. Furthermore, the RS algorithms are optimized for practical implementation as they enable a performance analysis as well as an effort adjustment which is determined by the maximum effort and limited by the available hardware resources. The method outperforms different common proposals resulting in an improved performance with less complexity. Moreover, the RS algorithms provide a soft output information for each transmitted bit applying simple probability calculations which presents a promising aspect for coded transmission.

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