Symbol Rate Upper-Bound on Distributed STBC with Channel Phase Information

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***Abstract*—Recently, single-symbol maximum-likelihood (ML) decodable distributed space-time block coding (DSTBC) has been developed for use in cooperative diversity networks. However, the symbol rate of the DSTBC decreases with the number of relays. This issue can be addressed if the channel phase information (CPI) of the first-hop is exploited, and such code is referred to as DSTBC-CPI. Some complex single-symbol decodable DSTBCs-CPI were reported in the literature. However, no upper-bound on the symbol rate of such DSTBC-CPI has been derived, although it is a fundamental issue. Furthermore, finding a tight (more preferably achievable) upper-bound is essential to check if any developed code is optimum or not. In this letter, we derive an upper-bound on the symbol rate of real single-symbol decodable DSTBC-CPI and show that the bound is independent of the number of relays in the network. Finally, we demonstrate that our derived bound is actually achievable.**

***Index Terms*—Channel phase information, DSTBC, symbol rate, upper-bound.**

I. INTRODUCTION

In a cooperative network, the relays cooperate to help the source transmit the information-bearing symbols to the destination [1],[2]. The cooperative strategy of the relays is crucial and it decides the performance of a cooperative network. A simple cooperative strategy is the *repetition-based cooperative strategy* [2]. This cooperative strategy achieves the full diversity order in the number K of relays. Furthermore, the maximum likelihood (ML) decoding at the destination is single-symbol ML decodable.[1] However, the repetition-based cooperative strategy has poor bandwidth efficiency, since its symbol rate[2] is just $1/K$. Many works have been devoted to improving the bandwidth efficiency of the cooperative networks, such as the cooperative beamforming [3] or the relay selection [4]. More attentions have been given to the *distributed space-time block code* (DSTBC) [5]–[14] and various issues have been studied for DSTBCs [15]–[17]. Although all those DSTBCs could improve the bandwidth efficiency, they were not single-symbol ML decodable in general, and hence, they had much higher decoding complexities than the repetition-based cooperative strategy.

Addressing the decoding complexity, some researchers have studied the DSTBC achieving the single-symbol ML decodability[3] and the full diversity order. In [20], Hua *et al.* found that most codes based on existing orthogonal designs were not single-symbol ML decodable any more. In [21] and [22], the authors used the generalized coordinate interleaved orthogonal designs and proposed the clifford unitary weight single-symbol decodable codes in cooperative networks. The codes were single-symbol ML decodable, but only for certain number of relays. Furthermore, the codes could achieve the full diversity order only for some specially designed constellations, not general constellations such as the standard quadrature amplitude modulation (QAM).

Recently, the DSTBC achieving the single-symbol ML decodability and full diversity for any number of relays and any general constellations was developed [23]. The associated matrices of those codes were row-monomial[4], and hence, they always generated uncorrelated noises at the destination, which lead to the single-symbol ML decodability and the full diversity order. Furthermore, an upper-bound onthe symbol rate of the DSTBC was derived [23]. This upper-bound revealed that the DSTBC had a much higher (approximately twice) symbol rate than the repetition-based cooperative strategy, irrespective of the number of relays

However, even the improved symbol rate of the DSTBC [23] decreases with the number K of relays, and it asymptotically approaches zero as K grows as in the repetition-based strategy. This issue can be addressed if the channel state information (CSI) of the channels from the source to the relays (the first-hop) is exploited by the relays as studied in [24] and independently in [22]. Note that such CSI of the first-hop is already available at the relay, and thus, exploiting the CSI does not imply any increase of pilot signaling or feedback overhead.[5] Specifically, the channel phase information (CPI) of the first-hop was used at the relays in order to further boost

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[1]A code or a scheme is single-symbol ML decodable, if its ML decoding metric can be written as a sum of multiple terms, each of which depends on at most one transmitted information-bearing symbol.

[2]In this paper, the symbol rate of a cooperative strategy or a distributed space-time code is defined to be the ratio of the number of transmitted information-bearing symbols to the number of time slots used by the relays to transmit all these symbols.

[3]Single symbol decodability was also studied for the conventional STBCs including quasi-orthogonal STBCs; for example, see [18] and [19].

[4]A matrix is said to be row-monomial (column-monomial) if the matrix has at most one non-zero entry on every row (column) [25].

[5]Note that the CSI of the channels from the relays to the destination (the second-hop) can be available at the relays *only* by using extra pilot signaling and feedback overhead. If such CSI of the second-hop were available at the relays, the distributed beamforming could be used across the multiple relays.

the symbol rate of the DSTBC and the codes were refereed to as DSTBC-CPI [22], [24]. However, *no* upper-bound on the symbol rate of DSTBC-CPI was derived in [22]; but deriving (preferably *achievable*) upper-bound is a fundamental issue and it is essential to test if any developed code is optimum or not. This issue is addressed in this letter.[6]

In this letter, as in [24] and [22], we consider the DSTBC-CPI. We derive an upper-bound on the symbol rate of the DSTBC-CPI. The derived upper-bound is much higher than that of the DSTBC of [23], especially for large K. In particular, the upper-bound on the symbol rate of DSTBC-CPI is independent of the number K of relays, which ensures the codes have good bandwidth efficiency even when there are many relays. Finally, we demonstrate that an existing STBC can be used as the proposed DSTBC-CPI and it satisfies the derived upper-bound; that is, we demonstrate that our obtained bound is actually an achievable bound.

Notation: Bold upper and lower letters denote matrices and row vectors, respectively. Also, $\text{diag}[x_1, \cdots, x_K]$ denotes the $K \times K$ diagonal matrix with $x_1, \cdots, x_K$ on its main diagonal; $\mathbf{0}$ the all-zero matrix; $\boldsymbol{I}$ the identity matrix; $(\cdot)^*$ the complex conjugate; $(\cdot)^H$ the Hermitian; $(\cdot)^T$ the transpose. Let $\boldsymbol{X} = [\boldsymbol{x}_1; \cdots; \boldsymbol{x}_K]$ denote the matrix with $\boldsymbol{x}_k$ as its k-th row, $1 \leq k \leq K$.

II. System Model

Consider a cooperative network with one source, K relays, and one destination. Every terminal has only one antenna and is of half-duplex. Denote the channel from the source to the k-th relay by h_k and the channel from the k-th relay to the destination by f_k, where h_k and f_k are spatially uncorrelated complex Gaussian random variables with zero mean and unit variance. As in many other publications on cooperative networks, the destination is assumed to have full CSI, i.e., it knows the instantaneous values of h_k and f_k by using pilot signals; while the source is assumed to have no CSI. Furthermore, we assume the relays are the so-called *partially-coherent relays* as defined in [22]. Specifically, the relays have partial CSI of the first-hop only. For example, the k-th relay has the CPI of the first-hop, i.e., it knows the phase θ_k of the channel coefficient h_k.[7] Note that this assumption does not imply more pilot signals compared to the assumption that the relays have no CSI of the first-hop. Actually, in order to make the destination have full CSI, the relays always need to forward the pilot signals from the source to the relays. Furthermore, the relays always need to transmit their own pilot signals to the destination [20]. Therefore, the same amount of pilot signals is needed in all circumstances. Furthermore, the assumption that the relays have the CPI of the first-hop does not imply any feedback overhead, because the relays do not need any CSI of the channels from themselves to the destination.

At the beginning, the source transmits N complex-valued information-bearing symbols over N consecutive time slots.[8] Let $\boldsymbol{s} = [s_1, \cdots, s_N]$ denote the information-bearing symbol vector transmitted from the source, where the power of s_n is E_s. The received signal vector $\boldsymbol{y}_k$ at the k-th relay is given by $\boldsymbol{y}_k = h_k \boldsymbol{s} + \boldsymbol{n}_k$, where $\boldsymbol{n}_k = [n_{k,1}, \cdots, n_{k,N}]$ is the additive noise at the k-th relay and it is uncorrelated complex Gaussian with zero mean and identity covariance matrix. All the relays are working in the amplify-and-forward mode and the amplifying coefficient ρ is $\sqrt{E_r/(1+E_s)}$ for every relay, where E_r is the transmission power at every relay. The k-th relay first obtains $\tilde{\boldsymbol{y}}_k$ by $\tilde{\boldsymbol{y}}_k = e^{-j\theta_k}\boldsymbol{y}_k$ and then builds the transmitted signal vector $\boldsymbol{x}_k$ as $\boldsymbol{x}_k = \rho(\tilde{\boldsymbol{y}}_k \boldsymbol{A}_k + \tilde{\boldsymbol{y}}_k^* \boldsymbol{B}_k)$. The matrices $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ are called the associated matrices of $\boldsymbol{X}$ and they have the dimension of $N \times T$. The received signal vector at the destination is given by $\boldsymbol{y} = \boldsymbol{w}\boldsymbol{X} + \boldsymbol{n}$, where $\boldsymbol{w} = [\rho f_1 |h_1|, \cdots, \rho f_K |h_K|]$, $\boldsymbol{X} = [\boldsymbol{s}\boldsymbol{A}_1 + \boldsymbol{s}^*\boldsymbol{B}_1; \cdots; \boldsymbol{s}\boldsymbol{A}_K + \boldsymbol{s}^*\boldsymbol{B}_K]$, and $\boldsymbol{n} = \sum_{k=1}^{K} (\rho f_k e^{-j\theta_k} \boldsymbol{n}_k \boldsymbol{A}_k + \rho f_k e^{j\theta_k} \boldsymbol{n}_k^* \boldsymbol{B}_k) + \boldsymbol{n}_d$. It is easy to see that the mean of $\boldsymbol{n}$ is zero and the covariance matrix $\boldsymbol{R}$ of $\boldsymbol{n}$ is given by

$$\boldsymbol{R} = \sum_{k=1}^{K} \left(|\rho f_k|^2 \left(\boldsymbol{A}_k^H \boldsymbol{A}_k + \boldsymbol{B}_k^H \boldsymbol{B}_k \right) \right) + \boldsymbol{I}. \quad (1)$$

III. Upper-Bound on Symbol Rate of DSTBC-CPI

As the DSTBC was defined in [23], we first mathematically define the DSTBC-CPI as follows.

Definition 1: A $K \times T$ code matrix $\boldsymbol{X}$ is called a DSTBC-CPI in variables $s_1, \cdots, s_N$ if the entries of $\boldsymbol{X}$ are 0, $\pm s_n$, $\pm s_n^*$, or multiples of these indeterminates by $\jmath = \sqrt{-1}$, and if the matrix $\boldsymbol{X}$ satisfies the following equality

$$\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^H = |s_1|^2 \boldsymbol{F}_1 + \cdots + |s_N|^2 \boldsymbol{F}_N, \quad (2)$$

where $\boldsymbol{F}_n = \text{diag}[F_{n,1}, \cdots, F_{n,K}]$ and $F_{n,1}, \cdots, F_{n,K}$ are non-zero. Furthermore, the associated matrices $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$, $1 \leq k \leq K$, are row-monomial.

It is easy to check that the DSTBC-CPI are single-symbol ML decodable. By using the technique in [23], it can be shown that the DSTBCs-CPI also achieve the full diversity order. In general, the covariance matrix $\boldsymbol{R}$ is not a diagonal matrix as we have seen in [23], and hence, the proposed DSTBC-CPI is fundamentally different from the conventional STBCs defined in [26] whose definition is based on $\boldsymbol{X}\boldsymbol{X}^H$. In fact, the existence of $\boldsymbol{R}^{-1}$ in the definition of the DSTBC-CPI makes the analysis of the code much more difficult. Fortunately, the covariance matrix $\boldsymbol{R}$ becomes a diagonal matrix if the associated matrices $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ are set to be row-monomial as shown in [23]. We adopt this setting in this letter and the analysis becomes possible. The objective of this paper is to find a tight (more desirably, achievable) upper-bound on the symbol rate on the DSTBC-CPI. To this end, we first prove the following lemma.

Lemma 1: Assume $\boldsymbol{X}$ is a DSTBC in variables $s_1, \cdots, s_N$, i.e., every row of $\boldsymbol{X}$ contains the information-bearing symbols

[6][24] is the conference version of this paper and some results of this paper was partially presented in [24]. However, no analytical proofs including Appendices A, B, and C of this paper were presented in [24].

[7]In this paper, we assume that the relays can estimate θ_k without any errors as in [12], [20]. It will be interesting to study the scenario when the relays do not have perfect estimations of θ_k; but it is beyond the scope of this paper.

[8]If the information-bearing symbols are real-valued, one can use the rate-one generalized real orthogonal design proposed by [26] in the cooperative networks without any changes. The codes achieve the single-symbol ML decodability and the full diversity order [20]. Therefore, we focus on the complex-valued symbols in this paper.

$s_1, \cdots, s_N$. Moreover, assume that the noise covariance matrix $\boldsymbol{R}$ of $\boldsymbol{X}$ is diagonal. After proper column permutations, we can partition $\boldsymbol{R}^{-1}$ into $\boldsymbol{R}^{-1} = \text{diag}[\boldsymbol{R}_1, \boldsymbol{R}_2, \cdots, \boldsymbol{R}_W]$ such that the main diagonal entries of $\boldsymbol{R}_w$ are all equal to R_w, $w = 1, \cdots, W$, and that $R_i \neq R_j$ for $i \neq j$. After the same column permutations, we can partition $\boldsymbol{X}$ into $\boldsymbol{X} = [\boldsymbol{X}_1, \cdots, \boldsymbol{X}_W]$. Let $\tilde{\boldsymbol{X}}_w$ denote a matrix composed of all the non-zero rows in $\boldsymbol{X}_w$.[9] Assume that $\tilde{\boldsymbol{X}}_w$ contains N_w different information-bearing symbols and they are $s_1^w, \cdots, s_{N_w}^w$.[10] Then $\boldsymbol{X}$ is a DSTBC-CPI if and only if every sub-matrix $\tilde{\boldsymbol{X}}_w$ is a DSTBC-CPI in variables $s_1^w, \cdots, s_{N_w}^w$.

Proof: See Appendix A. ■

This lemma implies that, when a DSTBC $\boldsymbol{X}$ generates uncorrelated noises at the destination, the code is single-symbol ML decodable as long as it can be partitioned into several single-symbol ML decodable codes. Lemma 1 is crucial to derive an upper-bound on the symbol rate of the DSTBC-CPI. This is because it enables us to analyze the symbol rate of every individual sub-matrix $\tilde{\boldsymbol{X}}_w$ instead of $\boldsymbol{X}$ itself. When $\tilde{\boldsymbol{X}}_w$ has one or two rows, it is easy to see that its symbol rate can be as large as one. When $\tilde{\boldsymbol{X}}_w$ has more than two rows, the following lemma shows that the symbol rate of $\tilde{\boldsymbol{X}}_w$ is exactly $1/2$.

Lemma 2: Assume $\boldsymbol{X}$ is a DSTBC-CPI and its noise covariance matrix is $\boldsymbol{R}$. By proper column permutations, we can partition $\boldsymbol{R}^{-1}$ into $\boldsymbol{R}^{-1} = \text{diag}[\boldsymbol{R}_1, \boldsymbol{R}_2, \cdots, \boldsymbol{R}_W]$ such that the main diagonal entries of $\boldsymbol{R}_w$ are all equal to R_w and $R_i \neq R_j$ for $i \neq j$. By the same column permutations, we can partition $\boldsymbol{X}$ into $\boldsymbol{X} = [\boldsymbol{X}_1, \cdots, \boldsymbol{X}_W]$. Let $\tilde{\boldsymbol{X}}_w$ denote all the non-zero rows in $\boldsymbol{X}_w$ and assume the dimension of $\tilde{\boldsymbol{X}}_w$ is $K_w \times T_w$. Then the symbol rate of $\tilde{\boldsymbol{X}}_w$ is exactly $1/2$ when $K_w > 2$.

Proof: See Appendix B. ■

Based on Lemmas 1 and 2, we now derive an upper-bound on the symbol rate of the DSTBC-CPI in the following theorem.

Theorem 1: When $K > 2$, the symbol rate $\mathcal{R}$ of the DSTBC-CPI satisfies the following inequality

$$\mathcal{R} = \frac{N}{T} \leq \frac{1}{2}. \tag{3}$$

Proof: See Appendix C. ■

We notice that the symbol rate of the DSTBC-CPI does not decrease with the number K of relays. Thus, the DSTBC-CPI have good bandwidth efficiency even in a cooperative network with many relays. Furthermore, compared to the DSTBC in [23], the DSTBC-CPI improve the bandwidth efficiency considerably, especially when the cooperative network has a large number of relays.

Remark 1: The improvement of the symbol rate is mainly because the relays exploit the CPI in the code construction. As we saw in [23], because the code matrix of a DSTBC contains the channel coefficient h_k, the associated matrices $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ must satisfy the following condition

$$\boldsymbol{A}_{k_1}\boldsymbol{R}^{-1}\boldsymbol{A}_{k_2}^H = \boldsymbol{B}_{k_1}\boldsymbol{R}^{-1}\boldsymbol{B}_{k_2}^H = \boldsymbol{0}, \qquad k_1 \neq k_2. \tag{4}$$

This condition severely limited the symbol rate of the DSTBC. On the other hand, by exploiting the CPI, the code matrix $\boldsymbol{X}$ of a DSTBC-CPI does *not* have any channel coefficients. Instead of the condition (4), therefore, the associated matrices $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ of a DSTBC-CPI just need to satisfy the following condition

$$\boldsymbol{A}_{k_1}\boldsymbol{R}^{-1}\boldsymbol{A}_{k_2}^H + \boldsymbol{B}_{k_1}\boldsymbol{R}^{-1}\boldsymbol{B}_{k_2}^H = \boldsymbol{0}, \qquad k_1 \neq k_2. \tag{5}$$

Thus, the symbol rate is greatly improved. Furthermore, recall that exploiting the CPI at the relays does not increase the pilot signals or require any feedback overhead.

Remark 2: Interestingly enough, the upper-bound $1/2$ derived in Theorem 1 is *achievable* by well-known codes, so-called the rate-halving codes in [26]. It is easy to prove that the rate-halving codes satisfy Definition 1. For example, when $N = 4$ and $K = 4$, the DSTBC-CPI achieving the upper-bound $1/2$ is given as follows:

$$\boldsymbol{X} = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & s_3 & -s_2 & s_1 & s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix}. \tag{6}$$

As another example, when $N = 4$ and $K = 3$, the DSTBC-CPI achieving the upper-bound $1/2$ is given as follows:

$$\boldsymbol{X} = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \end{bmatrix}. \tag{7}$$

Note that the upper-bound derived in Theorem 1 is the maximum achievable rate.

Remark 3: In [21] and [22], some single-symbol ML decodable DSTBCs-CPI were reported for certain number of relays and the symbol rates were higher than 1/2. However, those codes did not achieve full diversity order for general constellations such as the standard QAM. The codes [21], [22], were constructed based on so-called the coordinate interleaved orthogonal design (similar to the constellation rotation) and can be seen as complex single-symbol decodable DSTBCs. The codes could achieve full diversity *only* for some specially designed constellations. Furthermore, in [21] and [22], *no* upper-bound on the symbol rate of DSTBCs-CPI was derived, although this is a fundamental issue. In contrast to those works, the proposed DSTBCs-CPI should be considered as real single-symbol decodable DSTBCs. We do not make any special assumption on the constellation and we answer the fundamental question in Theorem 1 by deriving the upper-bound on the symbol rate. The obtained upper-bound is valid for *all* single-symbol ML decodable DSTBCs-CPI achieving full-diversity for *any* constellation including the standard QAM.

IV. Numerical Results

In this section, we present some numerical results to demonstrate the performance of the DSTBC-CPI. In our simulation, we assume uncorrelated flat Rayleigh fading channels among

[9] As an example, if $\boldsymbol{X} = \begin{bmatrix} s_1 & -s_2 & 0 & 0 \\ s_2^* & s_1^* & 0 & 0 \\ 0 & 0 & s_1 & s_2 \end{bmatrix}$, we have $W = 2$, $\boldsymbol{R}_1 = \text{diag}[1 + |\rho f_1|^2 + |\rho f_2|^2, 1 + |\rho f_1|^2 + |\rho f_2|^2]$, $\boldsymbol{R}_2 = \text{diag}[1 + |\rho f_3|^2, 1 + |\rho f_3|^2]$, $\boldsymbol{X}_1 = \begin{bmatrix} s_1 & -s_2 \\ s_2^* & s_1^* \end{bmatrix}$, and $\boldsymbol{X}_2 = [s_1 \;\; s_2]$.

[10] Note that $s_1^w, \cdots, s_{N_w}^w$ are all from the set $\boldsymbol{s} = [s_1, \cdots, s_N]$.

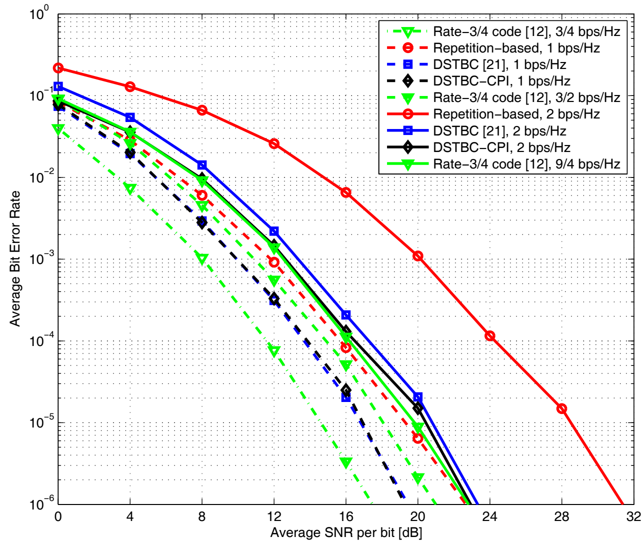

Fig. 1. Comparison of the rate-$3/4$ code [12], DSTBC [23], DSTBC-CPI, and the repetition-based cooperative strategy, $N = 4$, $K = 4$.

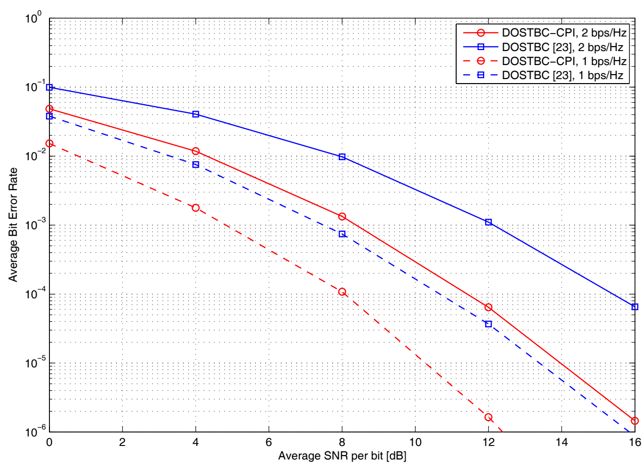

Fig. 2. Comparison of DSTBC [23] and DSTBC-CPI, $N = 8$, $K = 6$.

the terminals. The channel coefficients remain constant for the entire transmission of one code block. We define the average signal-to-noise ratio (SNR) per bit as the ratio of E_r to the logarithm of the size of the modulation scheme. Furthermore, we adopt the power allocation proposed in [10], i.e., $E_s = KE_r$. For comparison, the repetition-based cooperative strategy is chosen as the performance benchmark because it has single-symbol decodability and is widely studied. We also include the rate-$3/4$ code in [12] because it is a very well-known STBC. Please note that this code is not single-symbol ML decodable in cooperative networks. Lastly, the DSTBCs in [23] is considered, as we want to show the performance improvement by exploiting the CPI at the relays.

In Fig. 1, we let $N = 4$ and $K = 4$. We see that the average bit-error rate (BER) performance of the DSTBC-CPI is much better than that of the repetition-based cooperative strategy, especially when the bandwidth efficiency is 2 bps/Hz. The DSTBC and the DSTBC-CPI have almost the same performance. This is because, when $N = 4$ and $K = 4$, they have the same symbol rate $1/2$. Fig. 1 also demonstrates that the performance of the DSTBC-CPI is slightly worse than that of the rate-$3/4$ code proposed in [12]. But, note that the rate-$3/4$ code is *not* single-symbol ML decodable, and hence, its decoding complexity is much higher than that of the DSTBC-CPI. In Fig. 2, we set $N = 8$ and $K = 6$. For this case, the average BER performance of the DSTBC-CPI is now much better than that of the DSTBC. This is because, when $N = 8$ and $K = 6$, the symbol rate of the DSTBC-CPI is still $1/2$ by making use of the CPI; while the symbol rate of the DSTBC becomes $1/3$.

V. Conclusion

In this letter, we considered single-symbol ML decodable DSTBC achieving full diversity order and exploiting the channel phase information of the first-hop. For the code, we derived an upper-bound on the symbol rate of the code. The upper-bound on the symbol rate revealed that the DSTBC exploiting CPI has a much higher bandwidth efficiency than the DSTBC of [23]. In particular, the upper-bound on the symbol rate of the DSTBC exploiting CPI is independent of the number K of relays, and hence, the code has particularly better bandwidth efficiency with many relays than the DSTBC of [23] or the repetition-based transmission, because the symbol rate of the DSTBC decreases with the number of relays. We also demonstrated that a well-known existing STBC can be used as such DSTBC exploiting CPI and it achieves the upper-bound. That is, our derived upper-bound is the achievable bound.

Appendix A
Proof of Lemma 1

The sufficient part is easy to verify. Thus, we focus on the necessary part, i.e., if $\boldsymbol{X}$ is a DSTBC-CPI, all the sub-matrices $\boldsymbol{X}_w$ are also DSTBC-CPI. Assume that the dimension of $\boldsymbol{R}_w$ is $T_w \times T_w$.

Firstly, we show that $\boldsymbol{X}_w \boldsymbol{R}_w \boldsymbol{X}_w^H$ is a diagonal matrix. Because $\boldsymbol{X}_1, \cdots, \boldsymbol{X}_w$ contain all the non-zero entries in $\boldsymbol{X}$, it follows from (2) that, when $k_1 \neq k_2$, $[\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^H]_{k_1,k_2}$ is given by

$$[\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^H]_{k_1,k_2} = \sum_{w=1}^{W}\sum_{t=1}^{T_w}[\boldsymbol{X}_w]_{k_1,t}[\boldsymbol{X}_w]^*_{k_2,t}R_w = 0. \tag{A.1}$$

If all the terms in this summation are zero, it is trivial to show that $\sum_{t=1}^{T_w}[\boldsymbol{X}_w]_{k_1,t}[\boldsymbol{X}_w]^*_{k_2,t}R_w = 0$ for $1 \leq w \leq W$, which means $\boldsymbol{X}_w \boldsymbol{R}_w \boldsymbol{X}_w^H$ is a diagonal matrix.

If there is one term $[\mathbf{X}_{w_1}]_{k_1,t_1}[\mathbf{X}_{w_1}]^*_{k_2,t_1}R_{w_1} \neq 0$, some other terms must cancel this term in order to make (A.1) hold. Actually, the non-zero term $[\mathbf{X}_{w_1}]_{k_1,t_1}[\mathbf{X}_{w_1}]^*_{k_2,t_1}R_{w_1}$ must be cancelled by exactly one other term. This can be shown by contradiction. We assume that $[\mathbf{X}_{w_1}]_{k_1,t_1}[\mathbf{X}_{w_1}]^*_{k_2,t_1}R_{w_1}$ is cancelled by two other terms together, i.e.,

$$\begin{aligned} &[\mathbf{X}_{w_1}]_{k_1,t_1}[\mathbf{X}_{w_1}]^*_{k_2,t_1}R_{w_1} + [\mathbf{X}_{w_2}]_{k_1,t_2}[\mathbf{X}_{w_2}]^*_{k_2,t_2}R_{w_2} \\ + \; &[\mathbf{X}_{w_3}]_{k_1,t_3}[\mathbf{X}_{w_3}]^*_{k_2,t_3}R_{w_3} = 0. \end{aligned} \tag{A.2}$$

In order to make this equality hold, one of the following three equalities must hold: 1) $[\mathbf{X}_{w_2}]_{k_1,t_2} = \pm[\mathbf{X}_{w_1}]_{k_1,t_1}$; 2) $[\mathbf{X}_{w_3}]_{k_1,t_3} = \pm[\mathbf{X}_{w_1}]_{k_1,t_1}$; 3) $\pm[\mathbf{X}_{w_2}]_{k_1,t_2} = \pm[\mathbf{X}_{w_3}]_{k_1,t_3} =$

$[\mathbf{X}_{w_1}]^*_{k_2,t_1}$. However, those three equalities all contradict with our assumption that the covariance matrix $\boldsymbol{R}$ is diagonal. For example, we assume $[\mathbf{X}_{w_1}]_{k_1,t_1} = s_n^{w_1}$, $1 \le n \le N_{w_1}$, and the equality $[\mathbf{X}_{w_2}]_{k_1,t_2} = \pm[\mathbf{X}_{w_1}]_{k_1,t_1}$ holds. Thus, $[\mathbf{X}_{w_2}]_{k_1,t_2} = \pm s_n^{w_1}$ and $s_n^{w_1}$ is transmitted in the k_1-th row of $\boldsymbol{X}$ for at least twice. This makes the noise covariance matrix $\boldsymbol{R}$ non-diagonal, which contradicts with our assumption. If we assume $[\mathbf{X}_{w_1}]_{k_1,t_1}[\mathbf{X}_{w_1}]^*_{k_2,t_1}R_{w_1}$ is cancelled by more than two other terms, the same contradiction can be seen similarly. Thus, $[\mathbf{X}_{w_1}]_{k_1,t_1}[\mathbf{X}_{w_1}]^*_{k_2,t_1}R_{w_1}$ is cancelled by exactly one other term in the summation (A.1) and we have

$$[\mathbf{X}_{w_1}]_{k_1,t_1}[\mathbf{X}_{w_1}]^*_{k_2,t_1}R_{w_1} + [\mathbf{X}_{w_2}]_{k_1,t_2}[\mathbf{X}_{w_2}]^*_{k_2,t_2}R_{w_2} = 0. \tag{A.3}$$

Furthermore, because $R_i \ne R_j$ when $i \ne j$, (A.3) also implies that $R_{w_1} = R_{w_2}$ and $w_1 = w_2$. This means that, if one term in the summation (A.1) is non-zero, it must be cancelled by exactly one other term, which is from the same sub-matrix $\boldsymbol{X}_w$. Therefore, we have $\sum_{t=1}^{T_w}[\boldsymbol{X}_w]_{k_1,t}[\boldsymbol{X}_w]^*_{k_2,t}R_w = 0$, when $k_1 \ne k_2$, and $\boldsymbol{X}_w\boldsymbol{R}_w\boldsymbol{X}_w^H$ is a diagonal matrix.

Secondly, we show that the information-bearing symbols $s_1^w, \cdots, s_{N_w}^w$ are contained in every row of $\boldsymbol{X}_w$. Because every main diagonal entry of $\boldsymbol{R}_w$ is the same and $\boldsymbol{X}_w$ only contains non-zero rows, it follows from $\boldsymbol{R} = \sum_{k=1}^K \left(|\rho f_k|^2 \left(\boldsymbol{A}_k^H\boldsymbol{A}_k + \boldsymbol{B}_k^H\boldsymbol{B}_k\right)\right) + \boldsymbol{I}$ that $\boldsymbol{X}_w$ actually does not contain any zero entries. Then we assume that $[\boldsymbol{X}_w]_{k_1,t_1} = s_n^w$, $1 \le n \le N_w$. Because every entry in $\boldsymbol{X}_w$ is non-zero, we can find another non-zero entry $[\boldsymbol{X}_w]_{k_2,t_1}$, $k_1 \ne k_2$, from the t_1-th column of $\boldsymbol{X}_w$. Thus, $[\boldsymbol{X}_w\boldsymbol{R}_w\boldsymbol{X}_w^H]_{k_1,k_2}$ must contain the term $[\boldsymbol{X}_w]_{k_1,t_1}[\boldsymbol{X}_w]^*_{k_2,t_1}R_w$. Because $[\boldsymbol{X}_w\boldsymbol{R}_w\boldsymbol{X}_w^H]_{k_1,k_2} = 0$, $[\boldsymbol{X}_w]_{k_1,t_1}[\boldsymbol{X}_w]^*_{k_2,t_1}R_w$ must be cancelled by another term and we assume it is $[\boldsymbol{X}_w]_{k_1,t_2}[\boldsymbol{X}_w]^*_{k_2,t_2}R_w$, $t_1 \ne t_2$. In order to make $[\boldsymbol{X}_w]_{k_1,t_1}[\boldsymbol{X}_w]^*_{k_2,t_1}R_w + [\boldsymbol{X}_w]_{k_1,t_2}[\boldsymbol{X}_w]^*_{k_2,t_2}R_w = 0$, we must have $[\boldsymbol{X}_w]_{k_1,t_2} = \pm[\boldsymbol{X}_w]_{k_1,t_1}$ or $[\boldsymbol{X}_w]^*_{k_2,t_2} = \pm[\boldsymbol{X}_w]_{k_1,t_1}$. Due to the row-monomial condition, $[\boldsymbol{X}_w]_{k_1,t_2}$ can not be $\pm[\boldsymbol{X}_w]_{k_1,t_1}$, and hence, we have $[\boldsymbol{X}_w]_{k_2,t_2} = \pm[\boldsymbol{X}_w]^*_{k_1,t_1} = \pm s_n^{w*}$. This means that the k_2-th row contains the information-bearing symbol s_n^w as well. Taking a similar approach, we can show that the information-bearing symbols $s_1^w, \cdots, s_{N_w}^w$ are contained in every row of $\boldsymbol{X}_w$.

Because $\boldsymbol{X}_w\boldsymbol{R}_w\boldsymbol{X}_w^H$ is a diagonal matrix and every row of $\boldsymbol{X}_w$ contains all the information-bearing symbols $s_1^w, \cdots, s_{N_w}^w$, $\boldsymbol{X}_w\boldsymbol{R}_w\boldsymbol{X}_w^H$ can be written as

$$\boldsymbol{X}_w\boldsymbol{R}_w\boldsymbol{X}_w^H = |s_1^w|^2\boldsymbol{M}_1 + \cdots + |s_{N_w}^w|^2\boldsymbol{M}_{N_w}, \tag{A.4}$$

where $\boldsymbol{M}_n$ are diagonal and all the main diagonal entries are non-zero. Note that, if the relays only transmit $\boldsymbol{X}_w$ to the destination, $\boldsymbol{R}_w$ is actually the inverse of the noise covariance matrix at the destination. This is because $\boldsymbol{X}_w$ and $\boldsymbol{R}_w$ are obtained after the same column permutations. Therefore, (A.4) is equivalent to (2). Furthermore, since $\boldsymbol{X}_w$ is a sub-matrix of $\boldsymbol{X}$, it automatically satisfies conditions of Definition 1. Thus, we conclude that $\boldsymbol{X}_w$ is a DSTBC-CPI.

Appendix B
Proof of Lemma 2

From Lemma 1, every sub-matrix $\boldsymbol{X}_w$ is a DSTBC-CPI in variables $s_1^w, \cdots, s_{N_w}^w$. Furthermore, it is possible to show that the DSTBC-CPI satisfies

$$\boldsymbol{X}\boldsymbol{X}^H = |s_1|^2\boldsymbol{G}_1 + \cdots + |s_N|^2\boldsymbol{G}_N \tag{B.1}$$

where $\boldsymbol{G}_n = \text{diag}[G_{n,1}, \cdots, G_{n,K}]$ and $G_{n,1}, \cdots, G_{n,K}$ are strictly positive. Therefore, every sub-matrix $\boldsymbol{X}_w$ is also a generalized orthogonal design. We refer to any entry containing $s_{n_w}^w$ as the $s_{n_w}^w$-entry. Similarly, any entry containing $s_{n_w}^{w*}$ is referred to as the $s_{n_w}^{w*}$-entry.

By the row-monomial condition, any row in $\boldsymbol{X}_w$ can not contain more than one $s_{n_w}^w$-entry or $s_{n_w}^{w*}$-entry. Therefore, the rate of $\boldsymbol{X}_w$ is lower-bounded by $1/2$, which is achieved when every row contains exactly one $s_{n_w}^w$-entry and one $s_{n_w}^{w*}$-entry for $1 \le n_w \le N_w$.

Then we show that the rate can not be strictly larger than $1/2$ by contradiction. Without loss of generality, we assume the first row of $\boldsymbol{X}_w$ is $[s_1^w, \cdots, s_{N_w}^w, s_1^{w*}, \cdots, s_{N_w'}^{w*}]$, where $N_w' < N_w$. Hence, the rate of $\boldsymbol{X}_w$ is $N_w/(N_w + N_w')$ and it is strictly larger than $1/2$. Furthermore, because every entry in $\boldsymbol{X}_w$ is non-zero, every row in $\boldsymbol{X}_w$ contains exactly $N_w + N_w'$ non-zero entries. Because $s_{N_w'+1}^{w*}, \cdots, s_{N_w}^{w*}$ are not transmitted by the first row, the second row can not have any $s_{n_w}^w$-entries, $N_w' + 1 \le n_w \le N_w$. This can be shown by contradiction. For example, if the second row has $s_{N_w'+1}^w$ on the first column, the inner product of the first and second rows must have the term $s_1^w s_{N_w'+1}^{w*}$. Because $\boldsymbol{X}_w$ is a generalized orthogonal design, the inner product of any two rows must be zero. In order to cancel the term $s_1^w s_{N_w'+1}^{w*}$, the first row must have an $s_{N_w'+1}^{w*}$-entry, which contradicts our assumption. Thus, the second row can not contain any $s_{n_w}^w$-entries, $N_w' + 1 \le n_w \le N_w$. On the other hand, because the second row must contain exactly $N_w + N_w'$ non-zero entries, it must have the $s_{n_w}^w$-entries for $1 \le n_w \le N_w'$ and the $s_{n_w}^{w*}$-entries for $1 \le n_w \le N_w$.

Since $K_w > 2$, we can do further investigation on the third row of $\boldsymbol{X}_w$. The third row is decided by the first and the second row jointly. Because the first row does not have $s_{N_w'+1}^{w*}, \cdots, s_{N_w}^{w*}$, the third rows can not have any $s_{n_w}^w$-entries, $N_w' + 1 \le n_w \le N_w$. Furthermore, because the second row does not have any $s_{n_w}^w$-entries, $N_w' + 1 \le n_w \le N_w$, the third row can not have any $s_{n_w}^{w*}$-entries, $N_w' + 1 \le n_w \le N_w$. Hence, the third row can only have the $s_{n_w}^w$-entries and the $s_{n_w}^{w*}$-entries for $1 \le n_w \le N_w'$. There are at most $2N_w'$ non-zero entries in the third row and it contradicts with the fact that every row in $\boldsymbol{X}_w$ contains exactly $N_w + N_w'$ non-zero entries. This means that the rate of $\boldsymbol{X}_w$ can not be strictly larger than $1/2$. Because it has been shown that the rate of $\boldsymbol{X}_w$ is lower-bounded by $1/2$, we conclude that the rate of $\boldsymbol{X}_w$ is exactly $1/2$ when $K_w > 2$.

Appendix C
Proof of Theorem 1

Like in Lemmas 1 and 2, we partition $\boldsymbol{X}$ into $\boldsymbol{X}_1, \cdots, \boldsymbol{X}_w$, and we assume the dimension of $\boldsymbol{X}_w$ is $K_w \times T_w$. Because

$\boldsymbol{X}_w$ does not contain any zero entries, the total number of non-zero entries in $\boldsymbol{X}_w$ is $K_w T_w$. For convenience, we refer to any entry containing s_n as the s_n-entry. Similarly, any entry containing s_n^* is referred to as the s_n^*-entry.

We first consider the case that $K = 3$. Because $K = 3$, there is only one sub-matrix in $\boldsymbol{X}_1, \cdots, \boldsymbol{X}_w$ that contains three rows. Without loss of generality, we can assume that $\boldsymbol{X}_1$ is such sub-matrix, i.e., $K_1 = 3$. Then we partition $\boldsymbol{X}$ into $\boldsymbol{X} = [\boldsymbol{X}_1, \boldsymbol{X}^{'}]$. Note that every column in $\boldsymbol{X}^{'}$ can contain at most two non-zero entries; otherwise, this column should be included in $\boldsymbol{X}_1$. By Lemma 2, $\boldsymbol{X}_1$ is a DSTBC-CPI and its rate is exactly $1/2$. Furthermore, we assume $\boldsymbol{X}_1$ is in variables $s_1, \cdots, s_{N_1}$, $1 \leq N_1 \leq N$. By the proof of Lemma 2, every row of $\boldsymbol{X}_1$ contains exactly one s_n-entry and one s_n^*-entry, $1 \leq n \leq N_1$. Therefore, there is no s_n-entry or s_n^*-entry in $\boldsymbol{X}^{'}$; otherwise, there will be more than one s_n-entries or s_n^*-entries in a row of $\boldsymbol{X}$, which will make the noise covariance matrix $\boldsymbol{R}$ non-diagonal. Thus, the matrix $\boldsymbol{X}^{'}$ is actually a DSTBC-CPI in variables $s_{N_1+1}, \cdots, s_N$. Furthermore, because every column in $\boldsymbol{X}^{'}$ has at most two non-zero entries, it is easy to show that its rate can not be larger than $1/2$ by following the proof of Theorem 2 in [23].[11] Because the rate of $\boldsymbol{X}_1$ is exactly $1/2$ and the rate of $\boldsymbol{X}^{'}$ is less than $1/2$, the rate of $\boldsymbol{X} = [\boldsymbol{X}_1, \boldsymbol{X}^{'}]$ must be upper-bounded by $1/2$ when $K = 3$.

Secondly, we consider the case that $K > 3$. We assume the sub-matrices $\boldsymbol{X}_1, \cdots, \boldsymbol{X}_{W'}$ contain one or two rows and the sub-matrices $\boldsymbol{X}_{W'+1}, \cdots, \boldsymbol{X}_w$ contain three or more rows, i.e. $K_w \leq 2$ for $1 \leq w \leq W^{'}$ and $K_w \geq 3$ for $W^{'} + 1 \leq w \leq W$. Thus, the rate of $\boldsymbol{X}_w$ is exactly $1/2$ for $W^{'} + 1 \leq w \leq W$. This means, if an information-bearing symbol s_n appears in a row of $\boldsymbol{X}_w$,$W^{'} \leq w \leq W$, it appears exactly twice. On the other hand, (B.1) implies that every row of $\boldsymbol{X}$ must have the information-bearing symbol s_n for at least once, $1 \leq n \leq N$. Therefore, the following inequality holds

$$\sum_{w=1}^{W^{'}} K_w T_w + \sum_{w=W^{'}+1}^{W} \frac{K_w T_w}{2} \geq NK. \quad \text{(C.1)}$$

On the other hand, there are totally T_w columns in $\boldsymbol{X}_w$. Thus, the total number T of columns in $\boldsymbol{X}$ is given by $T = \sum_{w=1}^{W} T_w$. Then, from (C.1), it is easy to obtain $2N \leq T$ under the assumption that $K > 3$ and $K_w \leq 2$ for $1 \leq w \leq W^{'}$, and hence, the rate of $\boldsymbol{X}$ is upper-bounded by $1/2$ when $K > 3$.

[11]Note that $\boldsymbol{X}^{'}$ is a DSTBC-CPI in $N - N_1$ different variables. Furthermore, it only has three rows and every column in $\boldsymbol{X}^{'}$ has at most two non-zero entries. Therefore, when $N - N_1$ is even, we can follow Case III in the proof of Theorem 2 in [23] and show that the rate of $\boldsymbol{X}^{'}$ can not be larger than $1/2$. When $N - N_1$ is odd, the same conclusion can be made by following Case IV in the proof of Theorem 2 in [23].

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