Noncoherent Demodulation of Continuous Phase Modulated Signals using Extended Kalman Filtering

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***Abstract* - The paper addresses the problem of demodulating Continuous Phase Modulation (CPM) signals embedded in Gaussian noise. The paper introduces a description of CPM signals by a state-space model. Based on this model, a noncoherent demodulation method of CPM signals, that uses an Extended Kalman Filter (EKF), was developed. The performances of the demodulation method were tested on two types of CPM modulated signals, embedded in noise: Minimum Shift Keying (MSK) and Gaussian Minimum Shift Keying (GMSK). The results, obtained from simulations performed in MATLAB, leaded to the conclusion that, for moderately low Signal to Noise Ratios (SNR), the EKF algorithm provides satisfactorily even good results in noncoherent demodulation of CPM signals.**
Keywords: state space, CPM signals, noncoherent demodulation, Extended Kalman filtering

I. INTRODUCTION

Continuous Phase Modulation is a class of modulation schemes with the attractive property of a constant envelope. Constant envelope signals suffer less distortion in high power amplifiers and are preferred for wireless applications. Continuous Phase Modulation and related forms of digital phase modulation, such as Continuous Phase Frequency Shift Keying (CPFSK), MSK and GMSK, are nonlinear modulation schemes, efficient from both bandwidth and power perspectives [1], [2]. MSK is a special case of continuous phase frequency shift keying technique where modulation index is equal to 0.5 which results in a minimum frequency separation such that the modulation frequencies are still orthogonal, offers advantages in performance and ease of implementation. In MSK [3], the modulated carrier does not contain in phase discontinuities and frequency changes at carrier zero crossings. It is typical for MSK that the difference between the frequency 0's and 1's is equal to half the data rate. MSK modulation makes the phase change linear and limited to ±(π/2) over the symbol interval. Due to the linear phase change effect, better spectral efficiency is achieved.

In 1981 Murota and Hirade [4] proposed the use of a pre-modulation Gaussian low-pass filter to shape the spectrum of MSK. This filter removes the sudden transitions in the frequency modulation pulses of an MSK signal. The resulting Gaussian Minimum Shift Keying modulation achieves a narrower spectrum with attenuated sidelobes. The main advantages of GMSK are its spectral efficiency, its constant phase property, which allows it to be used with nonlinear power-efficient amplifier, as well as its robust performance. GMSK modulation is used in many important wireless communications standards. CPM signals can be demodulated by a variety of coherent and noncoherent demodulation techniques. Particularly, the differential demodulator, the limiter-discriminator and variations of these two schemes have been used and proposed for use with GMSK signaling. In this paper, we propose a noncoherent procedure that uses Extended Kalman Filtering (EKF) to demodulate the MSK and GMSK signals.

This paper is structured as follows. Section II introduces the state-space model of the CPM signal affected by additive Gaussian noise. In section III, we describe the Extended Kalman Filter algorithm, used in noncoherent demodulation of MSK and GMSK signals. Section IV provides simulation results. Finally, Section V gives the concluding remarks.

II. THE STATE SPACE REPRESENTATION OF CPM SIGNAL MODEL

A CPM signal, with constant amplitude embedded in additive noise $w[n]$, is expressed as:

$$y[n] = A\cos\varphi[n] + w[n] \tag{1}$$

where the positive real-valued A is the constant amplitude of the signal and $\varphi[n]$ is the signal phase. The additive noise is white and Gaussian, having zero-mean and variance σ_w^2.

A. *The state space model and the transition equation of CPM received signal*

We define the following 2×1 state vector $\mathbf{x}[n]$:

$$\mathbf{x}[n] = \begin{bmatrix}\varphi[n] & \Omega[n]\end{bmatrix}^T \tag{2}$$

with

$$\Omega[n] = \varphi[n] - \varphi[n-1] \tag{3}$$

where $\Omega[n]$ stands for the first order difference of the phase function $\varphi[n]$.

From (2), the signal phase can be written as:

$$\varphi[n] = \varphi[n-1] + \Omega[n-1] \tag{4}$$

Considering only the phase variations of a CPM signal, the state transition equation is written as:

$$\mathbf{x}[n] = \mathbf{F}\mathbf{x}[n-1] \quad (5)$$

where the 2×2 transition matrix $\mathbf{F}$ has the form:

$$\mathbf{F} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad (6)$$

We will assume that the frequency of the signal follows a random-walk model:

$$\Omega[n] = \Omega[n-1] + v[n] \quad (7)$$

where $v[n]$ is a sequence of independently and identically distributed random scalars with the distribution $N\left(0,\sigma_v^2\right)$.

The last equation must be added to (5) in order to obtain the complete description of the state evolution for the CPM received signal:

$$\mathbf{x}[n] = \mathbf{F}\mathbf{x}[n-1] + \mathbf{G}v[n] \quad (8)$$

where $\mathbf{G}$ is a 2×1 vector

$$\mathbf{G} = \begin{bmatrix} 0 & 1 \end{bmatrix}^T \quad (9)$$

As reveals (8), the state transition equation of CPM received signal model is linear.

B. The observation equation

In order to estimate the parameters of CPM signals corrupted by noise, a nonlinear observation equation is used. In this sense, the measured signal $y[n]$ is expressed as:

$$y[n] = A\cos\left(\mathbf{l}^T\mathbf{x}[n]\right) + w[n] \quad (10)$$

where

$$\mathbf{l}^T = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (11)$$

From (10), we can conclude that the observation equation is nonlinear, which means that, in order to use EKF, we apply the first order linearization procedure to $\mathbf{h}\left(\mathbf{x}[n]\right) = \cos\left(\mathbf{l}^T\mathbf{x}[n]\right)$ in (10) around the estimation of the state vector $\hat{\mathbf{x}}[n|n-1]$, [7]:

$$\mathbf{h}\left(\mathbf{x}[n]\right) = \mathbf{h}\left(\hat{\mathbf{x}}[n|n-1]\right) + \left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}[n|n-1]}\left(\mathbf{x}[n] - \hat{\mathbf{x}}[n|n-1]\right) \quad (12)$$

with:

$$\mathbf{H}[n] = \left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}[n|n-1]} = -\mathbf{l}\sin\left(\mathbf{l}^T\hat{\mathbf{x}}[n|n-1]\right) \quad (13)$$

The replacement of $\mathbf{h}\left(\mathbf{x}[n]\right)$ by its first order approximation has dramatic effects on the stability and the convergence of the EKF algorithm, which implies the occurrence, especially at very low SNR's, of "lack of convergence" cases.

III. THE EKF ALGORITHM

As far as the observation model is nonlinear, in order to apply the Kalman filtering procedure as it was shown, a first order linearization around estimated $\hat{\mathbf{x}}[n|n-1]$ is needed at each step of the standard Kalman algorithm. The procedure is known as the Extended Kalman Filter (EKF) algorithm [5] and it uses state-space equations (8) and (1), as well as the linearization of the observation function around the current vector estimate (10).

The initial state $\mathbf{x}[1]$, the observation noise $\mathbf{w}[n]$ and the state noise $v[n]$ are assumed jointly Gaussian and mutually independent. Let $\hat{\mathbf{x}}[n|n-1]$ and $\mathbf{R}[n|n-1]$ be the conditional mean and the conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{y}[1],\ldots,\mathbf{y}[n-1]$ and let $\hat{\mathbf{x}}[n|n]$ and $\mathbf{R}[n|n]$ be the conditional mean and conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{y}[1],\ldots,\mathbf{y}[n]$. Then [7]

Measurement update equations

$$\mathbf{H}[n] = -\mathbf{l}\sin\left(\mathbf{l}^T\hat{\mathbf{x}}[n|n-1]\right)$$

$$\mathbf{K}[n] = \mathbf{R}[n|n-1]\mathbf{H}^T[n]\left(\mathbf{h}[n]\mathbf{R}[n|n-1]\mathbf{H}^T[n] + \sigma_w^2\right)^{-1} \quad (14)$$

$$\hat{\mathbf{x}}[n|n] = \hat{\mathbf{x}}[n|n-1] + \mathbf{K}[n]\left(y[n] - \cos\left(\mathbf{l}^T\hat{\mathbf{x}}[n|n-1]\right)\right) \quad (15)$$

$$\mathbf{R}[n|n] = \mathbf{R}[n|n-1] - \mathbf{K}[n]\mathbf{H}[n]\mathbf{R}[n|n-1] \quad (16)$$

Time update equations

$$\hat{\mathbf{x}}[n+1|n] = \mathbf{F}\hat{\mathbf{x}}[n|n] \quad (17)$$

$$\mathbf{R}[n+1|n] = \mathbf{F}\mathbf{R}[n|n]\mathbf{F}^T + \mathbf{G}\mathbf{G}^T\sigma_v^2 \quad (18)$$

where $\mathbf{K}[n]$ is the Kalman gain matrix at moment n.

As initial conditions, the following equations are used:

$$\hat{\mathbf{x}}[1|0] = \begin{bmatrix} \Omega_0 & 0 \end{bmatrix}^T \quad (19)$$

$$\mathbf{R}[1|0] = \frac{\pi^2}{9}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (20)$$

IV. SIMULATION RESULTS

This section shows some preliminary results of the simulations performed for the noncoherent demodulation by the EKF algorithm of CPM signals in additive Gaussian noise.

In order to emulate the proposed demodulation method, a transmission system, that uses MSK or GMSK modulation, was simulated using the program MATLAB version 7.6. The message consists of a unipolar random sequence, generated by the MATLAB function *randsrc*. The CPM modulated signal propagates through a communication channel affected by additive, white and Gaussian noise (AWGN), having zero-mean and variance σ_w^2. The extended Kalman filter algorithm, presented in the previous Section, implements a noncoherent demodulation of the received signal. Simulations were performed for different values of the sampling rate (nSamp) and of the signal to noise ratio (SNR). The sampling rate indicates the number of samples by message bit that EKF uses for demodulation. It was found that the EKF algorithm performs satisfactorily even at low levels of SNR.

Fig. 1 presents a part of a binary message sequence, sent by MSK modulation, together with the demodulated signal given by the EKF filter output, the instantaneous frequency $\Omega[n]$. The sampling rate chosen for this example was $nsamp = 16$ samples/bit and the signal to noise ratio $SNR = 8\,dB$. In Fig. 1, it can be observed that, for the conditions mentioned above, the estimated signal concords to the message sent, even for fast bit changes in the sent message.

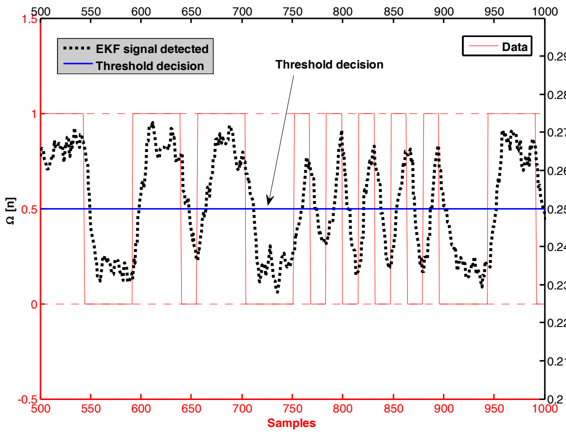

Fig. 1. MSK with SNR=8dB signal detected by EKF.

Performance evaluation of a data transmission system has, among the most important parameters, the bit error rate (BER). In order to test the capabilities of the noncoherent demodulation of CPM signals by EKF detection, an algorithm was developed that search the optimal value of the decision threshold in order to restore the received bits. The same procedure is used both for MSK and GMSK demodulation.

To make a comparative analysis of MSK and GMSK modulations, similar sets of measurements were taken in consideration. The performances of the Extended Kalman filter for the two types of modulation will be described below.

Fig. 2 illustrates the dependence of the MSK system BER performance on the sampling rate. Simulations were performed for four different values of the SNR (SNR = 2, 4, 6, 8dB) and for two different sampling rates (nSamp=4 and 8). It can be seen that, for SNR = 6dB and SNR = 8dB, the sampling rate variations do not affect dramatically the BER performance of MSK demodulation by EKF system.

Fig. 3 presents the dependence of performances of GMSK demodulation by EKF filtering on the sampling rate of the algorithm. Simulations were carried out under identical conditions with those used in the case of MSK detection. In

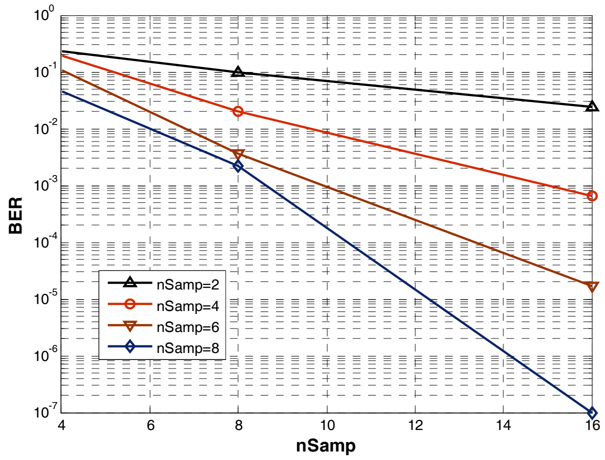

Fig. 2. The dependence of MSK demodulation performance on the sampling rate of the EKF algorithm for different values of SNR.

this case, simulations were performed for four different values of the sampling rate (nSamp = 4,8,16 and 24). A compromise between the signal to noise ratio and the sampling rate must be always searched for. As seen in the figure the performance increases as the sampling rate increases.

Fig. 4 shows, in the case of MSK signal detection by EKF, the dependence of BER performances on the signal to noise ratio (E_b/N_o), for three different values of the sampling rate, namely: nSamp = 4, 8 and 16. The results were compared with the theoretical curve.

As expected, the EKF demodulator performances improve with the sample rate increase. It was found that, for nSamp = 4, if $E_b/N_o < 4\text{dB}$, the extended Kalman filter's stability is no longer assured and, therefore, the algorithm becomes divergent. Consequently, the noncoherent demodulator by EKF algorithm can not be used under such conditions.

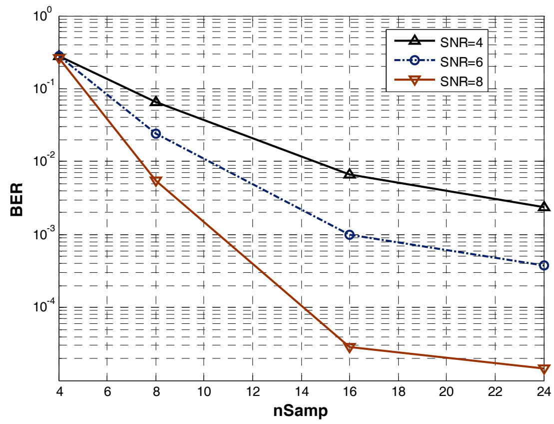

Fig. 3. The dependence of GMSK demodulation performance on the sampling rate of the EKF algorithm for different values of SNR in the case BT = 0.3.

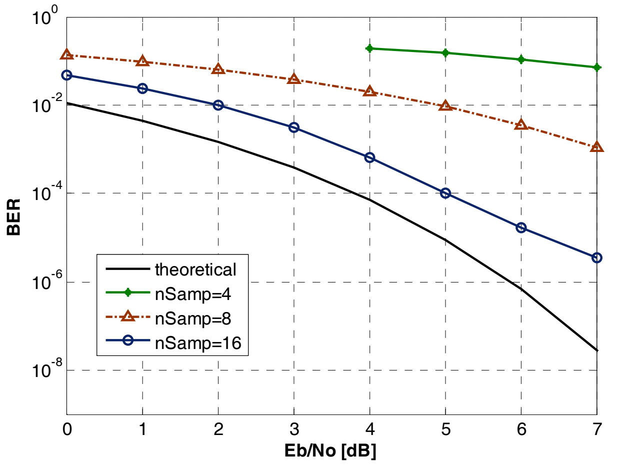

Fig. 4. The dependence of MSK demodulation performance on SNR for different values of sampling rate of EKF algorithm.

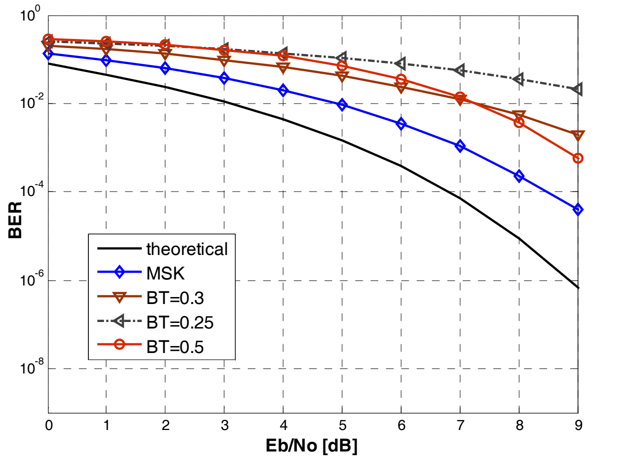

Fig.5. The comparison of performances of MSK and GMSK signal detection by EKF for the sampling rate nSamp =8.

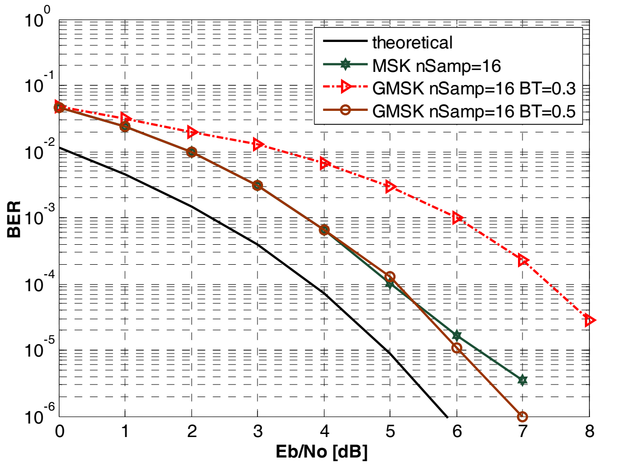

Fig.6. The comparison of performances of MSK and GMSK signal detection by EKF for the sampling rate nSamp =16.

Fig. 5 compares the BER performances of the EKF algorithm for the two types of CPM modulation (MSK and GMSK). We used the same sampling rate, nSamp=8, in both cases. As seen in the figure, the MSK signals demodulation gives better results than the GMSK signal demodulation. It is noted that the curves obtained by EKF for MSK modulation are closest to the ideal curve.

Fig. 6 compares the BER performances of the EKF algorithm for the MSK and GMSK modulation with the sampling rate nSamp=16. As seen in the figure, the curves obtained for MSK and GMSK demodulation with the BT=0.5 indicate similar performance for an E_b/N_o ratio less then 4dB. For an E_b/N_o ratio grater than 6dB, the performance of the GMSK is superior to that obtained by MSK modulation.

V. CONCLUSIONS

The coherent detection or the digital signal demodulation requires a receiver, able to determine or to estimate with acceptable errors both the frequency and the phase of the carrier frequency. Noncoherent techniques are generally less expensive and easier to build than coherent techniques and are often preferable, though they can degrade performance under certain channel conditions. Although generally coherent detection performs better than noncoherent detection, in AWGN channels, the carrier synchronization subsystems, necessary for coherent detection are more complex. That means that as long as the signal to noise ratio is sufficiently large, the noncoherent detection methods represent the simplest and practical approach.

In this paper we introduce a description of CPM signals by a state-space model, which permits the use of extended Kalman filter, in order to demodulate this kind of signals. We have studied the performances of the method for two types of signals from this class: MSK and GMSK modulations. We have compared their BER performances with the theoretical curve. The results, obtained from the simulations, leaded to the fact that, for moderately low SNR's, the EKF algorithm provides satisfactorily good results in noncoherent demodulation.

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