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# Concurrent Cognitive Task Modulates Coordination Dynamics

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### Abstract

Does a concurrent cognitive task affect the dynamics of bimanual rhythmic coordination? In-phase coordination was performed under manipulations of phase detuning and movement frequency and either singly or in combination with an arithmetic task. Predicted direction-specific shifts in stable relative phase from 0° due to detuning and movement frequency were amplified by the cognitive task. Nonlinear cross-recurrence analysis suggested that this cognitive influence on the locations of the stable points or attractors of coordination entailed a magnification of attractor noise without a reduction in attractor strength. An approximation to these findings was achieved through parameter changes in a motion equation in relative phase. Results are discussed in terms of dual-task performance as limited resources, dynamics rather than chronometrics, and reparameterization rather than degradation.

Keywords: Cognition; Coordination; Dynamics

# 1. Introduction

Everyday skilled actions are frequently accompanied by conversations that have no direct relation to the actions, problem solving that is focused on a domain different from the actions, and various orthogonal arithmetic operations conducted overtly or covertly. Measures of the influence on performance of analogous forms of cognitive activity have traditionally been chronometric, with larger reaction and movement times indicative of higher demands on attention (e.g., Posner, 1978). A primary lesson of more recent research is that a viable theory of the performance effects of irrelevant cognitive tasks is unlikely to develop from chronometric measures alone (Pellecchia & Turvey, 2001; Van Gemmert & Van Galen, 1997). Dynamical

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and biomechanical measures of performance are also needed. In large part the need arises because the adaptation of behavior to conditions of shared attention (SA) need not be simply an adjustment, or necessarily an adjustment, in planning and execution speed. It could be a reorganization, in terms of relevant degrees of freedom and parameter values. An inference from one class of theorizing (e.g., Logan & Gordon, 2001) is that a dual task composed of cognitive and behavioral activities—for example, problem solving and walking, respectively—entails a higher level process that controls the subordinate activities to ensure their successful concurrence. The control is achieved, on this theoretical account, by reconfiguring one or both tasks through the manipulation of their parameters.

In this article we measured the effects of task-unrelated cognitive activity on dynamical quantities defined at the level of coordination. Specifically, we examined the effect of arithmetic activity on the relatively simple task of synchronizing the rhythmic movements of the two hands (1:1 frequency locking) at a relative phase relation between the hands of  $0^\circ$ . The immediate advantages of this task are that it has been extensively investigated and that the relative phase,  $\phi$ , is a collective coordination variable that is known to satisfy an equation of motion, namely,

$$\dot{\phi} = \Delta \omega - a\sin\phi - 2b\sin 2\phi + \sqrt{Q}\zeta_t \tag{1}$$

The overdot in Equation 1 signifies the rate of change of  $\phi$ . The parameters of the sine functions in  $\phi$  and  $2\phi$  determine the relative strengths of the stable or attractive states at or near  $\phi = 0^{\circ}$  (in-phase coordination) and  $\phi = \pm 180^{\circ}$  (antiphase coordination). The parameter  $\Delta \omega$  is a detuning (Kelso, DelColle, & Schöner, 1990; Sternad, Collins, & Turvey, 1995) or imperfection parameter (Strogatz, 1994). Nonzero values of  $\Delta \omega$  shift the attractive states away from pure in-phase and antiphase patterns of coordination. The final right-hand term is a Gaussian white noise process  $\zeta_t$  of strength Q > 0 (Haken, Kelso, & Bunz, 1985; Schöner, Haken, & Kelso, 1986).

It is evident from Equation 1 that the stable values of relative phase (when the rate of change of  $\phi$  goes to zero) depend on the values of the coefficients *a* and *b* and the value of  $\Delta \omega$  that are current during an instance of interlimb rhythmic coordination. Values of *a* and *b* connect to movement frequency (the frequency at which the limbs oscillate together). Values of  $\Delta \omega$  connect to the arithmetic difference ( $\omega_L - \omega_R$ ) in hertz (or radians per second) between the preferred frequencies of the left and right limbs. Together, the parameters *a* and *b* and  $\Delta \omega$  determine how well  $\phi = 0^\circ$  and  $\phi = \pm 180^\circ$  are approximated.<sup>1</sup>

If the right-hand side of Equation 1 is plotted against  $\phi$ , the stable states of a movement system are found at the zero crossings, that is, those values of  $\phi$  where  $d\dot{\phi}/dt$  is zero. An attractor is specified when the slope of the curve at the zero crossing is negative. The standard deviation of  $\phi$  (*SD* $\phi$ ) around an attractor can be expressed in terms of the slope  $\lambda$  of the zero crossing and the strength *Q* of the stochastic force (Gilmore, 1981; Schöner et al., 1986; Schöner & Kelso, 1988):

$$SD\phi = \sqrt{\frac{Q}{2\,|\lambda|}} \,. \tag{2}$$

 $SD\phi$  has been applied extensively as an experimental index of attractor stability (e.g., Amazeen, Amazeen, & Turvey, 1998a; Park, Collins, & Turvey, 2001). Given the image of a

particle at the bottom of a potential well,  $\lambda$  indexes the steepness of the well and Q indexes the magnitude of the random kicks that displace the particle away from the well's nadir. The steeper the well, the more stable the pattern of coordination in the sense that the particle's displacement from the well bottom following a random kick of strength Q is smaller, and therefore  $SD\phi$  is smaller.

For a given detuning in combination with a given movement frequency, the solution of Equation 1 is a fixed point or attractor  $\phi^*$  with strength  $\lambda$ . Figure 1 shows the patterning of  $\phi^*$  and  $\lambda$  for three values of detuning in combination with three values of movement frequency. For convenience, we have shown the patterning of  $\phi^*$  in terms of deviation from  $\psi$  where  $\psi$  could be either 0° or 180°. Given Figure 1, one can ask whether these dynamical measures  $\phi^*$  and  $\lambda$ , measures of the stable states of interlimb coordination, are affected in addition by cogni-



Fig. 1. Predicted effects of detuning and movement frequency on relative phase (a), and attractor strength or  $\lambda$  (b), from Equation 1. Parameter values used to simulate low, medium, and high movement frequencies were *a* = 3 and *b* = 4, 2, and 1 for a/b of .75, 1.5, and 3, respectively.

tion. That is, are the effects of detuning and imposed movement frequency modulated by a simultaneous but coordination-irrelevant cognitive task?

### 1.1. Relation to previous research

In a series of experiments, the performance of arithmetic information-reduction tasks was found to affect the attractors of interlimb coordination (Pellecchia & Turvey, 2001). Information-reduction tasks are those in which the required transformation results in a reduction in information from stimulus to response (Posner, 1964; Posner & Rossman, 1965). For example, in a simple addition task, the stimulus might be the digits 6 and 8 and the response their sum, 14. The difference, measured in bits, between the information at input (6.5 bits in the simple addition task) and the information at output (3.8 bits), quantifies the size of the reduction. In the experiments in question, with  $\phi^*$  measured by the observed mean relative phase, coordination-irrelevant cognitive activity in the form of information reduction produced a significant shift in absolute relative phase  $|\phi^* - \psi|$  but not in signed relative phase  $(\phi^* - \psi)$ . Two of the experiments were restricted to interlimb coordination with zero detuning. One experiment involved a manipulation of movement frequency, the other involved a manipulation of the level of cognitive activity (0, 2.7, 4.5, or 5.9 bits of information reduction). In these experiments with zero detuning, the effect of coordination-irrelevant cognitive activity on  $|\phi^* - \psi|$  was found to be constant over movement speed and to increase with the level of activity indifferent to  $\psi$  (that is, whether  $\psi$  was 0° or 180°).

An additional experiment in the previously mentioned series involved a manipulation of detuning under conditions in which movement frequency was self-selected, and the level of cognitive activity was 4.4 bits reduction. As is evident from Figure 1A,  $\Delta \omega > 0$  should produce a positive shift in relative phase ( $\phi^* - \psi > 0^\circ$ ), and  $\Delta \omega < 0$  should produce a negative shift in relative phase ( $\phi^* - \psi < 0^\circ$ ). This expected outcome of  $\Delta \omega$  on  $\phi^*$  was observed in the additional experiment. The direction-specific phase shift brought about by  $\Delta \omega$ , however, was not magnified by the cognitive task. The effect of coordination-irrelevant cognitive activity in the additional experiment was limited to absolute relative phase  $|\phi^* - \psi|$  and was found to be identical in magnitude for detuned ( $\Delta \omega \neq 0$ ) and nondetuned ( $\Delta \omega = 0$ ) interlimb coordination at both  $\psi = 0^\circ$ and  $\psi = 180^\circ$ . Accommodating the results requires a mechanism for shifting the lead–lag relation of the two limbs (e.g., increase or decrease the directional bias) randomly from one trial or dynamical run to the next (Pellecchia & Turvey, 2001).

There is the possibility, however, that the failure to demonstrate an influence of coordination-irrelevant cognitive activity on  $(\phi^* - \psi)$  arose from methodological limitations. Although there was no statistically significant effect of cognitive activity on  $(\phi^* - \psi)$ , an amplification of the directional bias induced by  $\Delta \omega$  was evident in 7 of the 10 participants. This implied influence of cognitive activity on  $(\phi^* - \psi)$  might become more evident (more reliable) at a higher level of activity in experimental conditions that combine detuning and movement speed factorially in the manner of Figure 1A. Evaluating the latter possibility was the major goal of this experiment. Theoretically, a reliable demonstration of an increased directional bias with task-irrelevant cognitive activity would simplify the interpretation of the cognition–coordination relation.

### 1.2. Noise and cognition

It has been suggested that cognitive activities might permeate motor performance as enhanced levels of noise (Van Gemmert & Van Galen, 1997). The issue of noise in motor behavior was summarized recently by Riley and Turvey (2002). A prevailing conviction regarding biological movement is captured by the equation

$$X(t) = M(t) + N(t) \tag{3}$$

where X(t) is the times series of a motor behavior, M(t) is its deterministic part (the intended or ideal movement), and N(t) is its random part (white noise arising from activity within the neuromuscular processes). In the conventional interpretation of Equation 3, the closer N(t) is to zero, the closer is the actual movement X(t) to the intended or ideal movement M(t) (Newell & Corcos, 1993). Accordingly, N(t) is seen as problematic with undesirable consequences that have to be overcome. There are, however, a number of plausible alternatives to the conventional understanding of Equation 3, which make possible several different inferences about N(t) based on X(t) (Riley and Turvey, 2002). Simply, the measure of degree of randomness in a motor behavior X(t) is not identically a measure of the degree of randomness in the noise source or sources N(t) manifest in motor control. More variable does not necessarily mean more random.

In our research, the issue of noise was evaluated in two ways: through the standard deviation of relative phase,  $SD\phi$ , and the quantification of the cross-recurrence patterns of the left and right rhythmic units. Equation 2 expresses  $SD\phi$  as a function of Q and  $\lambda$ . Of these two factors only one is conceptually related to neuromotor noise, namely, Q. The other and most significant factor is attractor strength quantified by  $\lambda$ . Looking again at the deterministic part of Equation 1, altering either or both  $\Delta\omega$  and [a, b] leads to changes in  $\lambda$ . In which case,  $SD\phi$  varies on purely deterministic grounds. Reiterating the previous argument, variability in X(t) as measured by  $SD\phi$  is not of itself indicative of N(t).

In respect to Equation 2, to the extent that Q is constant over changes in detuning and movement frequency,  $SD\phi$  provides an index of attractor strength. To the extent that Q might vary with detuning and movement frequency, an important limitation of the measure of  $SD\phi$ becomes apparent—the inability to determine whether an observed change in  $SD\phi$  arises from change in Q, change in  $\lambda$ , or changes in both Q and  $\lambda$ .<sup>2</sup> In this experiment, cross-recurrence quantification (CRQ) analysis provides the means to measure  $\lambda$  and Q of interlimb coordination.

Recurrence quantification analysis was developed for the purpose of uncovering subtle time correlations and repetitions of patterns and involves no assumptions about data set size and distribution of data. The method of plotting recurring values of a single time series in time-delayed embedding space was introduced originally to facilitate the detection of signals in complicated time series (Casdagli, 1997; Eckmann, Kamphorst, & Ruelle, 1987). The subsequent development of methods for quantifying such plots (see Appendix A) has yielded an analytic tool that can distinguish signal from noise in very hard cases (e.g., the chaotic behavior of the logistic map limited to a series of 1,000 points; Zbilut, Giuliani, & Webber, 1998). In CRQ, time-delayed copies of the respective time series are used as surrogate dimensions required for embedding the data in higher dimensional space to examine recurrent structure between paired signals (Zbilut et al., 1998). Shockley, Butwill, Zbilut, and Webber (2002) demonstrated that CRQ was able to detect subtle

nonlinear dynamics of weakly coupled mechanical oscillators. Based on this demonstration, they hypothesized that CRQ should also be able to provide deeper insight than conventional methods into weakly coupled biological systems. Shockley, Santana, and Fowler (2003) found that CRQ could distinguish between the shared postural fluctuations of two speakers conversing on a mutual problem and the shared postural fluctuations of the same two speakers when they were simultaneously conversing with others.

The measures available in CRQ include %RECUR, which reflects shared locations in phase space (percentage of possible points where trajectories cross), and MAXLINE, which reflects stability of shared activity (convergence of nearby trajectories over time). We used measures of %RECUR and MAXLINE as indexes of Q and  $\lambda$ , respectively.

In summary, the attractors of interlimb rhythmic coordination are characterized formally by three quantities:  $\phi^*$  (location),  $\lambda$  (strength), and Q (noise magnitude) that can be expressed experimentally through the measures of mean relative phase  $\langle \phi \rangle$ , MAXLINE, and %RECUR, respectively. In respect to the broader issues of coordinated movement's adaptation to task-irrelevant cognitive activity, the question posed by this experiment was how these experimental indexes of  $\phi^*$ ,  $\lambda$ , and Q respond to a concurrently performed cognitive task.

### 2. Method

#### 2.1. Participants

Twelve students (4 men and 8 women) from the University of Hartford (Connecticut) provided informed consent and were paid \$20 for their participation. All participants classified themselves as right-handed and preferred to use the right hand for at least seven of eight items on a handedness inventory (write, hammer, throw ball, unscrew jar lid, use knife, use toothbrush, hold match while striking, hold racquet; Murphy & Peters, 1994).

### 2.2. Apparatus

Hand-held pendulums constructed of aluminum rods with wooden handles were 64, 48, and 34 cm in length with eigenfrequencies ( $\omega$ ) of .82, 1.01, and 1.29 Hz (or 5.18, 6.37, and 8.11 rad/sec), respectively. The eigenfrequency of an individual wrist-pendulum system is given by  $\omega = (g/l_e)^{1/2}$ , where g is the constant acceleration due to gravity, and  $l_e$  is the equivalent simple pendulum length, which can be calculated from the mass and length of the aluminum rod, the wooden handle, and the hand mass. The eigenfrequencies of the three pendulums were reported by Amazeen, Sternad, and Turvey (1996). The pendulums were combined according to  $\Delta\omega = (\omega_{left} - \omega_{right})$ , yielding three  $\Delta\omega$  values of -.47, 0, and .47 Hz (or -2.93, 0, and 2.93 rad/sec). Two 48-cm pendulums were used for the  $\Delta\omega = 0$  Hz condition. A 6DR/Ultratrak Pro (Skill Technologies, Phoenix, Arizona) motion analysis system was used to record movement trajectories of the pendulums. System hardware includes an electromagnetic transmitter and electromagnetic sensors that detect magnetic field vectors generated by the transmitter. One sensor was attached to the top of each pendulum. A digital signal processor computed each sensor's position and orientation relative to the transmitter. Signal sampling rate was 120 Hz.

### 2.3. Procedure

The coordination task consisted of in-phase wielding of the hand-held pendulums. The cognitive task consisted of counting backward by threes from a randomly chosen three-digit number, the 5.9 bits-reduction task used in an experiment noted previously.<sup>3</sup> Participants were each seated in a straight-backed wooden chair with forearms positioned midway between pronation and supination, resting on the armrests of the chair (see Figure 2). Hands extended over the front of the armrest to permit free movement of the wrist joint.

Participants were instructed to (a) grip the wooden handle of a pendulum firmly in each hand with the top of the handle flush with the top of the fist, (b) produce smooth, continuous oscillations of the pendulums in the sagittal plane through radial and ulnar deviations of the wrist, and (c) produce in-phase coordination. A digital auditory metronome, set to emit one beep per cycle, marked the pace of movement at 0.5, 1.0, or 1.5 Hz (3.14, 6.28, or 9.42 rad/sec).

Prior to the experiment proper, participants practiced the cognitive and bimanual coordination tasks separately and together, using all combinations of  $\Delta \omega$  and movement frequency for a minimum of 15 sec each. Participants were instructed to do their best on both tasks, that is, neither task was given priority. Participants were encouraged to practice as long as they needed to feel comfortable performing the tasks. The experiment consisted of three 30-sec trials each of



Fig. 2. Experimental setup used in this experiment. Participants swung two hand-held pendulums in in-phase coordination. Movement of the pendulums occurred in parasagittal planes through radial and ulnar deviation of the wrists.

18 experimental conditions—combinations of  $\Delta \omega$ , movement frequency, and cognitive activity (none or counting backward by threes)—for a total of 54 trials. The order of conditions was randomized. Participants' verbal responses to the cognitive task were audiotape recorded for subsequent analysis. The practice period and data collection lasted about 2 hr and was spread over two sessions, which were completed within 4 days of each other.

# 2.4. Data reduction

The displacement time series from the two sensors positioned at the top of each pendulum were used to calculate phase angles as

$$\theta_i = \arctan(x_i / \Delta x_i) \tag{4}$$

where *x* with overdot is the angular velocity at sample *i* (normalized in terms of the mean angular frequency for the trial) and  $\Delta x_i$  is the angular displacement of sample *i* (position at sample *i* minus the mean position for the trial). The time series of relative phase for each trial was computed as the difference between the left and right phase angles,  $\phi = (\theta_{\text{left}} - \theta_{\text{right}})$ . The mean difference in phase angles averaged across a time series yielded  $\langle \phi \rangle$  as the estimate of  $\phi^*$ . The variability in the phase-angle difference across a time series yielded  $SD\phi$ , as an estimate of  $\phi^*$ 's variability (for examples of the typical variation in a time series of interlimb relative phase, see Schmidt, Beek, Treffner, & Turvey, 1991). For in-phase coordination,  $\langle \phi \rangle$  is a measure of phase shift, a directionally specific shift of the attractor away from the required phase relation of 0°. Likewise,  $\langle |\phi| \rangle$  is a measure of absolute phase shift (where  $|\phi| = |(\theta_{\text{left}} - \theta_{\text{right}})|$ ), a directionally *nonspecific* shift of the attractor away from the required phase. For each participant, the values of  $\langle \phi \rangle$ ,  $\langle |\phi| \rangle$ ,  $SD\phi$ , and error rate were averaged across the three trials. Analyses were conducted on these average values.

### 2.5. CRQ analysis

Displacement time series were converted to unit interval ranges (amplitude converted to a range of 0 to 1) to ensure a common scale but not influencing the distribution of scores within each time series (Shockley et al., 2002; Webber, 2004). The general strategy in CRQ analysis is to determine if the data values of two different, but concurrently generated, time series share common values (i.e., are recurrent) in time-delayed embedding space. Two cross-recurrence measures of particular relevance to this work are %RECUR, the ratio of the number of recurrent points to the total number of possible recurrent points, and MAXLINE, the longest of the consecutive strings of recurrent points (see Figure 3). The first step in recurrence analysis procedures is to determine the appropriate values for the following input parameters: time delay, embedding dimensions, radius, and line length.

The *time delay* refers to the temporal offset between copies of the time series that are used as surrogate dimensions in reconstructed space. Ideally, a time delay should yield the minimum of correlated activity between points in the time series separated by that difference. The correlated activity can be determined using the first zero crossing of the autocorrelation function, or



Fig. 3. Example of two trajectories in the cross-recurrence phase space, one from the left rhythmic unit and one from the right rhythmic unit, that produce a line segment of length seven. Two points remain close during seven time steps within the limits of a moving ball of radius *r*. The longest such line in a recurrence plot is MAXLINE, a measure of the sensitivity of the system to small perturbations. (Adapted from Faure, P., & Korn, H. (1998). A new method to estimate the Kolmogorov entropy from recurrence plots: its application to neuronal signals. *Physica D*, *122*, 265–279.)

the first minimum of the average mutual information function (Abarbanel, 1996). The optimal delay for each condition in this experiment corresponds to approximately one quarter of a cycle of the dominant frequency for a given condition. The delays for each frequency condition corresponded to 21, 32, and 64 data points for conditions with metronome frequencies of 1.5, 1.0, and 0.5 Hz, respectively.

The number of *embedding dimensions* was selected using a false nearest neighbor analysis. *False neighbors* refer to points that are proximal to one another in a particular number of embedding dimensions that become separated when data are embedded in a higher number of dimensions (Abarbanel, 1996). A prescription for selecting the appropriate number of embedding dimensions in which to evaluate an unknown system is to select an embedding dimension at which the number of false neighbors reaches a minimum. Our selection of five embedding dimensions is consistent with the previously mentioned prescription and with previous choices for hand-held pendulum data under similar paradigms (Goodman, Riley, Mitra, & Turvey, 2000; Mitra, Riley, & Turvey, 1997).

The *radius* is an inclusion criterion for recurrence. That is, for cross-recurrence, the radius defines a distance around each value of trajectory x that, if another value of trajectory y falls within, will be considered a recurrent point. Selection of a radius is crucial for the successful application of recurrence quantification. Except for data generated from mathematical models, data values will rarely be identical in embedding space. The idea in recurrence analysis of physical and biological data is to select a radius large enough that it will capture recurrence that is just above the resolution threshold of the measurement instrument. Furthermore, the radius needs to be small enough to avoid global recurrence (e.g., all points recurrent). This parameter may be selected by computing recurrence measures for a range of radii. The selection should allow a reasonable amount of variability in recurrence measures across different observations for a given condition (i.e., %RECUR does not saturate at the floor of 0% or the ceiling of 100%). Furthermore, ranges are appropriate in which small changes in radius settings yield smooth (i.e., not large or discontinuous) changes in recurrence output measures. A large, abrupt change in recurrent values with small, steady changes in the radius likely indicates a boundary crossing in the local threshold that is required to detect changes at a particular scale. For example, periodic movements will yield a band of occupied area in embedding space of some roughly defined thickness. If the radius is set to some value within the maximum thickness of the band of occupied space, then modulating the radius in small steps will yield smooth, continuous changes in recurrent values (i.e., small increases in the radius will yield small increases in the number of recurrent points). However, as the radius is increased beyond the thickness of the occupied band, then recurrence will no longer increase steadily with steady increases in radius. There will be an abrupt decline in the rate of increase of recurrent values with corresponding increases in radius. With continued increases in radius, another region of some thickness of occupied space may be encountered (e.g., another location on the band), yielding another region of smooth changes in recurrence with small, steady changes in radius. The idea is to avoid selecting a radius that is too close to the size of the occupied band, which could produce artifactual changes in recurrence due to a change in degree of recurrent activity not related to experimental manipulations. The selection of a radius may be further constrained by plotting in log-log or semilog coordinates the obtained %RECUR for a range of radius values for a fixed time delay and embedding dimension. If the value of a radius is too low, the recurrence values in log coordinates will oscillate, indicating that the radius is set below the instrumentation resolution. An appropriate range should be at the beginning of where recurrence becomes roughly linear in these coordinates (Webber & Zbilut, 2005). When multiple samples are to be analyzed (e.g., multiple trials for multiple participants in various conditions), a certain percentage of the mean or maximum distance by which points are separated in the reconstructed phase space allows the recurrent inclusion criterion of each trial to be scaled intrinsically to the range of motion for the trial in question. Based on the previous criteria, our selection was a radius equal to 20% of the maximum distance separating points in reconstructed space for a given trial.

Finally, an appropriate value must be determined for *line length*. The number of consecutive recurrent points required to define a line segment should be at least two. The more points required reflect increasingly conservative estimates of the deterministic structure in the system. This may be appropriate if contamination of data is suspected. The minimal line segment in this study was considered to be two consecutive recurrent points.

CRQ was performed using cross-recurrence programs available free of charge from http://homepages.luc.edu/~cwebber/. The values noted previously for time delay, embedding dimensions, radius, and line length were used to compute cross-recurrence measures %RECUR and MAXLINE. For each participant, the values of %RECUR and MAXLINE were averaged across the three trials of each experimental condition. Subsequent data analyses were conducted on these mean values.

### 3. Results

# 3.1. 1:1 frequency locking

An analysis of variance (ANOVA) found no difference (p > .05) between left and right frequencies of oscillations (the mean values were identical, in all conditions, to two decimal places). That is, left and right rhythmic units were 1:1 frequency locked. The mean of the left and right frequency in a given condition defined the coupled movement frequency for that condition. Without the cognitive task, the metronome-specified movement frequencies of .5, 1.0, and 1.5 Hz were matched exactly by the coupled movement frequencies; with the cognitive task, the match was less perfect: .56, 1.03, and 1.45 Hz, respectively. ANOVA on the coupled movement frequency produced by the participants with factors of cognition and (metronome-paced) movement frequency found the interaction to be significant, F(2, 22) = 9.78, p <.001. Post hoc testing localized the interaction at the lowest movement frequency. Only at the metronome-paced frequency of .5 Hz was the coupled movement frequency produced under the cognitive task different from that produced in the absence of the cognitive task.

# 3.2. Task-irrelevant cognitive activity and coordination attractors $(\phi^*)$

Figure 4 shows the patterning of mean relative phase  $\langle \phi \rangle$ , the experimental estimate of  $\phi^*$ , as a function of detuning, movement frequency, and cognitive task. Comparison of Figure 4 with Figure 1A reveals that  $\phi^*$  was shifted by detuning and movement frequency in the manner expected from Equation 1. ANOVA confirmed the interaction of detuning and movement frequency, F(4, 44) = 73.31, p < .001 ( $\eta^2 = .87$ ), and the effect of detuning, F(2, 22) = 108.03, p < .001 ( $\eta^2 = .91$ ), (with each level of detuning significantly different from the other two levels).

More important, Figure 4 suggests that the cognitive task amplified the shift in  $\phi^*$ , dependent on both the sign and magnitude of detuning and as a function of movement frequency.<sup>4</sup> ANOVA confirmed the dependency of the influence of task-irrelevant cognitive activity on detuning and movement frequency, F(4, 44) = 3.90, p < .01 ( $\eta^2 = .26$ ). The cognitive task magnified the shift of  $\phi^*$  in the detuned ( $\Delta \omega \neq 0$ ) conditions but exerted no effect on  $\phi^*$  in the nondetuned conditions, F(2, 22) = 17.48, p < .001 ( $\eta^2 = .61$ ). Further, at the highest movement frequency, the cognitive task exaggerated the shift of  $\phi^*$  at  $\Delta \omega = -.47$  Hz and  $\Delta \omega = .47$  Hz ( $M = -25.72^\circ$  and 25.26°, respectively) relative to coordination in the absence of the cognitive task ( $M = -17.53^\circ$  and 18.21°, respectively).



Fig. 4. Mean relative phase as a function of detuning, movement frequency and cognitive activity.

### *3.3. Task-irrelevant cognitive activity and SD* $\phi$

The results for  $SD\phi$  are shown in Figure 5A. Consistent with expectations from Equation 1, analysis of  $SD\phi$  revealed a main effect of detuning, F(2, 22) = 16.31, p < .001 ( $\eta^2 = .60$ ). The mean values for the three levels of  $\Delta\omega$  were  $16.98^\circ$ ,  $13.68^\circ$ , and  $19.93^\circ$  for  $\Delta\omega = -.47$ , 0, and .47 Hz, respectively. All three values were significantly different from one another, contrary to Equation 1, which does not distinguish between values of  $\Delta\omega$  that differ only in sign. As expected from Equation 1, there was a main effect of movement frequency, F(2, 22) = 7.73, p < .01 ( $\eta^2 = .41$ ), with greater  $SD\phi$  at higher movement rates (mean values of  $14.98^\circ$ ,  $16.32^\circ$ , and  $19.29^\circ$  for 0.5, 1.0, and 1.5 Hz, respectively). A Detuning × Movement frequency interaction, F(4, 44) = 3.93, p < .01 ( $\eta^2 = .26$ ), showed, however, that the effect of movement frequency was restricted to  $\Delta\omega \neq 0$ .

The influence of the cognitive task on  $SD\phi$  was limited as suggested by Figure 5A. A significant interaction with detuning, F(2, 22) = 4.81, p < .05 ( $\eta^2 = .30$ ), indicated that the cognitive task magnified  $SD\phi$  at  $\Delta \omega = .47$  Hz (pendulum of higher eigenfrequency wielded in the left hand), but had no similar effect at  $\Delta \omega = -.47$  and 0 Hz (see Figure 5). A three-way (Detuning × Movement frequency × Cognitive task) ANOVA restricted to the detuned conditions ( $\Delta \omega = \pm .47$  Hz) and higher movement frequencies (1.0 and 1.5 Hz) revealed a main effect of cognitive task, F(1, 11) = 4.67, p = .05 ( $\eta^2 = .30$ ), with mean  $SD\phi$  values of 22.08° and 17.72° in the presence and absence of the cognitive task, respectively.

As noted in the introduction, and as is evident from Equation 2, changes in the measure of  $SD\phi$  may be due either to  $\lambda$  (strength of the coordination pattern) or Q (magnitude of stochastic force) or both  $\lambda$  and Q. Accordingly, the next phase in the analyses used the methods of CRQ to explore further the nature of interlimb coordination and to determine whether task-irrelevant cognition influenced  $\lambda$  and Q differentially. As suggested,  $\lambda$  can be defined by the measure MAXLINE (the inverse of which is a measure of sensitivity of the system to perturbations), and Q can be indexed by %RECUR. The quantity %RECUR provides a relative measure of noise strength. If %RECUR decreases from Manipulation A to Manipulation B, it may be inferred that Manipulation B results in more noise (in the sense of fewer shared points in the reconstructed phase space; see Appendix A, Fig. A2).

#### 3.4. Detuning, movement frequency, and CRQ

Figures 5B and 5C summarize the results. There are indications in the figures of a number of significant effects due to detuning and movement frequency acting singly or interactively but separate from the cognitive task. These effects are revealing of the basic recurrent structure of interlimb coordination and have implications for understanding the influence of task-irrelevant cognitive activity. %RECUR was reduced by detuning, F(2, 22) = 15.18, p < .001 ( $\eta^2 = .58$ ; 10.25% for  $\Delta \omega = 0$ , 9.13% for  $\Delta \omega = -.47$  Hz, and 8.09% for  $\Delta \omega = .47$  Hz), but was unaffected by movement frequency, F(2, 22) = 1.12, p > .05 ( $\eta^2 = .09$ ). As a further evaluation of the dependence of %RECUR on movement frequency, the ANOVA was repeated, restricted to the conditions of nonzero detuning at movement frequencies of 1 and 1.5 Hz. Again, no effect of movement frequency on %RECUR was in evidence (F < 1).

To reiterate, MAXLINE is the longest diagonal-line segment in the cross-recurrence plot (see Figure 3) and is correlated negatively with a system's sensitivity to small perturbations



Fig. 5. Standard deviation of relative phase or  $SD\phi$  (a), %RECUR (b), and MAXLINE (divided by 1,000 to simplify; c) as a function of movement frequency, detuning, and cognitive task.

(e.g., those that might arise from internal noise; Eckmann et al., 1987; Trulla, Giuliani, Zbilut, & Webber, 1996; see also Atay & Altintas, 1999). In respect to Equations 1 and 2, the strength of the local perturbations is expressed by Q and the sensitivity to Q for a given coordination (in-phase or antiphase), the inverse of  $\lambda$ , is determined by detuning and movement frequency (Amazeen, Amazeen, & Turvey, 1998b; Kelso, 1995).

On the basis of Figure 1B, it was expected that if MAXLINE indexed  $\lambda$ , then it should show a systematic dependency on detuning and movement frequency. Figure 5C suggests that MAXLINE varied with the two parameters in the expected manner. In respect to detuning, MAXLINE was 1603.24 for  $\Delta \omega = 0$  Hz, 1092.19 for  $\Delta \omega = .47$  Hz (left-hand lead), and 1305.85 for  $\Delta \omega = -.47$  Hz (right-hand lead), F(2, 22) = 25.49, p < .0001 ( $\eta^2 = .70$ ). As implied by the SD $\phi$  analysis: Left-leading conditions were characterized by weaker attractors than right-leading conditions for this group of right-handed participants. In respect to movement frequency, MAXLINE decreased systematically with increasing movement frequency, F(2, 22) = 21.32, p < .0001 ( $\eta^2 = .66$ ). A marginal Detuning × Movement frequency interaction suggests that the frequency modulation of MAXLINE was weaker for  $\Delta \omega = 0$ , F(4, 44) = 2.28, p = .08 ( $\eta^2 = .17$ ).

It is informative to consider Figure 5B and 5C in light of expectations for Q and  $\lambda$  in the absence of task-irrelevant cognitive activity. In respect to %RECUR, invariance over movement frequency is in agreement with basic assumptions about Q and the results of experiments assessing those assumptions (Fuchs & Kelso, 1994; Schöner et al., 1986). In agreement with the predicted behavior of  $\lambda$  shown in Figure 1B, MAXLINE reduced systematically with detuning and movement frequency.

### 3.5. Task-irrelevant cognitive activity and CRQ

An effect of task-irrelevant cognitive activity on %RECUR is suggested in Figure 5B. ANOVA confirmed that the cognitive task reduced %RECUR (from 9.60% to 8.70%), F(1, 11) = 14.65, p < .01 ( $\eta^2 = .57$ ), independently of detuning and movement frequency (two-way interaction Fs = 0.22 and 1.79, three-way interaction F = 1.79). Of significance to this finding is Figure A2 in Appendix A that shows how the adding of noise to each of two sine functions decreases the CRQ measure of %RECUR.

As shown in Fig 5C, the cognitive task exerted little influence on MAXLINE (F = 1.13), with mean values of 1370 and 1297 for no cognitive task and cognitive task conditions, respectively. Figure 5C also suggests that the cognitive task did not modulate the effects of detuning and movement frequency on MAXLINE (interaction Fs = 0.58 and 2.07). In sum, task-irrelevant cognitive activity affected %RECUR but not MAXLINE.<sup>5</sup>

A useful way to summarize the results of the cross-recurrence analyses is in terms of a multiple regression of the 18 mean values of %RECUR on the continuous variable of MAXLINE (divided by 1,000 to simplify) and the categorical variable of cognitive task (coded as 1/0). The regression  $R^2$  (17) was .77 with both variables significant. The coefficient of MAXLINE was 3.39, p < .0001, and the coefficient of cognitive task was -.65, p < .05. The multiple regression is plotted in Figure 6. The figure shows that %RECUR increased with MAXLINE and decreased with task-irrelevant cognitive activity. The implication of Figure 6 is that the cognitive reduction of recurrence in the reconstructed phase space (the cognitively induced increase in



Fig. 6. %RECUR as a function of MAXLINE and cognitive activity.

Q) was indifferent to attractor strength (the magnitude  $\lambda$  determined by detuning and movement frequency). Said differently, the cognitive task of counting backward by threes injected a fixed amount of noise into the coordination.

#### 3.6. Influence of the coordination conditions on cognitive task performance

A two-way ANOVA was used to examine the effects of detuning and movement frequency on mean error rate of the cognitive task. The proportional error was found to increase with movement frequency, F(2, 22) = 3.54, p < .05 ( $\eta^2 = .24$ ; magnitudes of 0.04, 0.05, and 0.08 at 0.5, 1.0, and 1.5 Hz, respectively) but was unaffected by detuning (p > .05). In addition, a similar two-way ANOVA showed the number of verbal responses to the cognitive task increased with movement frequency, F(2, 22) = 17.24, p < .001 ( $\eta^2 = .61$ ), with mean values of 17.9, 19.7, and 20.9 responses for metronome-specified rates of .5, 1.0, and 1.5 Hz, respectively.

# 4. Discussion

The attractors of interlimb rhythmic coordination are characterized by three quantities:  $\phi^*$  (location),  $\lambda$  (strength), and Q (noise magnitude). In this experiment interlimb rhythmic coordination was performed singly or in parallel with an arithmetic cognitive task. The results of this experiment suggest that the manipulation of coordination-irrelevant cognitive activity affected  $\phi^*$  and Q but not  $\lambda$ .<sup>6</sup>

The outcomes of dual-task performances are frequently interpreted in terms of competition for a limited single resource (e.g., Kahneman, 1973). With cognitive processes indexed by reaction time (RT), it has been observed that antiphase coordination lengthens RT relative to

in-phase coordination (Summers, Byblow, Bysouth-Young, & Semjen, 1998; Temprado, Zanone, Monno, & Laurent, 1999). Arguably, from the perspective of the central nervous system, the measure of task difficulty is stability (Schöner, 1994). That is to say, more difficult tasks are less stable tasks, and in terms of limited resources, less stable tasks are tasks that demand more resources. In respect to the aforementioned relation between RT and coordination pattern, antiphase is more prone to perturbation from internal noise (lower  $\lambda$  by Equation 1) than in-phase and, by hypothesis, is more resource demanding.

At first blush, the three-way interaction of this experiment in respect to  $\phi^*$  might seem to be indicative of a limited single resource: Less stable coordinated states (those of lower  $\lambda$ ) were shifted further by the arithmetic task than more stable coordinated states. Arguably, the demand on resources imposed by the arithmetic task limited the resources available for coordination, with coordinated states of lesser stability affected the most by the limitation. The foregoing account, however, is compromised by the evidence that the shift in  $\phi^*$  induced by the arithmetic task was not accompanied by a change in  $\lambda$ . To repeat, the effect of the cognitive task was to displace the attractor and make it noisier without affecting the sensitivity of the system to perturbations. This latter conclusion, of course, rests on the assumption that the MAXLINE quantity of CRQ is an appropriate index of  $\lambda$ . The assumption, as noted, is in accordance with current theory on recurrence patterns and is reinforced by the qualitative similarity of expected  $\lambda$  behavior (Figure 1B) and observed MAXLINE behavior (Figure 5C).

There is an alternative conceptualization of processing resource: It comes in multiple forms rather than a single bounded form. The single versus multiple contrast is best articulated within current investigations of working memory (WM). The view of WM as a single yet flexible cognitive resource of limited capacity (Daneman & Merikle, 1996; Just & Carpenter, 1992; Newell, 1990) is opposed by the view of WM as a collection of specialized cognitive subsystems organized by a central executive (Baddeley, 1986; Baddeley & Logie, 1999). The expectation from the single-resource perspective is that dual-task decrements should occur to some degree regardless of the tasks. The expectation from the multiple-resource perspective is that dual-task decrements should occur only when the tasks engage aspects of the same specialized cognitive subsystem (Cocchini, Logie, Della Sala, MacPherson, & Baddeley, 2002; Duff & Logie, 2001).

Within this dual-task experiment, participants were required to (a) perform identical-direction, rhythmic oscillations of the left and right hands and (b) generate successive three-digit numbers, where each number was three less than the number before. As is common in most investigations of dual-task performance within the WM literature (e.g., Cocchini et al., 2002), the participants in this experiment were not biased by instruction to emphasize one task at the expense of the other. The situation was one of SA. In terms of the architecture of WM (Baddeley & Logie, 1999), the arithmetic task involved verbal processing, storage, and executive control. In contrast, the rhythmic bimanual coordination task would seem to have involved only the latter aspect of the arithmetic task and then to a much lesser degree. The finding that the number of verbal responses increased as hand-held pendulums were oscillated at higher frequencies could reflect a coupling between cognitive and coordination tasks, perhaps through a common timekeeping mechanism. Given the minimal structural (or subsystem) overlap between the tasks, modest to zero changes in the characteristic properties of the coordination task would have been expected from the multiple-resource account (Baddeley & Logie, 1999; Cocchini et al., 2002). A multiple-resource account of this data might be made more plausible by the assumption that combining the two tasks involves the executive system and that this involvement comes at a cost (Duff & Logie, 2001). In sum, for both a single- and a multiple-resource account of this data the question is the same. How can "cost" be interpreted in a way that yields a significant change in  $\phi^*$ , dependent on the magnitudes of detuning and movement frequency, a significant change in Q independent from the magnitudes of detuning and movement frequency, but no significant change in  $\lambda$ ?

An answer to the preceding question is not readily forthcoming from Equation 1. Given their conventional interpretations, the parameters  $\Delta \omega$ , *a* and *b*, that shape the basic coordination pattern are unlikely sources of the observed effects on attractor location and attractor strength. To change detuning, the cognitive task would have to reduce or increase one of the two eigenfrequencies relative to the other. Or it would have to change both eigenfrequencies, but in opposite directions (increase one, decrease the other). Moreover, a cognitively induced change in  $\Delta \omega$  should affect both attractor location and attractor strength. Similarly, a cognitive-induced change in *a* and/or *b* should show up as a change in movement frequency accompanied by a change in both attractor location and attractor strength. Neither kinds of change were evident in the data.

The question of how to interpret "cost" is potentially addressable through an extension of Equation 1 derived empirically from investigations of handedness (i.e., detuning when  $\Delta \omega = 0$ ; Treffner & Turvey, 1995, 1996) and formally from considerations of the nonlinear coupling implications of subtle oscillator differences (Daffertshofer, van den Berg, & Beek, 1999):

$$\dot{\phi} = \Delta\omega - (a\sin\phi + 2b\sin 2\phi) - (c\cos\phi + 2d\cos 2\phi) + \sqrt{Q}\zeta_t$$
(5)

In respect to Equation 5's parameters, research suggests that  $\Delta\omega$ , *a* and *b*, are primarily responsible for the coordination pattern, with *c* and *d* playing a modulating role (Amazeen, Amazeen, Treffner, & Turvey, 1997; Riley, Amazeen, Amazeen, Treffner, & Turvey, 1997; Treffner & Peter, 2002; Treffner & Turvey, 1995, 1996). To date, experiments have linked *d* to the body's functional (left–right) asymmetry, with d > 0 and d < 0 encoding right- and left-handedness, respectively (Treffner & Turvey, 1995, 1996). In the application of Equation 5 to the question of interpreting "cost," it is assumed that cognitive activity is encoded by *c*, specifically by decreases in *c*.

The cited research permits the following assumptions for dual-task (bimanual rhythmic coordination and coordination-irrelevant cognition) performance. (1) For any given instance of interlimb rhythmic coordination performed alone (full attention [FA]), there is a level of attention encoded by c and dictated by  $\lambda$  (where  $\lambda$  reflects the values of  $\Delta\omega$ , a, b, and d). (2) For a given detuning and movement speed, parameters a, b, and d are fixed and independent of c. (3) Parameters c and d are small compared to parameters a and b. (4) In the general case (nonzero detuning), the sign of c conforms to the sign of  $\Delta\omega$ . (5) SA between rhythmic coordination and cognition reduces the magnitude of c. To a reasonable degree, simulations of the dynamics of Equation 5 under the previous assumptions reproduce the primary data patterns for  $\phi^*$  and  $\lambda$ (see Appendix B). These results suggest an additional assumption for the proposed dynamical model of this dual task: (6) Cognitive activity (or the withdrawal it makes on attentional resources) acts as a source of noise for interlimb rhythmic coordination. This assumption conforms to the hypothesis that cognition permeates motor behavior as noise (Van Gemmert & Van Galen, 1997), but some qualification might be required. Noise need not be a hindrance or a source of disorder. To the contrary, noise can induce new, more ordered regimes, can render unstable states stable, and can increase the degree of coherence among components (Anishenko, Neiman, Moss, & Schimansky-Geier, 1999; Riley & Turvey, 2002). The noise referred to in Assumption 6 might be constructive.

One major line of inquiry into noise's role in dynamical systems is in respect to the behavior of (deterministic) maps with control parameters perturbed by noise, either additive or multiplicative, either white or colored (e.g., Guiterraz, Iglesias, & Rodriguez, 1993; Zaikin, Garcia-Ojalvo, & Schimansky-Geier, 1999). With sufficient intensity this *parametric noise* changes the layout of a map's fixed points or the types (attractor, repeller) of its fixed points or both. The cognitive activity arising from coordination-irrelevant cognitive tasks may render Equation 5's parameters noisy. Colored (1/*f*) noise has been reported for cognitive activities (Gilden, 1997; Gilden, Thornton, & Mallon, 1995; Van Orden, Holden, & Turvey, 2003). An interesting possibility arises if different cognitive tasks generate noises of different colors and intensities and if cognitive-generated noise—as suggested by Van Gemmert and Van Galen (1997)—permeates the movement system. The possibility is that cognitive tasks may differ in their effects on concurrent interlimb rhythmic coordination simply because they generate different noises.

In summary, the proposed dynamical account of this data in the context of Equation 5 is that concurrent cognition modulates c and Q. The question posed previously of how to express cost within either a single-resource or multiple-resource account is answered in terms of "reparameterization." The magnitudes of specific parameters in the governing coordination dynamics under FA are not sustainable under SA (although they might become so with practice).

Although speculative and incomplete, the proposed dynamical account suggests how the classical cognitive concepts of resource allocation and structural interference (e.g., Kahneman, 1973; Navon & Miller, 1987; Pashler, 1994) might be examined and possibly interpreted through dynamical concepts (Temprado et al., 1999, Temprado, Zanone, & Monno, 2001). The proposal has given expression to dual-task performance in terms of (a) dynamics rather than chronometrics and (b) reparameterization rather than degradation. In respect to (b), it was not so much that interlimb rhythmic coordination in this experiment got worse under SA as it was that coordination became different. The response to SA was a reorganization of the coordination in terms of parameter values. The reparameterization idea dovetails with evolving cognitive notions about executive processes in the context of selective attention (e.g., Logan & Gordon, 2001; Norman & Shallice, 1986). Rephrasing remarks in the previous introduction, counting backward and rhythmically oscillating hand-held pendulums simultaneously may be said to entail a higher level process that controls the subordinate processes (arithmetic calculations, bimanual rhythmic coordination) to ensure their successful concurrence. It is suggested that the higher level exerts this control by reconfiguring the subordinate processes through adjustments of their parameters (Logan & Gordon, 2001).

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In conclusion, this research found direction-specific shifts in the stable relative phase due to detuning and movement frequency were amplified by a coordination-irrelevant cognitive task. Nonlinear cross-recurrence analysis suggested that this cognitive influence on the locations of the attractors of interlimb rhythmic coordination entailed a magnification of attractor noise without a reduction in attractor strength. Whether attractors would be weakened under dual-task conditions that involve a more difficult cognitive task or a less stable coordination pattern is unknown. These results and the associated dynamical analyses provide guidelines for future investigations into the interrelation between cognitive activity and movement coordination. Determination of the full scope of cognitive modulations of coordination must await further research.

### Notes

- 1. Formally, the relation between *a* and *b* depends on  $\Delta \omega$ . When  $\Delta \omega = 0$ , a given ratio of *a* to *b*, whatever its composition (e.g., 4:2, 2:1), has a fixed effect on the solutions of Equation 1. When  $\Delta \omega \neq 0$ , a given ratio's effect on the solutions of Equation 1 depends on the ratio's composition. That is, in the absence of detuning, *a* and *b* are linked quantities, whereas in the presence of detuning, *a* and *b* are unlinked quantities.
- 2. How does frequency affect the attractor of a single uncoupled hand-held pendulum? Phase space reconstruction applied to the displacement time series revealed that attractor strength was greatest and random noise was least when oscillatory movements were at the resonant frequency and that above and below resonance attractor strength declined and random noise magnified (Goodman et al., 2000).
- 3. The method described by Posner and Rossman (1965) was used to calculate the amount of information reduced in performing the cognitive task of counting back by threes. Participants started counting from a randomly chosen three-digit number. Counting backward reduces the population of numbers with which the participant deals. On average, the population of numbers over the course of the task is approximately 450.  $Log_2 450 = 8.81$ . Counting back by threes entails selecting one third of the information in the number series. Therefore, approximately 5.9 bits of information are reduced.
- 4. As a point of clarification, spatial and muscular definitions of relative phase are coincident in this task in which movements occur parallel to the sagittal plane. In this experiment, all three conditions of detuning (zero and two nonzero), all three conditions of movement frequency, and both conditions of cognitive task (none, counting backward) were conditions in which homologous muscles were concurrently active through the entirety of their respective cycles (zero detuning) or through most of their respective cycles (nonzero detuning). What Figure 4 shows is that detuning and movement frequency induced a shift from 0 degrees—perfect homologous muscle synchrony, perfect muscular in-phase—that was amplified by cognitive activity.
- So-called local dynamical models can be employed to determine parameter values and their changes. For example, Schmidt and colleagues (Schmidt, Bienvenu, Fitzpatrick, & Amazeen, 1998; Schmidt, Shaw, & Turvey, 1993; Schmidt & Turvey, 1994, 1995) have

used a truncated version of Equation 1,  $\phi = \Delta \omega - \kappa \sin \phi$  (Rand, Cohen, & Holmes, 1988), to evaluate local changes in the dynamics of relative phase. Regressing  $\Delta \omega$  on sin  $\langle \phi \rangle$  of all participants gives the parameter  $\kappa$  for steady state behavior, a parameter that is interpretable as a measure of cooperation. The interpretation follows from the understanding that the expression for the fixed point,  $\phi^* = \arcsin(\Delta \omega/\kappa)$ , highlights the stable value's dependence on the relative contributions of two opposing tendencies: competition ( $\Delta \omega$ ) and cooperation or coupling ( $\kappa$ ) between the two oscillators. For coordination,  $\Delta \omega < \kappa$ . Regression analysis of data from this experiment indicates that  $\kappa$ decreased with increasing frequency (as reported by Schmidt et al., 1993) and decreased further with cognitive activity. Specifically, values of  $\kappa$  in the absence of cognitive activity were 11.20 and 8.26 at 1.0 and 1.5 Hz, respectively; and in the presence of cognitive activity they were 8.73 and 5.96 at 1.0 and 1.5 Hz, respectively. What these changes in  $\kappa$  mean precisely is equivocal given recent investigations. It is apparent from the arcsine function previously mentioned that  $\kappa$  relates inversely to  $\phi^*$  or its experimental equivalent  $\langle \phi \rangle$  Accordingly, it is larger  $\langle \phi \rangle$  values (in this data, larger deviations from  $0^{\circ}$ ) for fixed  $\Delta \omega$  that lead to the inference of weaker cooperation and, perforce, lower attractor strength. What recent experiments have demonstrated, however, are circumstances in which a larger  $\langle \phi \rangle$  incurs a smaller SD $\phi$  (e.g., Amazeen et al., 1996; Mitra, Amazeen, & Turvey, 1998; Park, 2002; Park et al., 2001). These experiments indicate (minimally) that changes in attractor location and attractor strength are not necessarily commensurate and suggest, further, that a reduction in  $\kappa$  does not necessarily imply a reduction in attractor strength. To ascertain whether the latter has occurred requires other measures. In this article, %RECUR and MAXLINE are advanced as useful alternatives.

6. Earlier research in which information-reduction tasks were used to manipulate cognitive load provide evidence that changes in motor behavior are due to cognitive demand and not simply motor interference. In a study examining the effect of information-reduction tasks on standing posture, Pellecchia (2003) found that variability in anterior-posterior center of pressure was greater during digit classification (a 4.5-bit reduction task) than quiet standing. In contrast, digit reversal (0-bit reduction task) did not yield anterior-posterior center of pressure measures that were greater than the quiet standing condition. Stimuli (pairs of single digits), and interstimulus interval were the same for digit classification and reversal tasks. Of relevance to the issue of motor interference, the motor responses for both cognitive tasks required speaking two words. In a study by Pellecchia and Turvey (2001), participants performed bimanual coordination combined with digit reversal (0 bits reduced) and digit addition (2.7 bits reduced). Once again, stimuli and interstimulus interval were the same for digit addition and digit reversal tasks. Digit addition required a one-word response and digit reversal a two-word response, yet the addition task produced greater absolute relative phase shift in interlimb coordination than did the reversal task. Although the findings of these two experiments do not rule out motor interference as having an impact on interlimb coordination in this project, they strongly support the notion that the observed changes in motor behavior are, at least in part, attributable to cognitive load.

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# Appendix A

Recurrence analysis involves the determination of recurrent values in a reconstructed phase space (Webber & Zbilut, 1994, 1996; Zbilut & Webber, 1992). A phase space refers to a plot of the states of multiple dynamical variables with respect to one another, such that the coevolution of these variables may be evaluated. It has been demonstrated that between three and five embedding dimensions are required to fully reveal the contribution of the dynamical variables or "active degrees of freedom" supporting interlimb coordination (Goodman et al., 2000; Mitra et al., 1997). It is not yet clear, however, if these dynamical variables can be measured directly. When the underlying variables are unknown, time-delayed copies of a dynamical observable, x(t), may be used as surrogate dimensions in a reconstructed phase space (see Figure A1a). This space has been demonstrated to be isomorphic to the true phase space, and it is thereby useful in evaluating the preferred states of the system in the appropriate number of embedding dimensions (Takens, 1981). Points in phase space are considered recurrent if they fall within some Euclidean distance (radius) of one another.<sup>7</sup> Recurrent values within a single time series may be evaluated using a recurrence plot strategy by plotting only values of x(t) that recur in phase space, with x(i) along the abscissa and x(i) along the ordinate (such that a point is plotted when x(i) = x(i) within some radius). This recurrence plot strategy may also be used to evaluate the shared activity (recurrence) of two time series,  $x_l(t)$  and  $x_2(t)$  (see Figures A1b and A2). The latter approach to quantifying the recurrent structure of interlimb coordination, known as CRQ, may facilitate detection of the hypothesized noise due to cognitive activity.

The method of CRQ begins with effectively embedding the two time series in a common phase space. In practice, this corresponds to the comparison of a hyperdimensional scatter plot of embedded time series  $x_1$  to the hyperdimensional scatter plot of time series  $x_2$  (see Figure A1b). Recurrence comparisons refer to the comparison of the Euclidean distance of each value of the embedded time series  $x_1$  to each value of the embedded time series  $x_2$  (Shockley et al., 2002, 2003; Zbilut et al., 1998).<sup>8</sup> The percentage of embedded locations of  $x_1$  that recur in  $x_2$ (%RECUR) can be quantified as the ratio of the number of recurrent points to the number of possible recurrent points. In other words, %RECUR indexes shared locations in phase space (percentage of possible points where trajectories  $x_1$  and  $x_2$  cross). %RECUR is an index of the shared activity between the two signals. Accordingly, subtle disruptions to the shared activity as a result of increasing perturbations (*Q*) may be detected by %RECUR. The length of consecutive strings of recurrent points provide an index of how long the trajectories of the two time



Fig. A1. (a) Phase Space Reconstruction (PSR) of a sample data set. Time-delayed copies of the series x(t) (left) are used as surrogate dimensions, x(t + t) (middle), and x(t + 2t) (right) for 2-dimensional and 3-dimensional embedding spaces. (b) CRQ of two embedded time series. The series  $x_1$  (solid lines) and series  $x_2$  (dashed lines) are embedded in a reconstructed phase space. Intersecting trajectories are counted as recurrent points for %RECUR calculations and the longest parallel trajectory of  $x_1$  and  $x_2$  is MAXLINE.

series are coincident, which, in a recurrence plot, correspond to diagonal lines. MAXLINE refers to the longest such diagonal line segment in a recurrence plot (see Figure A1b). MAXLINE has been shown to correlate negatively with a system's *sensitivity* to small perturbations (Eckmann et al., 1987; Trulla et al., 1996; see also Atay & Altinas, 1999). In other words, in respect to Equation 2, whereas %RECUR provides a reasonable index of Q, the magnitude of noise in the system, MAXLINE provides a reasonable index of  $\lambda$ , the negative Lyapunov exponent that defines the sensitivity of the system to noise.

Figure A2 indicates how CRQ could be a useful tool for detecting the hypothesized "cognitive noise" affecting rhythmic movement. The figure shows the cross-recurrence structure of two synchronized periodic functions (sinusoids) for three levels of white noise—0, 2.5, and 5% of the cycle amplitude. As depicted in Figure A2, the addition of a small amount of noise to each periodic function perturbs the recurrence plot and is detected by CRQ as a reduction in %RECUR.



Fig. A2. Cross-recurrence plots of two synchronized sinusoids,  $x_1(i)$  and  $x_2(j)$ , with increasing noise from left to right of 0, 2.5, and 5% of the cycle amplitude. With *i* and *j* indexed along the abscissa and ordinate, respectively, pixels are darkened only when  $x_1(i) = x_2(j)$  within some radius. The %RECUR values from left to right are 1.003, 0.743, and 0.146, respectively.

# **Appendix B**

By Assumption 1 of the proposed dynamical model, if  $\Delta \omega = 3$ , a = 3, b = 3, d = .1 is an instance of interlimb rhythmic coordination, then it should be supposed that *c* under FA is similarly of nonzero value but smaller than *a*, *b* (by Assumption 3): for example, c = .3. Then, by Assumption 5, it can be supposed that in the presence of concurrent cognitive activity—that is, under SA—the parameter *c* is reduced: for example, c = .03. Solving Equation 5 for  $\phi^*$  and  $1/\lambda$  for these FA and SA conditions reveals that  $\phi^*$  increases from 9.8° (FA) to 10.9° (SA), and  $1/\lambda$  increases from 71.0 msec to 71.7 msec. A substantial reduction in attention (a reduction in the *c* parameter by a factor of 10) magnifies the fixed-point shift (11.12%) but does so with a minimal decrease (< 1%) in  $\lambda$ . For the corresponding coordination with parameters of  $\Delta \omega = -3$ , a = 3, b = 3, d = .1,  $\phi^*$  is greater and  $1/\lambda$  is smaller. In detail, the decrease in *c* from –.3 (FA) to –.03 (SA) increases  $\phi^*$  from 11.3° to 12.4° and  $1/\lambda$  from 70.9 msec to 71.5 msec.

What of instances of interlimb rhythmic coordination of the kind  $\Delta \omega = 0$ , a = 3, b = 3, d = .1? If c is scaled primarily by  $\Delta \omega$  (roughly "more detuning, more attention"), then for this instance of interlimb rhythmic coordination, c under FA may differ little from c under SA. The key hypothesis is that neither in the absence nor in the presence of cognitive activity does coordination with zero detuning warrant much attention. Given this hypothesis, c for FA should be small, for example, .020, and the corresponding c value for SA should be approximately the same, for example, .015. With  $\Delta \omega = 0$ , the sign of c is undefined. Solving for the coordination instance in question, with c signed identically to d, yields a decrease in  $\phi^*$  from FA (.84°) to SA (.82°) and zero change in  $1/\lambda$  (66.64 msec for each). Solving with c signed oppositely to d yields an increase in  $\phi^*$  from FA (.69°) to SA (.71°) and zero change in  $1/\lambda$  (66.65 msec for each). The results tend to comply with this second, opposite-sign convention. It is clear, however, that this convention alone will not accommodate the empirical observation that for  $\Delta \omega = 0$  the phase shift due to cognitive activity is more evident in  $\langle |\phi| \rangle$  (mean unsigned relative phase) than  $\langle \phi \rangle$ .

This latter issue aside, the  $\langle \phi \rangle$  data pattern of Figure 4 and the  $\lambda$  data pattern of Figure 5C can be approximated by Equation 5 through a simple generalization of the preceding notions. Combine the inverse relation of *a*, *b* magnitudes to  $\omega_c$  with the inverse relation, under FA, of *c* to  $\lambda$ . Then, for a fixed reduction of *c* by SA, simulations show that the SA–FA difference in  $\phi^*$ with decreasing [*a*, *b*] grows pronouncedly for a given  $|\Delta \omega| > 0$  and negligibly for  $\Delta \omega = 0$ . Likewise, for a fixed reduction of *c* by SA, simulations show that  $\lambda$  is slightly larger for FA than for SA, decreases for both SA and FA with [*a*, *b*], and more so for  $|\Delta \omega| > 0$  than for  $\Delta \omega = 0$ .

- 7. Values at any point in time in the series may be recurrent with values at any other point in time, provided that the points fall within the designated radius. That is, %RECUR is a time-independent measure.
- 8. Two different time series are used to generate a cross-recurrence plot. Thus, there is no ubiquitous diagonal line of identity. Accordingly, the recurrence measures are calculated over the entire recurrence plot, rather than within a single triangular region.