

Two-slot Channel Estimation for Analog Network Coding Based on OFDM in a Frequency-selective Fading Channel

Tomas Sjödin

Department of Computing Science
Umeå University
SE-901 87 Umeå Sweden

Haris Gacanin and Fumiyuki Adachi

Department of Electrical and Communication Engineering
Graduate School of Engineering, Tohoku University
Sendai, 980-8579 Japan
Email: harisg@mobile.ecei.tohoku.ac.jp

Abstract—Recently, broadband analog network coding (ANC) was introduced to utilize high-data rate transmission over the wireless – frequency selective fading – channel. However, ANC requires the knowledge of channel state information (CSI) for self-information removal and coherent signal detection. In this paper, we propose a two-slot pilot-assisted CE for bi-directional broadband ANC. In the first slot, two users transmit their respective pilots to the relay, where the users’ pilot signals are designed to avoid the interference and consequently, allow the relay to estimate the CSIs from both users. During the second slot the relay broadcast its pilot signal to both users that estimate the corresponding CSIs. It was shown by computer simulation that, even with imperfect CSI, the BER performance of broadband ANC gives a satisfactory performance for a low and moderate mobile terminal speed in a frequency-selective fading channel.

Index Terms—Broadband ANC, channel estimation, OFDM

I. INTRODUCTION

Next generation wireless networks should provide higher capacity for broadband services such as: video conferences, video on demand, mobile Internet, etc, to end-users. Network coding [1] has been used in wired networks to increase the network capacity, while in wireless networks it can exploit the broadcast nature of the wireless medium to further increase the capacity [2]. Recently [3], [4], it has been shown that network coding at the physical layer (PNC) can double the network capacity of bi-directional wireless communication. Narrowband analog network coding (ANC) introduced in [5] is a simpler implementation of PNC where the users signals are mixed in the wireless medium.

Broadband wireless communications experience a frequency-selective fading which distorts the transmitted signal. Recently [6], broadband ANC was introduced based on the assumption of perfect knowledge of channel state information (CSI). However, coherent detection and self-information removal in the broadband ANC scheme requires accurate channel estimation (CE). Unlike conventional (without relay) and cooperative (with relay) networks, where signals from different users are separated in time/frequency to avoid interference, in ANC the users’ signals interfere in the same time slot. Hence, in the case of pilot transmission, the relay cannot estimate CSIs from different users. To avoid

this problem a straightforward method is to allocate four time slots for pilot-assisted CE in ANC scheme. However, in a fast fading channel, where the pilot insertion rate must be increased to maintain the satisfactory bit error rate (BER) performance, the capacity benefit of ANC may be decreased. In [7], a complex maximum likelihood (ML) CE for narrowband (i.e., frequency-nonselective fading) ANC is presented. However, a knowledge of the noise variance and the channel cross-correlation channel coefficients is required a priori.

In this paper, we present a two-slot pilot-assisted CE scheme for broadband (i.e., frequency-selective fading) ANC based on OFDM radio access. In the first stage, both users transmit their pilots to the relay, where one of the pilot signals is cyclically shifted [8] to allow the relay to separate and estimate the CSIs from both users. This stage is named multiple-input single-output channel estimation (MISO-CE) due to its analogy to multiple-input multiple-output (MIMO) systems. We note here that the idea behind MISO-CE is also similar to cyclic delay diversity schemes presented in [9]. During the second stage the relay broadcast its pilot signal to the users, which estimate the corresponding CSIs. We evaluate the BER performance of broadband ANC with pilot-assisted CE by computer simulation and compared with the perfect CSI case.

The rest of the paper is organized as follows. In Section II, the network model is presented. Proposed CE scheme for ANC in a frequency-selective fading channel is presented in Section III. Computer simulations and discussions are presented in Section IV. We conclude our findings in Section V.

II. NETWORK MODEL

We consider a two-way relay network with the users, U_0 and U_1 , and the relay (R) as shown in Fig. 1. The users and the relay communicate using time division duplex (TDD) in two slots; 1) U_0 and U_1 transmit their respective signals to the relay, and 2) the relay broadcast the received signal to the users using an amplify-and-forward protocol (AF-P).

First slot: The j th user U_j data-modulated symbol sequence is represented by $\{d_j(n); n = 0 \sim N_c - 1\}$ for $j \in \{0, 1\}$. The symbol sequences are fed to an N_c -point inverse fast Fourier

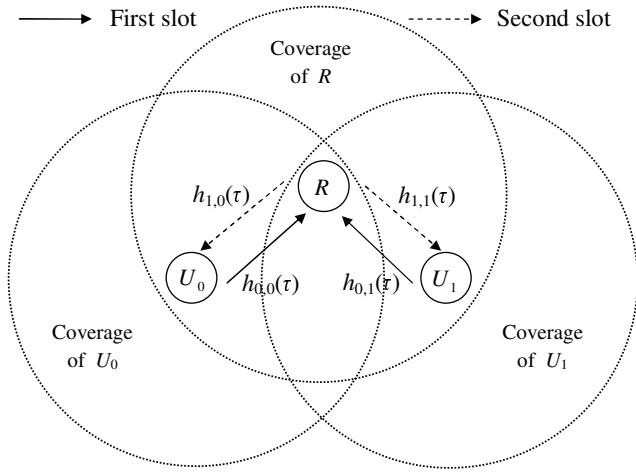


Fig. 1. Network model.

transform (IFFT) to generate the OFDM signal waveform for both users U_0 and U_1 . An N_g -sample guard interval (GI) is inserted and then, the signals from the users are transmitted over a frequency-selective fading channel.

The signal received at the relay can be expressed in the frequency domain as

$$R_r(n) = \sum_{j=0}^1 \sqrt{P} H_{0,j}(n) d_j(n) + N_r(n) \quad (1)$$

for $n = 0 \sim N_c - 1$, where $P (= E_s/T_c N_c)$, $H_{0,j}(n)$ and $N_r(n)$, respectively, denote the transmit signal power, channel gain between user U_j and the relay at the first stage and the additive white gaussian noise (AWGN) having the single-sided power spectral density N_0 . E_s and T_c denote the data-modulated symbol energy and the sampling period of IFFT, respectively.

Second slot: The relay amplifies the received signal by a factor of \sqrt{P} and broadcast $\{\tilde{R}_r(n) = \sqrt{P} R_r(n); n = 0 \sim N_c - 1\}$. After GI removal and N_c -point FFT, the signal received at the j th user U_j can be expressed as

$$R_j(n) = \tilde{R}_r(n) H_{1,j}(n) + N_j(n) \quad (2)$$

for $n = 0 \sim N_c - 1$, where $H_{1,j}(n)$ represents the channel gain between the j th user U_j and the relay during the second stage. The j th user U_j removes its self-information at the n th subcarrier as [6]

$$\tilde{R}_j(n) = R_j(n) - d_j(n) H_{0,j}(n) H_{1,j}(n) \quad (3)$$

for $n = 0 \sim N_c - 1$. One-tap zero forcing frequency domain equalization (ZF-FDE) is applied as

$$\hat{R}_j(n) = \tilde{R}_j(n) w_j(n), \quad (4)$$

where the j th user U_j equalization weight $w_j(n)$, is given by [6]

$$w_j(n) = \frac{H_{0,\bar{j}}^*(n) H_{1,\bar{j}}^*(n)}{|H_{0,\bar{j}}(n) H_{1,\bar{j}}(n)|^2}. \quad (5)$$

In Eq. (5), $(\cdot)^*$ denotes the complex conjugate operation and the bar over j signifies the unitary complement operation (i.e. "NOT" operation) that performs logical negation of j .

We note here that estimates of the channel gains are required to perform self-information removal and ZF-FDE given by Eqs. (3) and (5), respectively. The channel gains $\{H_{0,j}(n)\}$ and $\{H_{1,j}(n)\}$ are replaced by the channel gain estimates $\{H_{0,j}^e(n)\}$ and $\{H_{1,j}^e(n)\}$.

III. CHANNEL ESTIMATION

This section is devoted in part to the problem statement of conventional CE for ANC scheme and then, the proposed pilot-assisted CE scheme is presented.

A. Problem Statement

If a classical CE approach is to be used with ANC scheme the following problems arise:

- 1) To estimate all CSIs in an ANC network we need to allocate four time slots to separate different users' pilot signals. This significantly decreases the capacity of the ANC scheme in a fast fading channel since the pilot insertion rate must be increased (or the pilot insertion interval must be shortened).
- 2) In conventional (without relay) and cooperative (with relay) networks different users' pilot signals are separated by orthogonal frequencies or different time slots to avoid interference. However, in ANC the users transmit simultaneously during the first stage, and as a result of this, the pilot signals interfere with each other. Consequently, the relay cannot estimate the CSIs from different users during the first pilot transmission stage.

To address the above mentioned problems the proposed CE is presented below.

B. Two-slot CE for Broadband ANC

The ANC network model in Fig. 1 can be represented as; a multiple-input single-output (MISO) system during the first slot and two independent single-input single-output (SISO) systems during the second slot. Thereby, pilot-assisted CE for broadband ANC can also be represented in the same fashion as is illustrated in Fig. 2. The first stage of the CE process, illustrated in Fig. 2(a), is based on the MISO-CE principle since the signals from two users' antennas are received by a single antenna at the relay. The second stage of the CE process, illustrated in Fig. 2(b), is based on two independent SISO-CE schemes since the relay broadcasts its pilot signal using a single antenna while the signal is received by each user's antenna independently.

Figure 3 illustrates the pilot/data transmission frame structure of the two users, U_0 and U_1 , and the relay. The pilot and data frames are divided in two slots where each slot has a length of $N_c + N_g$ samples. The first slot of the pilot frame corresponds to the MISO-CE which is used to transmit the pilot signals, $p_0(t)$ and $p_1(t)$ from U_0 and U_1 , respectively, as illustrated in Fig. 2(a). The second slot is used by the relay during the SISO-CE to broadcast its pilot signal $p_0(t)$

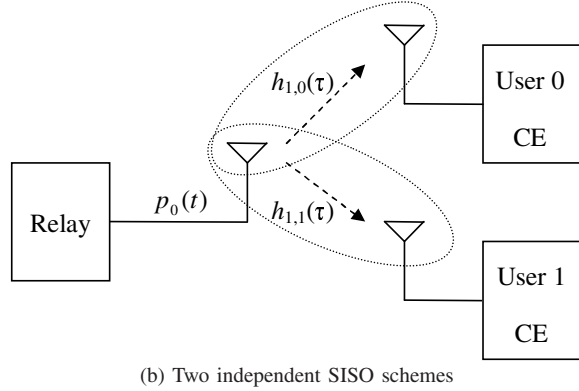
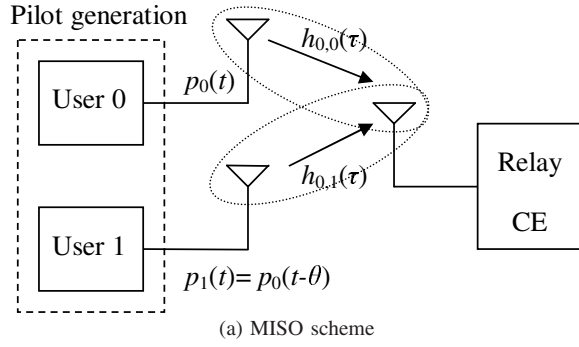
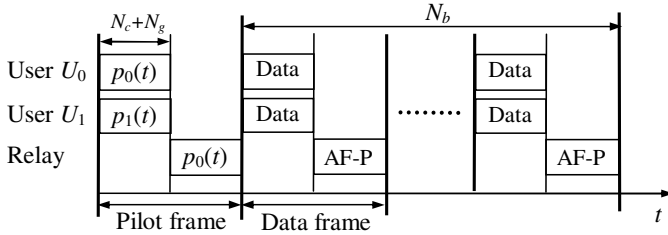


Fig. 2. Proposed CE scheme.



to the users as illustrated in Fig. 2(b). After the pilot frame transmission, N_b data frames are transmitted.

MISO-CE: The users, U_0 and U_1 , transmit their pilots to the relay over a frequency-selective fading channel during the first slot of a pilot frame as shown in Fig. 3. To avoid the problem resulting from overlapping of the channel impulse responses from different users during the first slot, the pilot signal $\{p_1(t)\}$ of U_1 is cyclicly shifted by θ samples relative to the pilot signal $\{p_0(t)\}$ of U_0 ; $\{p_1(t) = p_0(t - \theta); t = 0 \sim N_c - 1\}$ [8]. Consequently, the relay is able to estimate the CSIs from both users. This is explained in the following.

After GI removal and N_c -point FFT the pilot signal received at the relay can be represented as

$$R_{r,p}(n) = H_{0,0}(n)P_0(n) + H_{0,1}(n)P_0(n) \exp\left(-j2\pi n \frac{\theta}{N_c}\right) + N_r(n) \quad (6)$$

for $n = 0 \sim N_c - 1$, where we used the time shifting property of Fourier transform applied to $\{p_1(t) = p_0(t - \theta); t = 0 \sim N_c - 1\}$ [10]. The estimate of the channel gain is obtained by

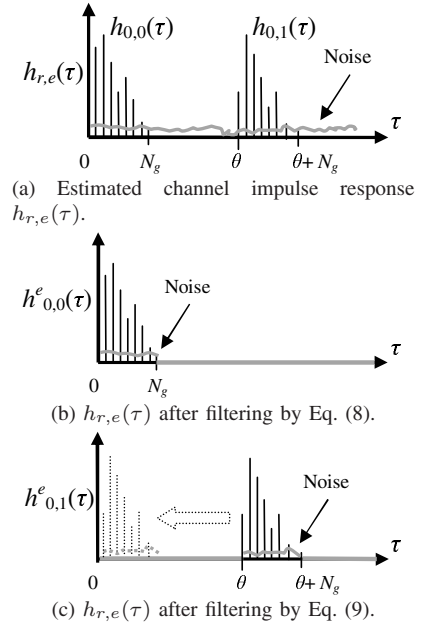


Fig. 4. Estimation of the channel impulse responses ($h_{0,0}^e$ and $h_{0,1}^e$ between the relay and the first and second user during the first transmission stage).

reverse modulation as

$$\begin{aligned} H_{r,e}(n) &= \frac{R_{r,p}(n)}{P_0(n)} \\ &= H_{0,0}(n) + H_{0,1}(n) \exp\left(-j2\pi n \frac{\theta}{N_c}\right) \\ &\quad + \tilde{N}_r(n) \end{aligned} \quad (7)$$

for $n = 0 \sim N_c - 1$, where $P_0(n) = \text{FFT}\{p_0(t)\}$ and $\tilde{N}_r(n) = N_r(n)/P_0(n)$. Then, N_c -point IFFT is applied to transform the estimated channel gain into the estimated channel impulse response $\{h_{r,e}(\tau); \tau = 0 \sim N_c - 1\}$, which is illustrated in Fig. 4(c). A filter is used to separate two channel gains, $h_{0,0}^e(\tau)$ and $h_{0,1}^e(\tau)$, as

$$h_{0,0}^e(\tau) = \begin{cases} h_0^e(\tau), & \text{where } \tau = 0 \sim N_g - 1, \\ 0, & \text{elsewhere.} \end{cases} \quad (8)$$

and

$$h_{0,1}^e(\tau) = \begin{cases} h_0^e(\tau - \theta), & \text{where } \tau = \theta \sim \theta + N_g - 1, \\ 0, & \text{elsewhere,} \end{cases} \quad (9)$$

as illustrated in Figs. 4(b) and 4(c), respectively. Note that the estimate of the channel impulse response $h_{0,1}(\tau)$ in Eq. (9) is shifted by θ .

Finally, an N_c -point FFT is applied to both $\{h_{0,0}^e(\tau)\}$ and $\{h_{0,1}^e(\tau)\}$ to obtain the estimate of the channel gains, $\{H_{0,0}^e(n); n = 0 \sim N_c - 1\}$ and $\{H_{0,1}^e(n); n = 0 \sim N_c - 1\}$ between the relay and the first and second user during the first stage, respectively.

SISO-CE: The relay broadcast its pilot sequence, $p_0(t)$, to both users U_0 and U_1 during the first slot of a pilot frame as shown in Fig. 3. Without loss of generality we focus on

TABLE I
SIMULATION PARAMETERS.

Transmitter (U_0, U_1)	Block size	$N_c = 256$
	GI	$N_g = 32$
	Data modulation	QPSK
Channel	L -path block Rayleigh fading with $\Delta = 1$	
Relay	Protocol	Amplify-and-forward
	Feedback	Perfect
Receiver (U_0, U_1)	FDE	ZF
	Channel estimation	Pilot-assisted

CE performed by the j th user U_j for $j \in \{0, 1\}$ as presented below.

The pilot signal received at the j th user U_j can be expressed as

$$R_{j,p}(n) = P_0(n)H_{1,j}(n) + N_j(n), \quad (10)$$

for $n = 0 \sim N_c - 1$. The estimate of the channel gain $\{H_{1,j}^e(n); n = 0 \sim N_c - 1\}$ is obtained by reverse modulation as

$$H_{1,j}^e(n) = \frac{R_{j,p}(n)}{P_0(n)} = H_{1,j}(n) + \tilde{N}_j(n) \quad (11)$$

for $n = 0 \sim N_c - 1$, where $j \in \{0, 1\}$ and $\tilde{N}_j(n) = N_j(n)/P_0(n)$. Then, N_c -point IFFT is applied to $\{H_{1,j}^e(n)\}$ to obtain the time domain channel impulse response $\{h_{1,j}^e(\tau); \tau = 0 \sim N_c - 1\}$. To reduce the noise, the noise filtering (i.e., zero-replacement) is applied to $\{h_{1,j}^e(\tau)\}$ for $\tau \geq N_g$ and finally, N_c -point FFT is applied to $\{h_{1,j}^e(\tau)\}$ as

$$H_{1,j}^e(n) = \sum_{\tau=0}^{N_g-1} h_{1,j}^e(\tau) \exp\left(-j2\pi n \frac{\tau}{N_c}\right) \quad (12)$$

for $n = 0 \sim N_c - 1$, to obtain the channel gain estimate between the relay and the j th user U_j during the second stage.

The estimates of CSI during the MISO-CE stage are required at the users terminals. In this paper, we assume that the channel gains, $\{H_{0,0}^e(n)\}$ and $\{H_{0,1}^e(n)\}$, are sent from the relay to the users by an ideal feedback channel. The channel gains $\{H_{0,j}^e(n)\}$ and $\{H_{1,j}^e(n)\}$ for $j \in \{0, 1\}$ are used by the users U_0 and U_1 to remove their self-information and detect the signal transmitted from the partner as described in Section II.

The combined channel gain $\{H_{0,j}^e(n)H_{1,j}^e(n)\}$, which is necessary for FDE, can be estimated at the j th destination U_j since the relay broadcasts the superimposed pilots by AF-P. Consequently, the users may estimate the combined channel gain by MIMO-CE scheme as shown in Sect. III-B. However, the accuracy of the estimates degrades since the noise is also amplified during broadcasting. To avoid this problem we estimate the channel gains $\{H_{0,j}^e(n)\}$ and $\{H_{1,j}^e(n)\}$, independently, to compute the combined channel gain.

IV. SIMULATION RESULTS

The parameters used in the simulations are summarized in Table I. We assume the OFDM system with $N_c = 256$ -subcarriers, $N_g = 32$ and ideal coherent quadrature

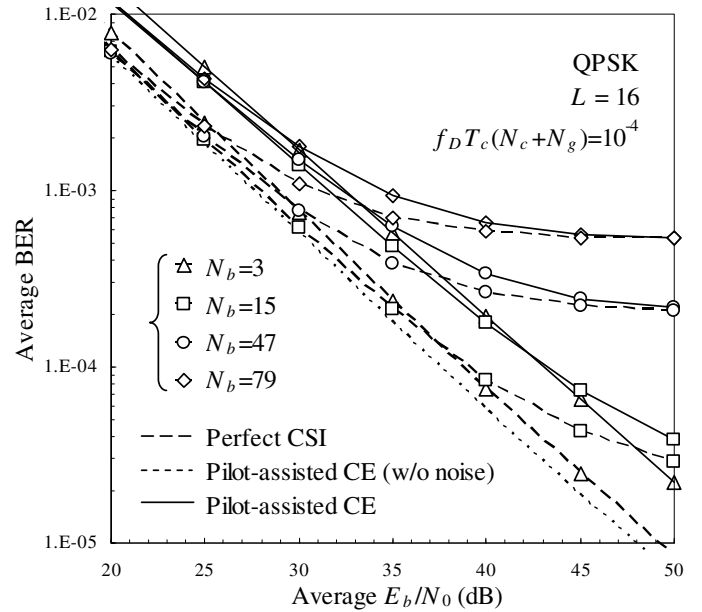


Fig. 5. BER performance.

phase-shift keying (QPSK) modulation and demodulation with $E[|d_j(n)|^2] = 1$. The propagation channel is $L = 16$ -path block Rayleigh fading channel, where the maximum time delay of the channel is less than the GI length and all paths are independent of each other. $f_D T_s$ represents the normalized Doppler frequency, where $1/T_s = 1/[T_c(1 + N_g/N_c)]$ is the transmission symbol rate ($f_D T_s = 10^{-4}$ corresponds to a mobile terminal speed of approximately 19 km/h for a transmission data rate of 100Msymbols/s and a carrier frequency of 2GHz). We assume error-free feedback channel from the relay to the user and no shadowing nor path loss. As a pilot we use a Chu-sequence given by $\{p_0(t) = \exp\{j\pi t^2/N_c\}; t = 0 \sim N_c - 1\}$ [11].

A. BER Performance

Figure 5 illustrates the BER performance as a function of the average signal energy per bit-to-AWGN power spectrum density ratio $E_b/N_0 (= 0.5 \times (E_s/N_0) \times (1 + N_g/N_c) \times (1 + 1/N_b))$ with the number of data frames N_b as a parameter for $f_D T_s = 10^{-4}$. The power loss due to GI and pilot insertion is taken into consideration. The terms "Perfect CSI" and "Pilot-assisted CE (w/o noise)" in Fig. 5, respectively, denote the perfect knowledge of CSI at all terminals and pilot-assisted CE without the noise effect during the estimation process (only tracking errors due to channel time selectivity are taken into consideration).

It can be seen from the figure that the two-slot pilot-assisted CE scheme achieves a satisfactory performance, while allocating only two slots for the proposed CE, which maintains a higher transmission data-rate in comparison with 4 slot pilot-assisted CE. The BER performance of the proposed channel estimator for broadband ANC degrades in comparison with the perfect CSI case; for $\text{BER} = 10^{-3}$ the E_b/N_0 degradation

is about 5, 4, 4.5, and 7 dB when $N_b = 3, 15, 47,$ and $79,$ respectively. This performance degradation is due to three factors: (i) the CE errors, (ii) the tracking errors and (iii) non-perfect self-information reduction due to presence of CE errors. In the case of CE without noise (long-dotted lines in Fig. 5), where only the propagation errors due to the channel time selectivity are considered, the E_b/N_0 degradation, for $\text{BER} = 10^{-3}$, is about 3.5, 3.7, 3.8 and 4 when $N_b = 3, 15, 47$ and $79,$ respectively. The figure shows that the BER performance with $N_b=79, 47$ and 15 improves over the $N_b=3$ case when $E_b/N_0 < 42, 34$ and 29 dB, respectively, because of smaller power loss due to pilot insertion. On the other hand, the tracking ability against the channel fading variations tends to be lost. It can be also seen from the figure that the BER performance with proposed CE, irrespective of $N_b,$ is the same for $E_b/N_0 < 25$ dB since the performance improvement is limited by the CE errors and the tracking errors. The BER floor is observed when $N_b=15, 47$ and $79.$ This is because in the case of larger N_b the pilot insertion interval becomes longer and the channel estimation cannot track against fading.

B. Impact of channel time-selectivity

The impact of $f_D T_s$ on the BER performance with practical CE is discussed below. As $f_D T_s$ increases, the tracking ability against the channel fading variations tends to be lost. Figure 6 illustrates the BER performance as a function of $f_D T_s$ for $E_b/N_0 = 15$ dB, 30 dB and 40 dB with $N_b = 3$ and $47.$ The BER performance of CE without noise effect is plotted as a reference.

It can be seen from the figure that for $f_D T_s = 10^{-5}$ (which corresponds to about 2 km/h mobile terminal speed) almost the same BER performance is achieved irrespective of $N_b.$ In the case of $f_D T_s = 10^{-4}$ (which corresponds to about 19 km/h mobile terminal speed), the BER performance slightly degrades irrespective of N_b and $E_b/N_0.$ On the other hand, as increases to $f_D T_s = 10^{-3}$ (which corresponds to about 190 km/h mobile terminal speed) the BER performance with $N_b = 16$ severely degrades since the tracking ability of the channel estimator against the channel time selectivity is lost.

Note that the estimated CSI at the relay must be fed back to the both destinations, which is not considered in this work.

V. CONCLUSION

In this paper, we presented a pilot-assisted CE scheme for broadband ANC. The proposed CE scheme is divided into two stages: (i) users transmit pilot simultaneously to the relay and (ii) the relay transmits to the users without loss of transmission data rate. The proposed CE scheme overcomes the problem of interfering pilots at the relay by cyclically shifting one of the pilots during the first stage. The BER performance of the proposed CE scheme for broadband ANC was evaluated by computer simulation. Our results show that the BER performance of broadband ANC with practical CE gives a satisfactory performance for a low and moderate mobile terminal speed in a frequency- and time-selective fading channel. In a fast fading, the ANC with conventional CE

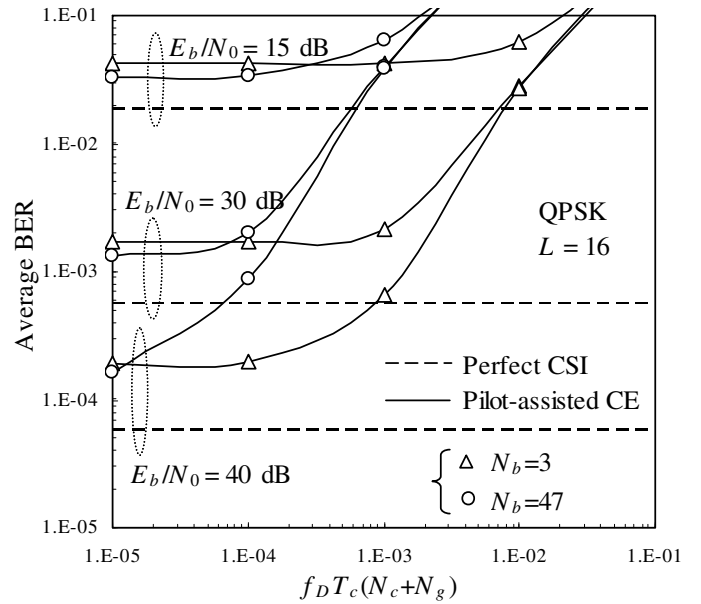


Fig. 6. Impact of $f_D T_s.$

requires four time slots and therefore, loose the tracking ability against fast fading, resulting in the poor BER performance and as a consequence lower network capacity. However, the ANC with the proposed CE requires two slots and can maintain the satisfactory capacity even in a fast fading environment.

The feedback of estimated CSIs at the relay to both destinations was not considered in this work. In addition, the tracking ability against a fast fading channel can be improved using the CE scheme with frequency division multiplex pilots. These are left as interesting future works.

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