

Design of variable structure power system stabilisers with desired eigenvalues in the sliding mode

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Abstract: A systematic approach for the design of variable structure power system stabilisers (VSPSS), with desired eigenvalues in the sliding mode is presented. A detailed sensitivity analysis reveals that the VSPSS is quite robust to wide variations in operating load and system parameters.

1 Introduction

High initial response, high gain excitation systems equipped with power system stabilisers (PSS) have been extensively used in modern power systems as an effective means of enhancing the overall system stability. A linear dynamic model of the system obtained by linearisation of a nonlinear model around a nominal operating point is usually adopted for the PSS design. Several stabilisation strategies have been proposed in the past to achieve improved dynamic performance of the system [1-8].

The PSS designed using a linearised model, provides an optimum performance for the operating point and system parameters considered. However, a fixed structure optimum PSS designed for a particular operating and system condition would provide suboptimum performance under variations in system parameters and operating conditions.

The application of adaptive PSS has been proposed [9-12] to counteract the problem of variations in the system parameters. The adaptive PSS provides an improved performance under variations in system parameters and operating conditions. However, for the realisation of such adaptive self-tuning PSS, online identification of system parameters, observation of system states and computation of feedback gains in a short sampling period is needed. Moreover, the system is generally represented by a low-order discrete-time dynamic model, the performance thereby becoming sub-optimal.

Variable structure control (VSC) theory has been applied for designing power system stabilisers [13, 14]. The variable structure controllers are quite insensitive to system parameter variations and their realisation is

simple. A systematic procedure for the selection of the proper switching vector is extremely important for the design of VSCs.

Hsu and Chan [13] have applied VSC theory for the design of PSS for a machine-infinite bus system. Following their approach, it is necessary to choose some of the components of the switching vector by trial and error. Although, this would ensure sliding mode operation on the switching hyperplane, the system performance would probably not be optimum.

Chan and Hsu [14] have further proposed an optimal VSPSS for a machine-infinite bus system as well as for a multimachine system. The proposed VSPSS is optimal in the sense that the switching hyperplane is obtained by minimising a quadratic performance index, in the sliding mode operation. The resulting switching vector and hence the switching hyperplane depends on the weighting matrices associated with the performance index, the optimum selection of which is extremely difficult.

Utkin and Yang [15] have suggested three alternative approaches for choosing a switching vector such that the sliding motion has desirable properties. In the first, a system is designed, in which the sliding mode is described by equations with favourably located eigenvalues; in the second, a quadratic performance index is minimised with respect to the state vector; the third procedure consists in minimising a quadratic performance index with respect to the state vector and also minimising the so-called equivalent control problem, which characterises the control costs in the sliding mode.

No attempt seems to have been made to design a VSPSS such that the resulting motion is described by equations with favourably placed eigenvalues. It should be noted that the desired location of the poles of a closed loop system can be more conveniently prescribed to achieve the desired dynamic performance, and hence the switching vector C , as compared to the selection of weighting matrices needed to achieve the desired dynamic performance and hence C as in case of optimum VSPSS.

2 System investigated

The system investigated, comprises a synchronous generator connected to an infinite bus through a double-circuit transmission line. An IEEE type-1 excitation system model [16], which neglects saturation of the exciter and voltage limits of amplifier output, has been considered.

3 PSS performance objectives

Two distinct types of system oscillations are usually encountered in an interconnected power system [8]. One type is associated with units at a generating station swinging with respect to the rest of the power system. Such oscillations are referred to as 'local plant mode oscillations'. The frequencies of these oscillations are typically in the range 0.8-2.0 Hz. The second type of oscillation is associated with the swinging of many machines in one part of the system against machines in other parts. These are referred to as 'interarea mode' oscillations, and have frequencies in the range 0.1-0.7 Hz. The basic function of the PSS is to add damping to both types of system oscillations.

It should be noted that only a local mode of oscillation is encountered in a simple machine-infinite bus system and hence the effectiveness of PSS in damping interarea modes of oscillations cannot be studied with a machine-infinite bus system.

The overall excitation control system (including PSS) is designed to

- (i) Maximise the damping of the local plant mode as well as interarea mode oscillations without compromising the stability of other modes
- (ii) Enhance system transient stability
- (iii) Not adversely affect system performance during major system upsets which cause large frequency excursions
- (iv) Minimise the consequences of excitation system malfunction due to component failures.

4 Variable structure systems (VSS)

The basic philosophy of the variable structure approach is simply obtained by contrasting it with the linear state regulator design for the single input-system

$$\dot{x} = Ax + bu \quad (1)$$

In the linear state regulator design, the structure of the state feedback is fixed as

$$u = K^T x \quad (2)$$

where, the state-feedback gain vector K is chosen according to various design procedures, such as eigenvalue placement or quadratic minimisation.

In VSSs, the control is allowed to change its structure, i.e., to switch at any instant from one to another member of a set of possible continuous functions of the state. The variable structure controller design problem is then to select the parameters of each of the structures and to define the switching logic [17]. A reward for introducing this additional complexity is the possibility of combining useful properties of each of the structures. Moreover, a VSS can possess new properties not present in any of the structures used. For instance, an asymptotically stable system may consist of two structures neither of which is asymptotically stable.

The change in structure of the controller takes place on the hyperplane

$$S = C^T x = 0 \quad (3)$$

where C is a constant vector. This hyperplane is also known as the switching hyperplane.

When the control signal u is a function of the state vector x undergoes discontinuities on the plane $S = 0$, the velocity vector also undergoes discontinuities on the same plane. If the state trajectories are directed towards

the plane, an $S = 0$, sliding mode will appear in this plane. The pair of inequalities,

$$L_t s_{-0} - S > 0 \quad \text{and} \quad L_t s_{+0} + S < 0 \quad (4)$$

are a sufficient condition for the sliding mode to exist.

The control signal is a piecewise linear function of x with discontinuous coefficients

$$u = -\sum_j i^T x \quad \psi_i = \begin{cases} a_i, & \text{if } X_i S > 0 \\ b_i, & \text{if } X_i S < 0 \end{cases} \quad (5)$$

where, a_i and b_i , are constants and $i = 1, 2, \dots, n$.

It should be noted that the switching of the state feedback gains occur on the discontinuity plane $S = 0$. The choice of controls should ensure that they give rise to the sliding mode on the discontinuity plane $S = 0$. The switching vector C is chosen so that the sliding motion has the desired properties.

4.1 Equations of the sliding mode with desired eigenvalues

Consider the dynamic model of the system in state-space form

$$\dot{x} = Ax + bu \quad (6)$$

Define a coordinate transformation

$$Z = Mx \quad (7)$$

where the transformation matrix M is chosen so that

$$MA = \begin{bmatrix} \Gamma^0 \\ b_2 \end{bmatrix} \quad (8)$$

The first $(n - 1)$ rows of the matrix M form the basis of a subspace orthogonal to the subspace spanned by the vector b , rank $M = n$ and b_2 is a nonzero scalar (n is the order of the system).

Substituting $x = M^{-1}Z$ from eqn. 7 into eqn. 6 we obtain

$$\dot{Z} = MAM^{-1}Z + Mbu \quad (9)$$

By partitioning Z such that $Z = [Z_1 \ Z_2]^T$ where Z_1 is a $(n - 1)$ column vector and Z_2 a scalar (i.e. the last element of Z), eqn. 9 reduces to

$$\dot{Z} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u \quad (10)$$

A_{11} , A_{12} , A_{21} and A_{22} are the block matrices making up the MAM^{-1} matrix with appropriate dimensions. From eqn. 10

$$\dot{Z}_1 = A_{11}Z_1 + A_{12}Z_2 \quad (11)$$

$$\dot{Z}_2 = A_{21}Z_1 + A_{22}Z_2 + b_2 u \quad (12)$$

Eqn. 11 may be regarded as describing the dynamics of an open-loop control system with state vector Z_1 and control signal Z_2 . Since the pair (A, b) is assumed to be controllable, the pair (A_{11}, A_{12}) is also controllable [15].

Substituting $x = M^{-1}Z$ in eqn. 3, the equation of the switching hyperplane reduces to

$$C^T M^{-1} Z = U \quad (13)$$

writing $C^T M^{-1} Z = [C_1 \ C_2]$ where C_1 is a $(n - 1)$ column vector and C_2 a scalar, eqn. 13 can be written as

$$C_1 Z_1 + C_2 Z_2 = 0 \quad (14)$$

Without loss of generality, we can assume that $C_2 = 1$ and the control signal Z_2 can be expressed as

$$Z_2 = -C_1^T Z_1 \quad (15)$$

Using eqns. 11 and 15 we obtain the equations of the sliding mode in closed loop form as

$$\dot{Z}_1 = (A_{11} - A_{12} C_1^T) Z_1 + A_c Z_1 \quad (16)$$

The eigenvalues of the matrix A_c may be placed arbitrarily in the complex plane, since the pair (A_{11}, A_{12}) is controllable, by a suitable choice of the vector C_1 .

The algorithm for the realisation of the switching vector and hence the switching hyperplane can be summarised as follows

- (i) Select the transformation matrix M (eqn. 7)
- (ii) Compute the vector C_y such that the eigenvalues k_1, \dots, k_{n-1} of the matrix A_c characterising the dynamics in the sliding mode have desirable placement
- (iii) Choose the equation of the hyperplane to be of the form

$$S = [C \quad Y] M x = 0 \quad (17)$$

5 Design of variable structure PSS with desired eigenvalues in the sliding mode for a machine-infinite bus system

The small perturbation transfer-function block diagram of the machine-infinite bus system [2], relating the pertinent variables of electrical torque, speed, angle, terminal voltage, field voltage and flux linkages, is shown in Fig. 1.

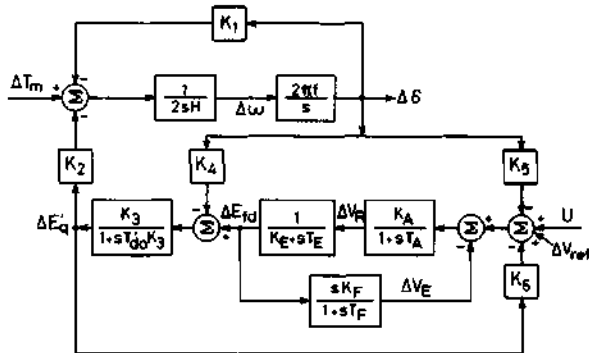


Fig. 1 Linearised small perturbation model of generator connected to infinite bus through transmission line

5.1 Evaluation of K constants

The initial d - q axis current and voltage components and torque angle needed for evaluating the K constants are obtained from the steady-state equations given in Section 9.1 using the system data given in Section 9.2. These are as follows

$$\begin{aligned} V_{d0} &= 0.8211 \text{ p.u.} & I_{i0} &= 0.8496 \text{ p.u.} \\ E'_{q0} &= 0.8427 \text{ p.u.} & V_{q0} &= 0.5708 \text{ p.u.} \\ I_{q0} &= 0.5297 \text{ p.u.} & V_o &= 1.0585 \text{ p.u.} \\ S_o &= 77.40^\circ \end{aligned}$$

The K constants evaluated using the relations given in Section 9.3 are

$$\begin{aligned} K_1 &= 1.15839 & K_2 &= 1.43471 & K_3 &= 0.36 \\ K_4 &= 1.83643 & K_5 &= -0.11133 & K_6 &= 0.31711 \end{aligned}$$

The dynamic model of the system is obtained from the transfer function model (Fig. 1) in state-space form as

$$\dot{x} = Ax + bu \quad (18)$$

where

$$x = [\Delta\omega \quad \Delta\delta \quad \Delta E'_q \quad \Delta E'_d \quad \Delta V_t \quad \Delta V_f]^T$$

and u is the stabilising signal obtained through VSPSS. The values of A and b are given in Section 9.4.

As discussed earlier the transformation matrix M should be chosen so that the first $(n-1)$ rows are orthogonal to vector b and the product of the n th row of M and b is nonzero. Accordingly, M is chosen as

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Applying the coordinate transformation $Z = Mi$, the block matrices A_{11} and A_{12} of the matrix MAM^{-1} for the system investigated, are:

$$A_{11} = \begin{bmatrix} 0.0 & -0.1158 & -0.1435 & 0.0 & 0.0 \\ 314.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.3061 & -0.463 & 0.1667 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.01 & -2.0 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 2.0 \\ 0.2 \end{bmatrix}$$

The dynamics of the system in the sliding mode is described by

$$\dot{Z}_1 = (A_{11} - A_{12} C_1^T) Z_1 + A_c Z_1 \quad (19)$$

The sliding mode operation which desired pole locations can easily be obtained by choosing the elements of C_1 appropriately. Pole placement technique is used to obtain C_1 .

To choose C_x so that the sliding motion has desired eigenvalues, the system described by eqn. 11 is transformed to phase canonical form

$$\dot{Z}_1 = A_{11p} Z_1 + A_{12p} Z_2 \quad (20)$$

where

$$A_{11p} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

$$A_{12p} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (21)$$

where $a_{51} = 0.6102$; $a_{52} = 1.4776$; $a_{53} = -72.0679$; $a_{54} = -37.053$; and $a_{55} = -2.363$. The closed-loop sliding mode equations can thus be written in phase-

canonical form as

$$\dot{Z}_1 = (A_{11p} - A_{12p} C_1^T) Z_1 \quad (22)$$

Expressing

$$C_1 = [C_{11} \ C_{12} \ C_{13} \ C_{14} \ C_{15}]^T$$

the characteristic equation for the system described by eqn. 22, is obtained as

$$s^5 + (C_{15} - a_{55})s^4 + (C_{14} - a_{54})s^3 + (C_{13} - a_{53})s^2 + (C_{12} - a_{52})s + (C_{11} - a_{51}) = 0 \quad (23)$$

For the system investigated, eqn. 23 reduces to

$$s^5 + (C_{15} + 2.363)s^4 + (C_{14} + 37.053)s^3 + (C_{13} + 72.068)s^2 + (C_{12} - 1.477)s + (C_{11} - 0.61) = 0 \quad (24)$$

A stabilising signal

$$u = -i/\wedge Aco - \int Aco \quad (25)$$

is considered. Such a proportional-integral stabiliser with speed deviation as its input signal is capable of providing the desired phase lead by appropriate selection of the gain settings, $\int J_1$ and i/\wedge . The sliding mode operation may be realised by switching the gain settings $ftri$ and j/\wedge according to the following logic

$$\psi_i = \begin{cases} \alpha_i & \text{if } x_i s > 0 \\ -\alpha_i & \text{if } x_i s < 0 \end{cases} \quad i = 1, 2 \quad (26)$$

Fig. 2 shows the schematic block diagram of the VSPSS.

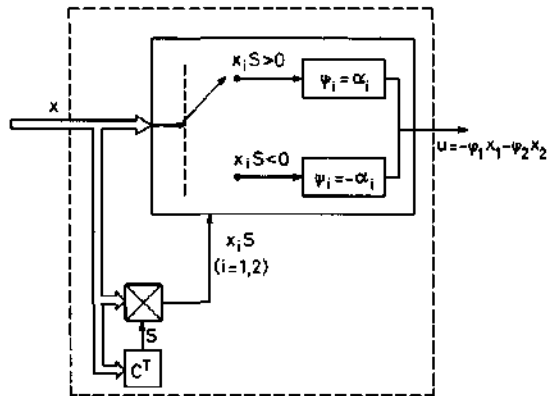


Fig. 2 Variable structure power system stabiliser

The desired location of the eigenvalues and the gain setting a , are obtained using the following step by step procedure:

Step 1: A set of real negative eigenvalues k_i ($i = 1, n$) are assumed.

Step 2: A performance index J given by

$$J = \int_0^{\infty} [\Delta\hat{\omega}^2(t) + A\hat{\omega}^2(t)] dt$$

where

$$\Delta\hat{\omega}(t) = A(o(t) - Aco(\omega))$$

$$\Delta\hat{\delta}(t) = A\zeta(0) - A\zeta(\omega)$$

is evaluated for a wide range of a considering a 1% step increase in mechanical torque. The value of a corresponding to J_{min} is chosen.

Step 3: All the eigenvalues are shifted to the left by a small step Δa .

Step 4: Repeat steps 2 and 3 sequentially until a minimum value of J_{min} is attained.

For the system studied, the minimum value of J_{min} was obtained for $\langle x_i = 15.0$ ($i = 1, 2$) and pole locations at $(-8.0, -8.5, -9.0, -9.5, -10.0)$. For these pole locations the characteristic equation of the system (eqn. 22) is

$$s^5 + 45.0s^4 + 808.75s^3 + 7256.25s^2 + 32501.5s + 58140.0 = 0 \quad (27)$$

Comparing the coefficients of eqns. 24 and 27 the vector C_1 thus obtained is

$$Q = [-82146.276 \quad 188.818 \quad 2647.961 \quad -32.674 \quad 539.925]^T$$

Hence, the switching vector $C = M[C_1 \ C_2]^T$ is given by

$$C = [-82146.276 \quad 188.818 \quad 2647.961 \quad -32.674 \quad 10 \quad 539.925]^T$$

Fig. 3 shows the dynamic responses for Aco and $A\zeta$ considering VSPSS following a 1% step increase in AT_m .

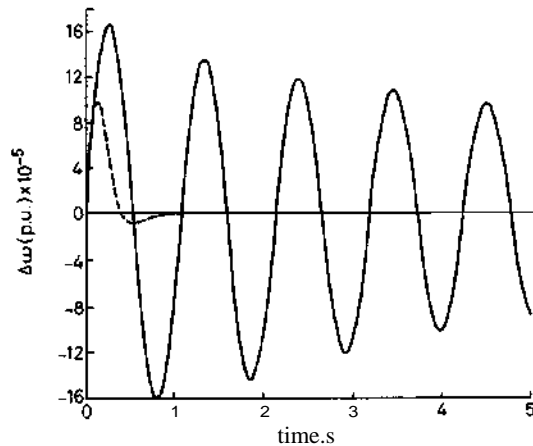
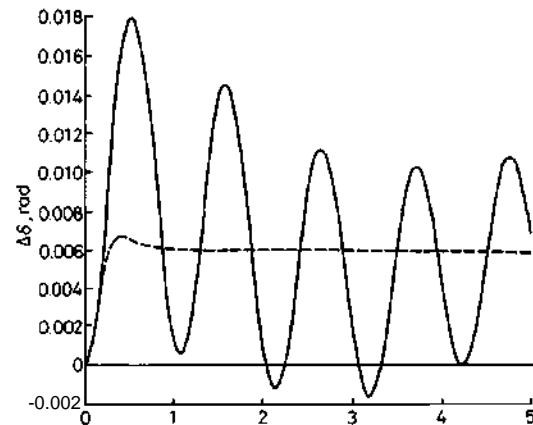


Fig. 3 Dynamic responses for Aco and AS
 — without PSS
 - - - VSPSS

Dynamic responses of the system without PSS are also plotted as a comparison. It can be clearly seen that the responses obtained with VSPSS are well damped.

6 Sensitivity analysis

A detailed sensitivity analysis is carried out to understand the sensitivity of the system with VSPSS to changes in significant system parameters viz. line reactance x_e , inertia constant H , field open-circuit time constant T'_{do} , AVR gain K_A etc. and loading conditions P and Q over a wide range, from their nominal values. The dynamic responses for A60 and A5 following a 1% step increase in $A T_m$ were obtained and analysed.

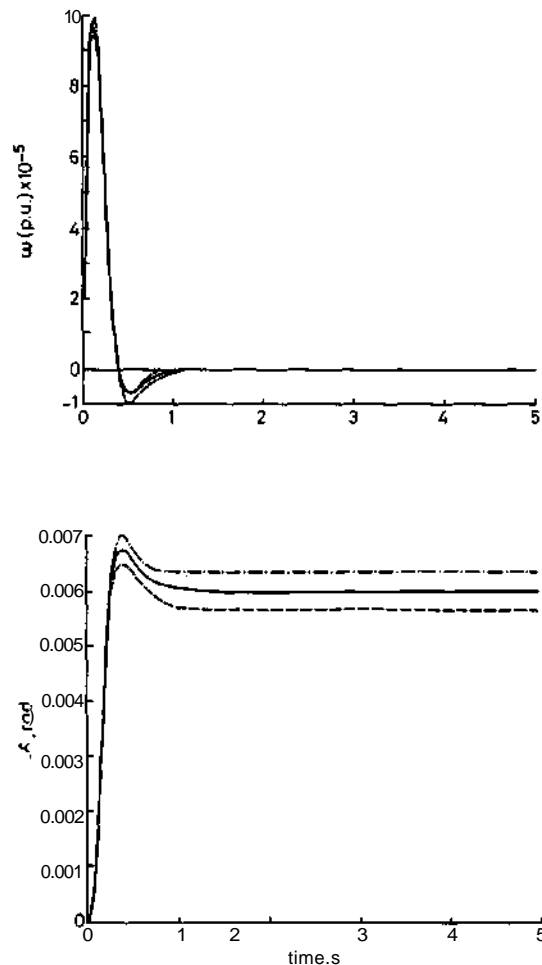


Fig. 4 Dynamic responses for $\pm 25\%$ change in line reactance x_e with VSPSS

- $x_e = 0.4$ p.u. (nominal)
- - - $x_e = 0.3$ p.u. (25% decrease)
- · - $x_e = 0.5$ p.u. (25% increase)

Fig. 4 shows the dynamic responses of the system with VSPSS for

- (i) $x_e = 0.4$ p.u. (nominal value)
- (ii) $x_e = 0.3$ p.u. (25% decrease from nominal)
- (iii) $x_e = 0.5$ p.u. (25% increase from nominal).

It can be clearly seen that the dynamic performance of the system with VSPSS is quite insensitive to $\pm 25\%$ variations in line reactance x_e from its nominal value.

Further, sensitivity analysis considering $\pm 25\%$ change in P , Q , K_A , H and T'_{io} from their nominal values revealed that the VSPSS is quite robust to wide variations in these parameters.

7 Conclusions

A systematic approach for the design of a VSPSS with desired eigenvalues in the sliding mode has been presented. The dynamic performance of the system with VSPSS is found to be well damped. A detailed sensitivity analysis considering VSPSS shows that the system dynamic performance is quite insensitive to wide changes in system parameters such as x_e , P , Q , T'_{iB} , H and K_A .

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9 Appendices

9.1: The steady-state values of the $d - q$ axis voltage and current components for the machine infinite-bus system for the nominal operating condition are given below [2]. These are expressed as functions of the steady-state ter-

minimal voltage V_{i0} and steady-state real and reactive load currents I_{P0} and I_{Q0} , respectively.

$$E_{q0} = [(V_{i0} + I_{Q0}x_q)^2 + (I_{P0}x_d)^2]^{1/2}$$

$$K = i(V_{i0} - I_{P0}r_e - I_{Q0}x_e)^2 + (I_{P0}x_e - I_{Q0}r_d)^2]^{1/2}$$

$$\sin \delta_0 = IV_{i0}I_{P0}(x_q + x_e) - r_ex_q(I_{P0}^2 + I_{Q0}^2) - V_{i0}I_{Q0}r_e/E_{q0}V_o$$

$$I_{Q0} = I_{P0}V_{i0}/E_{q0}$$

$$h_0 = UP_0X_q + I_{Q0}(V_{i0} + I_{Q0}x_q)yE_{q0}$$

$$V_{q0} = [(V_{i0} + I_{Q0}x_q)/E_{q0}]V_{i0}$$

$$V_{d0} = I_{Q0}x_q$$

$$E'_{q0} = V_{q0} + x'_d I_{d0}$$

where

I_d, I_q = direct and quadrature axis components of armature current
 V_d, V_q = direct and quadrature axis components of terminal voltage
 E'_q = voltage proportional to direct axis flux linkages
 β = angle between quadrature axis and infinite bus
 V_o = infinite bus voltage
 E_q = open-circuit terminal voltage

Subscript

o — steady-state value

9.2: The nominal parameters of the system and the operating conditions used for the sample problem investigated are given below. All data are given in per unit of value, except that H and time constants are in seconds.

Generator

$$H = 5.0 \text{ s } T_{i0} = 6.0 \text{ s}$$

$$x_d = 1.6 \text{ } x'_d = 0.32 \text{ } x_q = 1.55$$

IEEE type-I excitation system

$$K_A = 50.0 \text{ } T_A = 0.05 \text{ s}$$

$$K_E = -0.05 \text{ } T_E = 0.5 \text{ s}$$

$$K_F = 0.05 \text{ } T_F = 0.5 \text{ s}$$

Transmission line

$$x_e = 0.4 \text{ } r_e = 0.0$$

Operating condition

$$p = 1.0 \text{ } \delta = 0.05 \text{ } K_w = 1.0 \text{ } f = 50 \text{ Hz}$$

9.3: The constants K_1 - K_6 are evaluated using the relations given below [2] considering zero external resistance

i.e. $r_e = 0$ for the sample problem investigated,

$$K_1 = \frac{x'_d - x'_d}{x_e + x'_d} V_o \sin \delta_0 + \frac{E_{q0} V_o \cos \delta_0}{x_e + x_q}$$

$$K_2 = \frac{K_e \sin \delta_0}{x_e + x'_d}$$

$$K_3 = \frac{x'_d + x_e}{x_d + x_e}$$

$$K_4 = \frac{x_d - x'_d}{x_e + x'_d} V_o \sin \delta_0$$

$$K_5 = \frac{x_q}{x_e + x_q} \frac{V_{d0}}{V_{i0}} V_o \cos \delta_0 - \frac{x'_d}{x_e + x'_d} \frac{V_{d0}}{V_{i0}} V_o \sin \delta_0$$

$$K_6 = \frac{x_e}{x_e + x'_d} \frac{V}{V_{i0}}$$

9.4: The linear state-space model of the system [eqn. 18] is given by

$$\dot{x} = Ax + bu$$

where

$$x = [A_{\omega} \ A_{\delta} \ AE'_q \ AE_{fd} \ AV_R \ \Delta V_E]^T$$

$A =$

$$\begin{bmatrix} 0 & -\frac{K_1}{2H} & -\frac{K_2}{2H} & 0 & 0 & 0 \\ 2nf & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{E'_q}{T'_{do}} & -\frac{1}{T'_{do}K_3} & \frac{1}{T'_{do}} & 0 & 0 \\ 0 & 0 & 0 & \frac{K_E}{T_E} & \dot{1} & 0 \\ 0 & -\frac{K_A K_5}{T_A} & -\frac{\bar{K}AK\bar{\delta}}{T_A} & 0 & -\frac{\dot{1}}{T_A} & -\frac{\bar{K}A}{T_A} \\ 0 & 0 & 0 & -\frac{K_E K_F}{T_E T_F} & \frac{K_F}{T_E T_F} & -\frac{1}{T_F} \end{bmatrix}$$

$$b = [0 \ 0 \ 0 \ 0 \ *JT_A \ oy]$$