Analysis of Amplify-and-Forward DSTBCs over the Random Set Relay Channel

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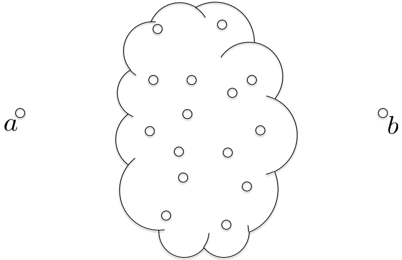
DSTBC over Uncoordinated Relay Pool



- Decode-and-forward DSTBC [Laneman'03, Barbarossa'04]
- Effect of node distribution on DF-DSTBC [Sadek'05]
- Best-relay AF [Bletsas'07, Krikidis'08]

[Laneman'03] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, Oct. 2003.

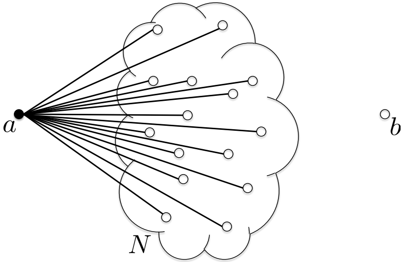
[Barbarossa'04] S. Barbarossa, L. Pescosolido, D. Ludovici, L. Barbetta, and G. Scutari, "Cooperative wireless networks based on distributed space-time coding," in *Proc. IEEE International Workshop on Wireless Ad-hoc Networks (IWWAN'04)*, Finland, 2004.

[Sadek'05] A. K. Sadek, W. Su, and K. J. R. Liu, "Clustered cooperative communications in wireless networks," in *IEEE Global Conference on Communications (Globecomm'05)*, U.S.A., 2005.

[Bletsas'07] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Commun.*, vol. 6, no. 9, pp. 3450–3460, 2007.

[Krikidis'08] I. Krikidis, J. Thompson, S. McLaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 235–237, Apr. 2008.

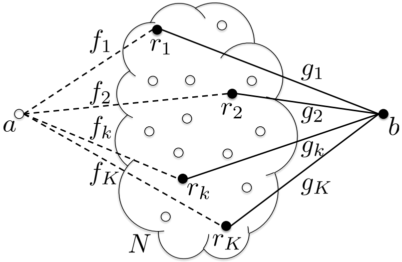
AF-DSTBC over Uncoordinated Relay Pool



- P_1 : Transmit power of the source
- f_k : Source-relay channel coefficient (Rayleigh, $\mathsf{E}[|f_k|^2] = 1$)
- γ_{ar_k}: Instantaneous SNR of source-to-r_k channel
- σ_1^2 : Average noise power at each relay

[Jing'06] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.

AF-DSTBC over Uncoordinated Relay Pool



- P_2 : Transmit power of *each* relay (constant)
- g_k : Source-relay channel coefficient (Rayleigh, $\mathsf{E}[|g_k|^2]=1$)
- $\gamma_{r_k b}$: Instantaneous SNR of r_k-to-destination channel
- σ_2^2 : Average noise power at destination

[Jing'06] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.

The Random Set Relay Channel

- Relay active iff $|f_k|^2 \geq \xi$
- Relay channel: $\mathcal{C} \triangleq \{f_1 g_1, \cdots, f_K g_K\}$ (random set)

$$p_K(\xi, N) = \binom{N}{K} e^{-K\xi}(1 - e^{-\xi})^{(N-K)}$$

$$p_K(\xi, \nu) = \lim_{N \to \infty} p_K(\xi, N) = \frac{\nu^K e^{-\nu}}{K!}$$

  - Assumption: f_k's are i.i.d
  - Using: $\Pr\{|f_k|^2 \geq \xi\}) = e^{-\xi}$

Amplification Strategies

- With CSI

$$\rho = \sqrt{\frac{P_2}{(1+\xi)P_1 + \sigma_1^2}} \cdot \frac{f}{|f|}$$

- Without CSI [Maham'08]

$$\rho = \sqrt{\frac{P_2}{(1+\xi)P_1 + \sigma_1^2}}$$

  - With linear DSTBC, achieves the same performance

[Maham'08] B. Maham, A. Hjorungnes, and G. Abreu, "Distributed GABBA space-time codes in amplify and forward relay networks," in *IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM'08)*, Darmstadt, Germany, Jul.21-23 2008.

Instantaneous Power at Receiver

Full-diversity DSTBC and MRC

$$\gamma_{ab} = \frac{\rho^2 P_1 \sum_{k=1}^{K} |f_k|^2 |g_k|^2}{\sigma_2^2 + \rho^2 \sigma_1^2 \sum_{k=1}^{K} |g_k|^2}$$

Instantaneous Power at Receiver

Full-diversity DSTBC and MRC

$$\gamma_{ab} = \alpha \sum_{k=1}^{K} |f_k|^2 |g_k|^2$$

$$\alpha = \frac{\bar{\gamma}_{ar}\bar{\gamma}_{rb}}{1 + (1+\xi)\bar{\gamma}_{ar} + \bar{\gamma}_{rb} \sum_{k=1}^{K} |g_k|^2}$$

Instantaneous Power at Receiver

Full-diversity DSTBC and MRC

$$\gamma_{ab} = \alpha \sum_{k=1}^{K} |f_k|^2 |g_k|^2$$

$$\alpha \approx \frac{\bar{\gamma}_{ar}\bar{\gamma}_{rb}}{1 + (1+\xi)\bar{\gamma}_{ar} + K\bar{\gamma}_{rb}}$$

$$K \gg 1 \Longrightarrow \sum_{k=1}^{K} |g_k|^2 \approx K$$

Statistics of the RSRC

- Define: $z = |f|^2 \cdot |g|^2$, $x \triangleq |f|^2$ and $y \triangleq |g|^2$
- Recall: $p_X(x) = e^{-x}$ and $p_Y(y) = e^{-y}$
- Then: $$\begin{aligned} F_Z(z|x \geq \xi) &= \int_{\xi}^{\infty} \mathrm{Pr}\{y \leq z/x\} \cdot p_X(x \geq \xi)\, dx \\ &= \int_{\xi}^{\infty} \left(1 - e^{-z/x}\right) \cdot e^{\xi - x}\, dx \\ &= 1 - e^{\xi} 2\sqrt{z} K_1(2\sqrt{z}) + e^{\xi} \int_0^{\xi} e^{-\frac{z}{x} - x}\, dx \end{aligned}$$

$$p_Z(z|x \geq \xi) = \frac{d}{dz} F_Z(z|x \geq \xi)$$

- Thus: $p_Z(z|x \geq \xi) = 2e^{\xi} K_0(2\sqrt{z}) - e^{\xi} \int\limits_0^{\xi} \frac{1}{x} e^{-\frac{z}{x} - x}\, dx$

- And: $\mu_z(-s; \xi) = \frac{1}{s} e^{\xi + \frac{1}{s}} E_1\left(\xi + 1/s\right)$

BER of Regular AF-DSTBC over the RSRC

- General expression

$$\bar{P}_{\mathsf{X}}(\bar{\gamma}_{ar},\bar{\gamma}_{rb};\xi,\eta,N,M)=\sum_{K=0}^{N} p_K(\xi,N)\cdot\bar{P}_{\mathsf{X}}(\bar{\gamma}_{ar},\bar{\gamma}_{rb};\xi,\eta K,M)$$

- With PSK:

$$\bar{P}_{\mathsf{PSK}}=\sum_{m=1}^{M-1}\frac{\bar{d}_{m:\mathsf{PSK}}}{2\log_2 M}\cdot\left[I(\delta_m^-,\alpha\cdot\Delta_{\mathsf{PSK}}(\delta_m^-),\eta K)-I(\delta_m^+,\alpha\cdot\Delta_{\mathsf{PSK}}(\delta_m^+),\eta K)\right]$$

- With QAM:

$$\bar{P}_{\mathsf{QAM}}=\sum_{m=1}^{\log_2\sqrt{M}}\left[\sum_{i=0}^{(1-2^{-m})\sqrt{M}-1}\frac{4d_{i:\mathsf{QAM}}}{\sqrt{M}\log_2\sqrt{M}}\cdot I\left(\tfrac{1}{2},\alpha\cdot\Delta_{\mathsf{QAM}}(i),\eta K\right)\right]$$

- Where: $I(\delta,\alpha\cdot\Delta,\eta K)=\frac{1}{\pi}\cdot\int_0^{\pi(1-\delta)}\left[\mu_z\left(-\frac{\alpha\cdot\Delta}{\sin^2(\theta)};\xi\right)\right]^{\eta K}d\theta,$

[Abreu'07] G. Abreu, "BER and mutual information of STBCs over fading channels with PSK/QAM modulations," in *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'07)*, Athens, Greece, Sep.3-7 2007.

Diversity of Regular AF-DSTBC over the RSRC

- Definition [Jafarkhani'05]:

$$G(K) \triangleq -\lim_{\bar{\gamma}\to\infty} \frac{\log\left(\mathsf{SER}(K)\right)}{\log(\bar{\gamma})}$$

- Symbol Error Rate

$$\mathsf{SER}(K) = c \cdot Q\left(\sqrt{\alpha \cdot \Delta \sum_{k=1}^{K} |f_k \cdot g_k|^2}\right) \leq \frac{c}{2}\mu_z^{\eta K}\left(-\frac{\alpha \cdot \Delta}{2}; \xi\right)$$

- Diversity gain

$$G(K) = -\lim_{\bar{\gamma}\to\infty} \frac{\eta K \left(-\log\left(\chi\right) + \frac{1}{\chi} + \log\left[\log(\chi)\right]\right)}{\log\left(\bar{\gamma}\right)}$$

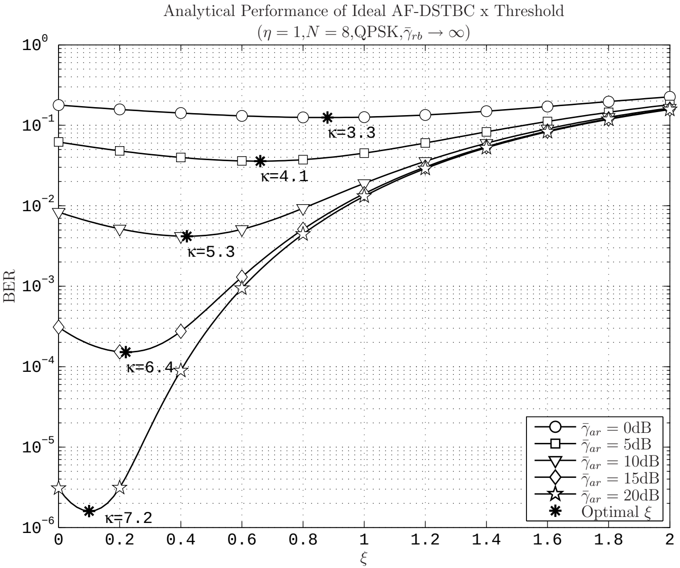
$$= \lim_{\bar{\gamma}\to\infty} \frac{\eta K \ \log(\chi)}{\log\left(\bar{\gamma}\right)} = \eta K, \quad \chi \propto \bar{\gamma}$$

- Use $E_1(x) = \varepsilon + \log(-x) + \sum_{n=1}^{\infty} \frac{x^n}{n\, n!}$

[Jafarkhani'05] H. Jafarkhani, Space-Time Coding Theory and Practice. Cambridge Academic Press, 2005.

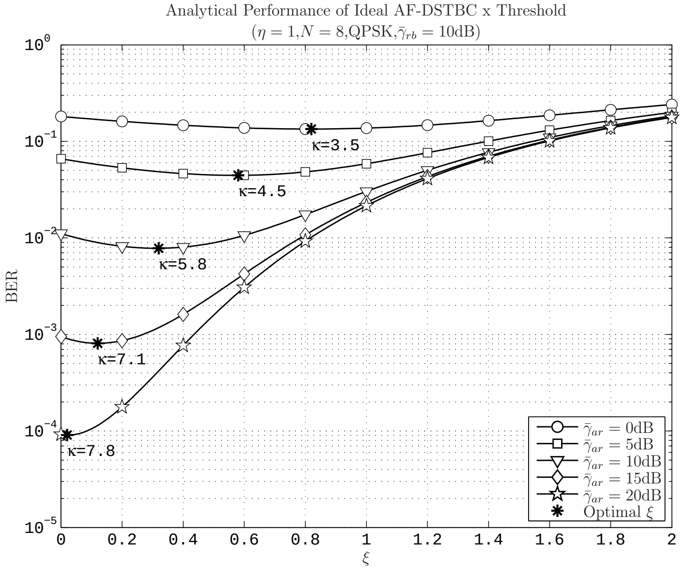
Performance of Ideal AF-DSTBC in RSRC

Relays Close to Destination



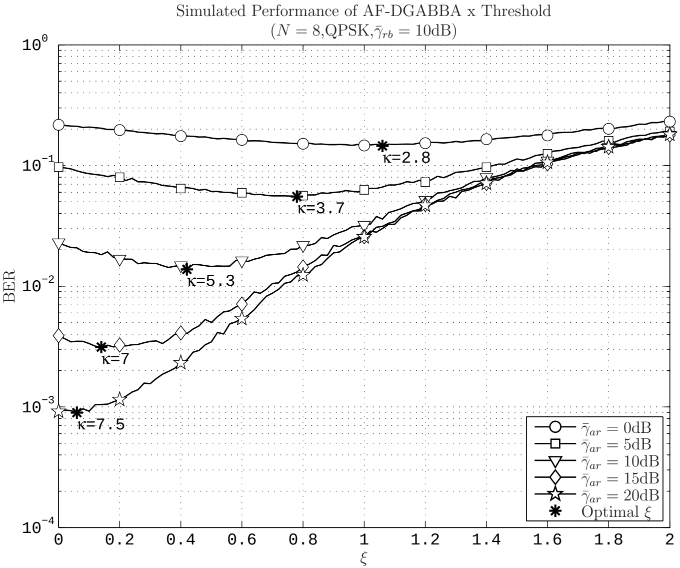
Performance of Ideal AF-DSTBC in RSRC

Relays in Midrange



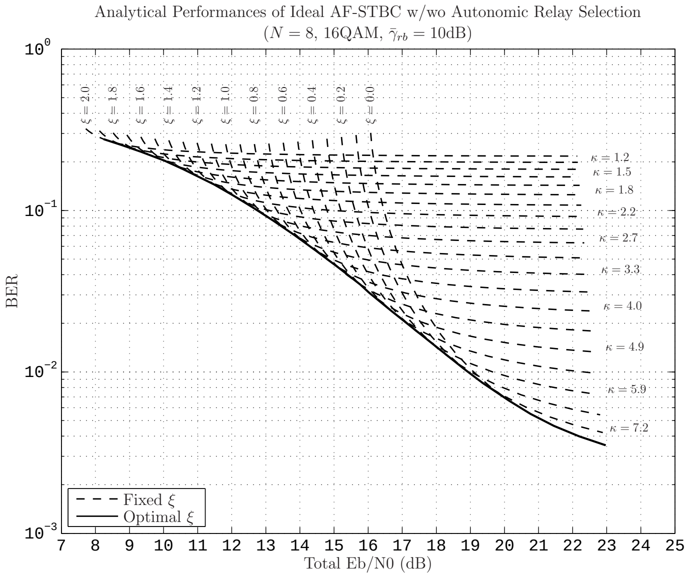
Performance of AF-GABBA with Genie

Relays in Midrange



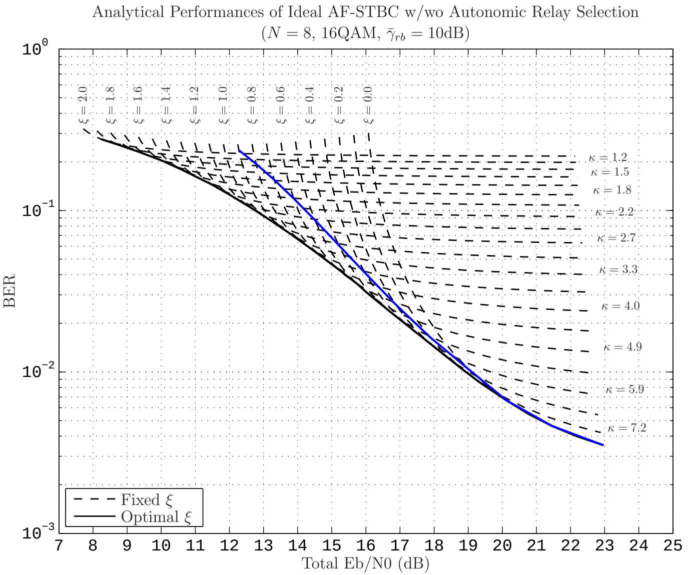
Spectral Efficiency of Ideal AF-DSTBC in RSRC

Small Pool



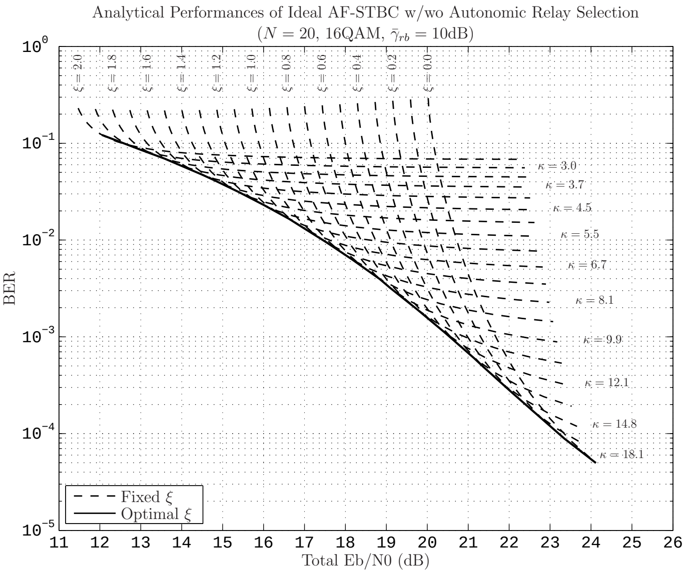
Spectral Efficiency of Ideal AF-DSTBC in RSRC

Small Pool



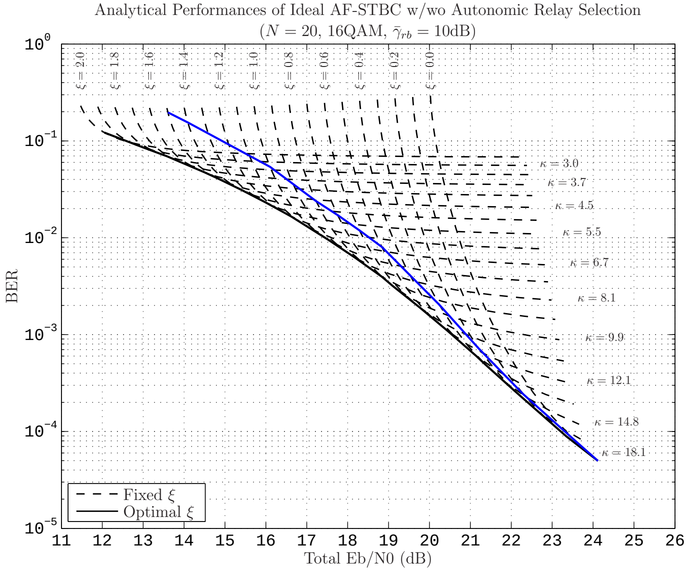
Spectral Efficiency of Ideal AF-DSTBC in RSRC

Large Pool



Spectral Efficiency of Ideal AF-DSTBC in RSRC

Large Pool



Average Mutual Information of AF-DSTBC

- Instantaneous SNR (without forward CSI):

$$\gamma_{ab} = \frac{\rho^2 P_1 \sum_{k=1}^{K} |f_k|^2 |g_k|^2}{\sigma^2 + \rho^2 \sigma^2 \sum_{k=1}^{K} |g_k|^2}$$

- Mutual Information (for a given $\{f_1 g_1, \cdots, f_K g_K\} \in \mathcal{C}$):

$$\mathcal{I} = \frac{1}{2} \log(1 + \gamma_{ab})$$

  - Where $\log$ is in base 2

- Average Mutual Information:

$$\bar{\mathcal{I}} = p_1(\xi, N) \cdot \bar{\mathcal{I}}_{K=1} + p_2(\xi, N) \cdot \bar{\mathcal{I}}_{K=2} + \cdots + p_N(\xi, N) \cdot \bar{\mathcal{I}}_{K=N}$$

Average Mutual Information: $(K = 1)$

- Instantaneous SNR (conditioned on $K = 1$):

$$\gamma_{ab}\big|_{K=1} = \frac{\rho^2 P_1 |f|^2 |g|^2}{\sigma^2 + \rho^2 |g|^2}$$

- Average Mutual Information:

$$\bar{\mathcal{I}}_{K=1} = \frac{1}{4\ln 2} \int_0^\infty \int_0^\infty \ln\left(1 + \frac{\bar{\gamma}_{ar}(x+\xi)y}{y+\varepsilon}\right) e^{-x} e^{-y}\, \mathrm{d}x\, \mathrm{d}y$$

$$= \frac{1}{4\ln 2}\left[e^{\varepsilon}\mathrm{Ei}(-\varepsilon) - e^{\frac{\varepsilon}{\bar{\gamma}_{ar}\xi+1}}\mathrm{Ei}\left(\frac{-\varepsilon}{\bar{\gamma}_{ar}\xi+1}\right) - e^{\frac{1+\bar{\gamma}_{ar}\xi}{\bar{\gamma}_{ar}}} \int_0^\infty \mathrm{Ei}\left(\frac{-\varepsilon-(\bar{\gamma}_{ar}\xi+1)y}{\bar{\gamma}_{ar}y}\right) e^{\frac{\varepsilon-\bar{\gamma}_{ar}y^2}{\bar{\gamma}_{ar}y}} \mathrm{d}y\right]$$

- Where: $\varepsilon \triangleq 1/\rho^2 = \frac{(1+\xi)P_1+\sigma^2}{P_2}$

Bounds on $\bar{\mathcal{I}}_{K=1}$

- Upper Bound:

$$\mathsf{E}[\log(1+x)] \leq \log(1+\mathsf{E}[x]) \Rightarrow \mathsf{E}[\log(1+\gamma_{ab})] \leq \log\left(1+\frac{\bar{\gamma}_{ab}K(1+\xi)}{K+\varepsilon}\right)$$

$$\bar{\mathcal{I}}_{K=1} \leq \frac{1}{4}\log\left(1+\frac{\bar{\gamma}_{ar}(1+\xi)}{1+\varepsilon}\right)$$

- Lower Bound:

$$\log(2\sqrt{x}) < \log(1+x)$$

$$\bar{\mathcal{I}}_{K=1} \geq \ln 2 + \frac{1}{2}[\ln\frac{\bar{\gamma}_{ar}\xi}{\varepsilon} + e^{\varepsilon}\,\mathrm{Ei}\,(-\varepsilon) - e^{\xi}\,\mathrm{Ei}\,(-\xi) - \mathcal{Z}]$$

- Where: $\mathcal{Z}$ is the Euler constant

Conditional Average Mutual Information: $(K \gg 1)$

$$K \gg 1 \Longrightarrow \sum_{k=1}^{K} |g_k|^2 \approx K$$

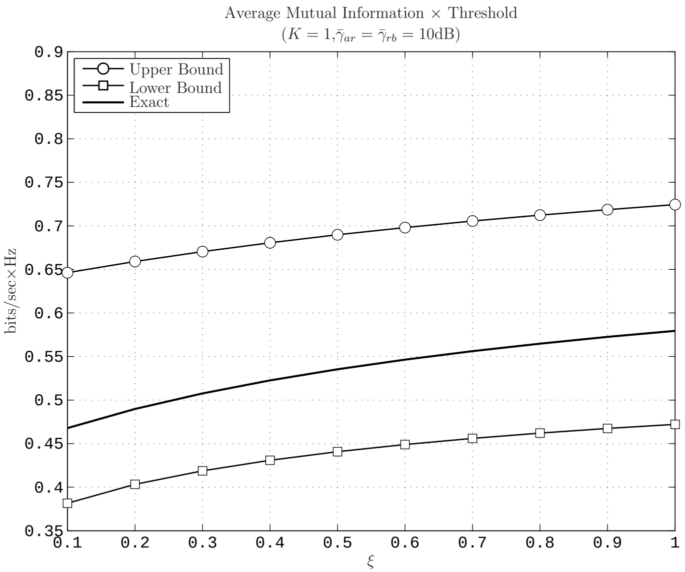
- Instantaneous SNR (conditioned on $K \gg 1$):

$$\gamma_{ab}\Big|_{K \gg 1} = \frac{P_1}{\sigma^2} \frac{K(1+\xi)}{\frac{(1+\xi)P_1+\sigma^2}{P_2} + K}$$

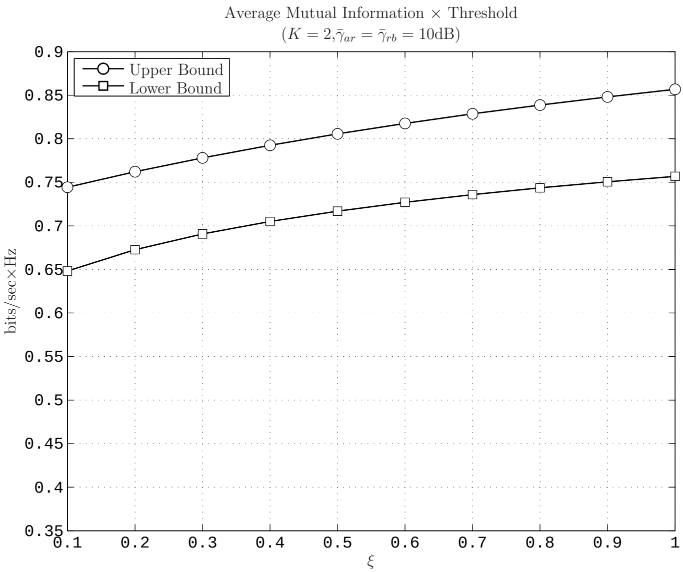
- Average Mutual Information:

$$\bar{\mathcal{I}}_{K \gg 1} = \frac{1}{4} \log\left(1 + \frac{\bar{\gamma}_{ar} K(1+\xi)}{K+\varepsilon}\right)$$

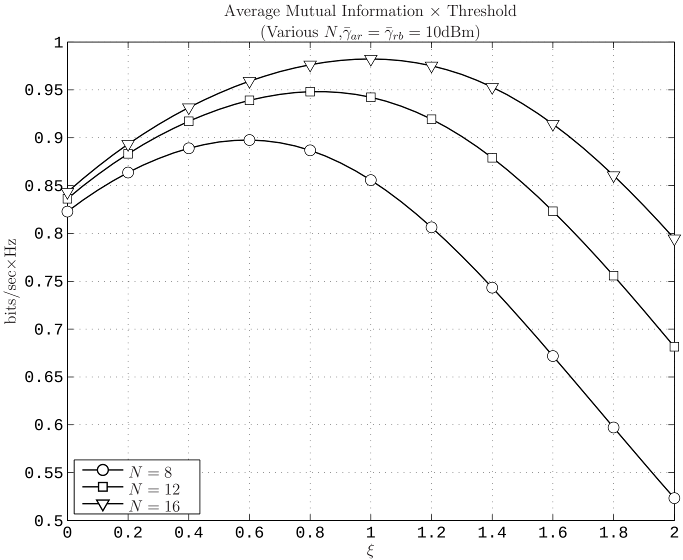
Conditional Average Mutual Information: $(K = 1)$



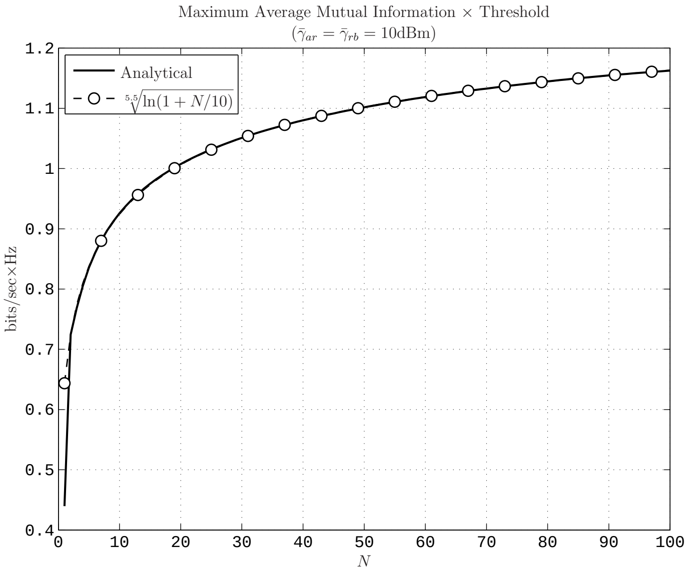
Conditional Average Mutual Information: $(K = 2)$



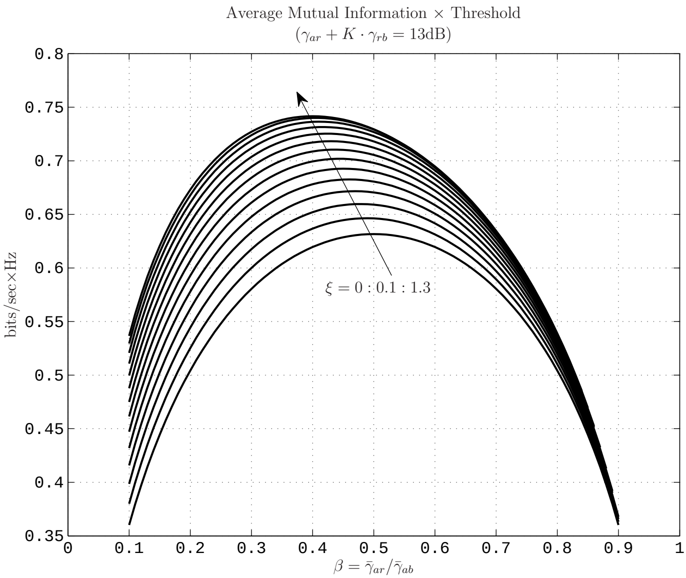
Average Mutual Information (Fixed N)



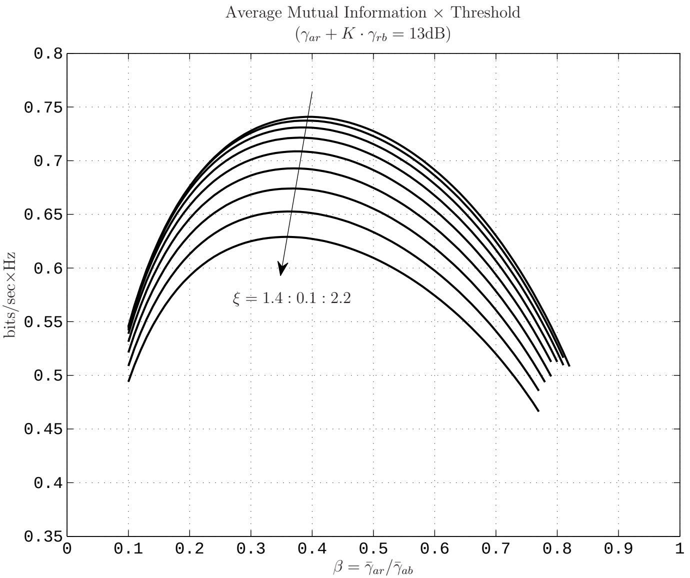
Maximum Average Mutual Information



Maximum Average Mutual Information



Maximum Average Mutual Information



Conclusions

Opportunistic Cooperation Works!

(Better BER, Efficiency and Mutual Information)