Novel Polyphase Training Sequence based Synchronization Estimator for OFDM

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***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) systems are sensitive to timing and frequency estimation errors. A DFT spread modified polyphase training sequence is proposed which has better merit factor and peak to average power ratio. By using the proposed training sequence two estimators are presented for timing and frequency synchronization. The CRLB expressions in closed form for these two estimators are also presented under the assumption of strict decoupling of timing and frequency offsets. Simulation results show that the MSE of the proposed estimators follow their respective CRLB.**

I. Introduction

OFDM systems are at the heart of the emerging communication standards, such as IEEE 802.11/16/22 and 3GPP-LTE. OFDM is used in the European Digital Audio/Video Broadcast (DAB/DVB) system and is being investigated for other mobile broadband digital communication systems.The OFDM receiver is realized in three stages: synchronization, channel estimation and forward error correction decoder. Synchronization is one of the crucial signal processing operations in OFDM system because of its sensitivity to timing and frequency errors [1]. There are two problems from the design point of view for OFDM receiver before channel estimation stage i.e., timing offset which occurs because of multipath fading and frequency offset which arises from the frequency mismatch between transceiver oscillators. In OFDM, ISI, which is caused by timing offset, can be avoided by inserting Cyclic Prefix (CP) of length greater than the Channel Impulse Response (CIR) length. The ICI, due to frequency offset, can be eliminated by maintaining the orthogonality of carriers, under the condition that the transmitter and the receiver have exactly same carrier frequency.

In Data Aided (DA) synchronization, the preamble is generally the first symbol of the frame and a training sequence is inserted in the preamble. All the work done so far on preamble design and DA synchronization refers to the classical papers of Schmidl and Cox [1]. Minn et al [2] and Park et al [3].

Seung et al [4] proposed a method which is robust for multipath fading environment and uses polyphase sequence which were proposed by Frank and Zadoff [5]. However, [5] has a limitation that the length of such sequences are restricted to perfect squares. They fail for any other length. This limitation was overcome by Chu [6], by modifying [5] to produce a sequence of any length. But sequences produced by Chu [6] have larger alphabet size. Liu and Fan [7] proposed modified Chu sequence with smaller alphabet size.

In this paper new sequence is proposed, taking [7] as the seed sequence and further processing it in order to get a perfect training sequence for a preamble that is robust to multipath fading environment. The proposed training sequence has better Merit Factor, better correlation properties and lower Peak to Average Power Ratio (PAPR) due to its polyphase nature. By using the proposed training sequence two estimators are presented for timing and frequency synchronization. The MSE of the proposed estimators follow their respective evaluated CRLB. The paper is arranged in this manner: Section II presents the proposed preamble structure and an analysis of its properties. Section III talks about the OFDM system and proposed synchronization estimators for timing and frequency offset estimation. Section IV presents the theoretical CRLB of proposed timing and frequency offset estimators. Section V explains the simulation parameters and the results. Finally, Section VI concludes.

II. A New Training Sequence and its Properties

The training sequences hold an important place in wireless technologies. In order to mitigate the bad effect of non-deterministic nature of wireless channel, training sequences are generally used for synchronization and channel estimation.

Consider a seed sequence $\mathbf{a}^r$, proposed by Liu and Fan, of length N_s and root index r, such that $\mathbf{a}^r = \{a_0^r, a_1^r, ..., a_{N_s-1}^r\}$. Each element of the seed sequence is defined as

$$a_n^r = \begin{cases} \exp\left(j\frac{2\pi}{N_s}\lfloor\frac{r.n^2}{2}\rfloor\right) & \text{if } N_s \text{ is even} \\ \\ \exp\left(j\frac{2\pi}{N_s}\frac{r.n(n+1)}{2}\right) & \text{if } N_s \text{ is odd} \end{cases} \tag{1}$$

where $0 \leq n \leq N_s - 1$, r is co-prime with N_s and takes one value from the range $1, 2, ..., (N_s - 2)$. Liu sequence is a constant amplitude sequence and the amplitude is unity, $|a_n^r|$ = 1.

A. Proposed Training Sequence and its Structure

A seed sequence is produced with the help of (1) for a length N_s, where N_s is even. We propose a new sequence $\mathbf{c}^r$

of length $2N_s$ by arranging the sequence in (1) as follows

$$\mathbf{c}^r = [(\mathbf{a}^r) \quad (-\mathbf{a}^{*r})] \tag{2}$$

We set the root index $r = 1$ in all further development and dispense with explicitly stating the superscript r. In order to produce a training sequence for preamble symbol of OFDM, take N point DFT where N is the number of subcarriers in one OFDM symbol and $N_s \ll N$, such that

$$\mathbf{C} = DFT[\mathbf{c}] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c_n \exp\left(-j\frac{2\pi kn}{N}\right) \tag{3}$$

B. Properties of Training Sequence

The training sequence $\mathbf{C}$ exhibits the following properties:

1) The proposed sequence have sharp autocorrelation peak with negligible amount of side lobes. The aperiodic autocorrelation of sequence $\mathbf{C}$ for a shift m is

$$R_{\mathbf{C}}(m) = \sum_{k=0}^{N-m-1} C_k C^*_{k+m} = \begin{cases} 2N_s & m = 0 \\ 0 & m \neq 0 \end{cases} \tag{4}$$

and the energy is E_C = $R_c(0)$. The periodic autocorrelation for polyphase sequences is exactly zero for $m \neq 0$ and it is non-zero for $m = 0$, whereas the PN periodic autocorrelation shows significant peaks, some above 10 % height of main lobe, at non-zero lags.

2) The absolute value of the cyclic cross-correlation function for the proposed sequence is a constant and equal to $1/\sqrt{2N_s}$. The cross-correlation value of $1/\sqrt{2N_s}$ at all lags is the theoretical minimum cross-correlation value for any two sequences that have ideal autocorrelation.

3) The main-lobe of autocorrelation function is of interest. The amount of energy contained in the main-lobe in relation to all side-lobes is defined as Merit Factor (MF) [8]. The proposed sequence being a polyphase sequence follows the property given by Antweiler [9]. The theoretical MF equation for the proposed sequence which is given by

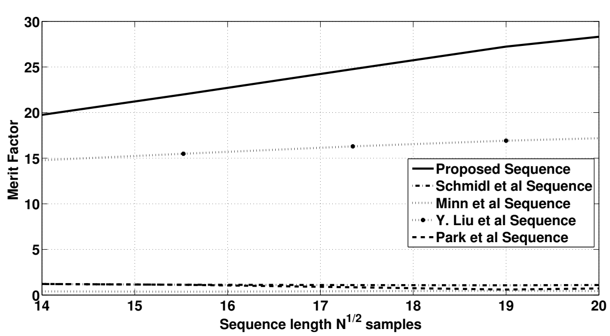

Fig. 1. Merit Factor of Different Training Sequences

$$MF_C = \alpha.\sqrt{N} + \beta \tag{5}$$

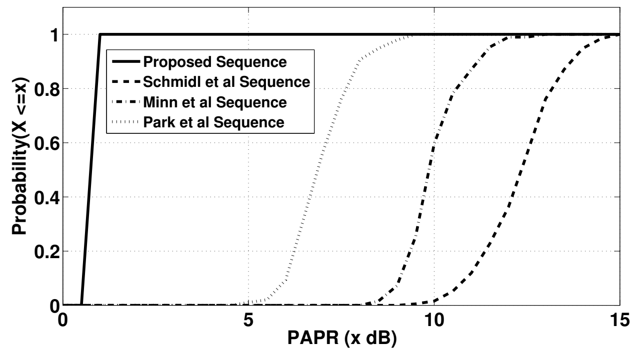

Fig. 2. CDF of PAPR for Training Sequences

where α = 1.38 and β = 0.457. The MF of proposed sequence is compared with a number of existing synchronization sequences in Fig. 1, which clearly shows that polyphase sequences have better MF, as their value always increases with the length of sequence; while in case of binary sequence their MF remains constant [9].

4) Training sequences must have very low PAPR to avoid signal clipping when a signal passes through a power amplifier. If N subcarriers are in phase (assuming the same symbols on all subcarriers), the peak power is N times of average power. For sampled signal, the PAPR for the interval $0 \leq n \leq (N-1))$ is defined as:

$$\mathcal{PAPR}_{\mathcal{C}} \equiv \frac{\max_{[0\leq n \leq N-1]} |\mathcal{IDFT}[\mathbf{C}_K]|^2}{E[|\mathcal{IDFT}\mathbf{C}_K|^2]} \tag{6}$$

The CDF of different training sequences are compared in Fig. 2, which clearly shows that polyphase sequences have better PAPR.

5) The proposed sequence can be generated for any length.

6) The proposed sequence has a very low alphabet size. The Liu sequence has an alphabet size of N_s whereas the Chu sequence has a larger alphabet size of $2N_s$ requiring more memory for implementation (1).

The improvements in MF and PAPR for the proposed sequence $\mathbf{C}$ is mainly due to the DFT operation which spreads the energy equally among all subcarriers and ensures better orthogonality of subcarriers.

III. Synchronization Estimator

A. The OFDM System

A preamble symbol is formed after appending the training sequence proposed in equation (3) with the data symbols and this combination of symbols is input to IFFT block of OFDM transmitter. CP is added at the output of IFFT block and the baseband signal is ready to be up-converted. The samples of the transmitted baseband OFDM signal are given by

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} C_k.\exp\left(j\frac{2\pi kn}{N}\right) \tag{7}$$

CP repeats the last N_g values of $\mathbf{c}$ at the beginning. The samples of the transmitted OFDM symbol $\mathbf{s}$ are given by

$$c_n = \begin{cases} s_{(N+n)}, & n \in [-N_g, -1] \\ s_n, & n \in [0, N-1] \end{cases} \quad (8)$$

The useful part of each OFDM symbol has a duration of T seconds. These samples are transmitted in a wireless fading environment with multipath with an effective CIR $\{h_l\}$ of length $l \in [0, L-1]$ affecting the signal. The CIR length L is assumed to be less than the CP length, $L \leq N_g$. The $N_g + N$ samples of the OFDM transmitted symbol pass through the channel of length L. The received waveform $r(t)$ at the receiver is sampled with period $T_s = T/N$ and its noise free samples are given by

$$r_n = \sum_{l=0}^{L-1} h_l c_{n-l} \quad (9)$$

The time domain received signal with timing offset is often modelled as a delayed signal and with frequency offset it is modelled as a phase distorted signal. The n-th sample of the received sequence in time domain is represented as

$$y_n = \exp\left(j\frac{2\pi n v}{N} + \phi\right) r_{n-\Delta\theta} + w_n \quad (10)$$

where $\Delta\theta$ is the timing offset, ϕ is a phase factor depending on $\Delta\theta$ and subcarrier spacing Δf, $v = \Delta f T_s N$ is the carrier frequency offset normalized by Δf and w_n is the time domain AWGN sample added to the OFDM symbol with mean zero and variance σ^2.

B. Timing Synchronization

The primary goal of pre-FFT processing is to provide a timing index to the FFT process. In order to get a sharp peak with proposed polyphase sequence at the output of correlation, it is better to correlate the received signal over a window size of N_s, which is the length of seed sequence, with a delayed replica of itself. Following Schmidl and Cox the delay set to half OFDM symbol length, $N/2$. This sample-by-sample correlation is performed at least over a length of one OFDM frame. For the received signal y_n, the correlation estimator for estimating the timing offset for the proposed training sequence (3) is

$$\widehat{\Delta\theta} = \underset{n}{\operatorname{argmax}} \left[\left| \sum_{i=0}^{N_s-1} \left(y^*_{n+i} y_{n+i+\frac{N}{2}}\right) \right|^2 \right] \quad (11)$$

where n is the total number of samples in one OFDM frame which consist of multiple OFDM symbol. The n-th sample at the output of the timing metric is defined by (11). The timing metric results in a pointed peak, whereas correlation output of Schmidl and Cox [1] results in a plateau and others have significant side lobes, as shown in Fig. 3. The best of best case is considered in Fig. 3, like proposed training sequence for proposed timing estimator and existing training sequence for timing estimators proposed for them. In the proposed estimator, peak of correlation output appears at the perfect time index, which exactly points out the starting point of FFT window. However, in case of all other preamble structures [1] - [4], it is necessary to average out and normalize the correlated output over multiple OFDM symbols in order to get acceptable results thus requiring the moving average filter. The proposed method results in a sharp correlation peak and does not require moving average filter.

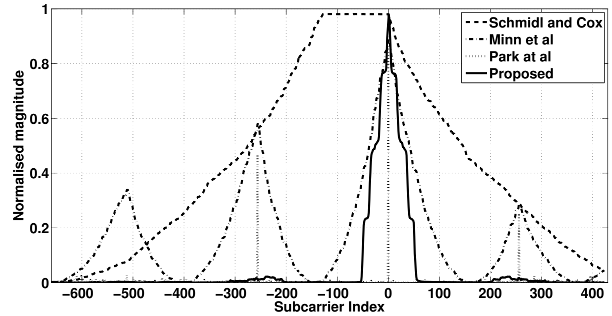

Fig. 3. Comparison of correlation peaks of existing and proposed training sequence for CP length=32 in AWGN, SNR = 20 dB

C. Coarse frequency offset estimation

Once the timing estimate is done, the correlation output (11) is used to estimate the frequency offset. The phase of the correlation output is equal to the phase drift between the samples that are $N/2$ OFDM samples apart. The estimate of the normalized fractional frequency offset following [10] is given by

$$\widehat{\Delta f}_{frac} = \pm\frac{1}{\pi}\angle\left[\sum_{i=0}^{N_s-1} y^*_i y_{i+\frac{N}{2}}\right] \quad (12)$$

The fractional frequency offset is compensated by multiplying the received samples y_n with the exponent as given below

$$y_{n_{\text{modified.frac.off}}} = y_n \exp\left(\frac{-j2\pi n\widehat{\Delta f}_{frac}}{N}\right) \quad (13)$$

IV. CRLB for Proposed Estimator

The Cramer-Rao Lower Bound (CRLB) provides a lower bound on the performance of an unbiased estimator [11]. We derive the CRLB assuming a perfectly decoupled problem with the received signal having either timing offset or frequency offset but not both simultaneously. This way of analysis gives optimistic lower bounds which will not be met in practice since the received signal has both kinds of offsets. However, the offset estimation and correction is normally done iteratively and one reaches a point during iteration where the signal has only timing offset or only frequency offset.

A. CRLB for Timing Offset

Let us put $\Delta f = 0$ in (10), then each of the M independent observations, y_n in the presence of noise is given by

$$y_n = r_{n-\Delta\theta} + w_n, \quad n = 0, \ldots, M-1 \quad (14)$$

Let $f(\mathbf{y};\Delta\theta)$ be the joint Probability Density Function (PDF) for the observation $\mathbf{y} = \{y_n\}$, parameterized by $\Delta\theta$ with the measurement noise being zero mean, complex Gaussian and variance N_0, the joint PDF is given by

$$f(\mathbf{y};\Delta\theta) = \left(\frac{1}{\sqrt{2\pi}\sigma_n}\right)^M \cdot \exp\left(-\frac{1}{2\sigma_n^2}[\sum_{n=0}^{M-1}(y_n - r_{n-\Delta\theta})(y_n - r_{n-\Delta\theta})^*]\right) \quad (15)$$

Taking natural log of (15) and second order partial derivative with respect to $\Delta\theta$, followed by the expectation operation and simplifying using the fact that $E[\omega_n] = 0$, we get

$$E\left[\frac{\partial^2 \ln f(\mathbf{y};\Delta\theta)}{\partial\Delta\theta^2}\right] = -\frac{1}{2\sigma_n^2} \cdot E\left[\sum_{n=0}^{M-1}\dot{r}_{n-\Delta\theta}\dot{r}^*_{n-\Delta\theta} + \sum_{n=0}^{M-1}\dot{r}_{n-\Delta\theta}\dot{r}^*_{n-\Delta\theta}\right] \quad (16)$$

where $\dot{r}_n$ is first order partial derivative. Substituting for r_n using (9) and using the orthogonal property of the CAZAC sequences and the result that $E\left[\sum_{m=0}^{L-1}|h_m|^2\right] = 1$, (16) can be simplified and approximated to,

$$E\left[\frac{\partial^2 \ln f(\mathbf{y};\Delta\theta)}{\partial\Delta\theta^2}\right] \approx -\frac{1}{\sigma_n^2} \cdot \left[\sum_{n=0}^{M-1}\dot{c}_{n-l-\Delta\theta}\dot{c}^*_{n-l'-\Delta\theta}\right] \quad (17)$$

And,

$$CRLB(\Delta\theta) = -E\left[\frac{\partial^2 \ln f(y_n;\Delta\theta)}{\partial\Delta\theta^2}\right]^{-1} \quad (18)$$

B. CRLB for Frequency Offset

Let us put $\Delta\theta = 0$ in (10). Then in the presence of noise, the received signal is given by

$$y_n = r_n \exp\left(\frac{j2\pi n v}{N}\right) + w_n \quad (19)$$

The inner product of the q-th and $(q+d)$-th OFDM symbols can be expressed as

$$P_n^q = \sum_{i=0}^{N_s-1}(y_{n+i}^q)^*(y_{n+i}^{q+d}) \quad (20)$$

Substitute for $\{y_n\}$ from (19) and for r_n from (9). Simplify the resulting equation using the orthogonal property of the CAZAC sequences. (20) approximates to,

$$P_n^q \approx \exp\left(\frac{j2\pi v d(N+N_g)}{N}\right) \cdot \sum_{i=0}^{N_s-1}\sum_{\substack{l=0\\ l=l'}}^{L-1}|h_l|^2(c_{n+i-l}^q)^* c_{n+i-l'}^{q+d} + \sum_{i=0}^{N_s-1} w_{1(n,i)}^q \quad (21)$$

where $w_{1(n,i)}^q$ is the modified noise term which is zero-mean and complex Gaussian.

We define the following variables [12].

$$\Phi = \sum_{i=0}^{N_s-1}\sum_{\substack{l=0\\ l=l'}}^{L-1}|h_l|^2(c_{n+i-l}^q)^* c_{n+i-l'}^{q+d} \quad (22)$$

$$\alpha_n^q = \Phi.\cos\left(\frac{2\pi v d(N+N_g)}{N}\right) + w_{I(n)} \quad (23)$$

$$\beta_n^q = \Phi.\sin\left(\frac{2\pi v d(N+N_g)}{N}\right) + w_{Q(n)} \quad (24)$$

where $w_{I(n)}$ and $w_{Q(n)}$ are the in-phase and q-phase components of the effective noise of the complex conjugate product respectively. Using these definitions, (21) can be written as

$$P_n^q = \alpha_n^q + j\beta_n^q \quad (25)$$

The joint probability density function of $\mathbf{P}^q = \{P_n^q\}$ based on Ψ independent observations is given by

$$f(\mathbf{P}^q; v) = \prod_{n=0}^{\Psi-1}\frac{1}{\sqrt{2\pi}\sigma_n} \cdot \exp\Bigg[-\frac{1}{2\sigma_n^2}\sum_{n=0}^{\Psi-1}\left(\alpha_n^q - \Phi\cos\left(\frac{2\pi v d(N+N_g)}{N}\right)\right)^2 + \left(\beta_n^q - \Phi\sin\left(\frac{2\pi v d(N+N_g)}{N}\right)\right)^2\Bigg] \quad (26)$$

Taking natural log, second order partial derivative w.r.t. v and simplifying with (23) and (24) we get

$$E\left[\frac{\partial^2 \ln f(\mathbf{P}^q; v)}{\partial v^2}\right] = -\frac{1}{\sigma_n^2}\left(\frac{2\pi d(N+N_g)}{N}\right)^2\sum_{n=0}^{\Psi-1}\Phi^2 \quad (27)$$

After solving the above equation we obtain the desired $CRLB(v)$ as

$$CRLB(v) = \frac{\sigma_n^2.N^2}{(2\pi d(N+N_g))^2\Phi^2\Psi} \quad (28)$$

V. Simulation Results

In the simulation, we used a fading generator based on Jake's model [13]. Rayleigh fading channel with ISI is used in simulations. The exponential power delay profile is assumed in all channels. The tap gains of the static ISI channel are fixed and the tap gain powers are the same as those of the Rayleigh fading channel. The MSE is simulated in multipath fading channel environment with simulation parameters: FFT/IFFT size is 512, CP length is 20, Bandwidth of 5 MHz, Length of Seed Sequence is 54, Total number of symbols per frame is 48, Number of Downlink data symbols is 14, Doppler frequency is 10 Hz, Number of subcarriers is 512, Correlation window size is 54, Delay used for correlation is 256 and Number of channel taps is 6.

The MF and CDF of PAPR is shown in Fig. 1 and Fig. 2 respectively. The proposed timing estimator's sharpness is captured in Fig. 3, where the correlation output of existing and proposed timing estimators are compared. A best of best case is considered for finding MSE for timing and frequency

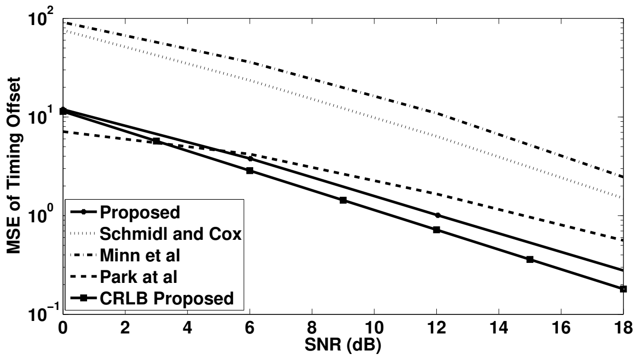

Fig. 4. MSE comparison for timing offset estimation

offset, like proposed training sequence for proposed estimators and existing training sequence for respective estimators. The MSE for timing and frequency offset estimators are given in Fig. 4 and Fig. 5 respectively. These MSE plots for timing and frequency offset estimation clearly show the practical MSE of proposed timing and frequency offset estimators have better performance compared to the performance of well-known estimators in fading channel. The practical MSE closely follows the CRLB for timing and frequency offset as calculated. The exponential power delay profile is assumed in all channels. The tap gains of the static ISI channel are fixed and the tap gain powers are the same as those of the Rayleigh fading channel. The mean square false alarm rate is also evaluated and shown in Fig. 6 for proposed timing offset estimator which is better than Park's. The false alarm rate for proposed timing estimator increases with increasing Doppler.

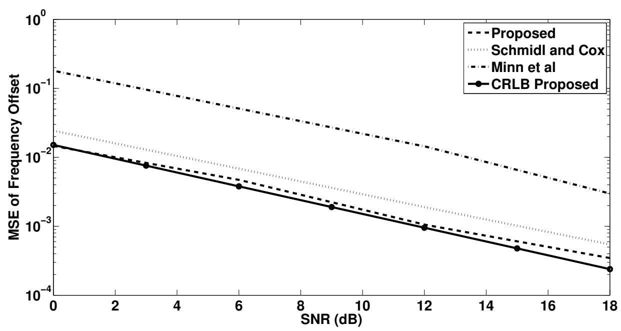

Fig. 5. MSE comparison for frequency offset estimation

VI. Conclusion

This paper proposes a novel DFT spread polyphase training sequence which has better merit factor and PAPR. A set of new estimators is proposed for timing and frequency offset estimation for OFDM system based on proposed training sequence whose MSE performance closely follows their CRLB expression and a closed form CRLB expression is also evaluated for these estimators. The proposed estimator performs better than a number of well-known estimators such as Schmidl et al, Minn et al and Park et al. The computational complexity of the proposed estimator is fairly low compared to these well-known estimators.

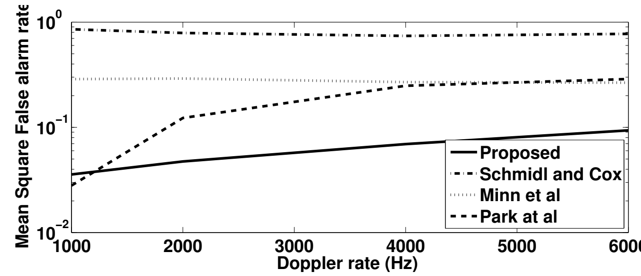

Fig. 6. Mean Square False Alarm rate

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