Adaptive sparse recovery of medical images with variational approach – preliminary study for CT stroke

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Abstract. Presented research was directed to effective signal recovery problem for computer-aided medical diagnosis. Extracted and visualized information covered in sensed data of imaging systems supports interpretation according to "second look" procedure. The integrated framework of compressive sensing was used to optimize CT acute stroke diagnosis. Previously studied nonlinear approximation of the sparse signals in adjusted dictionaries was extended with variational approach to extract more precisely the content components. Proposed methodology adjusts optimized fidelity norms and regularizing priors to semantic question of image-based diagnosis. Preliminary experimental study was performed to provide selected proof-of-concept results for designed CT hypodensity extractors.

Keywords: computer-aided diagnosis, image recovery, compressed sensing, variational image processing

1 Introduction

Images are naturally compressible in a sense that the sorted magnitudes of the transformed image coefficients decay quickly to zero according to the power law. In other words, images are approximately sparse in transform atoms (i.e. elementary signal-representing templates). Consequently, image and signal processing predominantly is based on sparse signal model founded on last decade achievements of harmonic analysis, approximation theory and wavelets. Sparse-Land has emerged as one of the leading concepts in a wide range of applications: denoising, restoration, feature extraction, detection, source separation, compression etc. [1].

Moreover, another competing or extending framework, i.e. variational image processing succeeded in developing an explosively fast speed [2], exploits signal sparsity as regularization prior. Rather than viewing images as being sparse in some basis, they are viewed as minimizers to certain energy functionals including adequate regularizers. The classic example is restoring a noisy image using total-variation regularization [3]. Variational image processing treats an image as a reality function whose sampling or sensing corresponds to the matrix form of a given discrete image. It enables use of useful concepts of functions, i.e. geometry, shapes, edges, smoothness etc., to achieve sub-pixel level accuracy in high-resolution image processing.

Compressive sensing (CS) integrates sparse signal models with variational image processing to reliably recover images acquired from just a few linear measurements. In general, CS (sometimes called sparse recovery) exploits the sparsity and smoothness/regularity of an unknown signals to be sensed by relatively small number of incoherent linear measurements selected a priori. Exhaustively developed theory assures, under respective conditions of high incoherence of measurement matrix, the signal recovery with relatively high resolution and enough accuracy. Generally, image recovery is optimization procedure (where output is a minimizer of certain functional constituting convex problem) with relaxation of important sparsity prior in terms of more computationally tractable norms, greedy alternatives and adaptively formulated semantic criteria of the accuracy.

Medical image recovery means diagnostic information recovered from data acquired in medical imaging systems to be clearly percepted by experts making correct interpretation. Sparse information recovery (generally based on image acquisition, preprocessing to improve the quality, extraction of imaged content possibly followed by automatic object/pattern recognition and even image interpretation) assumes images being sparse in some basis/frames to separate and reconstruct all necessary information to redundant image domain.

1.1 Acute stroke diagnosis

To verify presented concept of image content recovery for more effective diagnosis, two important applications of computer-aided diagnostic imaging were considered. Recognition of ischemic stroke symptoms in CT images and detection of directional breast cancer lesions in mammograms.

Stroke as one of the most frequent and most devastating event among human diseases is the first cause of permanent disability (over 40 years old) worldwide and the third most frequent cause of death. The diagnosis of acute stroke itself is clinical with assistance from the imaging techniques which determine the subtypes. The main method of instrumental stroke diagnosis is computed tomography (CT) widely applicable because of fast, accessible and inexpensive examinations. CT differentiates very well hemorrhagic and ischemic forms of stroke because fresh blood extravasated into brain parenchyma or pericerebral spaces is visible at once as seen at Figure 1. Moreover, CT remains an invaluable method for the detection of hemorrhagic complications, the intermediate signs of necrosis and the emerging threat of uncontrolled intracranial pressure increase followed by herniation and death.

However, there are serious problems with very early discovery of ischemic stroke in CT. In the first hours of ischemic stroke image interpretation is often ambiguous, indirect and not obvious. CT scans (without contrast enhancements; specificity$=96\%$) during the first 24 hours after the onset in the most cases with ischemic stroke is almost normal. Only direct finding enabling ischemia recognition is the extent of hypodense tissue on baseline CT significantly facilitating

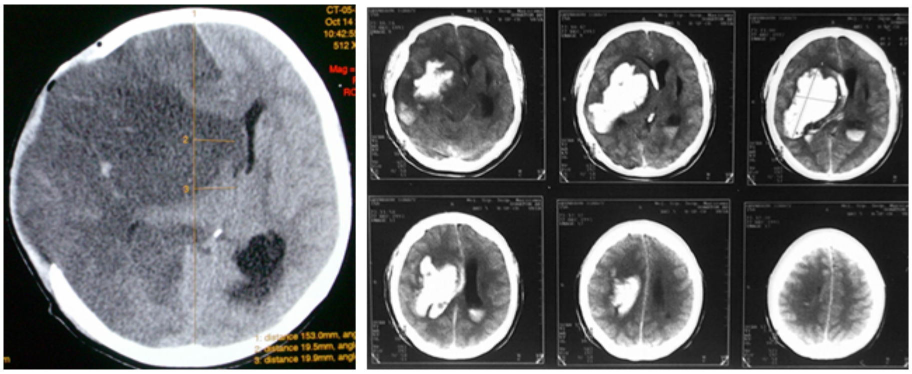

Fig. 1. Ischemic stroke (left) and haemorrhagic stroke (right) manifestation on CT scans.

stroke diagnosis. However, such noticeable lowering of brain attenuation coefficients is often perceptually hidden because of unstable technological conditions of image acquisition, distorted linearity, degraded contrast resolution, artifacts, noise etc. Therefore, acute stroke diagnosis needs computerized support to extract reliable signs of stroke, first of all hypodensity.

This introductory paper with preliminary results presents carried out research background as review of CS-based fundamental adjusted to CT stroke recovery problem. Possible optimization of the image recovery with high enough accuracy of diagnostic components was considered, according to reliable requirements of concrete diagnostic imaging. We proposed use of variational approach (VA) to recover separated components of information improving diagnosis of CT acute stroke. Following such concept, we designed and verified fundamental principles of computerized CT stroke diagnosis in order to separate the hypodensity as most important, direct sign of ischemia. Proposed concepts extends image model of sparsity in wavelets to variational recovering of hypodensity distribution.

Adaptation of the optimization criteria was directed to local minima of denoised image function (functional image model) to approximate a variation of tissue density in CT brain images. The framework of compressive sensing was used to verify several concepts of hypodensity extractors.

2 Framework of the proposed method

Let's start from compressive sensing problem to explain fundamentals of proposed method. The essential advantages of CS concept applied for medical image recovery are as follows: -reduced sensing cost (mostly radiation dose for CT because of reduced number of measurements), -sensing based on sparsifying dictionaries (possible content-dependent adaptivity to selected information components), -image recovery based on variational approach (possibly content-oriented), -image processing with the VA to approximate adaptively diagnostic components.

2.1 Sensing with sparse image models

Sparsity of unknown images is exploited to recover the image information from relatively small number of linear measurements, significantly fewer than required by Nyquist-Shannon theorem. Sparsity means that information contained in a signal/image is much smaller than its effective bandwith. However, in most cases of natural signals we have transform's economical signal representation in some dictionary (base, frame) $\mathbf{\Phi}$. Additional necessary condition is incoherence between sensing matrix (i.e. real imaging modality model) and sparsifying $\mathbf{\Phi}$. For example noiselets $\mathbf{\Xi}$ [4] are useful to measure sparse signals because they have pseudorandom dense representation and the signals compact in the wavelet domain are spread out in the noiselet domain.

Long term study has proved sparsity of CT brain images in wavelet or wavelet-like bases [5]. Selected sparsyfing transforms were applied to optimize sparsity with satisfactorily fidelity criterion. Further research was directed to other multiscale and local base/frame with nonseparable kernels in 2D/3D space were verified to optimize approximately sparse representation as content sparse representation. The lineal and nonlinear approximation procedures were used to extract target function of informative components like nonzeros of highly sparse representation. Unrepresented noise, artifacts and other uninformative signal components were excluded from reconstructed image information. Selected results were presented in section 4.

2.2 Problem to be solved

Ill-posed inverse problem of image recovery refers to noisy (η) sensing with full-rank measurement matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$ of

$$\mathbf{y} = \mathbf{A}\mathbf{x}^{(r)} + \eta \tag{1}$$

where compressed (e.g. low resolution or feature-oriented) or degraded measurement vector $\mathbf{y}$ of M observations is used to recover $\tilde{\mathbf{x}}$ with high enough resolution $N \gg M$ as reliable approximation of unknown real $\mathbf{x}^{(r)}$ (possibly with infinitive resolution physically but pragmatically limited to content clarity criteria). The measurement matrix $\mathbf{A}$ is linear operator of underdetermined system having infinitely many solutions. In case of noiselet-based sensing of the signals sparse in wavelet basis, we have $\mathbf{A} = \mathbf{\Xi}\mathbf{\Phi}$. In order to estimated unique and well-defined solution additional criteria are necessary. Moreover, such criteria could be fitted to content model and specific characteristics of the images.

2.3 Criteria of the solution

Inversion of the problem (1) is a variational minimization in the following adaptive form

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \kappa(\mathbf{x})F(\mathbf{x}) + \lambda(\mathbf{x})P(\mathbf{x}) \quad (2)$$

with κ-weighted criterion of the fidelity $F(\mathbf{x})$ to the observations (usually l_2 norm-based) and λ-weighted prior $P(\mathbf{x}) \in \mathbb{R}$ that determines regularity of the solution. The prior is low for the image features one is interested in.

Important optimization metrics to assess the efficacy of recovering procedures include: recovery accuracy, computational complexity (possible linear or convex programing) and convergence speed. Fidelity criteria need to take into account the highly structured features of natural/medical images and evident properties of the human visual system (HVS).

Strictly convex minimized function guarantees a unique and computationally tractable solution, what means that squared Euclidean norm l_2^2 (i.e. measure of energy) and l_1 are preferable. However, highly desired in signal processing is sparsity of the signals, especially for image recovery. It forces minimization of pseudo-norm $\|\mathbf{x}\|_0 = \#\{i; x_i \neq 0\}$ what is classical problem of combinatorial search but generally NP-hard. Imposing certain matrix $\mathbf{A}$ conditioning, i.e. incoherence and restricted isometry property (RIP), convex optimization with l_1 or simple greedy algorithms give the sparsest solution of l_0-based optimization. Other way of problem relaxation is solving l_p-minimizing problem where $\|\mathbf{x}\|_p = \sum_i \left(|x_i|^p\right)^{1/p}$ but every choice $0 < p < 1$ gives a concave functional [6]

Fidelity criteria are mostly designed with l_2 norm primarily but other norm are possible as follows

- typically least squares (LS), i.e. squared l_2 as $\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2$; integrated with TV prior is preferable for images corrupted by Gaussian noise with satisfactory edges preserving;
- recursive last squares (RLS) that minimizes weighted linear least squares cost function assuming deterministic signal model, able e.g. to estimate consistently the sparse signal's support and its nonzero entries [7];
- mean square error (MSE) or l_2 in the constrained form $\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \tau$ [8, 9];
- l_1-norm, used with TV prior is useful for removing impulsive noise [10, 11];
- general l_p fidelity constraint, i.e. $\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_p \leq \tau$ where $p \geq 1$ and τ is chosen depending on the noise pth moment $\mathbb{E}(\|\eta\|_p)$; for uniform quantization noise, $p = \infty$ is good choice [12];
- Dantzig selector [13] that uses indication function of $\|\mathbf{A}^*(\mathbf{y} - \mathbf{A}\mathbf{x})\|_\infty \leq \tau$, where $\mathbf{A}^*$ is adjoint of $\mathbf{A}$; the Dantzig selector is robust against measurement errors and more adaptive in a sense of fidelity criteria - with l_1 prior can be recast as a linear program.

Fundamental priors considered to recover medical image are as follows:

- based on derivatives of the image to impose some smoothness on the recovered image:
  - the Sobolev prior which is l_2 norm of the gradient approximated with a finite difference scheme $P(\mathbf{x}) = \sum_i \| \bigtriangledown x_i\|_2$; it favors uniformly smooth images;

- the total variation (TV) prior as $P(\mathbf{x}) = \sum_i \| \bigtriangledown x_i \|_1$; it assumes $\mathbf{x} \in l_1(\Omega)$ (Ω is image domain) and favors piecewise constant images with edge discontinuities of small perimeters [3]; TV functional is non-differentiable but still convex and causes sparsity of the solution [14];
- the weighted TV model (WTV) with certain discretization of TV with anisotropic weights defined for 5×5 support of 4/8/16 pixel neighborhood [15];

– sparsity prior while $\#(\sup \mathbf{x}) \leq K \ll M$ with unknown support distribution; NP_hard solution requires relaxation
  - to convex l_1 minimization (i.e. Laplacian prior) with computationally tractable implementations;
  - to $l_p: \ 0 < p < 1$ minimization;
  - in a form of greedy algorithms.

– sparsifying decompositions that produce sparse representation of natural signals; it is based on a reliable theory of sparse signal models what means that the signals can be well-approximated as a linear combination of only a few elements (vectors) from adaptively adjusted basis or dictionary; we have
  - orthogonal/biorthogonal bases of tensor wavelets, redundant curvelets with nonseparable kernels, contourlets, complex wavelets etc.;
  - SVD, KLT and others signular/independent vector extractors [16].

2.4 Recovery algorithms

The l_1 minimization is fundamental approach to recover a sparse signal from limited number of measurements. It provides a useful framework to perform accurate recovery by means of convex optimization problems. Stable signal recovery in noise is possible under a variety of common noise models, e.g. uniformly bounded noise or Gaussian noise. Both the RIP and coherence are useful to establish performance guarantees in noise. l_1 -based relaxation of l_0 pseudonorm is realized in standard decoder of basis pursuit (BP) with LS fidelity criterion for noiseless and noisy data (well known realizations of lasso or extended with fidelity criteria of the Dantzig selector).

Moreover, there is a variety of greedy methods to recover sparse signals. Greedy algorithms abandon exhaustive search for a series of locally optimal single-term updates. They rely on iterative approximation of the signal nonzeros (i.e. signal support with refined coefficients), either iterative identifying the support until a convergence criterion is fulfilled or subsequent signal estimation to provide matching to the measured data. Both essential approaches are applied, greedy pursuits (e.g. Orthogonal Matching Pursuit - OMP with iteratively adding new components that are estimated to be nonzeros) and greedy thresholding algorithms with element pruning steps (nonzero elements are removed iteratively from further analysis, e.g. the Iterative Hard Thresholding - IHT).

In greedy pursuit, starting from $\mathbf{x}^{(0)} = 0$ a k-term aproximant $\mathbf{x}^{(k)}$ is iteratively constructed by providing a set of active columns of $\mathbf{A}$ successively expanded at each next stage. The column selected at successive stage maximally reduces the residual l_2 error in approximating $\mathbf{y}$ from the currently active columns. An important example of such greedy strategy is the OMP, where the approximation for $\mathbf{x}$ is updated by projecting $\mathbf{y}$ orthogonally onto the columns of $\mathbf{A}$ representing current support estimate.

The Compressive Sampling Matching Matching Pursuit (CoSaMP) [17] keeps the nonzero support and either adds and remove alements in each iteration; new $\mathbf{x}$ estimate is restricted to new smaller support. Alternative approach for recovery of sparse signals is combinatorial algorithms [18].

2.5 Medical information recovery

Image edges, ridges and textures of specific ROI (Region of Interests) or denoised approximation signal (stroke case) tendency play decisive role in content-oriented medical image recovery. Accurate visual perception of extracted diagnostic information (lesion symptoms, signatures or any specific features experienced as direct or indirect sign of pathology) is key condition of correct image interpretation. Thus separation and noticeable extraction of diagnostic components significantly improves medical image recovery.

Instead of local image filtering or transform coefficient thresholding, one can use variational processing embedded in optimization procedure of semantic image recovery. Limited number of measurements in CS scheme adequately models acquisition limitations of real imaging systems with respect to potentially continues, noiseless image model as reliable function of interests. Random matrices simulating acquisition process enable modeling of acquisition limitations due to radiation dose, time and resolution-limits, movement and physical artifacts, technological noise, unstable detector sensitivity and contrast etc. Weighted priors of the iterative recovery are important tool to control semantic recovery of diagnostically improved medical images.

Adaptiveness of the recovery was mainly concentrated on models of sparse, locally limited signal segments of interests able to cope with the space-varying context; instead of an access to the whole sensed data to compute the solution, only local data are used to estimate adaptively and locally sparse solution [19].

3 Proposed method

Consequently, the design of adaptive image recovery was formulated according to the following adaptive optimization procedure

$$\tilde{\mathbf{x}} = \arg \min_{\mathbf{x},\mathbf{c}(\cdot)} \kappa(\mathbf{x}) \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_p + \mu(\mathbf{x}) \|\mathbf{\Phi}\mathbf{x}\|_1 + \lambda(\mathbf{x}) TV(\mathbf{x}) \tag{3}$$

with matrix $\mathbf{\Phi}$ sparsifying signal $\mathbf{x}$ and three signal-dependent criteria of adapted fidelity with priors of sparsity and smoothness represented by functional vector $\mathbf{c}(\cdot) = [\kappa(\cdot), \mu(\cdot), \lambda(\cdot)]$.

The fidelity metric $\|\mathbf{y} - \mathbf{Ax}\|_p$ should highly correlate to image accuracy for diagnostic and clinical procedures. First of all the specificity includes emphasized regions of interests or selected image components, e.g. high frequency representatives (e.g. signatures of breast cancer in mammograms) or local intensity maxima/minima (stroke case). Image fidelity criterion truncated to extracted a priori or a posteriori information was considered to be adaptive according to specific medical imaging problem (formal semantic model).

Furthermore, instead of fixed prior, the regularization could be enhanced by using a family of weighted prior model $P(x)^{(\mu,\lambda)}$ adapted to l_1 and TV specificity because of image edge, intensity and texture distribution related to content and diagnostic significance. Weighting parameters should be adapted to the noise level and reliability of estimated image model. The possible integration of sparsity estimate (l_1-like or greedy iteration) with TV smoothness prior (e.g. matched to 2D, 3D context models and edge specificity) and adjusted fidelity of reconstruction to optimize image recovery was studied. Initial results of such optimization realized in preliminary study were presented.

Adaptive recovery of diagnostic image content is based on two fundamental assumptions: a) as accurate as possible basic iterative image reconstruction according to respectively selected fidelity criterion and priors (a priori adaptation), b) semantic model of extracted information (a posteriori adaptation).

The following subsections shortly characterized possible realizations of the above general concept, adjusted to specificity of CT stroke diagnosis.

3.1 General scheme of extractors

General scheme of CT stroke extractors proposed in [5] includes: -image preprocessing to improve the quality, -initial analysis to select regions of interests susceptible to stroke hypodensity, -approximation of hypodensity image component, -optimization of visualized form of pathology extraction. Presented study concentrates only on approximation of hypodensity component. The following extractors were realized and compared in exemplar experiments.

Nonlinear approximation in sparsifying dictionaries. Such concept was realized in our previous study with adjusted bases of orthogonal/biorthogonal wavelets, Fourier transform (FT) and discrete cosine transform (DCT, and block version BDCT), and frames of curvelets, contourlets, surflets, shearlets and complex wavelets (CWT). Nonlinear approximation of decaying magnitudes of transform coefficients orders hypodensity among small set of the highest magnitudes. But the effects significantly depends on the matched dictionary.

VA-based image approximation. Variational approximation was realized according to global minimization of anisotropically discretized TV with weighted fidelity term related to input signal $\mathbf{x}^{(in)}$, measured or initially reconstructed, i.e. noisy and distorted, with important component to be approximated. Anisotropic TV operator with adjusted pixel neighborhood Ω_c is in a form of

$TV^{(aniso)}(\mathbf{x}) = \sum_{k,l\in\Omega_c} w_{k,l}|x_{i,j} - x_{i-k,j-l}|$, while optimisation procedure is as follows

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \kappa(\mathbf{x})\|\mathbf{x}^{(in)} - \mathbf{A}\mathbf{x}\|_1 + TV^{(aniso)}(\mathbf{x}) \quad (4)$$

where $\kappa(\mathbf{x})$ was optimized with combinatorial approach using simple criterion of maximized change of density variation estimate of ischemic region relate to normal region. Generally approximation of locally estimated lower frequency components is required.

Basic on algorithms presented in [20], different schemes of fidelity weighting to strengthen hypodensity extraction was verified. The weights were estimated as normalized factors of image intensity distribution or thresholded indicator of significant or insignificant areas.

CS-based image recovery. This hypodensity extractor extends the optimization procedure according to (4) with two elements:

- sparsity prior of minimized $\|\mathbf{\Phi x}\|_1$ with adjusted wavelet orthogonal base of coiflets with near symmetric wavelet functions and 12-tap filter banks; second-order cone program (SOCP) with a generic log-barrier algorithm (with Newton solver) was used as implementation[1];
- possible reduction of sensed data to verify possible measurement procedure limited to content-oriented sparsity of the images; however the required solution was possible adaptive concentration of the measurements to region (or component) of interests, only pseudo-random noiselets were verified with 1 to 10 reduction of the number of sensed data.

Therefore, realized concept of the image recovery with approximated hypodensity was as follows

a) data sensing with reduced number of measurements $M = N/10$ (basing on estimated sparsity of possible hypodensity component);
b) initial approximation of sensed data vector $\mathbf{x}^{(0)}$ with Conjugate Gradients solver of the set of linear equations (alternatively, additional fast reconstruction of Dantzig selector or CoSaMPare is possible);
c) performance of external iterative loop of l_1 sparsity prior with Dantzig selector (i.e. $\|\cdot\|_\infty$ fidelity term) and recast as linear program (or alternatively log-barrier with LS fidelity term); initiate iteration index $k = 1$ with standard stop condition:
   - calculate solution $\tilde{\mathbf{x}}^{(k)}$ of $l_1$dantzig as $\tilde{\mathbf{x}}^{(k)} = \arg\min_{\mathbf{x}} \|\mathbf{x}^{(k-1)}\|_1$ subject to $\|\mathbf{A}^T(\mathbf{y} - \mathbf{A}\mathbf{x})\|_\infty \leq \eta$,
   - embedded emphasize of hypodensity component in VA-based internal loop
     - solution of $\hat{\mathbf{x}}^{(k)} = \arg\min_{\mathbf{x}} \kappa_{int}(\mathbf{x})\|\tilde{\mathbf{x}}^{(k)} - \mathbf{A}\mathbf{x}\|_1 + TV^{(aniso)}(\mathbf{x})$ where $\kappa_{int}(\mathbf{x})$ are normalized intensities of successively recovered image $\tilde{\mathbf{x}}^{(k)}$, i.e. $\kappa_{int}(x) = \frac{x_{norm}+k}{k+1}$,

[1] http://users.ece.gatech.edu/ justin/l1magic/

- make substitution $\mathbf{x}^{(k)} = \hat{\mathbf{x}}^{(k)}$ and if continue, increment $k = k + 1$;

d) final VA-based approximation of hypodensity component $\mathbf{x}^{(final)} = \arg\min_{\mathbf{x}} \kappa_{final}(\mathbf{x}) \|\tilde{\mathbf{x}}^{(k)} - \mathbf{A}\mathbf{x}\|_1 + TV^{(aniso)}(\mathbf{x})$, where $\kappa_{final}(\mathbf{x})$ is normalized vector of recovered intensity distribution $\mathbf{x}^{(k)}$ thresholded to heuristically estimated probable hypodensity range;

e) adaptive histogram-based extraction of hypodense intensity range to visualize hypodensity component.

Because of weighted fidelity in variational procedure, sparsity prior was weighted due energy concentrated on selected image component. Sparsity adjusted to smoothing pattern controls the weights of fidelity in VA (we have greedy, integrated feedback in iterated recovery procedure). Final recovery was the integration of required component approximation with iterated reconstruction according to controlled sparsity and fidelity constraints.

4 Preliminary results

Exemplar, representative CT image with perceptible ischemia in the right hemisphere of the brain was used to characterize essential processing effects for three proposed hypodensity extractors. The preliminary results are presented on the following Figures: 2, 3 and 4, respectively.

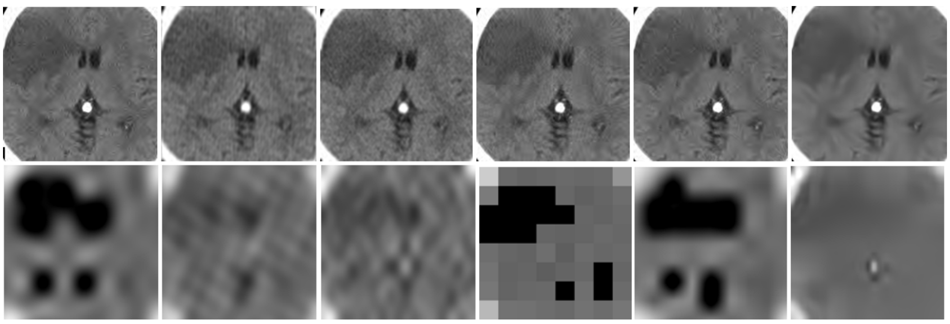

Fig. 2. Nonlinear approximation applied to exemplar, representative CT image of ischemic stroke (original is left image in Fig. 3); first row contains the results of reconstruction with 10% coefficients of wavelets, FT, DCT, BDCT, contourlets and CWT; the second row contains the reconstructions with 0.07% coefficients of the same transforms, respectively.

Presented sample of the effects is representative of the preliminary experiments conducted on a larger group of images. At lower level of the coefficient reduction (first raw in Figure 2), nonlinear approximation method allows image denoising (especially for CWT), but the perception enhancement of hypodense area is relatively insignificant. With very sparse image representation of the

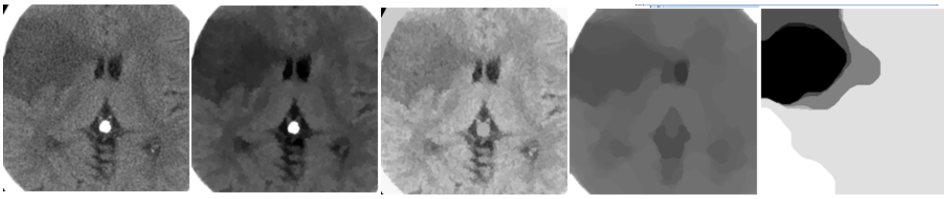

Fig. 3. VA-based image approximation applied to exemplar, representative CT image of ischemic stroke; from left to right the effects of hypodenstity extraction with different weights of fidelity term.

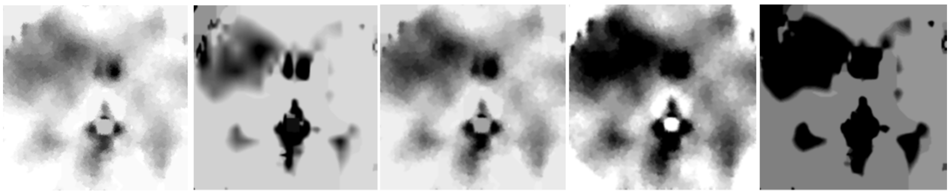

Fig. 4. CS-based image recovery with iterated VA approximation applied to exemplar, representative CT image of ischemic stroke (original is left image in Fig. 3); from left to right the effects of hypodenstity extraction with different combinations of integrated fidelity, sparsity and smoothness constraints.

biggest coefficients, the overall characteristics of reconstructed images is erased, the details are lost. Moreover, distinctly highlighted the area of ischemia (especially for wavelets, BDCT and contourlets) is strongly distorted, the resulting image is blurred, artificial, distant from the original.

In the case of the VA-based approximation, the reconstructed images preserve the details, wherein the enhancement of hypodense area is more significant, with clearly better perception. But at very strong approximation, only the outlines of the structures are reconstructed, the details are lost and image is percepted as artificial. However, smooth shape of ischemic area is reconstructed with a higher accuracy, more precisely, suggestively.

The concept of CS-based image recovery results in clearly exposed areas of tissue with reduced density, maintaining their shape with high precision of the details. Other unimportant components of content disappeared, and the images seem much better emphasize the hypodense nature of the area located in the right hemisphere of the brain. The essential features of this area appear to be significantly enhanced, their perception is definitely the best.

The advantages of the CS-based method was verified in selected several cases of early stroke. CT scans of early examinations with the lack of stroke symptoms and follow-up CT studies with convincing symptoms of hypodensity were processed and ad hoc visualized - see the examples in Figure 5. The pronounced extraction of the reduced density tissue was observed in each of the cases. Such confirmation of stroke occurrence at acute stage of symptom onset is really useful aid of stroke diagnosis. However, the method requires optimized visualization of extraction effects. Additionally, initial segmentation that limits effectively the areas of stroke susceptibility can give more explicit results to provide high enough specificity of such aided diagnosis.

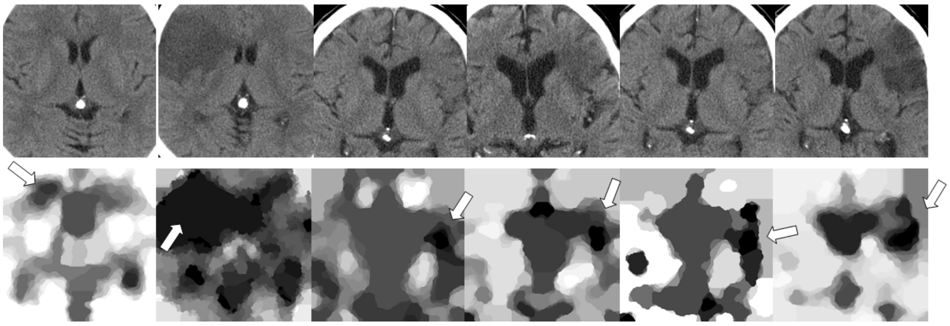

Fig. 5. The results of verification tests for CS-based image recovery. The upper row contains examples of chosen CT slices – three stroke couples of acute diagnosis and follow-up with confirmed symptom onset; lower raw shows their respective recovery with clearly extracted hypodensity, asymmetrically distributed around the axis of the brain, both for imperceptible early symptoms and late confirmations.

5 Conclusions

The advantage of image processing and recovery based on integrated CS framework is possibility of more complex, deeper or ”genetic” signal analysis to precisely select and reconstruct hidden components of high diagnostic importance. More degrees of freedom and possible forming of image components step-by-step, with adjusted adaptive regularization give opportunity to extract the hypodensity even in very difficult cases. However, design and optimization of such flexible model of whole image recovery is neither convex not numerically tractable problem. Instead of that we have heuristic integration problem of numerically tractable partial subproblems.

Future research will primarily focus on the specific, fast and convergent implementation of the outlined concepts. Especially, medical image applications tend toward perfected adaptation of optimization terms according to semantic models of diagnostic images.

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