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Fundamental limits on end-to-end throughput of network coding in multi-rate and multicast wireless networks

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ABSTRACT

This paper investigates the interaction between network coding and link-layer transmission rate diversity in multi-hop wireless networks. By appropriately mixing data packets at intermediate nodes, network coding allows a single multicast flow to achieve higher throughput to a set of receivers. Broadcast applications can also exploit link-layer rate diversity, whereby individual nodes can transmit at faster rates at the expense of corresponding smaller coverage area. We first demonstrate how combining rate-diversity with network coding can provide a larger capacity for data dissemination of a single multicast flow, and how consideration of rate diversity is critical for maximizing system throughput. Next we address the following question: given a specific topology of wireless nodes, what is the maximum rate that can be supported by the resultant network exploiting both network coding and multi-rate? We present a linear programming model to compute the maximal throughput that a multicast application can achieve with network coding in a rate-diverse wireless network. We also present analytical results where we observe noticeably better throughput than traditional routing. This suggests there is opportunity for achieving higher throughput by combining network coding and multi-rate diversity.

1. Introduction

There is an increasing interest in understanding the potential performance gains accruing from the use of network coding in multi-hop wireless environments. In particular, many military battlefield scenarios exhibit two characteristics that appear to motivate the use of network coding: (a) the reliance on bandwidth-constrained, ad hoc wireless links (e.g. using MANETs formed by vehicle-mounted radios in urban insurgencies) and (b) the need to disseminate information (e.g., maps, mission commands) to multiple recipients. The initial results on the power of network coding (**NC**), such as the original demonstration of Ahlswede et al. [1], of how in-network mixing of packets by intermediate nodes helps to achieve a communication capacity that is not achievable solely through routing were obtained for the case of a lossless, wireline network. More recently, several groups have investigated the potential performance gains realized by network coding for both unicast (e.g., [7]) and multicast (e.g., [11]) traffic in wireless environments, for a variety of application scenarios. All of these approaches fundamentally aim to exploit the wireless broadcast advantage (**WBA**) by using, whenever possible, a single link-layer broadcast transmission (of a packet formed by a linear combination of individual packets) to reach multiple neighboring nodes. By saving on the number of independent transmissions needed, network-coding approaches effectively reduce the fraction of time the wireless channel is held by a single transmitting node and thereby help to increase the overall network throughput.

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We believe that there is another degree of freedom in wireless environments, namely *link-layer rate diversity*, that network coding approaches have so far failed to exploit. Most commodity wireless cards are now capable of performing adaptive modulation to vary the link rate in response to the signal-to-interference levels at the receiver. Link rate diversity typically exhibits a *rate-range tradeoff*: if the same transmission power is used for all link transmission rates, then, in general, the faster the transmission rate, the smaller is the transmission range (although, the rate-distance variation in real life is somewhat irregular (e.g., see [2])). While this rate diversity has been extensively exploited for unicast traffic and is often standardized, its use in link-layer broadcasting is relatively limited. For example, while the current IEEE 802.11a/b/g standards mandate the transmission of the control frames (e.g. RTS/CTS/ACK) at the lowest rate (e.g., 6 Mbps for IEEE 802.11a), transmission rates for broadcast data are typically implementation-specific. Recently, however, there has been some work (e.g., [4]) that demonstrates that effective exploitation of such rate diversity by *routing* algorithms for link-layer broadcasts can result in significant (often 6-fold) reduction in the broadcast latency and increase in the achievable throughput.

In this paper, we investigate the impact that the use of such rate-diversity for link layer broadcasts may have on the performance of network coding. In addition, we shall also study the *relative importance* of network coding and link-layer transmission rate diversity. It is easy to conceptualize how the rate-range tradeoff inherent to all link-layer broadcasts might impact the performance of various network coding strategies. Without consideration of rate diversity, network coding algorithms operate on an implicit "more-is-better" assumption: since each broadcast transmission takes the same time, encoding a larger number of packets (for a correspondingly larger set of neighbors) into a single packet *always* results in a more efficient use of the wireless channel. In reality, the existence of the rate-range tradeoff often invalidates this assumption. For example, assume that a node n has a set of packets $\{P_1, P_2, \ldots, P_N\}$ targeted for its neighbors $\{n_1, n_2, \ldots, n_N\}$, where the neighbor indices are arranged in non-increasing order of the link transmission rates. Moreover, let R_i be the link rate between the node-pair (n, n_i). In this case, it is possible that combining the first i packets (transmitted at the rate R_i) proves to be more effective than combining the first $i+1$ packets, because the additional multiplexing gain achieved is negated by the need to use a disproportional smaller rate R_{i+1} for the packet broadcast. Our goals in this paper are thus to answer the following questions:

(1) If multi-rate diversity and network coding can improve together the throughput of the network.
(2) How does the consideration of transmission-rate diversity affect the maximum throughput that may be achieved by linear network coding in wireless environments, i.e., how sensitive are the achievable throughput curves to the impact of link-rate heterogeneity?
(3) How does the throughput achieved by a combination of rate-diverse transmissions and network coding differ in practice from that achieved by pure routing-based strategies that are rate-diversity aware?

Given the closely-coupled interactions between the degree of encoding, the resultant transmission rate and the contentions on the wireless channel, we focus in this paper on the case of single-source multicast problem. Note that the current paper is not *constructive*, i.e., it does not address the design of specific network-coding algorithms that are better at taking advantage of the rate diversity available in a specific network. Instead, our goal is to understand the fundamental interactions between transmission rate diversity and network coding.

1.1. Contributions of this paper

This paper makes the following contributions towards understanding the basic performance of network-coding for broadcast/multicast applications in wireless environments:

- It demonstrates that network coding and multi-rate diversity can improve together the throughput of the network.
- It provides a linear programming model developed for network coding with fixed rates to the variable rate case multicast.
- It uses this new model to compute the maximal throughput achieved with network coding in a rate-diverse wireless network.
- It presents the fundamental limits on the gain of network coding and multi-rate diversity.
- It presents analytical results showing the gain of network coding with multi-rate diversity.

These results demonstrate that it is possible to exploit link-rate diversity and wireless broadcast advantage to improve network performance. The linear programming model is important to measure how close to the optimal the protocols are and to help in the design of new protocols that exploit network coding and rate diversity.

The rest of the paper is organized as follows. Section 2 describes related work. Section 3 describes an initial result to motivate and establish the interplay between NC and rate diversity. In Section 4 we present the Linear Programming Model that finds the optimal rate the network can achieve. In Section 5, we present analytical results. In Section 6 we present the fundamentals limits on the throughput gain with Network Coding and Multi-rate Diversity. Finally, last section concludes the paper with our main conclusions and an enumeration of our current research directions.

2. Related work

The research around network coding was motivated by the seminal paper [1], which demonstrated that, in general, the use of in-network encoding of packets could attain an optimal capacity that cannot be realized via any feasible routing-only scheme. For multicast traffic, the 'capacity' is defined as the maximum data rate that a sender can send to *all members* of a set of receivers. It is given by the minimum of maximum flow (s, t) between sender s and each receiver t. It was shown that network coding can achieve multicast capacity. Li et al. [12] showed that it is sufficient for the encoding function to be linear. In addition to throughput, network coding offers additional benefits, such as robustness (by allowing nodes to receive potentially multiple copies of a single packet).

In wireless environments, network coding has been demonstrated to offer several benefits, such as improved energy efficiency [5] (by reducing the number of distinct transmissions) and higher throughput. For unicast applications, Katti et al. [7] have recently demonstrated that the judicious use of network coding can improve the overall wireless network throughput. Random linear coding for multi-hop wireless multicast applications has been studied in [14], which showed how such randomized coding could improve the overall download latency for file-sharing applications. A linear programming formulation was used in [15] to compute the theoretically maximum throughput that may be achieved for a wireless multicast flow under network coding and original multicast. However, all of these analysis do not consider the impact of transmission rate diversity at the link layer.

The use of such link-layer transmission rate diversity for broadcast and multicast routing was first explored in [4]. This paper introduced a rate-diversity aware broadcast tree construction heuristic, called WCDS (weighted connected dominating set), that was shown to reduce the broadcast latency (defined as the worst case dissemination delay of a packet to a group of receivers) by 3–5 times, compared to conventional diversity-unaware routing strategies. In [3], rate control is addressed at transport layer to adjust source rates. Khreishah et al. [8] developed distributed rate allocation algorithms and coding schemes, but for transmissions targeted towards multiple unicast flows.

We also present an analysis on the bound on the gain of network coding and multi-rate diversity. Liu et al. [13] demonstrated the bounds on the gain of network coding to be a factor of 2 for the single multicast case. In [16], it is shown that pairwise inter-session network coding can improve the throughput of routing-based solutions, regardless of whether perfect scheduling is used. Le et al. [9] derived a tighter upper bound on the throughput gain for a general wireless network based on the encoding number, i.e., the number of packets that can be encoded by a coding node in each transmission. For a single coding structure with n flows, the maximum throughput gain for both the coded and non-coded flows is upper bounded by $2n/(n+1)$. However, all of these analysis did not consider the impact of transmission rate diversity at the link-layer and instead assume that the transmission rate is independent of link distance.

Recently, Cui et al. [17] consider distributed scheduling of broadcast links. We provide a centralized solution that computes the optimal maximum throughput. The LP formulation can also give the scheduling rate at each link to achieve this optimum.

3. Motivation

We first use the classic 'butterfly' network example to understand the relative merits of rate diversity and network coding, and the potential gains that may accrue from a judicious combination of both. Consider the 5-node wireless topology in Fig. 1. The links between the nodes are all 11Mbps, except for the link between node 1 and 2 which is 1 Mbps. Node 1 wants to broadcast packet A and node 2 wants to broadcast packet B. Note that, in a wireless environment, the links are not independent; for example, node 1 uses the same interface to simultaneously reach both neighbors 3 and 4. For simplicity, let us assume that each packet is of size 11 Mbits.

A pure routing-based and rate diversity-*unaware* strategy tries to schedule the dissemination of the broadcast packets so as to avoid collisions among contending links. It is necessary to consider wireless interference. For example, when node 1 is sending a packet to node 4, node 2 cannot send a packet to node 3 because node 3 is on the same interference range from communication between node 1 and 4. It is easy to verify that an optimal transmission schedule consists of first having node 1 broadcast packet A to nodes 2, 3 and 4. To ensure that all these neighboring nodes are able to receive this packet, node 1 has to transmit with the lowest rate among the links (1–2, 1–3 and 1–4); in this case, 1 Mbps (link 1–2). Therefore, it spends 11 time units. Following that, node 2 transmits B to 1, 3 and 5. Node 3 subsequently transmits B to 4, following this with a transmission of packet A to 5. The scheduling solution is shown below. The total time unit is 24 (see Fig. 2).

We can improve it by using Network Coding. In the usual Network Coding solution, Node 3 sends A XOR B to node 4 and 5, in a single transmission. Node 4 has A and can recover B from A XOR B. Node 5 has B and can recover A from A XOR B. The scheduling solution is shown below. The total time unit is 23 instead of 24 (see Fig. 3).

Let us now consider the rate diversity aware case, where different nodes may employ different rates for their broadcast transmissions. First, consider a *pure routing* based approach, where, as before, the network must schedule the transmissions to avoid contentions among interfering links. It is then easy to verify that the entire

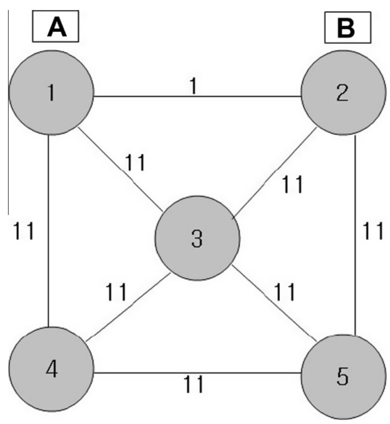

Fig. 1. A multi-rate network example.

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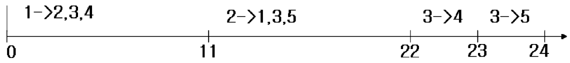

Fig. 2. Broadcast scheduling, rate diversity-*unaware* without NC.

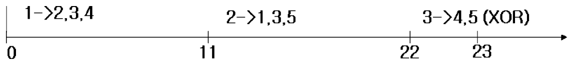

Fig. 3. Broadcast scheduling, rate diversity-*unaware* with NC.

broadcast dissemination can be completed in 4 time units. Namely, node 1 first transmits A to 3 and 4 (taking a total of 1 time unit). Node 2 then similarly transmits B to 3 and 5 (taking an additional 1 time unit). Following this, node 3 broadcasts A to 2 and 5 at 11 Mbps, and follows up with a broadcast of packet B to 1 and 4, again at 11 Mbps.

Interesting enough, combining network coding with rate diversity can reduce the overall transmission latency even further. To illustrate this, consider the following network coding-based transmission strategy. Node 1 first sends the packet A to 3 and 4 using the 11 Mbps transmission rate (node 2 cannot receive at this high rate). Next, node 2 sends packet B to 3 and 5 using the faster rate (11 Mbps). Then node 3 sends (at 11 Mbps) the XOR message to 4 and 5, and also to nodes 1 and 2. Node 1 will retrieve B by applying XOR (A, A XOR B), as it is already aware of its own packet A. An identical reasoning applies for node 2. Fig. 4 illustrates the transmission schedule in this case. Note that the total time consumed by this combination is 3 time units.

This canonical example serves to illustrate two important points. First, we have established that a combination of network coding and transmission rate diversity may prove to be mutually beneficial, resulting in an overall network throughput that is higher than that achievable by either strategy alone. Second, the example suggests that the gains from exploiting rate diversity (a reduction from 24 time units to 4) *may* be more spectacular than the gains accruing purely from network coding (a reduction from 24 time units to 23). Of course, we need to obtain the quantitative nature of the improvements in more practical, generalized topologies.

Next we introduce the linear programming model to compute the maximal achievable throughput and give illustrative examples. For a given topology, it allows us to compare how much we can improve. Finally we present the results.

4. The LP formulation for obtaining capacity of network coding under transmission rate diversity

In this section we describe our Linear Programming formulation for studying the optimal throughput achievable for the Multicast with Network Coding and for the Multicast with Network Coding and Multi-rate. We should notice that the model works for any given topology.

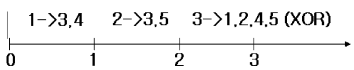

Fig. 4. Broadcast scheduling, rate diversity-aware with NC.

We represent the network with a directed connectivity hypergraph $H = (N,A)$, where N is the set of nodes and A is the set of hyperarcs. Hyperarc (i,J) represents that a packet transmitted by node i can be received by all nodes in J. For example, a transmission from node 1 to 3 and 4 can be represented as $(1,\{3,4\})$.

We assume the network is time-slotted, transmissions are limited to start at slot boundaries. We focus on maximizing the total throughput from a given source to one or more destination assuming that the source always has data to send and the receivers nodes are always ready to accept data.

We can incorporate wireless interference by constructing a conflict graph F. Vertices in F correspond to links in the connectivity hypergraph H. There is an edge between vertices l_{ij} and l_{pQ} in F if the links may not be active simultaneously.

An independent set is a set of vertices, such that there is no edge between any two vertices in the set. A maximal set is an independent set that is not a subset of any other independent set. We enumerate all maximal set of non-interfering hyperarcs of F as $A_k(\subset A)$, where $k = 1, \ldots, M$. M is the number of maximal sets and it varies with the network topology. An algorithm for computing all maximal independent set is given in [18].

Let z_{iJ} be the average rate at which packets are inject into hyperarc (i,J).

Let $c(i,l)$ be the capacity link from node i to node l.

The capacity c_k of hyperarc (i,J) is defined as

$$c_k(i,J) = \begin{cases} \min_{l \in J} c(i,l) & \text{if } (i,J) \in A_k \\ 0 & \text{otherwise.} \end{cases}$$

The case of multicast with Network Coding without multi-rate consists of a single rate and can also be evaluated by this.

The wireless medium contention is modeled as in [15].

$$\sum_k \lambda_k c_k(i,J) - z_{iJ} \geqslant 0, \forall (i,J) \in A,$$
$$\sum_k \lambda_k \leqslant 1,$$

where $\lambda_k \in [0,1]$ means the fraction of time allocated to A_k. Basically, it means that each independent set $A_k(\subset A)$ is activated λ_k time units. The total time of all independent sets is bounded to one. The average rate z_{iJ} is bounded by the hyperarc capacity times the time it is activated.

Let variable x^t_{iJj} indicates the amount of data flow from node i to node j with respect to destination t using hyperarc (i,J).

Time-share constraints ensure the average flow for each independent set is bounded by the amount of data flow from each node. Data-flow constraints ensures the amount of data from source node is f, from receivers is $-f$, and all other nodes is 0. Domain constraints ensures the range limits. Variables must be non-negative.

Network coding specific appears by achieving the maximum flow. The maximum throughput multicast f^*, from source s to non-empty set of terminal nodes T, with the proper constraints is formulated as follows:

maximize f

subject to

$$\sum_k \lambda_k c_k(i,J) - z_{iJ} \geqslant 0, \forall (i,J) \in A,$$
$$\sum_k \lambda_k \leqslant 1$$

where

$$c_k(i,J) = \begin{cases} \min_{l \in J} c(i,l) & \text{if } (i,J) \in A_k \\ 0 & \text{otherwise.} \end{cases}$$

Time-share constraint:

$$\sum_{\{P \subset J | P \cap K \neq \emptyset\}} z_{iJP} - \sum_{j \in K} x^t_{iJj} \geqslant 0, \quad \forall (i,J) \in A, K \subset J, t \in T.$$

Data-flow constraint:

$$\sum_{\{J|(i,J)\in A\}} \sum_{j\in J} x^t_{iJj} - \sum_{\{j|(i,I)\in A, i\in I\}} x^t_{jIi} = \begin{cases} f & \text{if } i = s, \\ -f & \text{if } i = t, \forall i \in N, t \in T \\ 0 & \text{otherwise.} \end{cases}$$

Domain constraint:

$$x^t_{iJj} \geqslant 0 \forall (i,J) \in A, j \in J, t \in T,$$
$$z_{iJ} \geqslant 0 \forall (i,J) \in A,$$
$$\lambda_k \geqslant 0, \forall k.$$

By constructing the conflict graphs, links in the same independent set do not interfere with each other and they can be activated simultaneously. By activating different independent sets at different times we can guarantee no interference. In addition, the constraints are used to impose a feasible scheduling over the network. Any solution satisfying these constraints is always feasible. This modeling approach is similar to [15] where it is proven that this constraints are also a sufficient and necessary condition for feasible scheduling. Observe that we are the first to exploit multi-rate diversity by having no constraint on the number of times a node can broadcast a packet.

It was proven by [6] that to find the optimal throughput under the protocol interference model is NP-Hard.

Subsequently, we use the LP formulations to compare the achievable network throughput with network coding, both in the absence of and using link rate diversity.

The maximum throughput multicast without network coding in wireless networks is achieved using the tree packing strategy, i.e., constructing multiple multicast trees each of which carries an independent flow such that the aggregated flow is maximized. A LP model is presented in [15] and this model is used to compute the throughput achieved in practice without network coding.

5. Analytical results for LP model

The LP model works for any topology. Moreover, it works for any network coding routing scheme: network coding multicast and variable rate multicast with network coding.

Here we present illustrative results for a set of very simple topologies. We used the Mosek LP solver [10]. The channel model used is TDMA. There is no packet collision since each node transmit at their scheduled time slot.

Consider the topologies shown in Figs. 5, 7 and 8. We have 12 nodes in a grid, one source and three destinations. For simplicity, let us assume we only can exploit a subset of the range of rate values available in the 802.11b standard. The full range is 1, 2, 5.5, 11 Mbps. We only have available the rates 1 and 11 Mbps, the latter only on selected links. The thin line represents a 1 Mbps link, and the thicker line represents a 11 Mbps link.

Consider the basic case, which is depicted in Fig. 5. We have the topology with 12 nodes, all links rate as 1. As shown in Fig. 6, previous LP results [15] show the maximum rate achieved by this network with traditional routing multicast is 0.4. Using network coding our LP Model shows that throughput is increased to 2/3.

Next, consider the topology in Fig. 7 which possesses two links with 11 Mbps. Although it has more links with a higher rate than the network in Fig. 5, when evaluated with the LP model shows the same ratio 2/3. This indicates that having a few better links does not always imply a gain in network overall throughput.

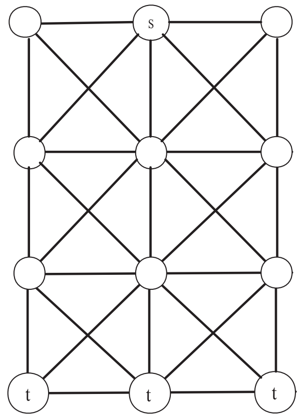

Fig. 5. Basic topology, all links have rate 1 Mbps.

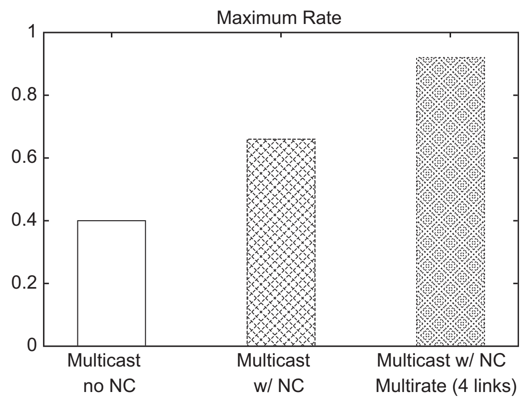

Fig. 6. Maximum throughput in 4 × 3 grid.

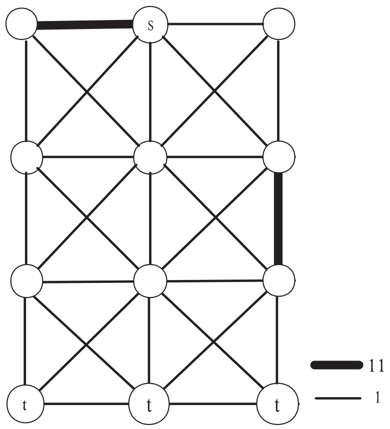

Fig. 7. Second topology, no improvement.

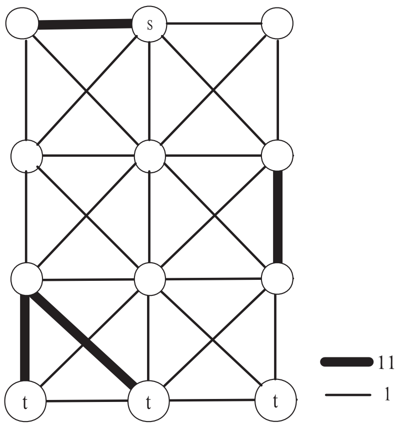

Fig. 8. Third topology, ratio improves.

Now, consider the topology shown in Fig. 8 which has four links at 11 Mbps.

By exploiting multi-rate with network coding, the LP model achieves a throughput = $\frac{22}{23}$. Thus, selecting the rate of 11, nodes can achieve a larger multicast rate and fully exploit the mesh model.

The schedule produced by the LP model is given in Fig. 9. After the network is filled, the scheduling cycles through part I, II and III. In part I, since all active hyperlinks have rate = 11 Mbps, 1 packet is transmitted in $\frac{1}{11}$ of time unit. In part II and III, nodes have rate = 1 Mbps, 1 packet is transmitted in 1 time unit each. In the end, each receiver gets 2 packets in $\frac{1}{11} + 1 + 1 = \frac{23}{11}$ time units, yielding a ratio of $\frac{2}{23/11} = \frac{22}{23}$.

6. Fundamentals limits

6.1. One coding structure

Consider the wheel topology, shown in Fig. 10(a). We select the wheel topology because it has been shown to provide the performance gain of network coding for different number of flows [7]. One node is placed on the center, $n-1$ nodes are spaced along a circle. Assume that all nodes can communicate with each other, except with the one exactly opposite to it. One example of a possible physical topology for such logical topology is shown in Fig. 10(b). We assume that the transmission of node i can be successfully received by all nodes along the circle except for node j. Each source node chooses its opposing node as its destination. All coding flows are relayed by the node at the center. We refer to this structure with at least one relay coding node and many neighbors nodes as *coding structure*. We assume that wireless nodes operate at half-duplex mode. Nodes that interfere with each other share the common channel bandwidth, denoted by T.

Without network coding, the maximum total throughput occurs when flow rate conservation is ensured at the relay node (i.e. the node in the center). The total bandwidth allocated to nodes 1 to n should be equal to the total bandwidth allocated to the node in the center. When this happens, the maximum total throughput is $T/2$.

With network coding, the maximum total throughput is realized when the transmission schedule follows a cyclic pattern, such as $1, 2, \ldots, n$, and relay node. In this case as well, the bandwidth is equally allocated among all nodes, resulting in a maximum total throughput of $nT/(n+1)$.

Lemma 6.1. *For n flows, the gain on Network Coding in throughput is upper bounded by* $2n/(n+1)$.

Proof. It follows by dividing the maximum total throughput without Network Coding $T/2$ by the maximum total throughput $nT/(n+1)$. □

Lemma 6.1 was first proven in [9].

Lemma 6.2. *Let m denotes the minimum rate and M the maximum rate. In a coding structure route-aware, the maximum gain from Multi-Rate is* $(m+M)/2m$.

Proof. In a coding structure, such as the wheel topology, which maximizes the network coding gain, the maximum gain is produced when one transmission at the minimum rate is replaced by one at the maximum rate. Therefore, the upper gain from Multi-Rate is $(m+M)/2m$. □

Thus we arrive at the Network Coding and Multi-Rate Fundamental Limit Theorem for one coding structure, first proven in [19].

Theorem 6.3. *In each coding structure, the throughput gain is bounded by* $2n/(n+1) * (m+M)/2m$*, where n is the number of coding flows, m is the minimum rate and M is the maximum rate.*

Proof. We have shown that the maximum gain in the wheel topology, which maximizes the throughput gain in a coding structure is given by Lemma 6.1. Let us assume that the gain of network coding is *independent* of multi-rate diversity gain, the best of both worlds. Therefore the gain is the product of both gains shown in Lemmas 6.1 and 6.2. Else, gain in one will jeopardize the gain in the other. The upper bound on the maximum gain still holds. □

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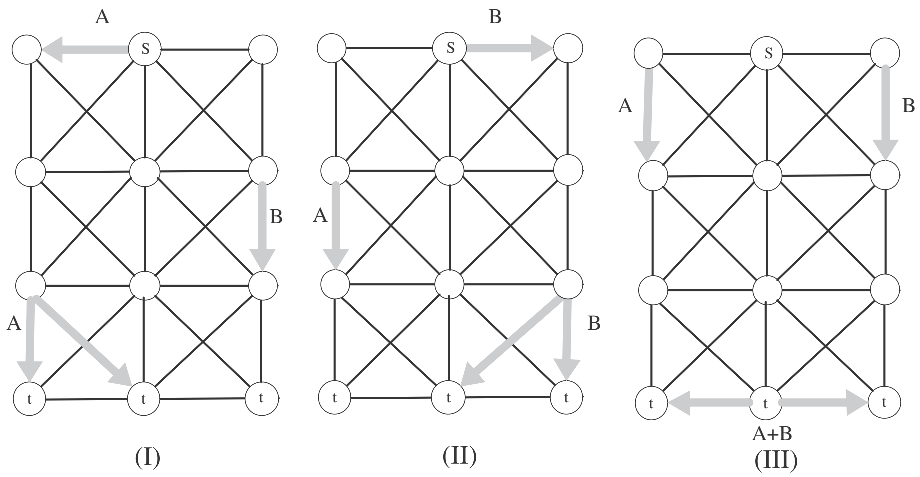

Fig. 9. Possible maximum ratio scheduling.

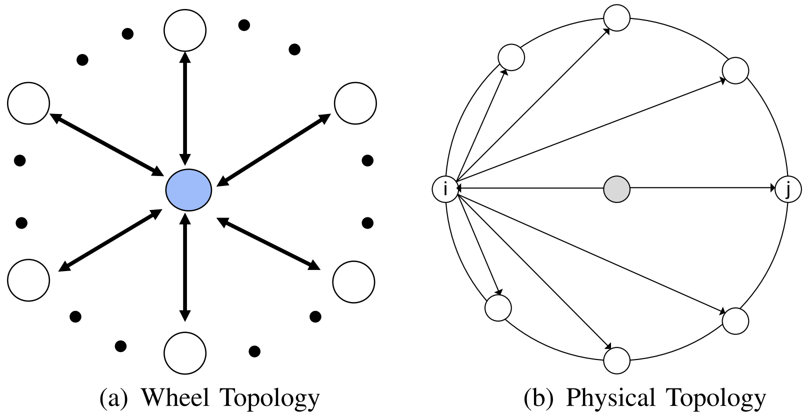

Fig. 10. Logic and Physical Topology.

6.2. General wireless network

Previously we characterize the throughput gain for the basic coding structure. Now we describe the throughput gain for a *general wireless network*. In this scenario, there can co-exist many coding structures in the network and there can also be non-coding flows causing interference with the coding flows.

Let G_c represents the throughput in the network using network coding and G_{nc} the same for a network without network coding. The throughput gain Γ is defined as G_c/G_{nc} The optimal values for G_c^* and G_{nc}^* may use different routes. We are interested in the maximum throughput gain $\Gamma*$ defined as the maximum value for G_c/G_{nc}, over all topologies, traffic demands and routing algorithms.

In the case there are non-coding flows causing interference with the coding flow, we use Lemma 6.4 proven in [9].

Lemma 6.4. *For a single coding structure with possible non-coding flows interfering with the coding node, the maximum throughput gain* Γ^* *is upper bounded by* $2n/(n+1)$.

In the case there are many coding structures in the network, we should note that the coding scheme only increases the bandwidth efficiency of the coding node. If a coding flow transverses several coding structures, its end-to-end throughput is upper bounded by the bottleneck coding structure. In other words, the throughput improvement is limited by the throughput gain in the transversed coding scheme with the *least* improvement.

Now we arrive at the Network Coding and Multi-Rate Fundamental Limit Theorem.

Theorem 6.5. *For a general network, the throughput gain* Γ^* *is bounded by* $2n/(n+1)*(m+M)/2m$, *where n is the number of coding flows, m is the minimum rate and M is the maximum rate.*

Proof. Theorem 6.3 shows that each coding structure *i* can provide a maximum throughput gain of $2n_i/(n_i+1)*(m+M)/2m$. Because we ensure the same throughput gain by all flows, the throughput gain is limited by one of the coding structures that provide the *least* throughput gain. □

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We can conclude that:

- Multi-Rate diversity gain does not depend on the number of flows n, while network coding solely does.
- The gain provided by multi-rate diversity is limited by the maximum rate, while network coding is limited by a factor 2, in accordance with previous work [13].

7. Conclusion

Our contribution is mainfold. We have demonstrated in this paper that multi-rate link layer broadcasts and network coding can be mutually combined to increase network throughput in multicast applications (at least for the case of a single broadcast or multicast flow).

We presented a linear programming model for studying the bounds achievable with NC and Multi-rate. We observed that NC and multi-rate can improve the network achievable rate. Also, we showed that this gain depends on topology and number and location of "variable links". Our analytical results show that more powerful links do not necessarily imply better throughput. These results are important to help in the design of new protocols that exploit network coding and rate diversity.

We have demonstrated in this paper that multi-rate link layer broadcasts and network coding can be mutually combined to increase network throughput in multicast applications. Our results are particularly useful for design and analysis of coding-aware MAC and routing protocols.

In addition, we provide the fundamental limits on the end-to-end throughput of Network Coding in multi-rate and multicast wireless networks.

Accordingly, as part of our future work, we plan to develop a rate-aware multicast routing protocol fully enhanced to take advantage not only of rate diversity but also network coding under mobile environments.

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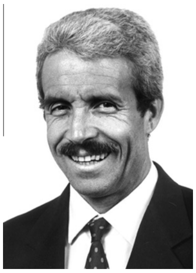

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