

Relay Beamforming Design with SIC Detection for MIMO Multirelay Networks with Imperfect CSI

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Abstract—In this paper, we consider a dual-hop multiple-input–multiple-output (MIMO) wireless multirelay network, in which a source–destination pair, both equipped with multiple antennas, communicates through multiple half-duplex amplify-and-forward (AF) relay terminals, which are also with multiple antennas. Since perfect channel-state information (CSI) is difficult to obtain in a practical multirelay network, we consider imperfect CSI for all channels. We focus on maximizing the signal-to-interference-plus-noise ratio (SINR) at the destination. We propose a novel robust linear beamforming at the relays, based on the QR decomposition filter at the destination node, which performs successive interference cancellation (SIC). Using the law of large numbers, we obtain the asymptotic rate in the presence of imperfect CSI, upon which the proposed relay beamforming is optimized. Simulation results show that the asymptotic rate matches with the ergodic rate well. Analysis and simulation results demonstrate that the proposed beamforming outperforms the conventional beamforming schemes for any power of CSI errors and SNR regions.

Index Terms—Channel state information (CSI), multiple-input–multiple-output (MIMO) relay, rate, relay beamforming, successive interference cancellation (SIC) detection.

I. INTRODUCTION

RELAY communications can extend the coverage of wireless networks and improve the spatial diversity of cooperative systems [1]. Meanwhile, the multiple-input–multiple-output (MIMO) technique is well verified to provide significant improvement in spectral efficiency and link reliability because of the multiplexing and diversity gains [2], [3]. Combining the relaying and MIMO techniques can make use of both advantages to increase the data rate in the cellular edge and extend the network coverage [4].

MIMO relay networks and MIMO broadcasting relay networks have been extensively investigated in [5]–[11]. In addition, MIMO multirelay networks have been studied in [12]–[18]. In [12], it is shown that the corresponding network capacity scales as $C = (M/2) \log(K) + O(1)$, where M is the

number of antennas at the source, and $K \rightarrow \infty$ is the number of relays, and a simple protocol to achieve the upper bound as $K \rightarrow \infty$ when perfect channel-state informations (CSIs) of both backward channels (BCs) and forward channels (FCs) are available at the relay nodes is also proposed. When CSIs are not available at the relays, a simple AF beamforming protocol is proposed at the relays, but the distributed array gain is not obtained. In [14] and [15], three relay beamforming schemes are designed based on matrix triangularization, which have superiority over the conventional zero-forcing (ZF) and amplify-and-forward (AF) beamformers. The proposed beamforming scheme can fulfill both the intranode gain and the distributed array gain. In [16], a beamforming scheme that achieves the upper bound of capacity with a small gap when K is significantly large is designed. However, it has a bad performance for small K , and the source needs CSI, which increases overhead. A unified algorithm is proposed in [17] for the optimal linear transceivers at the source and relays for both one- and two-way networks. In [18], two efficient relay beamformers for the dual-hop MIMO multirelay networks have been presented, which are based on matched filter (MF) and regularized ZF (RZF), and they utilize the QR decomposition (QRD) of the effective system channel matrix at the destination node [19]. The beamformers at the relay nodes can exploit the distributed array gain by diagonalizing both the BC and the FC. The QRD can exploit the intranode array gain by successive interference cancellation (SIC) detection. These two beamforming schemes have lower complexity because they only need one QRD at destination.

On the other hand, all the works for the multirelay MIMO system only consider perfect CSI to design beamformers at the relays or SIC matrices at the destination. For the multirelay networks, imperfect CSI is a practical consideration [12]. In particular, knowledge for the CSI of the FC at relays will result in large delay and significant training overhead, because the CSI of FCs at relays are obtained through feedback links to multiple relays [20]. The imperfect CSI of the BC at relays is also practical because of the channel estimation error.

For the works on imperfect CSI, the ergodic capacity and the bit error rate (BER) performance of the MIMO with imperfect CSI is considered in [21]–[23]. In [21], lower and upper bounds of mutual information under the CSI error are investigated. In [22], the authors studied the BER performance of the MIMO system under combined beamforming and maximal ratio combining (MRC) with imperfect CSI. In [23], the bit error probability is analyzed based on the Taylor approximation. Some optimization problem has been investigated with imperfect CSI in [24]–[26]. In [24], a lower bound of capacity by optimally

Manuscript received November 18, 2012; revised January 29, 2013 and March 20, 2013; accepted April 19, 2013. Date of publication April 24, 2013; date of current version October 12, 2013. This work was supported in part by the National 973 Project under Grant 2012CB316106 and Grant 2009CB824904 and in part by the National Natural Science Foundation of China under Grant 60972031 and Grant 61161130529. The review of this paper was coordinated by Dr. T. Taniguchi.

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Digital Object Identifier 10.1109/TVT.2013.2259857

configuring the number of antennas with imperfect CSI is maximized. In [26], the tradeoff between the accuracy of channel estimation and data transmission is studied, and it is shown the optimal number of training symbols is equal to the number of transmit antennas. In [27], the effects of channel estimation error on the receiver of MIMO AF two-way relaying networks is investigated.

In this paper, we propose new robust beamforming schemes for dual-hop MIMO multirelay networks under the condition of imperfect CSI. The SIC is also implemented at the destination by the QRD. The proposed beamformer at relay is based on the minimum mean square error (MMSE) receiver and the RZF precoder. We focus on optimizing the regularizing factors in them. We first optimize factor α^{MMSE} in the MMSE and then optimize factor α^{RZF} in RZF for a given α^{MMSE} . In the derivation, using the law of large numbers, we obtain the asymptotic rate capacity for the MMSE-RZF beamformer, based on which the performance of the beamformer for imperfect CSI can be easily analyzed. Simulation results show that the asymptotic rate capacity matches with the ergodic capacity well. The asymptotic rate also validates the scaling law in [12], when the imperfect CSI presents. Analysis and simulations demonstrate that the rate of MMSE-RZF outperforms other schemes whether CSI is perfect or not. The ceiling effect of the rate capacity and the situation that the CSI error increases with the number of relays are also discussed in this paper.

The remainder of this paper is organized as follows: In Section II, the system model of a dual-hop MIMO multirelay network is introduced. In Section III, we explain the MMSE-RZF-based beamforming scheme and the QRD. In Section IV, we optimize the MMSE-RZF and obtain the asymptotic rate of the system. Section V devotes to simulation results followed by conclusion in Section VI.

In this paper, the boldface lowercase and uppercase letters represent vectors and matrices, respectively. Notations $(\mathbf{A})_i$ and $(\mathbf{A})_{i,j}$ denote the i th row and the (i, j) th entry of matrix \mathbf{A} . Notations $\text{tr}(\cdot)$, $(\cdot)^\dagger$, $(\cdot)^*$, and $(\cdot)^H$ denote trace, pseudo-inverse, conjugate, and conjugate transpose operation of a matrix, respectively. Term \mathbf{I}_N is an $N \times N$ identity matrix. Term $\text{diag}\{\{a_m\}_{m=1}^M\}$ denotes a diagonal matrix with diagonal entries of a_1, \dots, a_M . $\|\mathbf{a}\|$ stands for the Euclidean norm of vector \mathbf{a} , and $\xrightarrow{\text{w.p.1}}$ represents convergence with a probability of 1. Finally, we denote the expectation operation by $E\{\cdot\}$.

II. SYSTEM MODEL

The considered MIMO multirelay network consists of single source and destination nodes both equipped with M antennas, and K N -antenna relay nodes distributed between the source–destination pair, as shown in Fig. 1. When the source node implements spatial multiplexing, the requirement $N \geq M$ must be satisfied if each relay node is supposed to support all the M independent data streams. We assume $M = N$ in this paper, while the proposed beamforming scheme and the results can be easily expanded to case $N > M$. We consider half-duplex nonregenerative relaying throughout this paper, where it takes two nonoverlapping time slots for the data to be transmitted from the source node to the destination node via

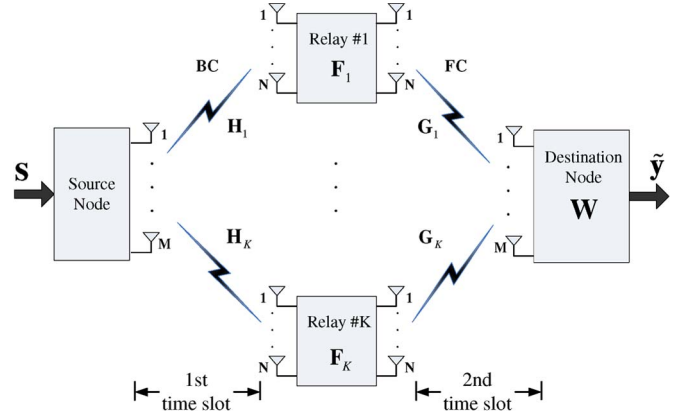


Fig. 1. System model of the dual-hop MIMO multirelay network with relay beamforming and SIC at the destination.

the BCs and the FCs. Due to deep large-scale fading effects that are produced by the long distance, we assume that there is no direct link between the source and the destination. In a practical system, each relay needs to transmit training sequences or pilots to acquire the CSI of all channels. Imperfect channel estimation and limited feedback are also practical considerations. Thus, in this paper, imperfect CSIs of the BC and the FC are assumed to be available at relay nodes. Assume that $\hat{\mathbf{H}}_k \in \mathbb{C}^{M \times M}$ and $\hat{\mathbf{G}}_k \in \mathbb{C}^{M \times M}$ stands for the available imperfect CSIs of the BC and the FC at the k th relay. We model the CSIs of the BC and the FC of the k th relay as

$$\mathbf{H}_k = \hat{\mathbf{H}}_k + e_1 \Omega_{1,k} \tag{1}$$

$$\mathbf{G}_k = \hat{\mathbf{G}}_k + e_2 \Omega_{2,k} \tag{2}$$

where $\mathbf{H}_k \in \mathbb{C}^{M \times M}$ and $\mathbf{G}_k \in \mathbb{C}^{M \times M}$ ($k = 1, \dots, K$) stand for the backward and forward MIMO-channel matrices of the k th relay node, respectively. $\Omega_{1,k}$ and $\Omega_{2,k}$ are matrices that are independent of \mathbf{H}_k and \mathbf{G}_k , respectively, whose entries are independently and identically distributed (i.i.d.) zero-mean complex Gaussian, with unity variance [21], [28]. Therefore, the powers of CSI errors of the BC and the FC are e_1^2 and e_2^2 , respectively. Since e_1^2 is the power of the channel estimation error, it can be made very small. e_2^2 is the power of the channel error majorly coming from channel quantization, which is bounded by $2^{-B/M}$ if B bits is used to do quantization. In this paper, we therefore assume $e_1^2 \ll 1$ and $e_2^2 < 1$, which are reasonable assumptions in a practical system. In this paper, all the relay nodes are supposed to be located in a cluster. Then, all channels $\mathbf{H}_1, \dots, \mathbf{H}_K$ and $\mathbf{G}_1, \dots, \mathbf{G}_K$ can be supposed to be i.i.d. and experience the same Rayleigh flat fading. Assume that the entries of \mathbf{H}_k and \mathbf{G}_k are zero-mean complex Gaussian random variables with a variance of 1. In the first time slot, the source node broadcasts the signal to all the relay nodes through BCs. Let $M \times 1$ vector \mathbf{s} be the transmit signal vector satisfying the power constraint $E\{\mathbf{s}\mathbf{s}^H\} = (P/M)\mathbf{I}_M$, where P is defined as the transmit power at the source node. Then, the corresponding received signal at the k th relay can be written as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k \tag{3}$$

where term \mathbf{n}_k is the spatiotemporally white zero-mean complex additive Gaussian noise vector, which is independent across k , with the covariance matrix $E\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_1^2\mathbf{I}_M$. Therefore, the noise variance σ_1^2 represents the noise power at each relay node.

In the second time slot, first, each relay node performs linear processing by multiplying \mathbf{r}_k with an $N \times N$ beamforming matrix \mathbf{F}_k . This \mathbf{F}_k is based on its imperfect CSIs $\hat{\mathbf{H}}_k$ and $\hat{\mathbf{G}}_k$. Consequently, the signal vector sent from the k th relay node is

$$\mathbf{t}_k = \mathbf{F}_k\mathbf{r}_k. \quad (4)$$

From a more practical consideration, we assume that each relay node has its own power constraint satisfying $E\{\mathbf{t}_k^H\mathbf{t}_k\} \leq Q$, which is independent of power P . Hence, a power constraint condition of \mathbf{t}_k can be derived as

$$\rho(\mathbf{t}_k) = \text{tr} \left\{ \mathbf{F}_k \left(\frac{P}{M} \mathbf{H}_k \mathbf{H}_k^H + \sigma_1^2 \mathbf{I}_N \right) \mathbf{F}_k^H \right\} \leq Q. \quad (5)$$

After linear relay beamforming processing, all the relay nodes simultaneously forward their data to the destination. Thus, the signal vector that is received by the destination can be expressed as

$$\mathbf{y} = \sum_{k=1}^K \mathbf{G}_k \mathbf{t}_k + \mathbf{n}_d = \sum_{k=1}^K \mathbf{G}_k \mathbf{F}_k \mathbf{H}_k \mathbf{s} + \sum_{k=1}^K \mathbf{G}_k \mathbf{F}_k \mathbf{n}_k + \mathbf{n}_d \quad (6)$$

where $\mathbf{n}_d \in \mathbb{C}^M$, which is satisfying $E\{\mathbf{n}_d\mathbf{n}_d^H\} = \sigma_2^2\mathbf{I}_M$, denotes the zero-mean white circularly symmetric complex additive Gaussian noise vector at the destination node with the noise power σ_2^2 .

III. RELAY BEAMFORMING DESIGN

Here, the QR detector at the destination node for SIC detection is introduced, and a relay beamforming scheme based on the MMSE receiver and the RZF precoder is proposed.

A. QRD and SIC Detection

The QRD detector is utilized as the destination receiver \mathbf{W} in this paper, which is proven to be asymptotically equivalent to that of the maximum-likelihood detector [19]. Let $\sum_{k=1}^K \hat{\mathbf{G}}_k \mathbf{F}_k \hat{\mathbf{H}}_k = \mathbf{H}_{SD}$ be the effective channel between the source and destination nodes, which can be estimated at the destination node by using the AF relay-channel estimation methods [29]–[31]. Then, (6) can be rewritten as

$$\mathbf{y} = \mathbf{H}_{SD}\mathbf{s} + \hat{\mathbf{n}} \quad (7)$$

where

$$\hat{\mathbf{n}} \cong \sum_{k=1}^K e_1 \hat{\mathbf{G}}_k \mathbf{F}_k \Omega_{1,k} \mathbf{s} + \sum_{k=1}^K e_2 \Omega_{2,k} \mathbf{F}_k \hat{\mathbf{H}}_k \mathbf{s} + \sum_{k=1}^K \mathbf{G}_k \mathbf{F}_k \mathbf{n}_k + \mathbf{n}_d \quad (8)$$

is the effective noise vector cumulated from the CSI errors, noise \mathbf{n}_k at the k th relay node, and noise vector \mathbf{n}_d at the destination. In the derivation, we omit the term including $e_1 e_2$. Even if we retain term $e_1 e_2$ in (8), it will result in some terms involving $e_1^2 e_2$, $e_1 e_2^2$ and $e_1^2 e_2^2$ when calculating the covariance of the effective noise $\hat{\mathbf{n}}$. The first two terms are always zero after taking expectation, while the only terms left are those involving $e_1^2 e_2^2$. Since $e_1^2 \ll 1$ and $e_2^2 < 1$, we have $e_1^2 e_2^2 \ll 1$. Therefore, it is reasonable to omit the term including $e_1 e_2$ in (8).

Finally, to cancel the interference from other antennas, the QRD of the effective channel is implemented as

$$\mathbf{H}_{SD} = \mathbf{Q}_{SD} \mathbf{R}_{SD} \quad (9)$$

where \mathbf{Q}_{SD} is an $M \times M$ unitary matrix, and \mathbf{R}_{SD} is an $M \times M$ right upper triangular matrix. Therefore, the QRD detector at the destination node is chosen as $\mathbf{W} = \mathbf{Q}_{SD}^H$, and the signal vector after QRD detection becomes

$$\tilde{\mathbf{y}} = \mathbf{Q}_{SD}^H \mathbf{y} = \mathbf{R}_{SD} \mathbf{s} + \mathbf{Q}_{SD}^H \hat{\mathbf{n}}. \quad (10)$$

A power control factor ρ_k is set with \mathbf{F}_k in (5) to guarantee that the k th relay transmit power is equal to Q . The transmit signal from each relay node after linear beamforming and power control becomes

$$\mathbf{t}_k = \rho_k \mathbf{F}_k \mathbf{r}_k \quad (11)$$

where the power control factor ρ_k can be derived from (5) as

$$\rho_k = \left(\frac{Q}{E \left[\text{tr} \left\{ \mathbf{F}_k \left(\frac{P}{M} \mathbf{H}_k \mathbf{H}_k^H + \sigma_1^2 \mathbf{I}_N \right) \mathbf{F}_k^H \right\} \right]} \right)^{\frac{1}{2}}. \quad (12)$$

B. Beamforming at Relay Nodes

The MF beamformer is used in [18] according to maximum ratio transmission and MRC, which are advantageous to the beamformers that are based on matrix decomposition in [15]. If the MF beamformer is used, then

$$\mathbf{F}_k^{\text{MF-MF}} = \hat{\mathbf{G}}_k^H \hat{\mathbf{H}}_k^H. \quad (13)$$

Another choice is to diagonalize the effective channel between the source and the destination, e.g.,

$$\mathbf{F}_k^{\text{ZF-ZF}} = \hat{\mathbf{G}}_k^\dagger \hat{\mathbf{H}}_k^\dagger = \hat{\mathbf{G}}_k^H \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H \right)^{-1} \left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k \right)^{-1} \hat{\mathbf{H}}_k^H. \quad (14)$$

MF-MF outperforms ZF-ZF in low SNR, while ZF-ZF outperforms MF-MF in high SNR [18]. However, these two schemes are not optimized.

In this paper, we propose a robust MMSE-RZF beamformer at the relay nodes. When MMSE-RZF is chosen, beamforming at the k th relay is

$$\mathbf{F}_k^{\text{MMSE-RZF}} = \hat{\mathbf{G}}_k^H \left(\hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H + \alpha_k^{\text{RZF}} \mathbf{I}_M \right)^{-1} \times \left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \hat{\mathbf{H}}_k^H. \quad (15)$$

Note that MF–MF and ZF–ZF are two extreme cases for $\alpha_k^{\text{MMSE}} = \alpha_k^{\text{RZF}} = \infty$ and $\alpha_k^{\text{MMSE}} = \alpha_k^{\text{RZF}} = 0$, respectively. Generally, if the α (either regularizing factor in MMSE or RZF) is too large, the effective channel matrix will far deviate from a diagonal matrix, which results in power consumption and interference across different data. If α is too small, the MMSE receiver and the RZF precoder will perform like a ZF receiver or precoder, which have the power penalty problem due to its inverse Wishart distribution term in its transmit power [32]–[34]. We aim to obtain the optimal α_k^{MMSE} and α_k^{RZF} to maximize the rate in this paper. However, to directly get the global optimal closed-form solution is difficult. In the following, we derive an optimized solution by two steps. We first derive an optimized α_k^{MMSE} by maximizing the signal-to-interference-plus-noise ratio (SINR) at the relay nodes, and then, we derive an optimized α_k^{RZF} dependent on the given optimized α_k^{MMSE} by maximizing the rate at the destination.

IV. ROBUST MINIMUM MEAN SQUARE ERROR REGULARIZED ZERO FORCING BEAMFORMER

Here, we derive the optimized α_k^{MMSE} and α_k^{RZF} in the MMSE-RZF beamformer by two steps. We first derive the optimized α_k^{MMSE} by maximizing the SINR at relay nodes and then derive the optimized α_k^{RZF} for a given α_k^{MMSE} based on the asymptotic rate. Although the derived solution is not global optimum, it is observed quite efficient in terms of the rate in the simulations.

A. Optimization of α_k^{MMSE}

We optimize α_k^{MMSE} by maximizing the SINR at relay nodes. For the k th relay, the signal vector after processed by an MMSE receiver is

$$\begin{aligned} \mathbf{v}_k &= \left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \hat{\mathbf{H}}_k^H \mathbf{r}_k \\ &= \left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k \mathbf{s} \\ &\quad + e_1 \left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \hat{\mathbf{H}}_k^H \boldsymbol{\Omega}_{1,k} \mathbf{s} \\ &\quad + \left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \hat{\mathbf{H}}_k^H \mathbf{n}_k. \end{aligned} \quad (16)$$

The first term in (16) is the signal vector, which contains interstream interference, because matrix $(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M)^{-1} \hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k$ is not diagonal if $\alpha_k^{\text{MMSE}} \neq 0$. Thus, we need to calculate the power of the desired signal and the interference. We use the diagonal decompositions in the following analysis, i.e.,

$$\begin{aligned} \hat{\mathbf{H}}_k \hat{\mathbf{H}}_k^H &= \mathbf{P}_k \text{diag}\{\theta_{k,1}, \dots, \theta_{k,M}\} \mathbf{P}_k^H \triangleq \mathbf{P}_k \boldsymbol{\Theta}_k \mathbf{P}_k^H \\ \hat{\mathbf{G}}_k \hat{\mathbf{G}}_k^H &= \mathbf{Q}_k \text{diag}\{\lambda_{k,1}, \dots, \lambda_{k,M}\} \mathbf{Q}_k^H \triangleq \mathbf{Q}_k \boldsymbol{\Lambda}_k \mathbf{Q}_k^H \end{aligned}$$

where \mathbf{P}_k and \mathbf{Q}_k are unitary matrices. To divide the interference from the desired signal, we introduce the following two lemmas.

Lemma 1: Assume that $\mathbf{A} \in \mathbb{C}^{M \times M}$ is a random matrix. If there is a diagonal decomposition $\mathbf{A} = \mathbf{Q} \boldsymbol{\Lambda} \mathbf{Q}^H$, where $\boldsymbol{\Lambda} = \text{diag}\{\lambda_1, \dots, \lambda_M\} \in \mathbb{R}^{M \times M}$ and matrix \mathbf{Q} is unitary, we have

$$E\{(\mathbf{A})_{m,m}^2\} = \frac{1}{M(M+1)} \left(\left(\sum_{\ell=1}^M \lambda_\ell \right)^2 + \sum_{\ell=1}^M \lambda_\ell^2 \right) \triangleq \mu(\lambda) \quad (17)$$

for any m , where the conditional expectation is taken with respect to distribution \mathbf{Q} conditioned on $\boldsymbol{\Lambda}$.

The proof of Lemma 1 can be directly obtained from [33], which considers perfect CSI. Although matrix \mathbf{A} in this paper is a multiplication of an imperfect channel matrix and its conjugate transpose, whose entries have covariance $1 - e_1^2$ or $1 - e_2^2$, the distribution of \mathbf{Q} is not changed and so is the expectation in Lemma 1. Note that the conditional expectation taken with respect to \mathbf{Q} that is conditioned on $\boldsymbol{\Lambda}$ is valid because \mathbf{Q} and $\boldsymbol{\Lambda}$ are independent [35].

Lemma 2: Assume that $\mathbf{A} \in \mathbb{C}^{M \times M}$ is a random matrix. If there is a diagonal decomposition $\mathbf{A} = \mathbf{Q} \boldsymbol{\Lambda} \mathbf{Q}^H$, with $\boldsymbol{\Lambda} = \text{diag}\{\lambda_1, \dots, \lambda_M\} \in \mathbb{R}^{M \times M}$ and unitary matrix \mathbf{Q} , we have

$$\begin{aligned} E\{ |(\mathbf{A})_{m,j}|^2 \} &= \frac{1}{(M-1)(M+1)} \sum_{\ell=1}^M \lambda_\ell^2 \\ &\quad - \frac{1}{(M-1)M(M+1)} \left(\sum_{\ell=1}^M \lambda_\ell \right)^2 \triangleq \nu(\lambda) \end{aligned} \quad (18)$$

for any $m \neq j$, where the conditional expectation is taken with respect to distribution \mathbf{Q} that is conditioned on $\boldsymbol{\Lambda}$.

Proof: Because \mathbf{A} is a conjugate symmetric matrix, the conditional expectation with respect to distribution \mathbf{Q} is

$$\begin{aligned} E \left\{ \sum_{j=1, j \neq m}^M |(\mathbf{A})_{m,j}|^2 \right\} &+ E\{(\mathbf{A})_{m,m}^2\} \\ &= E\{(\mathbf{A}\mathbf{A}^H)_{m,m}\} = E\{(\mathbf{Q}\boldsymbol{\Lambda}^2\mathbf{Q}^H)_{m,m}\} \\ &= \frac{1}{M} \sum_{\ell=1}^M \lambda_\ell^2. \end{aligned} \quad (19)$$

Since $E\{|(\mathbf{A})_{k,j}|^2\}$ are all equal for $j \neq k$, we have

$$\begin{aligned} E\{|(\mathbf{A})_{k,j}|^2\} &= \frac{1}{(M-1)} \left(\frac{1}{M} \sum_{\ell=1}^M \lambda_\ell^2 - E\{(\mathbf{A})_{m,m}^2\} \right) \\ &= \frac{1}{(M-1)(M+1)} \sum_{\ell=1}^M \lambda_\ell^2 \\ &\quad - \frac{1}{(M-1)M(M+1)} \left(\sum_{\ell=1}^M \lambda_\ell \right)^2. \end{aligned} \quad (20)$$

Now, we return to derive the SINR for each stream at each relay. The first term on the right-hand side of (16) can be

rewritten as

$$\begin{aligned} & \left(\widehat{\mathbf{H}}_k^H \widehat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \widehat{\mathbf{H}}_k^H \widehat{\mathbf{H}}_k \mathbf{s} \\ &= \mathbf{P}_k \frac{\boldsymbol{\Theta}_k}{\boldsymbol{\Theta}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_N} \mathbf{P}_k^H \mathbf{s}. \end{aligned} \quad (21)$$

Therefore, from Lemma 1, the power of the desired signal of the m th stream can be calculated by conditional expectation as

$$\begin{aligned} & E \left\{ \left| \left(\mathbf{P}_k \frac{\boldsymbol{\Theta}_k}{\boldsymbol{\Theta}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_N} \mathbf{P}_k^H \right)_{m,m} \mathbf{s}_m \right|^2 \right\} \\ &= \frac{P}{M} \mu \left(\frac{\theta_k}{\theta_k + \alpha_k^{\text{MMSE}}} \right) \end{aligned} \quad (22)$$

where θ_k denotes the set of all the diagonal entries in $\boldsymbol{\Theta}_k$. From Lemma 2, the interference from other streams by conditional expectation is

$$\begin{aligned} & E \left\{ \left| \sum_{j=1, j \neq m}^M \left(\mathbf{P}_k \frac{\boldsymbol{\Theta}_k}{\boldsymbol{\Theta}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_N} \mathbf{P}_k^H \right)_{m,j} \mathbf{s}_j \right|^2 \right\} \\ &= \frac{P(M-1)}{M} \nu \left(\frac{\theta_k}{\theta_k + \alpha_k^{\text{MMSE}}} \right). \end{aligned} \quad (23)$$

The effective noise of the m th stream is

$$\begin{aligned} \mathbf{n}_{\text{eff},k} &= e_1 \left(\widehat{\mathbf{H}}_k^H \widehat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \widehat{\mathbf{H}}_k^H \boldsymbol{\Omega}_{1,k} \mathbf{s} \\ &+ \left(\widehat{\mathbf{H}}_k^H \widehat{\mathbf{H}}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M \right)^{-1} \widehat{\mathbf{H}}_k^H \mathbf{n}_k \end{aligned} \quad (24)$$

whose covariance matrix by conditional expectation can be calculated as

$$\begin{aligned} & E \{ \mathbf{n}_{\text{eff},k} \mathbf{n}_{\text{eff},k}^H \} \\ &= (e_1^2 P + \sigma_1^2) \\ &\times E \left\{ \text{diag} \left\{ \left\{ \left(\mathbf{P}_k \frac{\boldsymbol{\Theta}_k}{(\boldsymbol{\Theta}_k + \alpha_k^{\text{MMSE}} \mathbf{I}_M)^2} \mathbf{P}_k^H \right)_{\ell,\ell} \right\}_{\ell=1}^M \right\} \right\} \\ &= \frac{e_1^2 P + \sigma_1^2}{M} \sum_{\ell=1}^M \frac{\theta_{k,\ell}}{(\theta_{k,\ell} + \alpha_k^{\text{MMSE}})^2} \mathbf{I}_M \end{aligned} \quad (25)$$

where we used the fact that $E\{\boldsymbol{\Omega} \mathbf{A} \boldsymbol{\Omega}^H\} = \text{tr}(\mathbf{A})$ for an $N \times N$ matrix \mathbf{A} and a unitary random matrix $\boldsymbol{\Omega}$. In (22)–(25), the conditional expectations are taken with respect to their respective unitary matrices. Combining (22)–(25), the SINR of the m th stream at the k th relay is

$$\begin{aligned} \text{SINR}_{k,m}^R &= \frac{\frac{P}{M} \mu \left(\frac{\theta_k}{\theta_k + \alpha_k^{\text{MMSE}}} \right)}{\frac{P(M-1)\nu \left(\frac{\theta_k}{\theta_k + \alpha_k^{\text{MMSE}}} \right)}{M} + \frac{\sum_{\ell=1}^M \frac{(e_1^2 P + \sigma_1^2) \theta_{k,\ell}}{(\theta_{k,\ell} + \alpha_k^{\text{MMSE}})^2}}{M}}. \end{aligned} \quad (26)$$

The derived SINR in (26) is neither the instantaneous SINR nor the average SINR. It is the average SINR over the channels corresponding to the fixed eigenmode $\boldsymbol{\Theta}$. To maximize the SINR expression, we introduce the following lemma, which is the conclusion of [33, App. B].

Lemma 3: For an SNR in terms of α

$$\text{SNR}(\alpha) = \frac{A \left(\sum_{\ell=1}^M \frac{\lambda_\ell}{\lambda_\ell + \alpha} \right)^2 + B \sum_{\ell=1}^M \frac{\lambda_\ell^2}{(\lambda_\ell + \alpha)^2}}{\sum_{\ell=1}^M \left[\frac{C \lambda_\ell}{(\lambda_\ell + \alpha)^2} + \frac{D \lambda_\ell^2}{(\lambda_\ell + \alpha)^2} + E \left(\frac{\lambda_\ell}{\lambda_\ell + \alpha} \right)^2 \right]} \quad (27)$$

is maximized by $\alpha = C/D$.

The optimum value of α can be obtained by differentiating (27) and setting it to be zero, which results in

$$\sum_{\ell > k} \frac{\lambda_\ell \lambda_k (\lambda_k - \lambda_\ell)^2 (C/D - \alpha)}{(\lambda_\ell + \alpha)^3 (\lambda_k + \alpha)^3} = 0. \quad (28)$$

Since the eigenvalues are not all equal, the SINR is maximized only when $\alpha = C/D$.

Substituting $\mu(\lambda)$ and $\nu(\lambda)$ into (26) and using Lemma 3, we obtain

$$\begin{aligned} \alpha_k^{\text{MMSE,opt}} &= \frac{\frac{e_1^2 P + \sigma_1^2}{M}}{\frac{1}{(M-1)(M+1)} \cdot \frac{P(M-1)}{M}} \\ &= (M+1) \left(e_1^2 + \frac{\sigma_1^2}{P} \right). \end{aligned} \quad (29)$$

We see that the derived $\alpha_k^{\text{MMSE,opt}}$ is a closed-form value independent of the instantaneous channel. It is a function of the power of the CSI error (e_1^2) and the SNR (P/σ_1^2) of the BC. $\alpha_k^{\text{MMSE,opt}}$ increases with e_1^2 , which means that a large regularization is needed to balance the desired signal and the additional noise that is inherited from the CSI error.

B. Optimization of α_k^{RZF}

To optimize α_k^{RZF} , we need to derive the rate of the system. In the rest of the analysis, we write $\mathbf{F}_k^{\text{MMSE-RZF}}$ in (15) as \mathbf{F}_k for simplicity. By adding the power control factor at the relays, we have

$$\mathbf{H}_{SD} = \sum_{k=1}^K \rho_k \widehat{\mathbf{G}}_k \mathbf{F}_k \widehat{\mathbf{H}}_k. \quad (30)$$

The effective noise vector in (8) is

$$\begin{aligned} \widehat{\mathbf{n}} &= \sum_{k=1}^K e_1 \rho_k \widehat{\mathbf{G}}_k \mathbf{F}_k \boldsymbol{\Omega}_{1,k} \mathbf{s} \\ &+ \sum_{k=1}^K e_2 \rho_k \boldsymbol{\Omega}_{2,k} \mathbf{F}_k \widehat{\mathbf{H}}_k \mathbf{s} + \sum_{k=1}^K \rho_k \mathbf{G}_k \mathbf{F}_k \mathbf{n}_k + \mathbf{n}_d \end{aligned} \quad (31)$$

which, after the QRD of the effective channel, has a covariance matrix of

$$\begin{aligned}
 & E \{ \widehat{\mathbf{n}} \widehat{\mathbf{n}}^H \} \\
 &= \text{diag} \left\{ \left\{ \left(e_1^2 P + \sigma_1^2 \right) \sum_{k=1}^K \left\| \rho_k \left(\mathbf{Q}_{SD}^H \widehat{\mathbf{G}}_k \mathbf{F}_k \right)_m \right\|^2 \right\}_{m=1}^M \right\} \\
 &+ \left(\frac{P e_2^2}{M} \sum_{k=1}^K \rho_k^2 \text{tr} \left(\mathbf{F}_k \widehat{\mathbf{H}}_k \widehat{\mathbf{H}}_k^H \mathbf{F}_k^H \right) \right. \\
 &\quad \left. + e_2^2 \sigma_1^2 \sum_{k=1}^K \rho_k^2 \text{tr} \left(\mathbf{F}_k \mathbf{F}_k^H \right) + \sigma_2^2 \right) \mathbf{I}_M \triangleq \mathbf{N}_{\text{cov}}. \quad (32)
 \end{aligned}$$

Finally, we obtain the SNR of the m th data stream at the destination after the QRD as

$$\text{SNR}_m^D = \frac{\frac{P}{M} |(\mathbf{R}_{SD})_{m,m}|^2}{\frac{P}{M} \sum_{j=m+1}^M |(\mathbf{R}_{SD})_{m,j}|^2 + (\mathbf{N}_{\text{cov}})_{m,m}}. \quad (33)$$

The ergodic rate is derived by summing up all the data rates on each antenna link, i.e.,

$$C = E_{\{\widehat{\mathbf{H}}_k, \widehat{\mathbf{G}}_k\}_{k=1}^K} \left\{ \frac{1}{2} \sum_{m=1}^M \log_2 \left(1 + \text{SNR}_m^D \right) \right\} \quad (34)$$

where the $\frac{1}{2}$ penalty is due to the two time-slot transmission. From (33), we see that it is difficult to directly obtain the optimal solution. We derive the asymptotic rate for large K and then get the optimized α^{RZF} . Since all terms in (30) and (31) include ρ_k except for \mathbf{n}_d , we first consider the expectation of ρ_k^{-2} . From (12), substituting the perfect CSIs with (1) and (2) and taking the conditional expectation with respect to \mathbf{P}_k and \mathbf{Q}_k and conditioned on λ and θ , we have (35). Here, we denote α , λ , and θ without subscripts k and m for simplicity, because all the channels for different relays are i.i.d. and λ_m (and θ_m) for every m are identically distributed. Equation (35) implies that the expectation of ρ_k^{-2} results in a uniform fixed ρ^{-2} for all relays. Therefore, we approximate ρ_k^{-2} by ρ^{-2} in the following analysis. The performance with such approximation varies little compared with using the dynamic power control factors [36]. Since all terms in the numerator and the denominator of (33) except for \mathbf{n}_d will generate ρ_k^2 , in the following analysis, we can omit ρ_k in calculation and multiply ρ^{-2} to σ^2 after calculating the power of \mathbf{n}_d in (43), i.e.,

$$\begin{aligned}
 & E \{ \rho_k^{-2} \} \\
 &= \frac{1}{Q} E \left\{ \frac{P}{M} \text{tr} \left(\mathbf{F}_k \left(\widehat{\mathbf{H}}_k \widehat{\mathbf{H}}_k^H + e_1^2 \Omega_{1,k} \Omega_{1,k}^H \right) \mathbf{F}_k^H \right) \right. \\
 &\quad \left. + \sigma_1^2 \text{tr} \left(\mathbf{F}_k \mathbf{F}_k^H \right) \right\} \\
 &= \frac{P}{QM} E \left\{ \text{tr} \left(\mathbf{Q}_k \frac{\Lambda_k}{(\Lambda_k + \alpha^{\text{RZF}} \mathbf{I}_M)^2} \mathbf{Q}_k^H \right. \right. \\
 &\quad \left. \left. \times \mathbf{P}_k \frac{\Theta_k^2}{(\Theta_k + \alpha^{\text{MMSE}} \mathbf{I}_M)^2} \mathbf{P}_k^H \right) \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{P e_1^2 + \sigma_1^2}{Q} E \left\{ \text{tr} \left(\mathbf{Q}_k \frac{\Lambda_k}{(\Lambda_k + \alpha^{\text{RZF}} \mathbf{I}_M)^2} \mathbf{Q}_k^H \right. \right. \\
 &\quad \left. \left. \times \mathbf{P}_k \frac{\Theta_k}{(\Theta_k + \alpha^{\text{MMSE}} \mathbf{I}_M)^2} \mathbf{P}_k^H \right) \right\} \\
 &= \frac{P}{Q} E \left\{ \frac{\theta^2}{(\theta + \alpha^{\text{MMSE}})^2} \right\} E \left\{ \frac{\lambda}{(\lambda + \alpha^{\text{MMSE}})^2} \right\} \\
 &+ \frac{(e_1^2 P + \sigma_1^2) M}{Q} \\
 &\times E \left\{ \frac{\theta}{(\theta + \alpha^{\text{MMSE}})^2} \right\} E \left\{ \frac{\lambda}{(\lambda + \alpha^{\text{MMSE}})^2} \right\} \triangleq \rho^{-2}. \quad (35)
 \end{aligned}$$

For the case of large K , using the law of large numbers, we have approximations (36). Note that

$$\begin{aligned}
 & (\mathbf{H}_{SD})_{i,i} \xrightarrow{\text{w.p.}} K \left(E \left\{ (\widehat{\mathbf{G}}_k \mathbf{F}_k \widehat{\mathbf{H}}_k)_{i,i} \right\} \right) \\
 &= K E \left\{ \left(\mathbf{Q}_k \frac{\Lambda_k}{\Lambda_k + \alpha^{\text{RZF}} \mathbf{I}_M} \right. \right. \\
 &\quad \left. \left. \times \mathbf{Q}_k^H \mathbf{P}_k \frac{\Theta_k}{\Theta_k + \alpha^{\text{MMSE}} \mathbf{I}_M} \mathbf{P}_k^H \right)_{i,i} \right\} \\
 &= K E \left\{ \left(\mathbf{Q}_k \frac{\Lambda_k}{\Lambda_k + \alpha^{\text{RZF}} \mathbf{I}_M} \mathbf{Q}_k^H \right)_{m,m} \right\} \\
 &\quad \times E \left\{ \left(\mathbf{P}_k \frac{\Theta_k}{\Theta_k + \alpha^{\text{MMSE}} \mathbf{I}_M} \mathbf{P}_k^H \right)_{n,n} \right\} \\
 &= \frac{K}{MN} E \left\{ \sum_{m=1}^M \frac{\theta_{k,m}}{\theta_{k,m} + \alpha^{\text{MMSE}}} \right\} E \left\{ \sum_{m=1}^M \frac{\lambda_{k,m}}{\lambda_{k,m} + \alpha^{\text{RZF}}} \right\} \\
 &= K E \left\{ \frac{\theta}{\theta + \alpha^{\text{MMSE}}} \right\} E \left\{ \frac{\lambda}{\lambda + \alpha^{\text{RZF}}} \right\} \quad (36)
 \end{aligned}$$

$$\begin{aligned}
 & E \left\{ (\widehat{\mathbf{G}}_k \mathbf{F}_k \widehat{\mathbf{H}}_k)_{i,j} \right\} \\
 &= E \left\{ \left(\mathbf{Q}_k \frac{\Lambda_k}{\Lambda_k + \alpha^{\text{RZF}} \mathbf{I}_M} \mathbf{Q}_k^H \mathbf{P}_k \frac{\Theta_k}{\Theta_k + \alpha^{\text{MMSE}} \mathbf{I}_M} \mathbf{P}_k^H \right)_{i,j} \right\} \\
 &= \sum_{\ell, m, n} E \left\{ (\mathbf{Q}_k)_{i,\ell} \left(\frac{\Lambda_k}{\Lambda_k + \alpha^{\text{RZF}} \mathbf{I}_M} \right)_\ell (\mathbf{Q}_k^H)_{\ell, m} \right. \\
 &\quad \left. \times (\mathbf{P}_k)_{m,n} \left(\frac{\Theta_k}{\Theta_k + \alpha^{\text{MMSE}} \mathbf{I}_M} \right)_n (\mathbf{P}_k^H)_{n,j} \right\} \\
 &= 0 \quad (37)
 \end{aligned}$$

for $i \neq j$, because $(\mathbf{Q}_k)_{i,\ell} (\mathbf{Q}_k^H)_{\ell, m} = 0$ for $i \neq m$ and $(\mathbf{P}_k)_{m,n} (\mathbf{P}_k^H)_{n,j} = 0$ for $m \neq j$. Then, we have

$$\frac{(\mathbf{H}_{SD})_{i,j}}{K} = \frac{\sum_{k=1}^K (\widehat{\mathbf{G}}_k \mathbf{F}_k \widehat{\mathbf{H}}_k)_{i,j}}{K} \xrightarrow{\text{w.p.}} E \left\{ (\widehat{\mathbf{G}}_k \mathbf{F}_k \widehat{\mathbf{H}}_k)_{i,j} \right\} = 0 \quad (38)$$

for large K . Therefore, from (36) and (38), we have

$$(\mathbf{H}_{SD})_{i,i} = O(K) \quad (39)$$

$$(\mathbf{H}_{SD})_{i,j} = o(K) \quad (40)$$

which results that $\frac{\mathbf{H}_{SD}}{K}$ is asymptotically diagonal for large K .

Thus, we have $\mathbf{Q}_{SD} \xrightarrow{w.p.} \mathbf{I}_M$ and $\mathbf{R}_{SD} \xrightarrow{w.p.} \mathbf{H}_{SD}$ for large K .

To obtain the power of interference, we calculate the non-diagonal entries of the effective channel matrix, which is included in (52) in the Appendix. Let us define the following expectations:

$$\begin{aligned} \mathcal{E}_1^\theta &\triangleq E \left\{ \frac{\theta}{(\theta + \alpha^{\text{MMSE}})} \right\} \\ \mathcal{E}_2^\theta &\triangleq E \left\{ \frac{\theta}{(\theta + \alpha^{\text{MMSE}})^2} \right\} \\ \mathcal{E}_3^\theta &\triangleq E \left\{ \frac{\theta^2}{(\theta + \alpha^{\text{MMSE}})^2} \right\} \\ \mathcal{E}_4^\theta &\triangleq E \left\{ \frac{\theta\theta'}{(\theta + \alpha^{\text{MMSE}})(\theta' + \alpha^{\text{MMSE}})} \right\} \\ \mathcal{E}_1^\lambda &\triangleq E \left\{ \frac{\lambda}{(\lambda + \alpha^{\text{RZF}})} \right\} \\ \mathcal{E}_2^\lambda &\triangleq E \left\{ \frac{\lambda}{(\lambda + \alpha^{\text{RZF}})^2} \right\} \\ \mathcal{E}_3^\lambda &\triangleq E \left\{ \frac{\lambda^2}{(\lambda + \alpha^{\text{RZF}})^2} \right\} \\ \mathcal{E}_4^\lambda &\triangleq E \left\{ \frac{\lambda\lambda'}{(\lambda + \alpha^{\text{RZF}})(\lambda' + \alpha^{\text{RZF}})} \right\}. \end{aligned}$$

Substituting (32), (35), (36), and (52) into (33) and (34), we obtain the asymptotic rate of the system as

$$C \xrightarrow{w.p.} \frac{M}{2} \log_2 \left(1 + \frac{\frac{P}{M} (K \mathcal{E}_1^\theta \mathcal{E}_1^\lambda)^2}{\mathcal{I}(\mathcal{E}^\lambda, \mathcal{E}^\theta) + \mathcal{N}(\mathcal{E}^\lambda, \mathcal{E}^\theta)} \right) \quad (41)$$

where

$$\begin{aligned} \mathcal{I}(\mathcal{E}^\lambda, \mathcal{E}^\theta) &= \frac{PK(M-1)(M+2)}{M(M+1)^2} \mathcal{E}_3^\theta \mathcal{E}_3^\lambda - \frac{PK(M-1)}{M(M+1)^2} \mathcal{E}_4^\theta \mathcal{E}_3^\lambda \\ &\quad - \frac{PK(M-1)}{M(M+1)^2} \mathcal{E}_3^\theta \mathcal{E}_4^\lambda - \frac{PK(M-1)M}{M(M+1)^2} \mathcal{E}_4^\theta \mathcal{E}_4^\lambda \quad (42) \end{aligned}$$

is the power of interference, and

$$\begin{aligned} \mathcal{N}(\mathcal{E}^\lambda, \mathcal{E}^\theta) &= (e_1^2 P + \sigma_1^2) K \mathcal{E}_2^\theta \mathcal{E}_3^\lambda + PK e_2^2 \mathcal{E}_3^\theta \mathcal{E}_4^\lambda \\ &\quad + e_2^2 \sigma_1^2 KM \mathcal{E}_2^\theta \mathcal{E}_2^\lambda + \sigma_2^2 \rho^{-2} \quad (43) \end{aligned}$$

is the asymptotic power of noise for large K , which can be calculated by taking expectations over \mathbf{P}_k and \mathbf{Q}_k to (32). Generally, the expectations in the asymptotic rate are difficult to obtain. Fortunately, if we write the expectations by the arithmetic mean with random samples $\lambda(\ell)$ for $\ell = 0, \dots, \infty$ as

$$\mathcal{E}_1^\lambda = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{\ell=1}^L \frac{\lambda(\ell)}{\lambda(\ell) + \alpha^{\text{RZF}}} \quad (44)$$

$$\mathcal{E}_2^\lambda = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{\ell=1}^L \frac{\lambda(\ell)}{(\lambda(\ell) + \alpha^{\text{RZF}})^2} \quad (45)$$

$$\mathcal{E}_3^\lambda = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{\ell=1}^L \frac{[\lambda(\ell)]^2}{(\lambda(\ell) + \alpha^{\text{RZF}})^2} \quad (46)$$

$$\mathcal{E}_4^\lambda = \lim_{L \rightarrow \infty} \frac{1}{L(L-1)} \left(\left(\sum_{\ell=1}^L \frac{\lambda(\ell)}{\lambda(\ell) + \alpha^{\text{RZF}}} \right)^2 - \sum_{\ell=1}^L \frac{[\lambda(\ell)]^2}{(\lambda(\ell) + \alpha^{\text{RZF}})^2} \right) \quad (47)$$

the asymptotic rate can be maximized by using Lemma 3. Finally, we obtain

$$\begin{aligned} \alpha^{\text{RZF,opt}} &= \left(PK e_2^2 + \frac{\sigma_2^2 P}{Q} \right) \mathcal{E}_3^\theta + \left(e_2^2 \sigma_1^2 KM + \frac{(e_1^2 P + \sigma_1^2) M \sigma_2^2}{Q} \right) \mathcal{E}_2^\theta \\ &= \frac{(e_1^2 P + \sigma_1^2) K \mathcal{E}_2^\theta + \frac{PK(M-1)(M+2)}{M(M+1)^2} \mathcal{E}_3^\theta - \frac{PK(M-1)}{M(M+1)^2} \mathcal{E}_4^\theta}{(e_1^2 P + \sigma_1^2) K \mathcal{E}_2^\theta + \frac{PK(M-1)(M+2)}{M(M+1)^2} \mathcal{E}_3^\theta - \frac{PK(M-1)}{M(M+1)^2} \mathcal{E}_4^\theta}. \quad (48) \end{aligned}$$

C. Remarks

From (48), we can observe that $\alpha^{\text{RZF,opt}}$ is independent of the instantaneous CSIs. This is very practical because, once we have $\alpha^{\text{MMSE,opt}}$ in terms of SNRs of the BC and the FC, which we call as PNR ($= P/\sigma_1^2$) and QNR ($= Q/\sigma_2^2$), i.e., e_1^2 and e_2^2 , $\alpha^{\text{RZF,opt}}$ can be easily calculated, although the solution is not in closed form. The derived $\alpha^{\text{MMSE,opt}}$ in (29) and $\alpha^{\text{RZF,opt}}$ in (48) are monotonically increasing functions of e_1 and e_2 , respectively. Thus, the robust MMSE-RZF balances the desired signal and the additional noise inherited from CSI error through larger regularizing factors. On the other hand, the proposed scheme is of low complexity because it needs only one QRD at the destination, while in the work of [15], K or $2K$ QRD is needed at the relay nodes.

From the asymptotic rate in (41), we see that they satisfy the scaling law in [12], i.e., $C = (M/2) \log(K) + O(1)$ for large K . Therefore, the proposed scheme achieves the intranode array gain M and the distributed array gain K . The intranode array gain is the gain that is obtained from the introduction of multiple antennas in each node of the dual-hop networks. The distributed array gain results from the implementation of multiple relay nodes and needs no cooperation among them. Note that, although the MMSE-RZF with QR SIC is optimized for large K , it also has efficient performance for small K , which is validated by the simulations. Moreover, from (41), it is observed that, when QNR grows to infinity for a fixed PNR, the rate will reach a limit. When PNR and QNR both grow to infinity, the capacity will linearly grow with PNR and QNR (in decibels) for perfect CSIs or reach a limit for imperfect CSIs. The limit of the rate performance is always referred as the ‘‘ceiling effect’’ [28] and will be confirmed by simulations.

Consider the case where the CSI error varies with the number of relays (K). Generally, the CSI errors of the BC are caused by

the estimation error. The CSI errors of the FC are caused by the estimation error, the quantization error, and the feedback delay. As in [21], [26], and [28], we assume that

$$e_1^2 = \sigma_e^2 = \frac{1}{1 + \frac{\rho_\tau T_\tau}{M}} \quad (49)$$

$$e_2^2 = \sigma_e^2 + \sigma_q^2 + \sigma_d^2 = \frac{1}{1 + \frac{\rho_\tau T_\tau}{M}} + 2^{-B/M} + \left(1 - \mathcal{J}_0\left(\frac{K+1}{2} \cdot 2\pi f_D \tau\right)\right) \quad (50)$$

where σ_e^2 denotes the estimation error during the training phase for the TDD mode, σ_q^2 denotes the quantization error due to limited bits of feedback, and σ_d^2 denotes the error weight caused by the feedback delay in each relay. Term $\frac{K+1}{2}$ scales the average delay for each relay when there is K relay nodes. ρ_τ is the SNR of pilot signals in the training phase, T_τ is the duration of the training phase, B is the number of feedback bits for each relay, \mathcal{J}_0 is a Bessel function of the first kind of order 0, f_D is the maximum Doppler shift, and τ is the feedback delay. Note that, in (49) and (50), the calculations are approximations for analysis and numerical simulations. Substituting such e_1^2 and e_2^2 into the asymptotic capacities and dividing the numerator and the denominator by K^2 , we see that the denominator will first decrease when e_2^2 is small and then increase when e_2^2 becomes large, which implies that the denominator will reach a minimum value at some K . Therefore, there exists an optimal number of relays to maximize the asymptotic capacities, which will be confirmed by simulations.

V. SIMULATION RESULTS

Here, numerical results are carried out to validate what we draw from the analysis in the previous sections for the proposed beamforming design. We compare the robust MMSE-RZF with MF and MF-RZF in [18] and QR in [14]. The α^{RZF} of MF-RZF in [18] is fixed to 1. ZF that is mentioned in Section III is also plotted for reference. In all these figures, we set $M = N = 4$ and focus on the performance of rates for various numbers of relays, SNRs of the BC and the FC, and powers of CSI errors. The ergodic rates are plotted by simulations through 1000 different channel realizations.

A. Capacity Versus Number of Relays

In Fig. 2, we compare the ergodic and asymptotic rate capacities of the MMSE-RZF beamforming schemes for various regularizing factors. We set $\text{PNR} = \text{QNR} = 10$ dB and $e_1^2 = e_2^2 = 0.01$. The curves are the asymptotic rates, and the dots are the ergodic rates that are obtained from simulation. The ergodic rate converges to the derived asymptotic rate for various α^{MMSE} and α^{RZF} values. In Fig. 3, we compare the rate performance of the five beamforming schemes. The advantage of the proposed robust MMSE-RZF can be observed. Note that MF, MF-RZF, and ZF are all special cases of MMSE-RZF, which are not optimized with the system condition. The bad performance of QR can be explained as that its effective

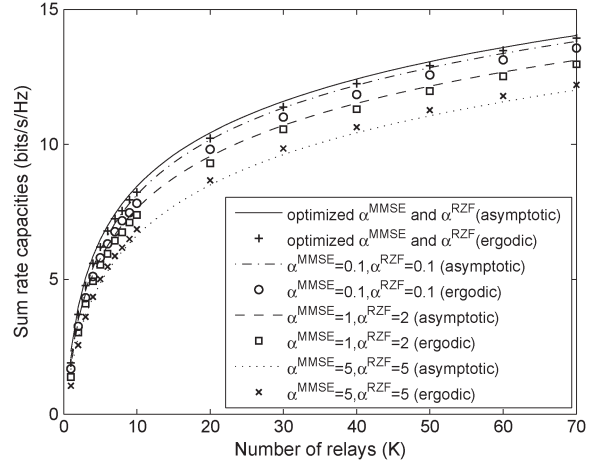


Fig. 2. Ergodic rates and asymptotic rates versus K for various α^{MMSE} and α^{RZF} .

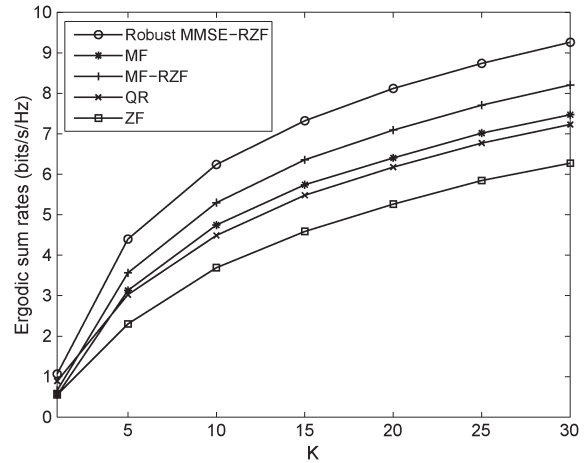


Fig. 3. Ergodic rates versus K . $\text{PNR} = 10$ dB, and $\text{QNR} = 10$ dB. $e_1^2 = e_2^2 = -10$ dB.

channel matrix is not diagonal but upper triangular, which results in power consumption. The poor performance of ZF comes from the inverse Wishart distribution term in its power control factor at the relays, particularly when $M = N$. We find that the ergodic capacities still satisfy the scaling law in [12], i.e., $C = (M/2) \log(K) + O(1)$ for large K in the presence of CSI errors. This is also consistent with the asymptotic capacities that are derived for MMSE-RZF with QR SIC detection.

B. Capacity Versus Power of CSI Error

Fig. 4 compare the ergodic rate capacities versus the power of CSI error. We set $K = 3$ and $\text{PNR} = 10$ dB and $\text{QNR} = 10$ dB. In this case, MMSE-RZF also outperforms others as the power of CSI error increases. The superiority of the MMSE-RZF compared with MF and MF-RZF decreases as e_1 and e_2 increase, and ZF outperforms MF for small $e_1(e_2)$ and underperforms MF for big $e_1(e_2)$. This is because, when e_1 and e_2 are small, the optimal α^{MMSE} and α^{RZF} should be small, e.g., $\alpha^{\text{MMSE,opt}} = 0.5$ for $\text{PNR} = 10$ dB and $e_1 = 0$, while in the MF and MF-RZF, $\alpha^{\text{MMSE}} = \infty$ and $\alpha^{\text{RZF}} = 1$,

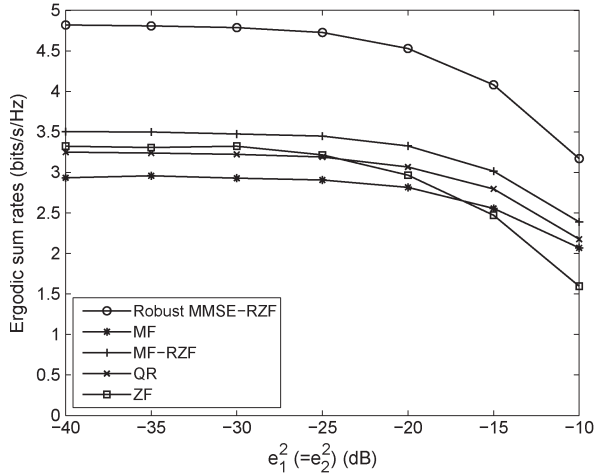


Fig. 4. Ergodic rates versus $e_1^2 (= e_2^2)$. PNR = 10 dB, and QNR = 10 dB.

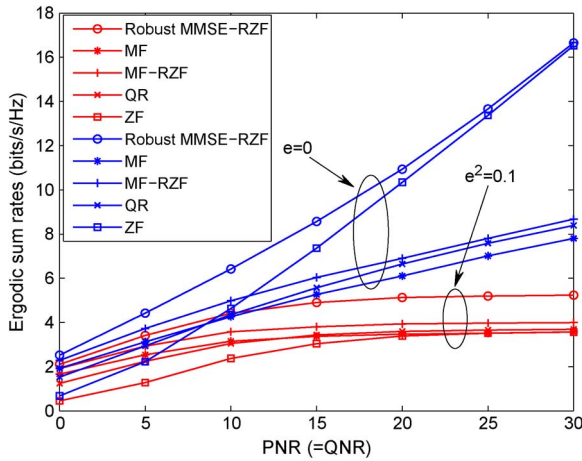


Fig. 5. Ergodic rates versus PNR(= QNR). $e_1^2 = e_2^2 = 0$ or $e_1^2 = e_2^2 = -10$ dB.

which are far away from the optimal values. When e_1 and e_2 grow, $\alpha^{\text{MMSE,opt}}$ and $\alpha^{\text{RZF,opt}}$ increase, which get close to MF and MF-RZF.

C. Capacity Versus PNR and QNR

In Fig. 5, we simultaneously increase PNR and QNR for $K = 5$. When CSIs are perfect, the rates of all the five beamformings linearly grow with PNR (= QNR) in decibels. When the CSI error occurs, we see the capacity limits. This is the “ceiling effect” that is discussed in Section V. This can be also seen in (41). If CSIs are perfect, the SNR of each stream linearly grows with PNR (= QNR); thus, the rate linearly grows with PNR (= QNR) in decibels. When CSI is imperfect, the numerator and the denominator will simultaneously grow, resulting in a limit of the SNR and the rate. The rate of the ZF beamformer converges to the proposed robust MMSE-RZF beamformer at high SNR (PNR and QNR) for perfect CSI. This can be explained by the derived $\alpha^{\text{MMSE,opt}}$ in (29) and $\alpha^{\text{RZF,opt}}$ in (48), which both converge to zero at a high SNR (large P/σ_1^2 and Q/σ_2^2) and perfect CSI ($e_1 = e_2 = 0$). Since ZF is known to be the optimal beamforming for high SNR, the

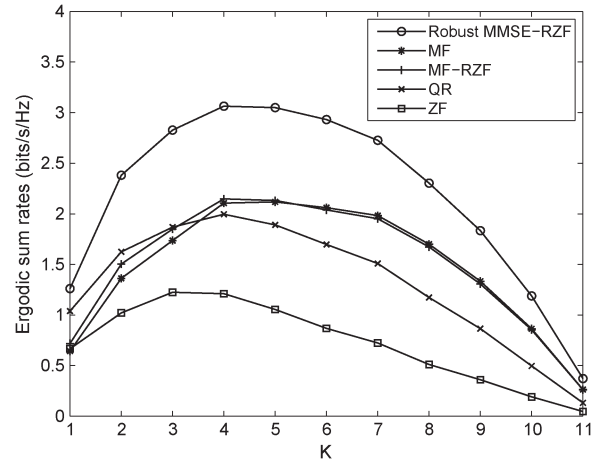


Fig. 6. Ergodic rates versus K . PNR = QNR = 10 dB. e_1 and e_2^2 are changed with number of relays K , as defined in (49) and (50).

proposed MMSE-RZF is asymptotically optimal at a high SNR with the QR SIC at the destination.

D. Capacity Versus Relay Number for Dynamic CSI Error

It is interesting to see Fig. 6, which shows the rates versus the relay number K when CSI errors vary with K . This is a very practical scenario when we assume that the error that is caused by channel estimation is $\sigma_e^2 = 0.05$, $B = 24$ for feedback, $f_D = 10$ Hz for $f = 2.4$ GHz, and $v = 4.5$ Km/h for pedestrian speed $f_D = vf/C$; C denotes the speed of light; and $\tau = 5$ ms, which is available in practical transmission. The feedback is based on the Lloyd vector-quantization algorithm as in [28]. It is observed that the rates achieve maximum at some optimal relay number in the presence of the CSI error. For the assumption in this figure, the optimal K is 4. For multirelay networks, the processing and feedback delay is always large. Thus, we can save power and improve performance by only choosing the optimal number of relays to forward the signal. The selected relays can be constant or based on the instantaneous CSI.

E. Capacity Versus Number of Feedback Bits

In Fig. 7, using the same model as in Fig. 6, we focus on the effect of limited feedback. As can be observed, the rates increase fast with the number of feedback bits at the beginning but slowly when the bits are enough to restore the CSIs. In Fig. 8, we further compare the performance under different numbers of feedback bits with the perfect CSI case. It is observed that, as the number of feedback bits increases, the rate approaches that of the perfect CSI case, and the rate is very close to that of the perfect CSI case when the feedback bits are 12.

VI. CONCLUSION

In this paper, based on the linear beamformer at the relay and QR SIC at the destination, we have proposed a robust MMSE-RZF beamformer optimized in terms of rate in a dual-hop MIMO multirelay network with AF relaying protocol in the

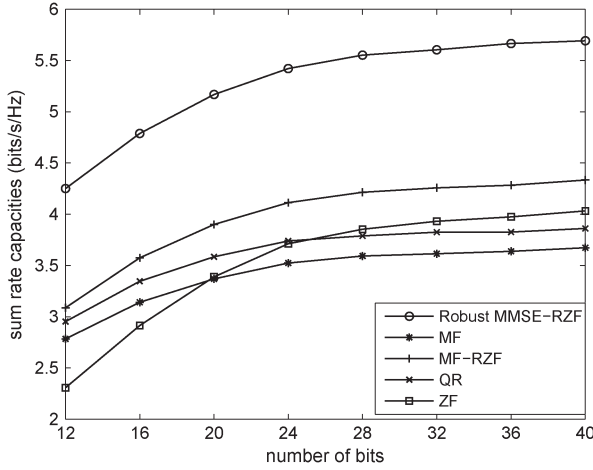


Fig. 7. Ergodic rates versus number of feedback bits. PNR = QNR = 10 dB. $e_1 = 0$, and e_2^2 is defined as in (50).

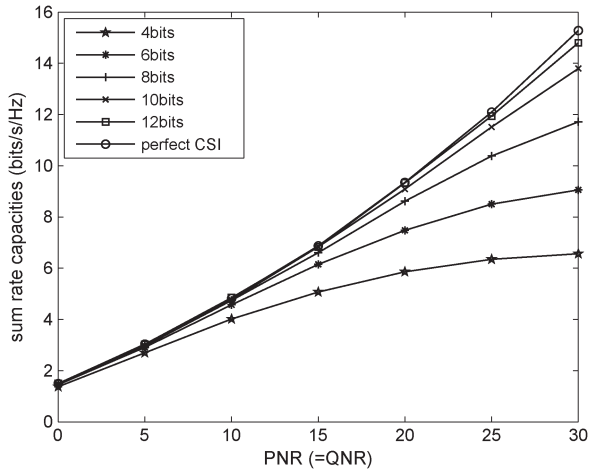


Fig. 8. Ergodic rates versus number of feedback bits. PNR = QNR = 10 dB. $e_1 = 0$, and e_2^2 is defined as in (50).

presence of imperfect CSI. Since it is difficult to obtain the global optimal MMSE-RZF beamformer, we have solved the optimization in two steps. The MMSE receiver has been optimized by maximizing the SINR at relay nodes. The RZF precoder has been optimized by maximizing the asymptotic rate that is derived upon a given MMSE receiver using the law of large numbers. Simulation results have shown that the asymptotic rate matches with the ergodic rate. Analysis and simulations have demonstrated that the proposed robust MMSE-RZF outperforms other coexistent beamforming schemes.

APPENDIX

We calculate the square of the norm of the nondiagonal as

$$\begin{aligned} & |(\mathbf{H}_{SD})_{(i,j)}|^2 \\ &= \left| \left(\sum_{k=1}^K \mathbf{Q}_k \frac{\boldsymbol{\Lambda}_k}{\boldsymbol{\Lambda}_k + \alpha^{\text{RZF}} \mathbf{I}_M} \mathbf{Q}_k^H \mathbf{P}_k \frac{\boldsymbol{\Theta}_k}{\boldsymbol{\Theta}_k + \alpha^{\text{MMSE}} \mathbf{I}_M} \mathbf{P}_k^H \right)_{(i,j)} \right|^2 \end{aligned}$$

$$\begin{aligned} &= \sum_k \left| \sum_{\ell,m,n} (\mathbf{Q}_k)_{i,\ell} \frac{\lambda_{k,\ell}}{\lambda_{k,\ell} + \alpha^{\text{MMSE}}} \right. \\ &\quad \left. \times (\mathbf{Q}_k)_{m,\ell}^* (\mathbf{P}_k)_{m,n} \frac{\theta_{k,n}}{\theta_{k,n} + \alpha^{\text{RZF}}} (\mathbf{P}_k)_{j,n}^* \right|^2 \\ &= \sum_{k,\ell,n,r,t} \sum_{m \neq i,j} \frac{\lambda_{k,\ell}}{\lambda_{k,\ell} + \alpha^{\text{MMSE}}} \frac{\lambda_{k,r}}{\lambda_{k,r} + \alpha^{\text{MMSE}}} \frac{\theta_{k,n}}{\theta_{k,n} + \alpha^{\text{RZF}}} \\ &\quad \times \frac{\theta_{k,t}}{\theta_{k,t} + \alpha^{\text{RZF}}} (\mathbf{Q}_k)_{i,\ell} (\mathbf{Q}_k)_{m,\ell}^* (\mathbf{Q}_k)_{i,r}^* (\mathbf{Q}_k)_{m,r} \\ &\quad \times (\mathbf{P}_k)_{m,n} (\mathbf{P}_k)_{j,n}^* (\mathbf{P}_k)_{m,t}^* (\mathbf{P}_k)_{j,t} \\ &\quad + \sum_{k,\ell,n,r,t} \frac{\lambda_{k,\ell}}{\lambda_{k,\ell} + \alpha^{\text{MMSE}}} \frac{\lambda_{k,r}}{\lambda_{k,r} + \alpha^{\text{MMSE}}} \frac{\theta_{k,n}}{\theta_{k,n} + \alpha^{\text{RZF}}} \\ &\quad \times \frac{\theta_{k,t}}{\theta_{k,t} + \alpha^{\text{RZF}}} |(\mathbf{Q}_k)_{i,\ell}|^2 |(\mathbf{Q}_k)_{i,r}|^2 (\mathbf{P}_k)_{i,n} \\ &\quad \times (\mathbf{P}_k)_{j,n}^* (\mathbf{P}_k)_{i,t}^* (\mathbf{P}_k)_{j,t} \\ &\quad + \sum_{k,\ell,n,r,t} \frac{\lambda_{k,\ell}}{\lambda_{k,\ell} + \alpha^{\text{MMSE}}} \frac{\lambda_{k,r}}{\lambda_{k,r} + \alpha^{\text{MMSE}}} \frac{\theta_{k,n}}{\theta_{k,n} + \alpha^{\text{RZF}}} \\ &\quad \times \frac{\theta_{k,t}}{\theta_{k,t} + \alpha^{\text{RZF}}} (\mathbf{Q}_k)_{i,\ell} (\mathbf{Q}_k)_{j,\ell}^* (\mathbf{Q}_k)_{i,r}^* (\mathbf{Q}_k)_{j,r} \\ &\quad \times |(\mathbf{P}_k)_{j,n}|^2 |(\mathbf{P}_k)_{j,t}|^2 \end{aligned} \quad (51)$$

$$\begin{aligned} & |(\mathbf{H}_{SD})_{(i,j)}|^2 \\ & \xrightarrow{\text{w.p.}} \frac{K(M+2)}{(M+1)^2} E \left\{ \frac{\lambda^2}{(\lambda + \alpha^{\text{RZF}})^2} \right\} E \left\{ \frac{\theta^2}{(\theta + \alpha^{\text{MMSE}})^2} \right\} \\ & \quad - \frac{K}{(M+1)^2} E \left\{ \frac{\lambda \lambda'}{(\lambda + \alpha^{\text{RZF}})(\lambda' + \alpha^{\text{RZF}})} \right\} \\ & \quad \times E \left\{ \frac{\theta^2}{(\theta + \alpha^{\text{MMSE}})^2} \right\} - \frac{K}{(M+1)^2} E \left\{ \frac{\lambda^2}{(\lambda + \alpha^{\text{RZF}})^2} \right\} \\ & \quad \times E \left\{ \frac{\theta \theta'}{(\theta + \alpha^{\text{MMSE}})(\theta' + \alpha^{\text{MMSE}})} \right\} \\ & \quad - \frac{KM}{(M+1)^2} E \left\{ \frac{\lambda \lambda'}{(\lambda + \alpha^{\text{RZF}})(\lambda' + \alpha^{\text{RZF}})} \right\} \\ & \quad \times E \left\{ \frac{\theta \theta'}{(\theta + \alpha^{\text{MMSE}})(\theta' + \alpha^{\text{MMSE}})} \right\}. \end{aligned} \quad (52)$$

Since the fact in [33] says that

$$E \{ |(\mathbf{Q}_r)_{i,k}|^2 |(\mathbf{Q}_r)_{\ell,k}|^2 \} = \frac{2}{M(M+1)}, \quad \text{if } i = \ell \quad (53)$$

$$E \{ |(\mathbf{Q}_r)_{i,k}|^2 |(\mathbf{Q}_r)_{\ell,k}|^2 \} = \frac{1}{M(M+1)}, \quad \text{if } i \neq \ell \quad (54)$$

then, when $i \neq m, \ell \neq r$, we have

$$\begin{aligned}
& E \left\{ (\mathbf{Q}_k)_{i,\ell} (\mathbf{Q}_k)_{m,\ell}^* (\mathbf{Q}_k)_{i,r}^* (\mathbf{Q}_k)_{m,r} \right\} \\
&= \frac{1}{M(M-1)} E \left\{ \sum_{\ell,r=1, \ell \neq r}^M (\mathbf{Q}_k)_{i,\ell} (\mathbf{Q}_k)_{m,\ell}^* (\mathbf{Q}_k)_{i,r}^* (\mathbf{Q}_k)_{m,r} \right\} \\
&= \frac{1}{M(M-1)} E \left\{ \sum_{\ell,r=1}^M (\mathbf{Q}_k)_{i,\ell} (\mathbf{Q}_k)_{m,\ell}^* (\mathbf{Q}_k)_{i,r}^* (\mathbf{Q}_k)_{m,r} \right\} \\
&\quad - \frac{1}{M(M-1)} E \left\{ \sum_{\ell=1}^M |(\mathbf{Q}_k)_{i,\ell}|^2 |(\mathbf{Q}_k)_{m,\ell}|^2 \right\} \\
&= \frac{E \left\{ \left(\sum_{\ell=1}^M (\mathbf{Q}_k)_{i,\ell} (\mathbf{Q}_k)_{m,\ell}^* \right) \left(\sum_{r=1}^M (\mathbf{Q}_k)_{i,r} (\mathbf{Q}_k)_{m,r}^* \right) \right\}}{M(M-1)} \\
&\quad - \frac{1}{M(M-1)} \cdot \frac{1}{(M+1)} \\
&= -\frac{1}{(M-1)M(M+1)}. \tag{55}
\end{aligned}$$

Substituting (53)–(55) into (51), and through some manipulation, we have (52), where λ and λ' , and θ and θ' are different singular values within one decomposition. Obviously, (52) is equal to zero when α^{MMSE} and α^{RZF} are zero.

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