SPECTRUM SENSING OF ORTHOGONAL SPACE-TIME BLOCK CODED SIGNALS WITH MULTIPLE RECEIVE ANTENNAS

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ABSTRACT

We consider detection of signals encoded with orthogonal space-time block codes (OSTBC), using multiple receive antennas. Such signals contain redundancy and they have a specific structure, that can be exploited for detection. We derive the optimal detector, in the Neyman-Pearson sense, when all parameters are known. We also consider unknown noise variance, signal variance and channel coefficients. We propose a number of GLRT based detectors for the different cases, that exploit the redundancy structure of the OSTBC signal. We also propose an eigenvalue-based detector for the case when all parameters are unknown. The proposed detectors are compared to the energy detector. We show that when only the noise variance is known, there is no gain in exploiting the structure of the OSTBC. However, when the noise variance is unknown there can be a significant gain.

1. INTRODUCTION

Cognitive radio is a new concept of using spectrum holes, that occur in licensed spectrum. Introducing cognitive radios in a primary network inevitably creates increased interference to the primary users. Secondary users must sense the spectrum and detect primary user signals at very low SNR [1], to avoid causing too much interference. Thus, spectrum sensing is one of the most essential elements of cognitive radio.

One of the simplest and most widely used sensing schemes is the energy detector [2]. This detector is optimal if both the signal and the noise are Gaussian, and the noise power is known. It is known that the structure imposed by modulation with a finite alphabet constellation does not appreciably improve performance over energy detection [3]. However, if the noise power is unknown, it is impossible to set the detection threshold, and the energy detector does not work at all. Even if the noise power is known, or estimated, to some accuracy, it is known that the performance of the energy detector is severely degraded with the noise uncertainty.

All manmade signals introduce redundancy to the signal in a controlled manner, for example by modulation, channel coding and space-time coding. Much literature is concerned with detectors that exploit structure of signals, either to obtain better performance than the energy detector, or to circumvent the known-noise power assumption. Structure of the signal that results in periodic mean and autocorrelation, for example channel coding and OFDM modulation, introduce cyclostationarity to the signal. Detection based on the cyclostationarity property was proposed in [4]. Detectors based on ratios of the eigenvalues of the sample covariance matrix were proposed in [5], and shown to perform well when the signals to be detected are highly correlated.

We are interested in detecting signals encoded with orthogonal space-time block codes (OSTBC). Such signals contain redundancy and they have a specific structure. We propose a number of detectors that can exploit this structure, under different circumstances. We first consider a genie detector, where all parameters are known. This is an unrealistic scenario, but serves as an upper bound on the detector performance. Then, we consider the cases of completely unknown and partially unknown parameters. We propose detectors based on a GLRT approach, that exploits the known structure of the received sample covariance matrix. The GLRT is not necessarily optimal. Thus, for the case of completely unknown parameters we also propose an alternative detector based on an eigenvalue ratio test.

2. MODEL AND PROBLEM FORMULATION

We consider a system where the signal is encoded with an OSTBC. Assume that there are n_r receive antennas and n_t transmit antennas. The OSTBC code matrix $\mathbf{X} \in \mathbb{C}^{n_t \times N}$ is a linear function of n_s symbols $s_1, \ldots, s_{n_s}$ and their complex conjugates. The coded symbols (columns of $\mathbf{X}$) are transmitted over N time intervals. Let $\mathbf{Y} \in \mathbb{C}^{n_r \times N}$ be the received matrix that consists of the space-time coded signal plus noise, i.e.

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{E}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix, and $\mathbf{E} \in \mathbb{C}^{n_r \times N}$ is a matrix of noise. Here, we have assumed perfect time and frequency synchronization. This is not practically feasible, but the detectors that we will propose serves as an upper bound on detection performance. The noise is assumed to be complex white zero-mean Gaussian with variance σ^2. That is, the real and imaginary parts of the entries of $\mathbf{E}$ are i.i.d $\mathcal{N}\left(0, \sigma^2/2\right)$. For the special case of the well known Alamouti code, (6.3.1) in [6], we have the following code matrix

$$\mathbf{X} = \begin{bmatrix} s_1 & s_2^* \\ s_2 & -s_1^* \end{bmatrix},$$

where s_1 and s_2 are the two ($n_s = 2$) complex symbols transmitted over two ($N = 2$) time intervals by two ($n_t = 2$) antennas. Using n_r receive antennas, the channel matrix in this special case is

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ \vdots & \vdots \\ h_{n_r1} & h_{n_r2} \end{bmatrix} \in \mathbb{C}^{n_r \times 2}.$$

Now we will rewrite (1), so that we get the received data ex-

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pressed explicitly as a linear function of the symbols s_i. We denote by vec $(\mathbf{A})$, the vector obtained by stacking the columns of the matrix $\mathbf{A}$ on top of each other. Furthermore, we denote the real and imaginary parts of a matrix $\mathbf{B}$ by $\overline{\mathbf{B}}$ and $\widetilde{\mathbf{B}}$ respectively. The same notation $\overline{(\cdot)}$ and $\widetilde{(\cdot)}$ is also used for vectors and scalars. Now, since $\mathbf{X}$ is a linear space-time block code, we get from equation (7.1.8) in [6] that there exists a matrix $\mathbf{F} \in \mathbb{C}^{n_r N \times 2n_s}$ such that

$$\text{vec}\,(\mathbf{HX}) = \mathbf{Fs}, \tag{2}$$

where

$$\mathbf{s} = \left[\overline{s}_1, \ldots, \overline{s}_{n_s}, \widetilde{s}_1, \ldots, \widetilde{s}_{n_s}\right]^T \in \mathbb{R}^{2n_s \times 1}.$$

When $\mathbf{X}$ is also orthogonal, then (7.4.14) of [6] states that the matrix $\mathbf{F}$ has the property

$$\text{Re}\left(\mathbf{F}^H\mathbf{F}\right) = \|\mathbf{H}\|^2\mathbf{I}.$$

Since $\mathbf{s}$ is real-valued, we can rewrite (2) as

$$\begin{bmatrix} \text{vec}\left(\overline{\mathbf{HX}}\right) \\ \text{vec}\left(\widetilde{\mathbf{HX}}\right) \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{F}} \\ \widetilde{\mathbf{F}} \end{bmatrix} \mathbf{s}.$$

Let

$$\mathbf{G} \triangleq \begin{bmatrix} \overline{\mathbf{F}} \\ \widetilde{\mathbf{F}} \end{bmatrix} \in \mathbb{R}^{2n_r N \times 2n_s}.$$

Then $\mathbf{G}$ has the property

$$\mathbf{G}^T\mathbf{G} = \overline{\mathbf{F}}^T\overline{\mathbf{F}} + \widetilde{\mathbf{F}}^T\widetilde{\mathbf{F}} = \text{Re}\left(\mathbf{F}^H\mathbf{F}\right) = \|\mathbf{H}\|^2\mathbf{I}.$$

Now, we can rewrite (1) as

$$\mathbf{y} \triangleq \begin{bmatrix} \text{vec}\left(\overline{\mathbf{Y}}\right) \\ \text{vec}\left(\widetilde{\mathbf{Y}}\right) \end{bmatrix} = \mathbf{Gs} + \mathbf{e}, \quad \mathbf{y} \in \mathbb{R}^{2n_r N \times 1},$$

where

$$\mathbf{e} \triangleq \begin{bmatrix} \text{vec}\left(\overline{\mathbf{E}}\right) \\ \text{vec}\left(\widetilde{\mathbf{E}}\right) \end{bmatrix} \in \mathbb{R}^{2n_r N \times 1}.$$

Thus, $\mathbf{G}$ is a generator matrix for the space-time block code represented by the code matrix $\mathbf{X}$. Returning to the Alamouti code, it can be shown that the generator matrix in that case can be written

$$\mathbf{G} = \begin{bmatrix} \bar{h}_{11} & \bar{h}_{12} & -\tilde{h}_{11} & -\tilde{h}_{12} \\ \vdots & & & \vdots \\ \bar{h}_{n_r1} & \bar{h}_{n_r2} & -\tilde{h}_{n_r1} & -\tilde{h}_{n_r2} \\ -\bar{h}_{12} & \bar{h}_{11} & -\tilde{h}_{12} & \tilde{h}_{11} \\ \vdots & & & \vdots \\ -\bar{h}_{n_r2} & \bar{h}_{n_r1} & -\tilde{h}_{n_r2} & \tilde{h}_{n_r1} \\ \tilde{h}_{11} & \tilde{h}_{12} & \bar{h}_{11} & \bar{h}_{12} \\ \vdots & & & \vdots \\ \tilde{h}_{n_r1} & \tilde{h}_{n_r2} & \bar{h}_{n_r1} & \bar{h}_{n_r2} \\ -\tilde{h}_{12} & \tilde{h}_{11} & \bar{h}_{12} & -\bar{h}_{11} \\ \vdots & & & \vdots \\ -\tilde{h}_{n_r2} & \tilde{h}_{n_r1} & \bar{h}_{n_r2} & -\bar{h}_{n_r1} \end{bmatrix} \in \mathbb{R}^{4n_r \times 4}.$$

Now consider K space-time blocks $\mathbf{Y}_k$, or equivalently K vectors $\mathbf{y}_k$, received in a sequence. Moreover, we assume that the channel is slow fading, such that the generator matrix $\mathbf{G}$ remains the same during the whole time of reception. In spectrum sensing we wish to detect whether there is a signal present or not. That is, we want to discriminate between the two hypotheses:

$$H_0: \; \mathbf{y}_k = \mathbf{e}_k, \; k = 1, \ldots, K,$$
$$H_1: \; \mathbf{y}_k = \mathbf{Gs}_k + \mathbf{e}_k, \; k = 1, \ldots, K.$$

We assume that the elements of $\mathbf{s}_k$ are i.i.d. $\mathcal{N}\left(0, \frac{\gamma^2}{2}\right)$. The motivation for this is that capacity optimal signals are Gaussian, and the symbols will have zero mean and have equal variance in the real and imaginary parts if they come from a modulation which is symmetric in both the real and imaginary parts. This is the case for example for M-PSK and M-QAM modulations, but not for BPSK. Then the elements of $\mathbf{y}_k$ are also zero-mean Gaussian and

$$\mathbf{y}_k | \left\{H_0, \sigma^2\right\} \sim \mathcal{N}\left(\mathbf{0}, \frac{\sigma^2}{2}\mathbf{I}\right)$$
$$\mathbf{y}_k | \left\{H_1, \sigma^2, \gamma^2, \mathbf{G}\right\} \sim \mathcal{N}\left(\mathbf{0}, \frac{\gamma^2}{2}\mathbf{G}\mathbf{G}^T + \frac{\sigma^2}{2}\mathbf{I}\right).$$

Let $\mathbf{Q}_0 \triangleq \frac{\sigma^2}{2}\mathbf{I}$ and $\mathbf{Q}_1 \triangleq \frac{\gamma^2}{2}\mathbf{G}\mathbf{G}^T + \frac{\sigma^2}{2}\mathbf{I}$. The matrix $\mathbf{G}$ has low rank ($2n_s$), provided that $n_r N > n_s$. For the Alamouti code, this is the case if $n_r \geq 2$. Thus, $\mathbf{Q}_1$ is a low rank matrix plus an identity matrix and $\mathbf{Q}_0$ has full rank. We can write the likelihood functions for the received vectors under both hypotheses as

$$p\left(\mathbf{y}_1, \ldots, \mathbf{y}_K | \left\{H_i, \sigma^2, \gamma^2, \mathbf{G}\right\}\right) = \prod_{k=1}^{K} \frac{\exp\left(-\frac{1}{2}\mathbf{y}_k^T\mathbf{Q}_i^{-1}\mathbf{y}_k\right)}{\sqrt{2\pi \det\left(\mathbf{Q}_i\right)}}$$
$$= \frac{1}{\left(2\pi \det\left(\mathbf{Q}_i\right)\right)^{K/2}} \exp\left(-\frac{1}{2}\sum_{k=1}^{K}\mathbf{y}_k^T\mathbf{Q}_i^{-1}\mathbf{y}_k\right).$$

3. SIGNAL DETECTION

In the following we will propose a number of detectors for the cases of known, partially known and completely unknown parameters σ^2, γ^2 and $\mathbf{G}$.

3.1. Optimal Genie Detection

The optimal Neyman-Pearson test, when $\mathbf{Q}_0$ and $\mathbf{Q}_1$ are known, is to compare the likelihood ratio with a threshold. That is,

$$L \triangleq \frac{p\left(\mathbf{y}_1, \ldots, \mathbf{y}_K | H_1, \sigma^2, \gamma^2, \mathbf{G}\right)}{p\left(\mathbf{y}_1, \ldots, \mathbf{y}_K | H_0, \sigma^2\right)}$$
$$= \left(\frac{\det\left(\mathbf{Q}_0\right)}{\det\left(\mathbf{Q}_1\right)}\right)^{K/2} \exp\left(-\frac{1}{2}\sum_{k=1}^{K}\mathbf{y}_k^T\left(\mathbf{Q}_1^{-1} - \mathbf{Q}_0^{-1}\right)\mathbf{y}_k\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{3}$$

where η is a detection threshold.

3.2. Unknown Parameters, GLRT Approach

In general the parameters $\mathbf{G}$, σ^2 and γ^2 are unknown. In that case we can construct a generalized likelihood ratio test (GLRT):

$$L_{\text{GLRT}} \triangleq \frac{p\left(\mathbf{y}_1, \ldots, \mathbf{y}_K | H_1, \widehat{\mathbf{Q}_1}\right)}{p\left(\mathbf{y}_1, \ldots, \mathbf{y}_K | H_0, \widehat{\mathbf{Q}_0}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{GLRT}}, \tag{4}$$

where $\widehat{\mathbf{Q}_1}$ and $\widehat{\mathbf{Q}_0}$ are the maximum-likelihood (ML) estimates of $\mathbf{Q}_1$ and $\mathbf{Q}_0$ under H_1 and H_0 respectively. It is generally hard to find the ML-estimates of $\mathbf{Q}_0$ and $\mathbf{Q}_1$. However we will propose a few methods to obtain near-ML estimates of these covariance matrices.

Estimation of $\mathbf{Q}_1$: We know that the matrix $\mathbf{G}$ has low rank ($2n_s$), provided that $n_rN > n_s$. Thus, under H_1, the covariance matrix $\mathbf{Q}_1$ will have $2n_s$ eigenvalues equal to $\frac{\sigma^2+\|\mathbf{H}\|^2\gamma^2}{2}$ and the rest $2n_rN - 2n_s$ equal to $\frac{\sigma^2}{2}$. This structure can be used to obtain near-ML estimates of $\mathbf{Q}_1$ under H_1. More specifically, we can write the eigenvalue decomposition of the covariance matrix as

$$\mathbf{Q}_1 = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T, \tag{5}$$

where

$$\mathbf{\Lambda} \triangleq \text{diag}\left(\underbrace{\frac{\sigma^2+\|\mathbf{H}\|^2\gamma^2}{2},\ldots,\frac{\sigma^2+\|\mathbf{H}\|^2\gamma^2}{2}}_{2n_s},\underbrace{\frac{\sigma^2}{2},\ldots,\frac{\sigma^2}{2}}_{2n_rN-2n_s}\right) \tag{6}$$

is a diagonal matrix, and $\mathbf{U}$ is orthonormal so that $\mathbf{U}\mathbf{U}^T = \mathbf{U}^T\mathbf{U} = \mathbf{I}$. The diagonal of $\mathbf{\Lambda}$ contains the eigenvalues sorted in descending order. This property will be used to estimate the covariance matrix. Let

$$\widehat{\mathbf{Q}} \triangleq \frac{1}{K}\sum_{k=1}^{K}\mathbf{y}_k\mathbf{y}_k^T,$$

be the sample covariance matrix of $\{\mathbf{y}_k\}$. This is an unbiased and consistent estimate of $E\left[\mathbf{y}_k\mathbf{y}_k^T\right]$. In fact, if there is no further prior information about the structure of the covariance matrix, $\widehat{\mathbf{Q}}$ is the ML-estimate of $\mathbf{Q}_1$. We write the eigenvalue decomposition of the covariance matrix estimate:

$$\widehat{\mathbf{Q}} = \widehat{\mathbf{U}}\widehat{\mathbf{\Lambda}}\widehat{\mathbf{U}}^T,$$

where the eigenvalues $\widehat{\lambda}_i$ in $\widehat{\mathbf{\Lambda}}$ are sorted in descending order, and $\widehat{\mathbf{U}}\widehat{\mathbf{U}}^T = \widehat{\mathbf{U}}^T\widehat{\mathbf{U}} = \mathbf{I}$. To estimate $\mathbf{Q}_1$, we will use the estimated eigenvectors contained in $\widehat{\mathbf{U}}$ but smoothen the eigenvalues. Now, let

$$\lambda^+ \triangleq \frac{1}{2n_s}\sum_{i=1}^{2n_s}\widehat{\lambda}_i \quad \text{and} \quad \lambda^- \triangleq \frac{1}{2\left(n_rN-n_s\right)}\sum_{i=2n_s+1}^{2n_rN}\widehat{\lambda}_i. \tag{7}$$

Then λ^+ is an estimate of $\frac{\sigma^2+\|\mathbf{H}\|^2\gamma^2}{2}$ and λ^- is an estimate of $\frac{\sigma^2}{2}$. Because of the property (6), we can now estimate the covariance matrix under H_1, near-ML [7], by

$$\widehat{\mathbf{Q}_1} \triangleq \widehat{\mathbf{U}}\widehat{\mathbf{\Lambda}_1}\widehat{\mathbf{U}}^T, \tag{8}$$

where

$$\widehat{\mathbf{\Lambda}_1} \triangleq \text{diag}\left(\underbrace{\lambda^+,\ldots,\lambda^+}_{2n_s},\underbrace{\lambda^-,\ldots,\lambda^-}_{2n_rN-2n_s}\right). \tag{9}$$

The matrix $\widehat{\mathbf{\Lambda}_1}$ contains the smoothened eigenvalue estimates on the diagonal.

If the noise variance σ^2 is known, the correct value should be used in the covariance matrix estimate. A straight-forward approach is to simply insert the correct value instead of the estimated value into $\widehat{\mathbf{\Lambda}_1}$. That is,

$$\widehat{\mathbf{\Lambda}_1} \triangleq \text{diag}\left(\lambda^+,\ldots,\lambda^+,\frac{\sigma^2}{2},\ldots,\frac{\sigma^2}{2}\right). \tag{10}$$

Estimation of $\mathbf{Q}_0$: Under H_0, the covariance matrix is $\mathbf{Q}_0 = \frac{\sigma^2}{2}\mathbf{I}$. Thus, we only need to estimate the noise variance σ^2. We will propose two possible estimates $\widehat{\sigma^2}$ of σ^2. Then, we take

$$\widehat{\mathbf{Q}_0} = \frac{\widehat{\sigma^2}}{2}\mathbf{I}. \tag{11}$$

The first proposal is to use the ML-estimate under H_0:

$$\frac{\widehat{\sigma^2}}{2} = \frac{1}{2n_rNK}\sum_{k=1}^{K}\|\mathbf{y}_k\|^2. \tag{12}$$

In the second proposal, we also consider the structure of the covariance matrix. More specifically, when there is a signal present (H_1) we know that the expected value of the $2n_s$ largest eigenvalues is equal to $\frac{\sigma^2+\|\mathbf{H}\|^2\gamma^2}{2}$. Thus, the ML-estimate of σ^2 will be contaminated with the signal. We also know that the expected value of all other eigenvalues is equal to $\frac{\sigma^2}{2}$, whether there is a signal present or not. Thus, using

$$\frac{\widehat{\sigma^2}}{2} = \lambda^-, \tag{13}$$

would yield a better estimate if there is a signal present, and only incurs a small loss in accuracy if there is only noise since we use only $2n_rN - 2n_s$ samples instead of $2n_rN$.

If the noise variance σ^2 is known, clearly the covariance matrix $\widehat{\mathbf{Q}_0} = \frac{\sigma^2}{2}\mathbf{I}$ is completely known.

3.3. Unknown Parameters, Eigenvalue-Based Detection

The GLRT is not optimal. Here we propose an alternative approach, based on comparisons between the eigenvalues of $\widehat{\mathbf{Q}}$. Our approach is inspired by [5], who considered the detection of a completely *unknown*, but correlated signal. Reference [5] used the ratio between the largest and smallest eigenvalue of the sample covariance matrix as a test statistic, and as an alternative, the ratio of the average eigenvalue to the smallest one. This performed well when the signal to be detected had a significant correlation structure. Signals encoded by an OSTBC are strongly correlated. Additionally, we know the eigenvalue structure of $\mathbf{Q}$ explicitly under both H_0 ($\mathbf{Q} = \frac{\sigma^2}{2}\mathbf{I}$) and H_1 (see (5)-(6)). Hence, we can exploit much more information about the signal than what a direct application of the detectors in [5] would do. We propose the test

$$L_{\text{eig}} \triangleq \frac{\lambda^+}{\lambda^-} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{eig}}, \tag{14}$$

where λ^+ and λ^- are given by (7). This detector does not require any knowledge about the parameters σ^2, γ^2 and $\mathbf{G}$.

3.4. Energy Detection

The energy detector [2] measures the energy of the received signal, and compares it to a threshold:

$$L_{\text{energy}} \triangleq \sum_{k=1}^{K}\|\mathbf{y}_k\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{energy}}. \tag{15}$$

One drawback with the energy detector is that the noise variance σ^2 must be known at the detector, to set the threshold. On the other hand it does not require, and therefore does not exploit, any knowledge about the signal. The energy detector will serve as a baseline for detector performance.

Detector	Statistic	$\mathbf{Q}_0$ (σ^2)	$\mathbf{Q}_1$ (σ^2, γ^2, $\mathbf{G}$)
(i) Optimal Genie	(3)	known	known
(ii) Energy	(15)	known	not needed
(iii) GLRT	(4)	known	(8),(10)
(iv) GLRT	(4)	(11), (12)	(8)-(9)
(v) GLRT	(4)	(11), (13)	(8)-(9)
(vi) Eigenvalue	(14)	requires only (7)	

Table 1. Summary of proposed detectors.

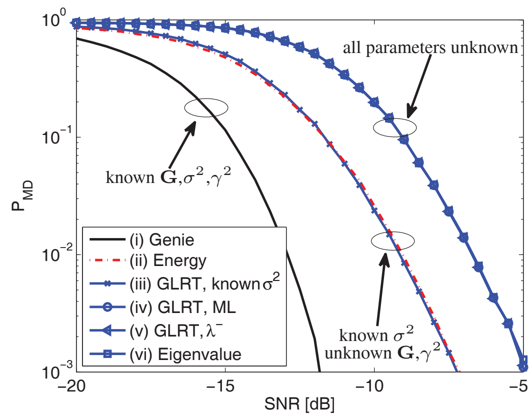

Fig. 1. Probability of missed detection P_{MD} versus SNR for different schemes. $P_{\text{FA}} = 0.05$, $K = 100$, $n_r = 4$.

4. NUMERICAL RESULTS

We show some numerical results for the proposed detection schemes, exemplified by the Alamouti code. All results were obtained by Monte-Carlo simulation. All simulations were run for 50000 realizations at each SNR value. The SNR in dB is defined as $10 \log_{10}(\gamma^2/\sigma^2)$. Performance is given as the probability of missed detection, P_{MD}, as a function of SNR. The noise variance was set to $\sigma^2 = 1$ and the SNR was varied. The channel coefficients were drawn from a complex circularly symmetric $\mathcal{N}(0,1)$ distribution. The probability of false alarm P_{FA} was fixed to decide the decision threshold. Then, the probability of missed detection P_{MD} was computed based on this threshold for each SNR value. A summary of the proposed detectors is given in Table 1.

Figure 1 shows the results for $P_{\text{FA}} = 0.05$, $K = 100$ and $n_r = 4$. In terms of performance, we observe three groups of detectors. Firstly, it is shown that the optimal genie detector is significantly better than the other detectors. Thus, knowing the channels would yield a significant gain. Secondly, we note that the schemes that assume known noise variance, (ii) energy detection and (iii) GLRT with known σ^2, perform almost identically. Thirdly, the detectors which do not know the noise variance ((iv)-(vi)) perform worst, and almost identically.

Figure 2 shows the same as Figure 1, but the number of receive antennas is increased to $n_r = 8$. The performance relation of the different detectors is similar to the previous case. We observe a gain of approximately $3-5$ dB SNR for all detectors by using 8 antennas instead of 4. It is worth noting that the gain in using more antennas is larger for the detectors that exploit the signal structure ((i) and (iii)-(vi)), than for the energy detector (ii). This is owing to the fact that the more receive antennas there are, the more correlated is the received signal.

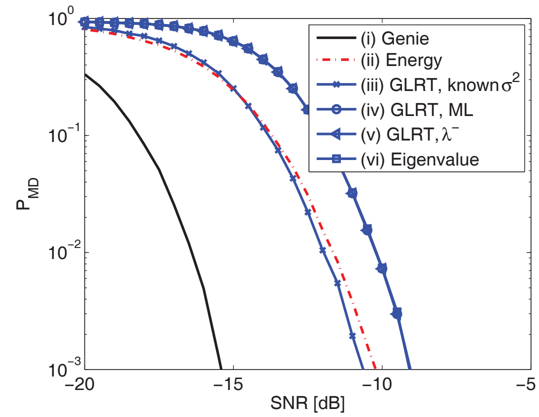

Fig. 2. Same as Figure 1, but with $n_r = 8$.

We have compared the performance of the eigenvalue-based detector in Section 3.3 with a detector that uses the eigenvalue ratios proposed in [5] instead of (14). We observed that using our proposed eigenvalue ratio (14) outperforms the eigenvalue ratios proposed in [5] with about $1-5$ dB SNR, for the cases in Figures 1-2. The reason is that our detectors exploit more information about the structure of the signal. We omit more detailed results due to space limitations.

5. CONCLUDING REMARKS

In this work we assumed perfect time and frequency synchronization. This is not realistic in practice, so the results are an upper bound for the detector performance. The problem of imperfect synchronization is a topic for future studies.

Moreover, in this work we proposed to estimate the unknown parameters. Perhaps the problem of unknown parameters could also be dealt with using a Bayesian approach, imposing a prior distribution on the unknown parameters.

6. REFERENCES

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