# Asymptotically Optimal Transmit Strategies for the Multiple Antenna Interference Channel

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***Abstract*— We consider the interference channel with multiple antennas at the transmitter. We prove that at high signal-to-noise ratio (SNR), the zero-forcing transmit scheme is optimal in the sum-rate sense. Furthermore we prove that at low SNR, maximum-ratio transmission is optimal in the sum-rate sense. We also provide a discussion of the connection to classical results on spectral efficiency in the wideband regime. Finally, we propose a non-convex optimization approach based on monotonic optimization to solve the sum rate maximization problem.**

## I. Introduction

The interference channel (IFC) is a classic object of study in information theory [1], [2]. While its capacity is an open problem, a number of results on achievable rates were established a fairly long time ago [3], [4]. In particular, it is known that strong interference should be decoded and subtracted off the received data whenever possible, and that weak interference should be treated as noise. More recently, there has been renewed interest in the IFC, as witnessed by a number of contributions in the research literature ( [5], [6], [8], for example). The principal driving motivation for this interest is that the IFC is a sound model for the spectrum sharing scenario in wireless communications, where multiple independent radio links coexist and interfere in the same spectral band and therefore interfere with each other.

In this work we are concerned with the IFC for the case when the transmitter has multiple antennas. We refer to this as the multiple-input single-output (MISO) IFC. A sketch of the MISO IFC, with two transmitter-receiver pairs, is given in Figure 1. The importance of using multiple antennas is twofold. First, it provides the usual rate and diversity gains [9]. Second, if (partial) channel state information is available at the transmitters, then this can be exploited to minimize the interference that one system generates to the other system.

Some previous work on the MISO IFC is available. The MISO IFC is a special case of the multiple-input multiple-output (MIMO) IFC [5], [6], and hence many results therein can be specialized to the MISO case. A characterization of the MISO IFC from a game-theoretic (both non-cooperative and cooperative) point of view was presented by [11]. An explicit parameterization of the achievable rate region was given in [10].

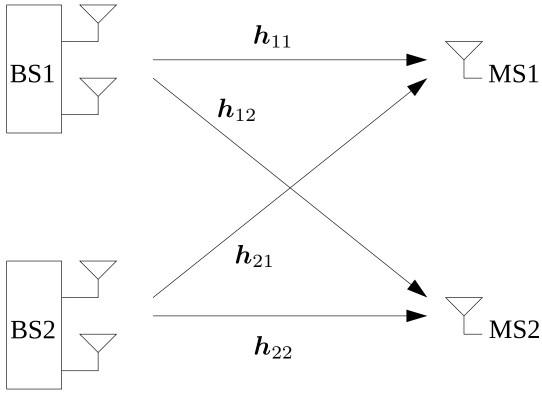


Fig. 1. The two-user MISO interference channel under study (illustrated for $n = 2$ transmit antennas).

**Contribution:** This paper is concerned with the characterization of sum-rate optimal transmit strategies for the MISO IFC. We provide two main results. First we prove that at high SNR, the zero-forcing (ZF) strategy is optimal in the sum-rate sense. Second, we show that at low SNR, the sum-rate-optimal strategy becomes maximal-ratio transmission (MRT). The results are discussed in the context of spectral efficiency in the wideband regime [12].

## II. Model

We consider the 2-user MISO IFC in Figure 1. We shall assume that transmission consists of scalar coding followed by beamforming,[1] and that all propagation channels are frequency-flat. This leads to the following basic model for the matched-filtered, symbol-sampled complex baseband data received at $\mathrm{MS}_1$ and $\mathrm{MS}_2$:

$$y_1 = \boldsymbol{h}_{11}^T \boldsymbol{w}_1 s_1 + \boldsymbol{h}_{21}^T \boldsymbol{w}_2 s_2 + e_1$$

$$y_2 = \boldsymbol{h}_{22}^T \boldsymbol{w}_2 s_2 + \boldsymbol{h}_{12}^T \boldsymbol{w}_1 s_1 + e_2$$

where $s_1$ and $s_2$ are transmitted symbols, $\boldsymbol{h}_{ij}$ is the (complex-valued) $n \times 1$ channel-vector between $\mathrm{BS}_i$ and $\mathrm{MS}_j$, and $\boldsymbol{w}_i$ is the beamforming vector used by $\mathrm{BS}_i$. The variables $e_1$, $e_2$ are noise terms which we model as i.i.d. Gaussian with zero mean and variance $\sigma^2$.

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[1]Single-stream transmission (scalar coding followed by beamforming) is optimal under certain circumstances, for example provided that $\mathrm{BS}_i$ knows $\boldsymbol{h}_{ii}$ and $\mathrm{MS}_1$, $\mathrm{MS}_2$ treat the interference as Gaussian noise [7], [9].

Under these assumptions the following rates are achievable:

$$R_1(\boldsymbol{w}_1, \boldsymbol{w}_2) = \log_2\left(1 + \frac{|\boldsymbol{w}_1^T \boldsymbol{h}_{11}|^2}{|\boldsymbol{w}_2^T \boldsymbol{h}_{21}|^2 + \sigma^2}\right) \qquad (1)$$

for the link $\text{BS}_1 \rightarrow \text{MS}_1$, and

$$R_2(\boldsymbol{w}_1, \boldsymbol{w}_2) = \log_2\left(1 + \frac{|\boldsymbol{w}_2^T \boldsymbol{h}_{22}|^2}{|\boldsymbol{w}_1^T \boldsymbol{h}_{12}|^2 + \sigma^2}\right) \qquad (2)$$

for $\text{BS}_2 \rightarrow \text{MS}_2$. For fixed channels $\{\boldsymbol{h}_{ij}\}$, and under the power constraint

$$\|\boldsymbol{w}_i\|^2 \leq 1, \quad i = 1, 2, \qquad (3)$$

we define the achievable rate region as

$$\mathcal{R} = \bigcup_{\boldsymbol{w}_1, \boldsymbol{w}_2, \|\boldsymbol{w}_i\|^2 \leq 1} (R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_1, \boldsymbol{w}_2)).$$

## III. Main results

Define the sum-rate as follows

$$R(\boldsymbol{w}_1, \boldsymbol{w}_2) \triangleq R_1(\boldsymbol{w}_1, \boldsymbol{w}_2) + R_2(\boldsymbol{w}_1, \boldsymbol{w}_2).$$

Note that $R$ is a function of $\boldsymbol{h}_{ij}$ and $\boldsymbol{w}_i$ (where the notation makes the latter dependence explicit).

*Proposition 1:* At high SNR, ZF is sum-rate optimal. More precisely

$$\lim_{\sigma \to 0} \text{argmax}_{\|\boldsymbol{w}_1\|^2 \leq 1, \|\boldsymbol{w}_2\|^2 \leq 1} R(\boldsymbol{w}_1, \boldsymbol{w}_2) = (\boldsymbol{w}_1^{\text{ZF}}, \boldsymbol{w}_2^{\text{ZF}}),$$

where

$$\boldsymbol{w}_1^{\text{ZF}} = \frac{\Pi^{\perp}_{\boldsymbol{h}^*_{12}} \boldsymbol{h}^*_{11}}{\left\|\Pi^{\perp}_{\boldsymbol{h}^*_{12}} \boldsymbol{h}^*_{11}\right\|}$$

and

$$\boldsymbol{w}_2^{\text{ZF}} = \frac{\Pi^{\perp}_{\boldsymbol{h}^*_{21}} \boldsymbol{h}^*_{22}}{\left\|\Pi^{\perp}_{\boldsymbol{h}^*_{21}} \boldsymbol{h}^*_{22}\right\|}.$$

*Proposition 2:* At low SNR, MRT is sum-rate optimal. More precisely

$$\lim_{\sigma \to \infty} \text{argmax}_{\|\boldsymbol{w}_1\|^2 \leq 1, \|\boldsymbol{w}_2\|^2 \leq 1} R(\boldsymbol{w}_1, \boldsymbol{w}_2) = (\boldsymbol{w}_1^{\text{MRT}}, \boldsymbol{w}_2^{\text{MRT}}),$$

where

$$\boldsymbol{w}_1^{\text{MRT}} = \frac{\boldsymbol{h}^*_{11}}{\|\boldsymbol{h}_{11}\|} \quad \text{and} \quad \boldsymbol{w}_2^{\text{MRT}} = \frac{\boldsymbol{h}^*_{22}}{\|\boldsymbol{h}_{22}\|}.$$

*Proof:* (of Propositions 1 and 2) We note first from Corollary 1 in [10] that for any rate point on the Pareto boundary of the achievable region $\mathcal{R}$, we must have

$$\boldsymbol{w}_1 = \alpha_1 \frac{\Pi_{\boldsymbol{h}^*_{12}} \boldsymbol{h}^*_{11}}{\|\Pi_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\|} + \sqrt{1 - \alpha_1^2} \frac{\Pi^{\perp}_{\boldsymbol{h}^*_{12}} \boldsymbol{h}^*_{11}}{\|\Pi^{\perp}_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\|} \qquad (4)$$

and

$$\boldsymbol{w}_2 = \alpha_2 \frac{\Pi_{\boldsymbol{h}^*_{21}} \boldsymbol{h}^*_{22}}{\|\Pi_{\boldsymbol{h}_{21}} \boldsymbol{h}_{22}\|} + \sqrt{1 - \alpha_2^2} \frac{\Pi^{\perp}_{\boldsymbol{h}^*_{21}} \boldsymbol{h}^*_{22}}{\|\Pi^{\perp}_{\boldsymbol{h}_{21}} \boldsymbol{h}_{22}\|}, \qquad (5)$$

where $\alpha_1, \alpha_2$ are real-valued and $0 \leq \alpha_1, \alpha_2 \leq 1$. Hence, the beamforming vectors corresponding to the optimal sum-rate point have the form of (4)–(5). Also, the point $(\alpha_1, \alpha_2) = (0, 0)$ corresponds to zero-forcing: $\boldsymbol{w}_1 = \boldsymbol{w}_1^{\text{ZF}}$ and $\boldsymbol{w}_2 = \boldsymbol{w}_2^{\text{ZF}}$. Furthermore,

$$(\alpha_1, \alpha_2) = \left(\frac{\|\Pi_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\|}{\|\boldsymbol{h}_{11}\|}, \frac{\|\Pi_{\boldsymbol{h}_{21}} \boldsymbol{h}_{22}\|}{\|\boldsymbol{h}_{22}\|}\right)$$

corresponds to maximum-ratio transmission: $\boldsymbol{w}_1 = \boldsymbol{w}_1^{\text{MRT}}$ and $\boldsymbol{w}_2 = \boldsymbol{w}_2^{\text{MRT}}$.

Let

$$\begin{aligned}
\gamma_1 &\triangleq \|\Pi_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\|, \\
\gamma_2 &\triangleq \|\Pi_{\boldsymbol{h}_{21}} \boldsymbol{h}_{22}\|, \\
\xi_1 &\triangleq \left\|\Pi^{\perp}_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\right\|, \\
\xi_2 &\triangleq \left\|\Pi^{\perp}_{\boldsymbol{h}_{21}} \boldsymbol{h}_{22}\right\|, \\
\lambda_1 &\triangleq \frac{|\boldsymbol{h}_{21}^H \boldsymbol{h}_{22}|^2}{\left\|\Pi_{\boldsymbol{h}^*_{21} \boldsymbol{h}^*_{22}}\right\|}, \\
\lambda_2 &\triangleq \frac{|\boldsymbol{h}_{12}^H \boldsymbol{h}_{11}|^2}{\left\|\Pi_{\boldsymbol{h}^*_{12} \boldsymbol{h}^*_{11}}\right\|}.
\end{aligned}$$

Then we can write

$$\begin{aligned}
R(\boldsymbol{w}_1, \boldsymbol{w}_2) &= R(\alpha_1, \alpha_2) \\
&= \log\left(1 + \frac{(\alpha_1 \gamma_1 + \sqrt{1 - \alpha_1^2} \xi_1)^2}{\sigma^2 + \alpha_2^2 \lambda_1}\right) \\
&\quad + \log\left(1 + \frac{(\alpha_2 \gamma_2 + \sqrt{1 - \alpha_2^2} \xi_2)^2}{\sigma^2 + \alpha_1^2 \lambda_2}\right)
\end{aligned} \qquad (6)$$

Applying Lemma 2 (see Appendix) proves Proposition 1. Next, observe that

$$\|\boldsymbol{h}_{11}\| = \sqrt{\|\Pi_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\|^2 + \left\|\Pi^{\perp}_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\right\|^2}$$

and

$$\|\boldsymbol{h}_{22}\| = \sqrt{\|\Pi_{\boldsymbol{h}_{21}} \boldsymbol{h}_{22}\|^2 + \left\|\Pi^{\perp}_{\boldsymbol{h}_{21}} \boldsymbol{h}_{22}\right\|^2}.$$

Applying Lemma 3 (see Appendix) proves Proposition 2. ■

## IV. Illustration

To illustrate the results, we make use of the following variant of the Pareto boundary parameterization. This parameterization is proven in [10] and as explained therein it also has a game-theoretic interpretation.

*Proposition 3:* Any point on the Pareto boundary is achievable with the beamforming strategies

$$\begin{aligned}
\boldsymbol{w}_1(\lambda_1) &= \frac{\lambda_1 \boldsymbol{w}_1^{\text{MRT}} + (1 - \lambda_1) \boldsymbol{w}_1^{\text{ZF}}}{\|\lambda_1 \boldsymbol{w}_1^{\text{MRT}} + (1 - \lambda_1) \boldsymbol{w}_1^{\text{ZF}}\|} \quad \text{and} \\
\boldsymbol{w}_2(\lambda_2) &= \frac{\lambda_2 \boldsymbol{w}_2^{\text{MRT}} + (1 - \lambda_2) \boldsymbol{w}_2^{\text{ZF}}}{\|\lambda_2 \boldsymbol{w}_2^{\text{MRT}} + (1 - \lambda_2) \boldsymbol{w}_2^{\text{ZF}}\|}
\end{aligned} \qquad (7)$$

for some $0 \leq \lambda_1, \lambda_2 \leq 1$.

The parameterization of the Pareto boundary of the two-user MISO IFC in Proposition 3 can be interpreted in the following sense: The strategy $\lambda = 0$ corresponds to a completely selfish behavior. On the other hand, $\lambda = 1$ corresponds to a completely altruistic behavior. By choosing a certain $\lambda$, the transmitter can choose its level of selfishness or altruism. Note that there is a one-to-one mapping between $\lambda_k$

and $\alpha_k$ in Section III. In particular, for the parameterization in (7), $\lambda_k = 0$ corresponds to ZF transmission while $\lambda_k = 1$ corresponds to MRT.

In the numerical results, we will use the parameterization in (7) to illustrate the complete achievable rate region. In this region, we mark the ZF and MRT point. As the SNR is increased for a certain set of channel realizations, we can observe that the ZF and MRT rate points behave as predicted theoretically in the last section.

In Figure 2, we show the achievable rate region, along with the MRT and ZF points for a two-user MISO IFC with two antennas at each transmitter, and for fixed but randomly chosen channel realizations. Illustrations are provided for different SNRs in the range $\{-30, -10, 0, 10, 30\}$ dB. The asymptotic optimality of MRT for small SNR and ZF for high SNR can be clearly observed. The path of these two operating modes cross at an SNR of about 0 dB.

## V. Discussion

We provide an interpretation and alternative derivation of Proposition 2 using the results of [12]. Specifically, [12] analyzed the low-SNR regime for a communication link and introduced two performance measures, namely the $\left(\frac{E_b}{N_0}\right)_{\min}$ and the wideband slope $S_0$. Reference [12] then showed that the system bandwidth $B$, the transmission rate $R$, the transmit power $P$ and the spectral efficiency $C(\frac{E_b}{N_0})$ satisfy the fundamental limit

$$\frac{R}{B} \leq C\left(\frac{E_b}{N_0}\right). \tag{8}$$

The function $C\left(\frac{E_b}{N_0}\right)$ is directly related to the common capacity expression $\mathrm{C}(\mathrm{SNR})$, i.e. $C\left(\frac{E_b}{N_0}\right) = \mathrm{C}(\mathrm{SNR})$ for the SNR which solves

$$\frac{E_b}{N_0}\mathrm{C}(\mathrm{SNR}) = \mathrm{SNR}.$$

At low SNR, the function $C(\frac{E_b}{N_0})$ can be expressed (see [12]) as

$$C\left(\frac{E_b}{N_0}\right) \approx \frac{S_0}{3\mathrm{dB}}\left(\frac{E_b}{N_0}\Big|_{\mathrm{dB}} - \frac{E_b}{N_0}_{\min}\Big|_{\mathrm{dB}}\right), \tag{9}$$

with

$$\left(\frac{E_b}{N_0}\right)_{\min} = \frac{\log_e 2}{\dot{\mathrm{C}}(0)} \quad \text{and} \quad S_0 = \frac{2\left[\dot{\mathrm{C}}(0)\right]^2}{-\ddot{\mathrm{C}}(0)}. \tag{10}$$

The closer $\frac{E_b}{N_0}$ gets to $\left(\frac{E_b}{N_0}\right)_{\min}$ the better is the approximation in (9). Note, that the first and second derivative in (10) are taken of the function common capacity function $\mathrm{C}(\mathrm{SNR})$.

The low SNR approximation was recently extended to the multiple access networks in [13] in terms of its (robust) slope region. For the multiple access channel, there is basically no difference in terms of minimum $\frac{E_b}{N_0}$ between MAC and single-user channels. The following result generalizes this to the MISO interference channel. Define $\mathrm{SNR} = \frac{1}{\rho}$.

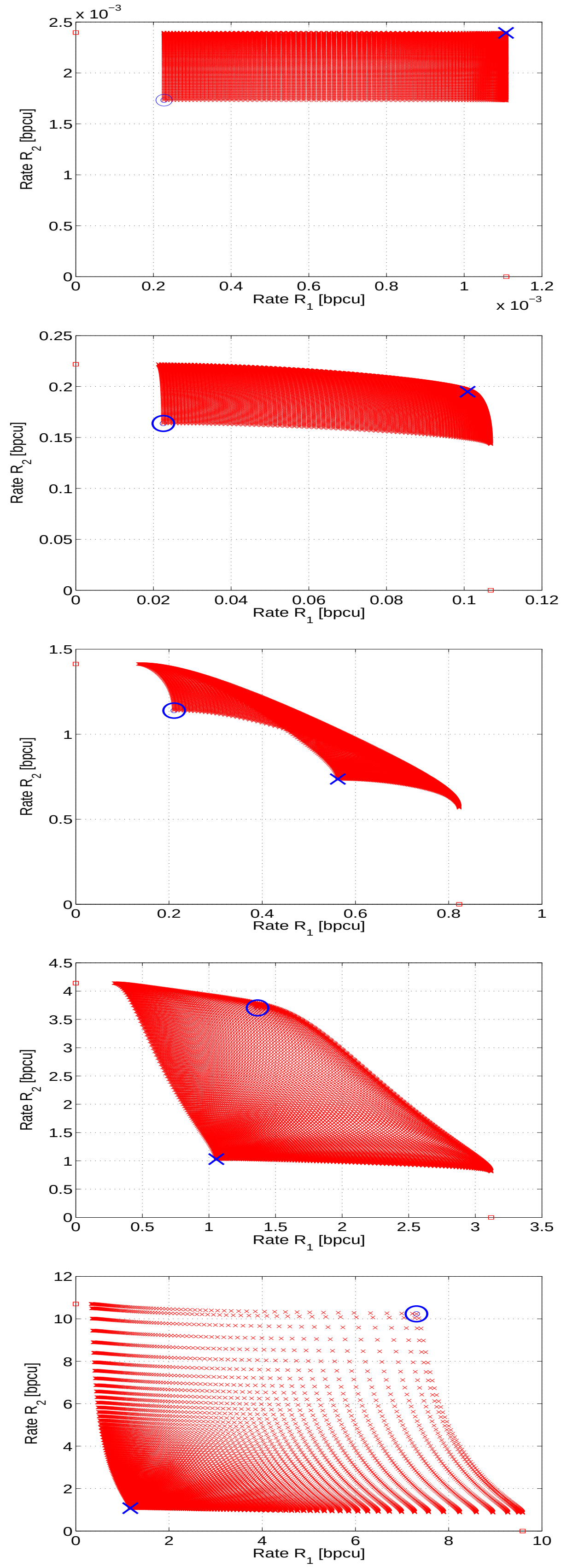


Fig. 2. Examples of achievable rate regions for the two-user two-antenna MISO IFC. The red points correspond to the parameterization in (7). The red squares are the two single-user points. The blue circles correspond to ZF beamforming rates. The blue crosses correspond to the MRT beamforming rates. The SNR is $\{-30, -10, 0, 10, 30\}$ dB.

*Proposition 4:* Let $C_k(\mathrm{SNR}) = \log\left(1 + \mathrm{SNR}|\boldsymbol{h}_{kk}^T \boldsymbol{w}_k|^2\right)$ for $1 \le k \le 2$. Then, even in interference channels, the following minimum $\frac{E_b}{N_0}$ is achievable

$$\frac{E_b}{N_{0\,\min}} = \left[\frac{\partial C_k(0)}{\partial \mathrm{SNR}}\right]^{-1}. \tag{11}$$

*Proof:* We follow closely the proof of [12, Theorem 8] and start with an upper bound on the achievable rate where we assume that the receiver knows the codewords of the other users perfectly and subtract them before decoding the intended user. The capacity for user $k$, $1 \le k \le 2$, in this genie-aided setting is

$$\overline{C}_k(\mathrm{SNR}) = \log\left(1 + \mathrm{SNR}|\boldsymbol{h}_{kk}^T \boldsymbol{w}_k|^2\right), \tag{12}$$

whose derivative at SNR $= 0$ is equal to the expression in (11). To lower-bound the capacity we apply a receiver which treats the interference of the other user as additive noise. The lower bound is

$$\underline{C}_k(\mathrm{SNR}) = \log\left(1 + \frac{\mathrm{SNR}|\boldsymbol{h}_{kk}^T \boldsymbol{w}_k|^2}{\sigma^2 + I_j \mathrm{SNR}}\right), \tag{13}$$

where $I_j = |\boldsymbol{h}_{jk}^T \boldsymbol{w}_j|^2$ is the interference caused by the other user. The function in (13) has a derivative at SNR $= 0$ which is identical to that in (11). This completes the proof because upper and lower bound converge to the same minimum $\frac{E_b}{N_0}$. ■

One interesting observation is that the optimal (in terms of minimum $\frac{E_b}{N_0}$) receiver at the mobiles is the receiver which treats the interference simply as additional additive noise.

With this background we are able to restate Proposition 2 and provide an alternative proof.

*Proposition 5:* The beamforming vectors which optimize the minimum $\frac{E_b}{N_0}$ correspond to the MRT beamformers.

*Proof:* The result follows directly from the characterization in (11) because

$$\frac{E_b}{N_{0\,\min,1}} = \frac{1}{|\boldsymbol{h}_{11}^T \boldsymbol{w}_1|^2} \text{ and } \frac{E_b}{N_{0\,\min,2}} = \frac{1}{|\boldsymbol{h}_{22}^T \boldsymbol{w}_2|^2}$$

which is minimized by the MRT beamforming vectors. ■

## VI. Sum-rate maximization by monotonic optimization

In applications, one is typically interested in finding specific points on the Pareto boundary, a notable example being the sum-rate point. The main difficulty with solving the sum-rate maximization problem is that the problem is non-convex. Using the parameterization in Proposition 3, this maximization problem can be posed as

$$\max_{0 \le \lambda_1, \lambda_2 \le 1} \left\{R_1(\boldsymbol{\lambda}) + R_2(\boldsymbol{\lambda})\right\}, \tag{14}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2]$. In this section, we shall illustrate that problem (14) can be solved using polyblock algorithm for monotonic optimization [14]. Effectively the approach is to transform the original nonconvex objective function (14) into a strictly increasing function over a constraint set that is normal. The price to pay is that the dimension of the variable space must be enlarged from two (corresponding to $\lambda_1, \lambda_2$) to three. We give a brief description in what follows; more details are available in [15].

The first result relates the sum-rate maximization problem to the area of monotonic optimization.

*Proposition 6:* The maximum sum-rate problem

$$\max_{\boldsymbol{\lambda} \in [0,1]^2} \left\{R_1(\boldsymbol{\lambda}) + R_2(\boldsymbol{\lambda})\right\}$$

is a difference of monotonic functions (d.m.) programming problem.

*Proof:* The result follows as a corollary of Lemma 4 in Appendix II, because the objective function can be rewritten as

$$\begin{aligned} R_1(\boldsymbol{\lambda}) + R_2(\boldsymbol{\lambda}) &= [f_1(\boldsymbol{\lambda}) - g_2(\boldsymbol{\lambda})] + [f_2(\boldsymbol{\lambda}) - g_1(\boldsymbol{\lambda})] \\ &= \underbrace{f_1(\boldsymbol{\lambda}) + f_2(\boldsymbol{\lambda})}_{\phi(\boldsymbol{\lambda})} - [\underbrace{g_2(\boldsymbol{\lambda}) + g_1(\boldsymbol{\lambda})}_{\psi(\boldsymbol{\lambda})}], \end{aligned}$$

where the functions $f_i(\boldsymbol{\lambda})$ and $g_i(\boldsymbol{\lambda})$, $i = 1, 2$ are defined in Appendix II. By Lemma 4 both functions $\phi(\cdot)$ and $\psi(\cdot)$ are monotonically increasing. ■

As a consequence of Proposition 6, problem (14) can be formulated as the following general d.m. problem

$$\max_{\boldsymbol{\lambda} \in [0,1]^2} \{\phi(\boldsymbol{\lambda}) - \psi(\boldsymbol{\lambda})\} \tag{15}$$

with strictly increasing functions $\phi(\cdot)$ and $\psi(\cdot)$. Next, we substitute $\psi(\boldsymbol{\lambda}) = \psi(\mathbf{1})(1 - t)$ in (15) and obtain the equivalent programming problem with $\boldsymbol{x} \triangleq [\lambda_1, \lambda_2, t]$

$$\max\{\underbrace{\phi(\boldsymbol{x}) + \psi(\mathbf{1})(x_3 - 1)}_{\Phi(\boldsymbol{x})}\} \quad \text{s.t.} \quad \boldsymbol{x} \in \mathcal{D} \tag{16}$$

with the constraint set

$$\mathcal{D} = \left\{\boldsymbol{x} \in \mathbb{R}_+^3 : x_1 \le 1, x_2 \le 1, x_3 \le 1 - \frac{\psi(x_1, x_2)}{\psi(\mathbf{1})}\right\}. \tag{17}$$

Two key observations allow us to proceed. First, we note that the function $\Phi(\boldsymbol{x})$ is strictly increasing. Second, we have the following result about the constraint set.

*Lemma 1:* The set $\mathcal{D}$ defined in (17) is normal.

*Proof:* Choose the vector $\boldsymbol{x} \in \mathcal{D}$ and choose any vector $\boldsymbol{y}$ such that $0 \le \boldsymbol{y} \le \boldsymbol{x}$. We need to verify that $\boldsymbol{y} \in \mathcal{D}$. First note that $0 \le y_1 \le x_1 \le 1$ and $0 \le y_2 \le x_2 \le 1$. Also, since $\psi(\cdot)$ is strictly increasing in $x_1, x_2$, we have $\psi(x_1, x_2) \ge \psi(y_1, y_2)$. Since $y_3 \le x_3$, it follows that $\boldsymbol{y} \in \mathcal{D}$, too. ■

Furthermore, the constraint set is compact, bounded, and connected.

We have found that the programming problem in (15) is equivalent to maximization of a strictly increasing function over a normal set. This is a monotonic optimization problem in standard form [14]. Therefore, we can apply the outer polyblock approximation algorithm [14] to solve the sum-rate maximization problem (14). For a detailed description of the monotonic optimization framework and how it can be applied to solve the sum-rate maximization problem (14), see [15].

## VII. Conclusions

The parameterization of the achievable rate region of the two-user MISO IFC leads to a simple structured but non-convex optimization problem. In this work, we try to understand the asymptotic behavior of the optimal beamforming solution at high and low SNR. It turns out that for small SNR, MRT is optimal and the interpretation in terms of minimum $\frac{E_b}{N_0}$ justifies the analysis. For high SNR values, the optimal strategy is ZF beamforming.

## Appendix I
## Low and high SNR results

*Lemma 2:* Let $\gamma_1, \gamma_2, \xi_1, \xi_2, \lambda_1$ and $\lambda_2$ be positive real numbers. For any $\alpha_i \in [0,1]$, $i \geq 0$ and any positive number $\sigma$, consider $R(\alpha_1, \alpha_2)$ in (6). We have that

$$\lim_{\sigma \to 0} \arg\max_{S^2} R_\sigma(\alpha_1, \alpha_2) = (0,0),$$

where $S^2$ is the unit square in $\mathbb{R}^2$ defined as

$$S^2 = \{(\alpha_1, \alpha_2) : 0 \leq \alpha_1, \alpha_2 \leq 1\}.$$

*Proof:* Consider the function

$$\begin{aligned} f_\sigma(\alpha_1,\alpha_2) &= 2^{R_\sigma(\alpha_1,\alpha_2)} \\ &= \frac{(\sigma^2+\xi_1^2)(\sigma^2+\xi_2^2)}{\sigma^4[\sigma^2+\lambda_1\alpha_2^2+(\gamma_1\alpha_1+\xi_1\sqrt{1-\alpha_1^2})^2]} \times \\ &\quad \frac{(\sigma^2+\lambda_1\alpha_2^2)(\sigma^2+\lambda_2\alpha_1^2)}{\sigma^2+\lambda_2\alpha_1^2+(\gamma_2\alpha_2+\xi_2\sqrt{1-\alpha_2^2})^2}. \end{aligned} \tag{18}$$

Since the function $2^x$ is continuous and strictly increasing we have that

$$\arg\min_{S^2} R_\sigma(\alpha_1,\alpha_2) = \arg\min_{S^2} f_\sigma(\alpha_1,\alpha_2).$$

We define

$$a_1 = \max_{S^2} \lambda_2\alpha_1^2 + (\gamma_2\alpha_2 + \xi_2\sqrt{1-\alpha_2^2})^2$$

and

$$a_2 = \max_{S^2} \lambda_1\alpha_2^2 + (\gamma_1\alpha_1 + \xi_1\sqrt{1-\alpha_1^2})^2.$$

Since the numerator in the function $f_\sigma(\alpha_1,\alpha_2)$ is always non-negative, we obtain

$$f_\sigma(\alpha_1,\alpha_2) \geq \frac{(\sigma^2+\xi_1^2)(\sigma^2+\xi_2^2)(\sigma^2+\lambda_1\alpha_2^2)(\sigma^2+\lambda_2\alpha_1^2)}{\sigma^4(\sigma^2+a_1)(\sigma^2+a_2)}.$$

We observe that $f_\sigma(0,0) = 1$ and compare the value of $f_\sigma(\alpha_1,\alpha_2)$ with the value $f_\sigma(0,0)$. Consequently we get

$$\begin{aligned} f_\sigma(\alpha_1,\alpha_2) - 1 &\geq \frac{(\sigma^2+\xi_1^2)(\sigma^2+\xi_2^2)(\sigma^2+\lambda_1\alpha_2^2)(\sigma^2+\lambda_2\alpha_1^2)}{\sigma^4(\sigma^2+a_1)(\sigma^2+a_2)} - 1 \\ &= \frac{(\xi_1^2+\xi_2^2+\lambda_1\alpha_2^2+\lambda_2\alpha_1^2-a_1-a_2)\sigma^2}{(\sigma^2+a_1)(\sigma^2+a_2)} \\ &\quad + \frac{(\xi_1^2+\xi_2^2)(\lambda_1\alpha_2^2+\lambda_2\alpha_1^2)+\xi_1^2\xi_2^2+\lambda_1\lambda_2\alpha_1^2\alpha_2^2-a_1a_2}{(\sigma^2+a_1)(\sigma^2+a_2)} \\ &\quad + \frac{\xi_1^2\xi_2^2(\lambda_1\alpha_2^2+\lambda_2\alpha_1^2)+(\xi_1^2+\xi_2^2)\lambda_1\lambda_2\alpha_1^2\alpha_2^2}{\sigma^2(\sigma^2+a_1)(\sigma^2+a_2)} \\ &\quad + \frac{\xi_1^2\xi_2^2\lambda_1\lambda_2\alpha_1^2\alpha_2^2}{\sigma^4(\sigma^2+a_1)(\sigma^2+a_2)} \\ &\geq -\frac{(a_1+a_2)\sigma^2}{(\sigma^2+a_1)(\sigma^2+a_2)} - \frac{a_1a_2}{(\sigma^2+a_1)(\sigma^2+a_2)} \\ &\quad + \frac{\xi_1^2\xi_2^2(\lambda_1\alpha_2^2+\lambda_2\alpha_1^2)}{\sigma^2(\sigma^2+a_1)(\sigma^2+a_2)} \\ &= \frac{-(a_1+a_2)\sigma^4-a_1a_2\sigma^2+\xi_1^2\xi_2^2(\lambda_1\alpha_2^2+\lambda_2\alpha_1^2)}{\sigma^2(\sigma^2+a_1)(\sigma^2+a_2)}. \end{aligned}$$

For an arbitrarily chosen positive number $\varepsilon$ we can find a $\sigma_\varepsilon > 0$ such that

$$(a_1+a_2)\sigma^4 + a_1a_2\sigma^2 \leq \varepsilon/2$$

for all $\sigma \in (0, \sigma_\varepsilon]$. Now for every point $(\alpha_1,\alpha_2) \in S^2$ such that

$$\lambda_1\alpha_2^2 + \lambda_2\alpha_1^2 \geq \varepsilon\xi_1^{-2}\xi_2^{-2}$$

and every $\sigma \in (0, \sigma_\varepsilon]$ we have

$$\begin{aligned} f_\sigma(\alpha_1,\alpha_2) - 1 &\geq \frac{-(a_1+a_2)\sigma^4-a_1a_2\sigma^2+\xi_1^2\xi_2^2(\lambda_1\alpha_2^2+\lambda_2\alpha_1^2)}{\sigma^2(\sigma^2+a_1)(\sigma^2+a_2)} \\ &\geq \frac{-\varepsilon/2+\varepsilon}{\sigma^2(\sigma^2+a_1)(\sigma^2+a_2)} \\ &= \frac{\varepsilon}{2\sigma^2(\sigma^2+a_1)(\sigma^2+a_2)} > 0. \end{aligned}$$

Thus for any $\sigma \in (0, \sigma_\varepsilon]$ the minimum of $f_\sigma(\alpha_1,\alpha_2)$ at a point of $S^2$ within the elipse defined by the equation

$$\lambda_1\alpha_2^2 + \lambda_2\alpha_1^2 = \varepsilon\xi_1^{-2}\xi_2^{-2}.$$

Since $(0,0)$ is within this ellipse for each $\varepsilon > 0$ and its diameter tends to zero when $\varepsilon$ goes to zero, we have that

$$\lim_{\sigma \to 0} \arg\min_{S^2} f_\sigma(\alpha_1,\alpha_2) = (0,0),$$

which proves the lemma. ∎

*Lemma 3:* Let $\gamma_1, \gamma_2, \xi_1, \xi_2, \lambda_1$ and $\lambda_2$ be positive real numbers. For any $\alpha_i \in [0,1]$, $i \geq 0$ and any positive number $\sigma$, consider $R(\alpha_1, \alpha_2)$ in (6). We have that

$$\lim_{\sigma \to +\infty} \arg\max_{S^2} R_\sigma(\alpha_1,\alpha_2) = \left(\frac{\gamma_1}{\sqrt{\gamma_1^2+\xi_1^2}}, \frac{\gamma_2}{\sqrt{\gamma_2^2+\xi_2^2}}\right),$$

where $S^2$ is the unit square in $\mathbb{R}^2$ defined as

$$S^2 = \{(\alpha_1, \alpha_2) : 0 \leq \alpha_1, \alpha_2 \leq 1\}.$$

*Proof:* As in the proof of Lemma 2 we consider the function $f_\sigma(\alpha_1,\alpha_2)$ defined by Equation (18). Clearly, for any positive $\sigma$ the function $f_\sigma(\alpha_1,\alpha_2)$ is continuous on $S^2$ and differentiable infinitely many times on $\mathrm{int}(S^2)$ which is the set of all interior points of $S^2$. We can calculate the first partial derivatives to be

$$\begin{aligned} \frac{\partial f_\sigma}{\partial \alpha_1}(\alpha_1,\alpha_2) &= 2f_\sigma(\alpha_1,\alpha_2) \times \\ &\left( \frac{\lambda_2\alpha_1(\gamma_2\alpha_2+\xi_2\sqrt{1-\alpha_2^2})^2}{(\sigma^2+\lambda_2\alpha_1^2)[\sigma^2+\lambda_2\alpha_1^2+(\gamma_2\alpha_2+\xi_2\sqrt{1-\alpha_2^2})^2]} \right. \\ &\left. - \frac{(\gamma_1\alpha_1+\xi_1\sqrt{1-\alpha_1^2})(\gamma_1-\xi_1\alpha_1/\sqrt{1-\alpha_1^2})}{\sigma^2+\lambda_1\alpha_2^2+(\gamma_1\alpha_1+\xi_1\sqrt{1-\alpha_1^2})^2} \right) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial f_\sigma}{\partial \alpha_2}(\alpha_1,\alpha_2) &= 2f_\sigma(\alpha_1,\alpha_2) \times \\ &\left( \frac{\lambda_1\alpha_2(\gamma_1\alpha_1+\xi_1\sqrt{1-\alpha_1^2})^2}{(\sigma^2+\lambda_1\alpha_2^2)[\sigma^2+\lambda_1\alpha_2^2+(\gamma_1\alpha_1+\xi_1\sqrt{1-\alpha_1^2})^2]} \right. \\ &\left. - \frac{(\gamma_2\alpha_2+\xi_2\sqrt{1-\alpha_2^2})(\gamma_2-\xi_2\alpha_2/\sqrt{1-\alpha_2^2})}{\sigma^2+\lambda_2\alpha_1^2+(\gamma_2\alpha_2+\xi_2\sqrt{1-\alpha_2^2})^2} \right). \end{aligned}$$

Suppose now that $\sigma > 0$ and that $(x_1, x_2) \in \mathrm{int}(S^2)$ is a local optimum of $f_\sigma(\alpha_1,\alpha_2)$. We must then have that

$$\frac{\partial f_\sigma}{\partial \alpha_1}(x_1,x_2) = \frac{\partial f_\sigma}{\partial \alpha_2}(x_1,x_2) = 0.$$

This is equivalent to

$$\gamma_1 - \frac{\xi_1 x_1}{\sqrt{1-x_1^2}} = \frac{\lambda_2 x_1(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})^2}{(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})(\sigma^2+\lambda_1 x_2^2)}\times \frac{\sigma^2+\lambda_1 x_2^2+(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})^2}{\sigma^2+\lambda_2 x_1^2+(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})^2} \tag{19}$$

and

$$\gamma_1 - \frac{\xi_2 x_2}{\sqrt{1-x_2^2}} = \frac{\lambda_1 x_2(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})^2}{(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})(\sigma^2+\lambda_2 x_1^2)}\times \frac{\sigma^2+\lambda_2 x_1^2+(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})^2}{\sigma^2+\lambda_1 x_2^2+(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})^2}. \tag{20}$$

Let us now define

$$a_i = \max_{x_i\in[0,1]} \gamma_i x_i + \xi_i\sqrt{1-x_i^2}$$

and

$$b_i = \min_{x_i\in[0,1]} \gamma_i x_i + \xi_i\sqrt{1-x_i^2}$$

for $i=1,2$. For the right hand side of Equation (19) we have that

$$\frac{\lambda_2 x_1(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})^2}{(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})(\sigma^2+\lambda_1 x_2^2)}\times \frac{\sigma^2+\lambda_1 x_2^2+(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})^2}{\sigma^2+\lambda_2 x_1^2+(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})^2} \geq 0$$

and

$$\frac{\lambda_2 x_1(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})^2}{(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})(\sigma^2+\lambda_1 x_2^2)}\times \frac{\sigma^2+\lambda_1 x_2^2+(\gamma_1 x_1+\xi_1\sqrt{1-x_1^2})^2}{\sigma^2+\lambda_2 x_1^2+(\gamma_2 x_2+\xi_2\sqrt{1-x_2^2})^2} \leq \frac{\lambda_2 a_2^2(\sigma^2+\lambda_1+a_1^2)}{b_1\sigma^2(\sigma^2+b_2^2)}.$$

Since

$$\lim_{\sigma\to+\infty} \frac{\lambda_2 a_2^2(\sigma^2+\lambda_1+a_1^2)}{b_1\sigma^2(\sigma^2+b_2^2)} = 0$$

we have that for an arbitrary $\varepsilon>0$, there exists $\sigma_\varepsilon^{(1)}$ such that

$$\frac{\lambda_2 a_2^2(\sigma^2+\lambda_1+a_1^2)}{b_1\sigma^2(\sigma^2+b_2^2)} \leq \varepsilon$$

and thus

$$0 \leq \gamma_1 - \frac{\xi_1 x_1}{\sqrt{1-x_1^2}} \leq \varepsilon,$$

whenever $\sigma \geq \sigma_\varepsilon^{(1)}$. In a similar fashion we conclude that there exists $\sigma_\varepsilon^{(2)}$ such that

$$0 \leq \gamma_2 - \frac{\xi_2 x_2}{\sqrt{1-x_2^2}} \leq \varepsilon,$$

whenever $\sigma \geq \sigma_\varepsilon^{(2)}$. Choosing $\sigma_\varepsilon$ to be equal to $\max\{\sigma_\varepsilon^{(1)},\sigma_\varepsilon^{(2)}\}$, we obtain that for $\sigma\geq\sigma_\varepsilon$ we have

$$0 \leq \gamma_i - \frac{\xi_i x_i}{\sqrt{1-x_i^2}} \leq \varepsilon, \quad i=1,2. \tag{21}$$

Since the functions $\gamma_i - \xi_i x/\sqrt{1-x^2}$ are continuous for $x\in[0,1)$, we have that for a sufficiently small $\varepsilon$, the inequalities (21) are equivalent to

$$\frac{\gamma_i}{\sqrt{\gamma_i^2+\xi_i^2}} - g(\varepsilon) \leq x_i \leq \frac{\gamma_i}{\sqrt{\gamma_i^2+\xi_i^2}}, \quad i=1,2.$$

Here the function $g(\varepsilon)$ is such that $\lim_{\varepsilon\to 0} g(\varepsilon)=0$. This simply means that for an arbitrary $\delta>0$ we can choose $\varepsilon>0$ and consequently $\sigma_\varepsilon$ such that

$$\frac{\gamma_i}{\sqrt{\gamma_i^2+\xi_i^2}} - \delta \leq x_i \leq \frac{\gamma_i}{\sqrt{\gamma_i^2+\xi_i^2}}, \quad i=1,2, \tag{22}$$

for all $\sigma\geq\sigma_\varepsilon$. We observe that when $\sigma\geq\sigma_\varepsilon$ we have

$$\frac{\partial f_\sigma}{\partial \alpha_i}(\alpha_1,\alpha_2) > 0, \text{ if } \alpha_i > \frac{\gamma_i}{\sqrt{\gamma_i^2+\xi_i^2}}$$

and

$$\frac{\partial f_\sigma}{\partial \alpha_i}(\alpha_1,\alpha_2) < 0, \text{ if } \alpha_i < \frac{\gamma_i}{\sqrt{\gamma_i^2+\xi_i^2}} - \delta,$$

for $i=1,2$. Since for any $\sigma>0$, the function $f_\sigma(\alpha_1,\alpha_2)$ is continuous on $S^2$, this implies that for an arbitrary point $(\alpha_1,\alpha_2)\in S^2$, there exists a point $(x_1,x_2)$ in the square defined by (22) such that $f_\sigma(\alpha_1,\alpha_2)\geq f_\sigma(x_1,x_2)$. Moreover, this inequality is strict if the point $(\alpha_1,\alpha_2)$ does not belong to the square defined by (22). Thus if $(x_1,x_2)$ is a point, where a global minimum of $f_\sigma(\alpha_1,\alpha_2)$ is achieved, then $x_1$ and $x_2$ satisfy (22) for any sufficiently large $\sigma$. This shows that

$$\lim_{\sigma\to+\infty} \arg\min_{S^2} f_\sigma(\alpha_1,\alpha_2) = \left(\frac{\gamma_1}{\sqrt{\gamma_1^2+\xi_1^2}}, \frac{\gamma_2}{\sqrt{\gamma_2^2+\xi_2^2}}\right),$$

This completes the proof of the lemma.

■

## APPENDIX II
## MONOTONIC OPTIMIZATION RESULTS

Let us define the following quantities for further use

$$\begin{aligned} \gamma_{11} &= ||\boldsymbol{h}_{11}||, \qquad \gamma_{12} = ||\boldsymbol{h}_{11}^H \Pi_{\boldsymbol{h}_{12}}^{\perp}||, \\ \gamma_{22} &= ||\boldsymbol{h}_{11}||, \qquad \gamma_{21} = ||\boldsymbol{h}_{22}^H \Pi_{\boldsymbol{h}_{21}}^{\perp}||. \end{aligned}$$

Obviously, the inequalities

$$\gamma_{11} \geq \gamma_{12} \geq 0 \qquad \text{and} \qquad \gamma_{22} \geq \gamma_{21} \geq 0 \tag{23}$$

hold. Define further the functions

$$\begin{aligned} f_1(\boldsymbol{\lambda}) &= \log\left(\sigma_n^2 + |\boldsymbol{w}_1(\lambda_1)^T \boldsymbol{h}_{11}|^2 + |\boldsymbol{w}_2(\lambda_2)^T \boldsymbol{h}_{21}|^2\right), \\ f_2(\boldsymbol{\lambda}) &= \log\left(\sigma_n^2 + |\boldsymbol{w}_2(\lambda_2)^T \boldsymbol{h}_{22}|^2 + |\boldsymbol{w}_1(\lambda_1)^T \boldsymbol{h}_{12}|^2\right), \\ g_1(\boldsymbol{\lambda}) &= \log\left(\sigma_n^2 + |\boldsymbol{w}_1(\lambda_1)^T \boldsymbol{h}_{12}|^2\right), \\ g_2(\boldsymbol{\lambda}) &= \log\left(\sigma_n^2 + |\boldsymbol{w}_2(\lambda_2)^T \boldsymbol{h}_{21}|^2\right). \end{aligned}$$

Finally, let $f(\boldsymbol{\lambda}) = f_1(\boldsymbol{\lambda}) + f_2(\boldsymbol{\lambda})$ and $g(\boldsymbol{\lambda}) = g_1(\boldsymbol{\lambda}) + g_2(\boldsymbol{\lambda})$.

*Lemma 4:* The functions $f_1(\boldsymbol{\lambda}), f_2(\boldsymbol{\lambda}), f(\boldsymbol{\lambda})$ as well as $g_1(\boldsymbol{\lambda}), g_2(\boldsymbol{\lambda}), g(\boldsymbol{\lambda})$ are strictly increasing in $\lambda_1$ and $\lambda_2$.

*Proof:* All six functions depend on $\lambda_1$ or $\lambda_2$ via the following terms

$$\begin{aligned}\alpha_1(\lambda_1) &= |\boldsymbol{w}_1^T(\lambda_1)\boldsymbol{h}_{11}|^2\\ &= \frac{|\left(\lambda_1 \boldsymbol{w}_1^{\text{MRT}} + (1-\lambda_1)\boldsymbol{w}_1^{\text{ZF}}\right)^T \boldsymbol{h}_{11}|^2}{||\lambda_1 \boldsymbol{w}_1^{\text{MRT}} + (1-\lambda_1)\boldsymbol{w}_1^{\text{ZF}}||^2}\\ &= \frac{\left(\lambda_1||\boldsymbol{h}_{11}|| + \frac{(1-\lambda_1)}{||\Pi^\perp_{\boldsymbol{h}_{12}}\boldsymbol{h}_{11}||}(\boldsymbol{h}_{11}^H\Pi^\perp_{\boldsymbol{h}_{12}}\boldsymbol{h}_{11})\right)^2}{\lambda_1^2 + (1-\lambda_1)^2 + 2\lambda_1(1-\lambda_1)\frac{||\boldsymbol{h}_{11}^H\Pi^\perp_{\boldsymbol{h}_{12}}||}{||\boldsymbol{h}_{11}||}}\\ &= \frac{(\lambda_1\gamma_{11} + (1-\lambda_1)\gamma_{12})^2}{1 - 2\lambda_1(1-\lambda_1)(1-\frac{\gamma_{12}}{\gamma_{11}})}.\end{aligned}$$

Similarly, we obtain

$$\begin{aligned}\alpha_2(\lambda_2) &= |\boldsymbol{w}_2^T(\lambda)\boldsymbol{h}_{22}|^2 = \frac{(\lambda_2\gamma_{22} + (1-\lambda_2)\gamma_{21})^2}{1-2\lambda_2(1-\lambda_2)(1-\frac{\gamma_{21}}{\gamma_{22}})},\\ \beta_1(\lambda_1) &= |\boldsymbol{w}_1^T(\lambda)\boldsymbol{h}_{12}|^2 = \frac{\lambda_1^2\gamma_{11}^2}{1-2\lambda_1(1-\lambda_1)(1-\frac{\gamma_{12}}{\gamma_{11}})},\\ \beta_2(\lambda_2) &= |\boldsymbol{w}_2^T(\lambda)\boldsymbol{h}_{21}|^2 = \frac{\lambda_2^2\gamma_{22}^2}{1-2\lambda_2(1-\lambda_2)(1-\frac{\gamma_{21}}{\gamma_{22}})}.\end{aligned}$$

Next, the first derivatives with respect to $\lambda_1$ or $\lambda_2$ are computed directly as

$$\begin{aligned}\frac{d\alpha_1(\lambda_1)}{d\lambda_1} &= \frac{2\gamma_{11}(\lambda_1\gamma_{11} + (1-\lambda_1)\gamma_{12})}{(\gamma_{11} - 2\lambda_1(1-\lambda_1)(\gamma_{11}-\gamma_{12}))^2} \times\\ &\quad (\gamma_{11}^2 - \gamma_{12}^2)(1-\lambda_1) \geq 0, \qquad (24)\end{aligned}$$

where the last inequality follows from (23). The monotonicity of $\alpha_2(\lambda_2)$ follows similarly. The first derivative of $\beta_1(\lambda_1)$ with respect to $\lambda_1$ is given by

$$\frac{d\beta_1(\lambda_1)}{d\lambda_1} = \frac{2\lambda_1\gamma_{11}^3(\gamma_{11}(1-\lambda_1) + \lambda_1\gamma_{12})}{(\gamma_{11} - 2\lambda_1(1-\lambda_1)(\gamma_{11}-\gamma_{12}))^2} \geq 0. \qquad (25)$$

Since $f(\lambda_1, \lambda_2)$ and $g(\lambda_1, \lambda_2)$ can be expressed as

$$\begin{aligned}f(\boldsymbol{\lambda}) &= \log(\sigma_n^2 + \alpha_1(\lambda_1) + \beta_2(\lambda_2))\\ &\quad + \log(\sigma_n^2 + \alpha_2(\lambda_2) + \beta_1(\lambda_1))\end{aligned}$$

and

$$g(\boldsymbol{\lambda}) = \log(\sigma_n^2 + \beta_2(\lambda_2)) + \log(\sigma_n^2 + \beta_1(\lambda_1))$$

the result in Lemma 4 follows from (24) and (25). ■

## REFERENCES


[1] R. Ahlswede, “The capacity region of a channel with two senders and two receivers,” *Ann. Prob.*, vol. 2, pp. 805–814, 1974.

[2] A. B. Carleial, “Interference channels,” *IEEE Trans. on Inf. Theory*, vol. 24, no. 1, pp. 60–70, Jan. 1978.

[3] T. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” *IEEE Trans. on Information Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[4] M. H. M. Costa, “On the Gaussian interference channel,” *IEEE Trans. on Inf. Theory*, vol. 31, pp. 607–615, 1985.

[5] X. Shang, B. Chen, and M. J. Gans, “On the achievable sum rate for MIMO interference channels,” *IEEE Trans. on Inf. Theory*, vol. 52, pp. 4313–4320, 2006.

[6] S. A. Jafar and M. Fakhereddin, “Degrees of freedom for the MIMO interference channel,” *IEEE Trans. on Inf. Theory*, vol. 53, pp. 2637–2642, 2007.

[7] X. Shang and B. Chen, “Achievable rate region for downlink beamforming in the presence of interference,” *Proc. IEEE Asilomar*, 2007.

[8] S. Vishwanath and S. A. Jafar, “On the capacity of vector Gaussian interference channels,” *IEEE ITW*, 2004.

[9] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.

[10] E. Jorswieck, E. G. Larsson and D. Danev, “Complete characterization of the Pareto boundary for the MISO interference channel,” *IEEE Transactions on Signal Processing*. To appear.

[11] E. G. Larsson and E. A. Jorswieck, “Competition and cooperation on the MISO interference channel”, *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 7, pp. 1059–1069, Sept. 2008.

[12] S. Verdú, “Spectral efficiency in the wideband regime,” *IEEE Trans. on Information Theory*, vol. 48, no. 6, pp. 1319–1343, June 2002.

[13] T. Muharemovic, A. Sabharwal, and B. Aazhang, “Policy-based multiple access for decentralized low power systems,” *IEEE Trans. on Wireless Communication*. To appear.

[14] H. Tuy, “Monotonic optimization: Problems and solution approaches,” *SIAM Journal on Optimization*, vol. 11, no. 2, pp. 464–494, 2000.

[15] E. Jorswieck and E. G. Larsson, “Monotonic optimization framework for the two-user MISO interference channel,” *in preparation*, 2008.