On Cellular Capacity with Base Station Cooperation

Li Ping, Peng Wang,
Department of Electronic Engineering,
City University of Hong Kong, Hong Kong SAR
{eeliping, pengwang}@cityu.edu.hk

Hao Wang, and Xiaokang Lin
Department of Electronic Engineering,
Tsinghua University, China
hao-wang04@mails.tsinghua.edu.cn,
linxk@mail.sz.tsinghua.edu.cn

***Abstract*—This paper is concerned with cellular systems with rate constraints in fading environments. We consider a simple but asymptotically optimal matched filtering/successive interference cancellation (MF-SIC) scheme. We derive several concise and closed-form bounds for the power and spectral efficiencies of various base station cooperation strategies, which provide some useful insights into cellular systems.**

I. INTRODUCTION

Cellular capacity is an intriguing problem in wireless communication systems. The conventional approach to this problem treats interference as additive noise [1] and is strictly sub-optimal. Global joint detection with full base station (BS) cooperation is required to achieve optimal performance but then the problem becomes much more complicated. The Wyner model [2] has been widely used to compare the performance of different multiple-access technologies (such as time-division multiple-access (TDMA) and wideband (WB)). This model is based on two assumptions: (i) inter-cell interference is from adjacent cells only; and (ii) all the same-cell users experience the same path loss. These two assumptions, though not very practical, greatly simplify the problem, based on which many insights can be gained [2]-[5]. There have also been efforts to apply multiple-input multiple-output (MIMO) modeling to cellular systems [6]. However, the distributed nature of BSs in cellular systems makes it difficult to derive closed-form capacity expressions. Overall, the explicit characterization of cellular capacity with general channel conditions remains a challenging problem.

This paper addresses the uplink cellular capacity problem using a reasonably realistic system model incorporating path loss, lognormal fading and Rayleigh fading. We employ a simple matched filtering/successive interference cancellation (MF-SIC) strategy, based on which very concise performance formulas are derived. We show that the MF-SIC approach is asymptotically optimal as the average number of simultaneous users in each cell (denoted by K below) increases. The results derived in this paper provide both upper and lower bounds for the true cellular capacity. These bounds are quite tight for reasonably large K values (e.g., $K > 16$). We show that very significant gains are available by introducing BS cooperation.

II. PRELIMINARY

Consider an uplink cellular system with L hexagon cells where L is assumed to be large enough such that edge effects can be ignored. Denote by K the density of simultaneous users per cell, i.e., on average there are K users transmitting simultaneously in each cell. Note that the actual user density supported by the system can be higher than K based on, e.g., TDMA. The results of this paper point to the advantage of allowing K to be as large as possible.

The base station (BS) located at each cell center is equipped with M receive antennas. For convenience, we assume a single transmit antenna at each user side. However, all the results in this paper can be extended to the case of multiple antennas at the user side based on a similar treatment as that used in [7].

Denote by x_k the transmitted signal of user k ($k = 1, 2, \ldots, LK$). Let $\boldsymbol{h}_k = [h_{k,1}, h_{k,2}, \ldots, h_{k,LM}]^T$ where $(\cdot)^T$ denotes transpose and $h_{k,m}$ is the channel coefficient from user k to antenna m. The received signal (considering all BSs) is written as

$$\boldsymbol{y} = \sum\nolimits_{k=1}^{LK} \boldsymbol{h}_k x_k + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{n}$ is a vector of complex additive white Gaussian noise (AWGN) samples with mean zero and variance N_0. Perfect channel state information (CSI) at both transmitters and receivers is assumed.

Assume that each user has the same rate constraint of R/K where R is the system sum rate per cell. This assumption is most applicable to delay-sensitive services such as speech and video. We assume a quasi-static fading environment where scheduling is not a suitable option. (However, TDMA-type time sharing techniques are allowed.) Some detailed assumptions for channel conditions are as follows.

(i) The channel is quasi-static, i.e., the fading coefficients remain constant during each frame;
(ii) Each channel coefficient contains three multiplicative factors, i.e., path loss, lognormal and Rayleigh fading;
(iii) The Rayleigh fading components from each user to different BS antennas are independent and identically distributed (i.i.d.). For each user, the lognormal fading components are equal for the antennas in the same BS and i.i.d. for those in different BSs;
(iv) All users are independent and uniformly distributed (i.u.d.) in the entire cellular area. Hence the distribution of the channel coefficients is the same for every user.

In this paper, we are interested in verifying if the rate constraint for each user can be supported for a given BS cooperation strategy and in finding the corresponding long-term average power and spectral efficiency. In [7], power efficiency is derived for a single-cell MIMO channel. We are now interested in extending the results in [7] to multi-cell environments.

III. CELLULAR CAPACITY WITH FULL BS COOPERATION

Consider a cellular system with full BS cooperation (FBSC). We adopt a sub-optimal matched filtering/successive interference cancellation (MF-SIC) strategy below. Denote by $\|\cdot\|$ the 2-norm of a vector. Let $\boldsymbol{h}_k = \sqrt{g_k}\,\boldsymbol{u}_k$ where $g_k = \|\boldsymbol{h}_k\|^2$ and $\boldsymbol{u}_k$ is a unit vector. The decoding order of SIC applied at the receiver side is determined by the relative values of $\{g_k\}$, i.e., the signal of the user with the largest g_k is decoded first. To

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avoid deep fading, we assume that a user does not transmit if its channel gain is below a threshold G_0 that corresponds to an outage probability ε. This implies that

(i) the power corresponding to the users in outage should be set to zero and not included in computing ATSP;
(ii) active users see less interference.

For convenience, we define an indicator function $I(i, j)$ for any two users i and j as

$$I(i,j)=\begin{cases}0, & \text{if } g_j<G_0 \text{ or } g_i\le g_j;\\ 1, & \text{if } G_0<g_j<g_i.\end{cases} \quad (2)$$

To decode x_k, we use $\boldsymbol{u}_k$ to correlate the received signal, i.e.,

$$\begin{aligned}\boldsymbol{u}_k^*\cdot\boldsymbol{y}&=\boldsymbol{u}_k^*\cdot\left(\sum\nolimits_{i=1}^{K}\boldsymbol{h}_i I(k,i)x_i+\boldsymbol{n}\right)\\ &=\sqrt{g_k}\,x_k+\left(\sum\nolimits_{i=1}^{K}\sqrt{g_i}\,\boldsymbol{u}_k^*\boldsymbol{u}_i I(k,i)x_i+\boldsymbol{u}_k^*\boldsymbol{n}\right)\end{aligned} \quad (3)$$

where $(\cdot)^*$ denotes conjugate transpose. Note that in (3), user k only sees the signals from users $\{i, I(i,j) = 1\}$ as interference (those from users $\{i, I(i,j) = 0\}$ are either in outage or have been successfully decoded and peeled out by SIC). The signal-to-noise ratio (SNR) for user k in (3) (denoted by SNR_k) is

$$\begin{aligned}SNR_k&=\frac{p_k g_k}{\sum\nolimits_{i=1}^{K}p_i g_i\left|\boldsymbol{u}_k^*\boldsymbol{u}_i\right|^2 I(k,i)+N_0}\\ &=\frac{p_k g_k}{\sum\nolimits_{i=1}^{K}p_i g_i\phi_{k,i}I(k,i)+N_0}\end{aligned} \quad (4)$$

where $\phi_{k,i}\equiv\left|\boldsymbol{u}_k^*\boldsymbol{u}_i\right|^2$ and $p_k = \mathrm{E}[|x_k|^2]$ is the transmitted power of user k. Assume rate-R/K ideal coding for each user that achieves Shannon capacity $R/K = \log_2(1+SNR_k)$, $\forall k$. We rewrite (4) as

$$p_k=(2^{R/K}-1)\left(\sum\nolimits_{i=1}^{K}p_i g_i\phi_{k,i}I(k,i)+N_0\right)/g_k . \quad (5)$$

Then $\{p_k\}$ can be calculated recursively using (5) for each channel realization $\{\boldsymbol{h}_k\}$.

Lemma 1: Assume that the amplitudes of $\{h_{k,m}\}$ in (1) are identically distributed and their phases are i.u.d.. Then

$$\mathrm{E}\left[\phi_{k,i}\right]=1/LM,\ \forall\, k\neq i. \quad (6)$$

The proof of Lemma 1 is given in the Appendix. Note that Lemma 1 is applicable to systems with either co-located or distributed receive antennas provided that the related assumptions hold.

Now Let us focus on a user k with channel gain $g_k = g$. Our derivation includes the following steps.

First, we fix $\{g_i\}$ (and $\{I\{k,i\}$ as well) and take averages on both sides of (5) with respect to $\{\phi_{k,i}\}$. From lemma 1, we have

$$\mathrm{E}\left(p_k\right)=(2^{R/K}-1)\left(\sum_{i=1}^{K}\frac{\mathrm{E}\left(p_i\right)g_i I(k,i)}{LM}+N_0\right)g^{-1} \quad (7)$$

or in a non-recursive form

$$\mathrm{E}\left(p_k\right)=N_0\left(2^{R/K}-1\right)\prod_{i=1}^{K}\left(1+\left(2^{R/K}-1\right)I(k,i)/LM\right)g^{-1}. \quad (8)$$

Here (8) gives the average transmission power for a user with channel gain g and decoding order $\{I(k,i)\}$, which is independent of the detailed channel gains of others users.

Second, we take averages on both sides of (8) with respect to $\{I(k,i)\}$, i.e., the decoding order. Denote by $F(\cdot)$ the cumulative distribution function (CDF) of g. For user k under consideration with channel gain g, the probability that user i's channel gain is small than g is simply $F(g)$, which implies

$$\mathrm{E}(I(k,i))=F(g)-\varepsilon . \quad (9)$$

Here the term ε is related to the situation when user i is in outage (thus no interference from user i). From (9) we have

$$\mathrm{E}\left(p_k\right)=N_0\left(2^{R/K}-1\right)\prod_{i=1}^{K}\left(1+\left(2^{R/K}-1\right)\left(F(g)-\varepsilon\right)/LM\right)g^{-1}. \quad (10)$$

Hereafter we will always assume $L\to\infty$. Define

$$p(g)\equiv\lim_{L\to\infty}\mathrm{E}\left(p_k\right)=N_0(2^{R/K}-1)e^{K(2^{R/K}-1)\left(F(g)-\varepsilon\right)/M}g^{-1}. \quad (11)$$

Clearly, $p(g)$ is a deterministic function of g. It is the average transmission power for a user who sees channel gain g.

Finally, the average of p(g) with respect to channel gain g is

$$\begin{aligned}\mathrm{E}[p(g)]&=\int_{G_0}^{\infty}p(g)f(g)dg\\ &=N_0\int_{G_0}^{\infty}(2^{R/K}-1)e^{K(2^{R/K}-1)F(g)/M}g^{-1}f(g)dg\end{aligned} \quad (12)$$

where $f(g) = F'(g)$ is the probability density function (PDF) of g.

Note: $\mathrm{E}[p(g)]$ is the long-term average transmission power of each user in a block fading environment over all possible block-wise channel realizations, which is different from the definition of ergodic performance for fast fading channels.

From assumption (iv) in Section II, all users have the same $f(g)$. Hence the long-term average sum-power is K times of that of a single user. Denote by $P^{MF\text{-}SIC}(R, K)$ the average transmitted sum power (ATSP) per cell for the MF-SIC strategy. Then

$$\begin{aligned}P^{MF-SIC}(R,K)&=K\mathrm{E}[p(g)]\\ &=N_0\int_{G_0}^{\infty}K(2^{R/K}-1)e^{K(2^{R/K}-1)\left(F(g)-\varepsilon\right)/M}g^{-1}f(g)dg .\end{aligned} \quad (13)$$

The term inside the integration in (13) is continuous and uniformly convergent provided that $f(g)$ is continuous. Therefore when $K\to\infty$, we can exchange the order of limit and integration. This leads to

$$P^{MF-SIC}(R,\infty)=N_0\int_{G_0}^{\infty}R\ln 2\cdot 2^{R\left(F(g)-\varepsilon\right)/M}g^{-1}f(g)dg . \quad (14)$$

As MF-SIC is a specific detection strategy, so

$$P^{Opt}(R,K)\le P^{MF-SIC}(R,K) \quad (15)$$

where $P^{Opt}(R, K)$ is the minimum ATSP per cell achieved by the optimal strategy.

Similar to the conclusion in [7], we can show the following.

Lemma 2: The MF-SIC strategy is asymptotically optimal when $K\to\infty$, i.e.,

$$P^{Opt}(R,\infty)=P^{MF-SIC}(R,\infty). \quad (16)$$

Lemma 3: $P^{Opt}(R, K)$ is a monotonically decreasing function of K.

The proofs of lemmas 2 and 3 are similar to their counterparts for single-cell MIMO systems derived in [7] and omitted here due to space limitations. Combining (12)-(15), we have the following theorems.

Theorem 1: For a cellular system with FBSC, the minimum ATSP per cell is lower- and upper-bounded by

$$P^{MF-SIC}(R,\infty) \le P^{Opt}(R,K) \le P^{MF-SIC}(R,K)\ . \qquad (17)$$

These two bounds converge when $K\rightarrow\infty$.

Theorem 2: Given sum power P per cell, the asymptotic cellular capacity for $K\rightarrow\infty$ is the solution of C (in terms of bits/channel-use/cell) in the following equation.

$$P = N_0 \int_{G_0}^{\infty} C \ln 2 \cdot 2^{C(F(g)-\varepsilon)/M} g^{-1} f(g) dg\ . \qquad (18)$$

We now present some examples with the following channel conditions. The edge-length of every cell is normalized to 1. We only consider the channel coefficients from a user to the BS antennas within 20 cycles (i.e., the nearest 187 cells). Path loss follows a fourth power law. The standard deviation for lognormal fading is 8. The outage probability is set at $\varepsilon = 0.01$. Each BS is equipped with a single receive antenna, i.e., $M = 1$.

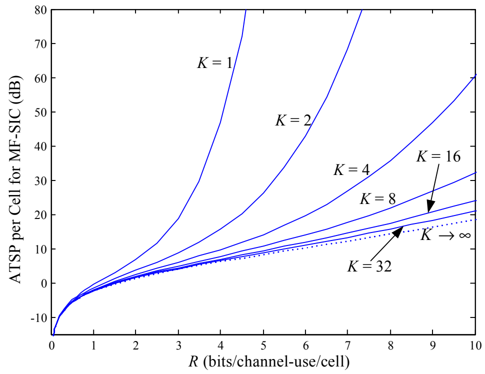

Fig. 1. The power efficiency of the MF-SIC strategy in a cellular system with FBSC and different densities of simultaneous users K. $M = 1$.

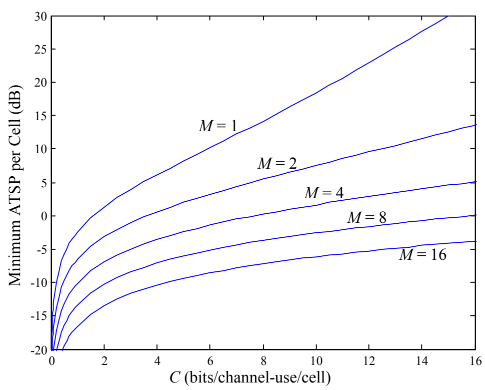

Fig. 2. The capacities of cellular systems with FBSC and different M. The density of simultaneous users is assumed to be high enough ($K\rightarrow\infty$).

Fig. 1 plots the upper bound $P^{MF\text{-}SIC}(R, K)$ and lower bound $P^{MF\text{-}SIC}(R, \infty)$ in Theorem 1. When $K = 16$, these two bounds are already very close. They provide a reasonably accurate estimation of $P^{Opt}(R, K)$ following Theorem 1.

We can clearly see from Fig. 1 the significant power savings due to the increase of simultaneous user density K, indicating that allowing more simultaneous users is advantageous. This agrees with the observation made in [3].

Fig. 2 compares the asymptotic cellular capacities for different M using Theorem 2. We can see that increasing M is a very effective way to save power. It is interesting to see that $P^{Opt}(R, \infty)$ is almost flat when both C and M are large. In this case, throughput can be significantly enhanced using minimal additional transmission power.

IV. Partial Signal Utilization

The discussion in Section III is based on full utilization of signals at all BSs. We now consider a more realistic approach with partial signal utilization. More specifically, we focus on a partially matched filtering/SIC (PMF-SIC) strategy in which the signal for each user is collected only from a limited number of BSs. We have seen from Figs. 1 and 2 that cellular capacity is power limited with MF-SIC. In the following, we will see that cellular capacity becomes interference limited with PMF-SIC, which is caused by interfering signals that cannot be suppressed by SIC. Note that in general, PMF-SIC still requires global cooperation among all BSs, but it can be implemented in a distributed way (e.g., using the technique in [8]).

A. *Partially Matched Filtering (PMF)*

For each channel realization, we decompose the channel vector $\boldsymbol{h}_k$ of user k to $\boldsymbol{h}_k = \boldsymbol{h}_k^{used} + \boldsymbol{h}_k^{unused}$ where $\boldsymbol{h}_k^{used}$ and $\boldsymbol{h}_k^{unused}$ represent the components used and unused in decoding x_k, respectively. Thus $\boldsymbol{h}_k^{unused} x_k$ represents interference from user k that is not cancelled by SIC. In practice, $\boldsymbol{h}_k^{used}$ can be selected using a variety of criteria. In this paper, we select several BSs that have the highest channel gains with respect to user k to form $\boldsymbol{h}_k^{used}$. The rest BSs then form $\boldsymbol{h}_k^{unused}$.

We rewrite (1) using the above decomposition as

$$\boldsymbol{y} = \sum_{k=1}^{LK} \boldsymbol{h}_k^{used} x_k + \sum_{k=1}^{LK} \boldsymbol{h}_k^{unused} x_k + \boldsymbol{n} = \sum_{k=1}^{LK} \boldsymbol{h}_k^{used} x_k + \boldsymbol{\xi} \qquad (19)$$

where $\boldsymbol{\xi} = \sum_{k=1}^{LK} \boldsymbol{h}_k^{unused} x_k + \boldsymbol{n}$ represents the interference and noise component that cannot be removed by SIC. Note that (19) has a similar form as (1). However, the covariance of $\boldsymbol{\xi}$ is no longer a diagonal matrix for a finite K.

The PMF-SIC strategy is essentially the same as MF-SIC in section III, except that matched filtering is applied to $\{\boldsymbol{h}_k^{used} x_k\}$ only and the signals $\{\boldsymbol{h}_k^{unused} x_k\}$ are treated as un-resolvable interference. Provided that interference signals are known, the power allocation in (5) can be employed to obtain $\{p_k\}$. However, interference signals are in turn determined by $\{p_k\}$. We may therefore use the following iterative method to estimate $\{p_k\}$. First, ignore interference signals and compute $\{p_k\}$. Then estimate interference using $\{p_k\}$, based on which $\{p_k\}$ can be re-computed. This process continues iteratively and may lead to two consequences: a) $\{p_k\}$ converge to some finite values; or b) $\{p_k\}$ diverge to infinity. (It can be shown that $\{p_k\}$ and interference power are non-reducing during the iteration and so they will not oscillate.) We can use the converged cases to obtain an upper bound of the cellular power efficiency and claim a system transmission failure in the diverged cases.

Fig. 3 shows the numerical results for the system transmission failure probability of a cellular system with finite K based on the above iterative process. Each BS is equipped with only a single receive antenna ($M = 1$) and only the best BS with respect to user k is selected for decoding x_k. Other conditions are the same as those used in Figs. 1 and 2. In this case, the system effectively reduces to a non-BS cooperation

(NBSC) one and it further reduces to conventional TDMA when K = 1. From Fig. 3 we can see that allowing more users to transmit simultaneously can greatly increase the achievable system throughput. For example, when the system transmission failure probability is 10^{-2}, the system throughput achieved by K = 32 is about 1.25 bits/channel-use/cell, which is about three times of that achieved by K = 1 (i.e., 0.4 bit/channel-use/cell). The dashed line is for the limiting case of $K\to\infty$, which will be discussed in detail below.

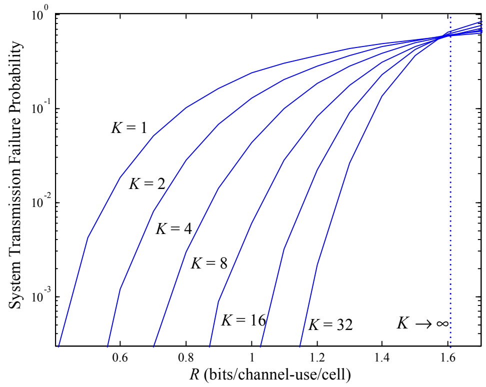

Fig. 3. The system transmission failure probability versus the sum rate per cell in a cellular system with NBSC. M = 1.

B. Asymptotic Performance with $K\to\infty$

So far, we are unable to obtain a closed-form characterization of the power and spectral efficiencies for PMF-SIC with finite K. However, the problem becomes manageable when $K\to\infty$. We now model ξ in (19) as a vector of i.i.d. AWGN samples with zero mean and equal variance. The validity of this model depends on the power allocation method that affects the interference component in ξ. For the PMF-SIC strategy, this modeling has the following consequences when $K\to\infty$:

(i) Every BS antenna receives the same level of interfering power.

(ii) The interference signals at different BS antennas are independent.

These two consequences are consistent with (and so validate) the original modeling of ξ. The detailed derivations of (i) and (ii) are omitted due to space limitation. They are basically based on the law of large numbers and the symmetric property of BS antennas.

Denote by $I_0 + N_0$ the variance of each entry in ξ, where I_0 represents the contribution of interference. Without confusion, we still use $F(\cdot)$ and $f(\cdot)$ to denote the CDF and PDF of channel gain $\| \boldsymbol{h}_k^{used} \|^2$, respectively. Based on the i.i.d. AWGN modeling of ξ above and following a similar procedure as that from (3) to (11), we have

$$p(g) = (I_0 + N_0)(2^{R/K} - 1)e^{K(2^{R/K}-1)(F(g)-\varepsilon)/M} g^{-1} \quad (20)$$

and the ATSP per cell for the PMF-SIC strategy is given by

$$P^{PMF-SIC}(R,\infty) = (I_0 + N_0)\int_{G_0}^{\infty} R\ln 2 \cdot 2^{R(F(g)-\varepsilon)/M} g^{-1} f(g)dg \,. \quad (21)$$

Next we consider the calculation of I_0. Given p_k, the transmitted power of user k, the interference power contributed by this user is $p_k \| \boldsymbol{h}_k^{unused} \|^2$. Considering all LK users in the system, the total interference power is $\sum_{k=1}^{LK} p_k \| \boldsymbol{h}_k^{unused} \|^2$ that is shared by a total number of LM BS antennas. Therefore

$$\begin{aligned} I_0 &= \frac{1}{LM}\sum_{k=1}^{LK} \mathrm{E}\left(p_k \| \boldsymbol{h}_k^{unused} \|^2\right) \\ &\overset{(a)}{=} \frac{LK}{LM} \mathop{\mathrm{E}}_{g}\left(\mathrm{E}\left(p_k \| \boldsymbol{h}_k^{unused} \|^2 \middle| g\right)\right) \\ &\overset{(b)}{=} \frac{K}{M} \mathop{\mathrm{E}}_{g}\left(\mathrm{E}\left(p_k \middle| g\right)\mathrm{E}\left(\| \boldsymbol{h}_k^{unused} \|^2 \middle| g\right)\right) \\ &\overset{(c)}{=} \frac{K}{M} \mathop{\mathrm{E}}_{g}\left(p(g)\mathrm{E}\left(\| \boldsymbol{h}_k^{unused} \|^2 \middle| g\right)\right). \end{aligned} \quad (22)$$

In the above, the equality (a) holds since $\mathrm{E}(\alpha) = \mathop{\mathrm{E}}_{\beta}(\mathrm{E}(\alpha|\beta))$ for any two random variables α and β, (b) holds since p_k and $\| \boldsymbol{h}_k^{unused} \|^2$ are independent for a given signal gain $\| \boldsymbol{h}_k^{used} \|^2 = g$ and (c) follows the definition in (20).

Define $\varphi(g) = \mathrm{E}\left(\| \boldsymbol{h}_k^{unused} \|^2 \middle| g\right)$, which is a deterministic function of g and can be obtained by the Monte-Carlo method. Substituting $\varphi(g)$ and (20) into (22) and letting $K\to\infty$, we obtain

$$\begin{aligned} I_0 &= \lim_{K\to\infty} \frac{K}{M}\int_{G_0}^{\infty} p(g)\varphi(g)f(g)dg \\ &= \lim_{K\to\infty} \frac{I_0+N_0}{M}\int_{G_0}^{\infty} K(2^{\frac{R}{K}}-1)2^{K(2^{\frac{R}{K}}-1)(F(g)-\varepsilon)/M} g^{-1}\varphi(g)f(g)dg \\ &= \frac{I_0+N_0}{M}\int_{G_0}^{\infty} R\ln 2\cdot 2^{R(F(g)-\varepsilon)/M} g^{-1}\varphi(g)f(g)dg \end{aligned} \quad (23)$$

or equivalently

$$I_0 = \frac{N_0\int_{G_0}^{\infty} R\ln 2\cdot 2^{R(F(g)-\varepsilon)/M} g^{-1}\varphi(g)f(g)dg}{M - \int_{G_0}^{\infty} R\ln 2\cdot 2^{R(F(g)-\varepsilon)/M} g^{-1}\varphi(g)f(g)dg}. \quad (24)$$

Note that I_0 should be strictly non-negative. Thus the maximum achievable transmission rate R by PMF-SIC can be evaluated by setting the denominator of (24) to zero. It serves as a lower bound for the true capacity of the cellular system with partial signal utilization.

Theorem 3: For a cellular system with $K\to\infty$ and partial signal ultization, the cellular capacity in each cell is lower-bounded by the solution of R in the following equation.

$$\int_{G_0}^{\infty} R\ln 2\cdot 2^{R(F(g)-\varepsilon)/M} g^{-1}\varphi(g)f(g)dg = M \quad (25)$$

and the ATSP per cell of the PMF-SIC strategy for an achievable throughput R is given by (21) and (24).

Similar to the asymptotical optimality of MF-SIC in Section III, we conjecture that the lower bound above achieved by PMF-SIC is asymptotically tight when $K\to\infty$. However, we have no rigorous conclusion so far.

An interesting conclusion from Theorem 3 is that the number of antennas M plays an important role in the power efficiency and maximum achievable throughput of cellular systems with PMF-SIC. To make this clearer, let us re-write (25) as

$$\int_{\varepsilon}^{1} \frac{R}{M}\ln 2\cdot 2^{\frac{R}{M}(t-\varepsilon)} \cdot \frac{\varphi(F^{-1}(t))}{F^{-1}(t)} dt = 1 \quad (26)$$

where $F^{-1}(\cdot)$ is the inverse of $F(\cdot)$. Note that $\varphi(F^{-1}(t))$ and $F^{-1}(t)$

are both functions of M. We have verified by numerical analysis that the ratio $\varphi(F^{-1}(t))/F^{-1}(t)$ remains almost unchanged with M for the channel model considered here. Thus we expect that the solution of R/M in (26) is approximately a constant for different M. This is indeed the case, as to be seen from the example below.

Denote by B the number of BSs selected for decoding each user's signal. The FBSC scenario is a special case here with $B = \infty$. Another extreme scenario is $B = 1$ where SIC is applied in each cell individually without BS cooperation. Fig. 4 shows the power efficiencies of such NBSC systems with different M when $K\rightarrow\infty$. The corresponding maximum achievable throughputs (computed using (25)) are also plotted for reference. From this figure we can see that the maximum achievable throughput is approximately a linear function of M, with an increase rate of about 1.6 bits/channel-use/cell per additional BS antenna.

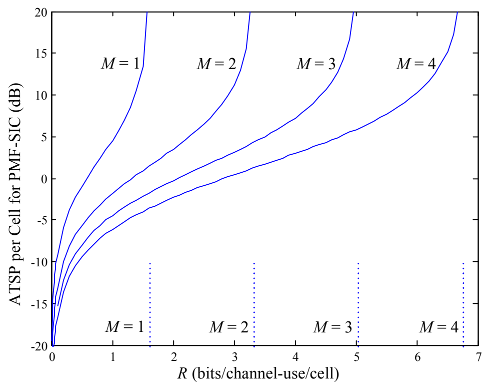

Fig. 4. The power efficiencies of various cellular systems with NBSC and different M (solid curves) when $K\rightarrow\infty$. The corresponding maximum achievable throughputs by PMF-SIC (dashed lines) are also plotted.

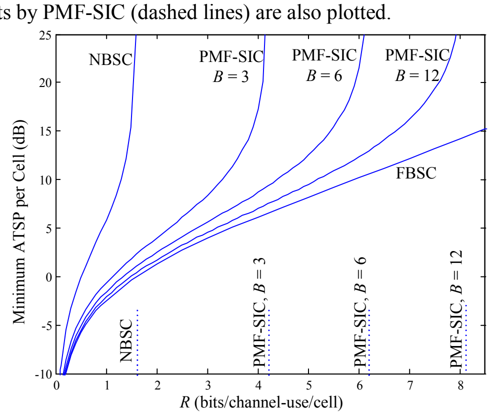

Fig. 5. Performance comparison among cellular systems with various BS cooperation strategies. $M = 1$ and $K\rightarrow\infty$. The corresponding maximum achievable throughputs by PMF-SIC (dashed lines) are also plotted.

Fig. 5 compares the performance of cellular systems with various BS cooperation strategies when $K\rightarrow\infty$. From Fig. 5, we can see that BS cooperation can provide significant power and spectral advantages over conventional NBSC cellular systems. When B is large, the sub-optimal PMF-SIC strategy can achieve performance similar to that of the optimal FBSC strategy.

V. Conclusions

In this paper, we adopt a simple and asymptotically optimal MF-SIC strategy and derive the closed-form expression for the corresponding required sum power, based on which some useful upper and lower bounds are obtained to estimate the capacity and power efficiency of cellular systems. We show that allowing more users to transmit simultaneously and increasing the number of BS antennas at each cell are two efficient ways to improve the cellular system performance with various BS cooperation strategies. We also show that the adaptive PMF-SIC scheme (that can be implemented using the message-passing algorithm introduced in [8]) is a promising approach to the tradeoff between the performance and implementation complexity.

Appendix: Proof of Lemma 1

Proof: Let $\boldsymbol{u}_k = [u_{k,1}, u_{k,2}, \ldots, u_{k,LM}]^T$. By definition, conditioned on given g_k and g_i (for notational simplicity, we do not show this condition explicitly below), we have

$$\mathrm{E}\left[\phi_{k,i}\right] = \mathrm{E}\left[\left|\sum_{m=1}^{LM} u_{k,m}^{*} u_{i,m}\right|^2\right]$$

$$= \mathrm{E}\left(\sum_{m=1}^{LM} |u_{k,m}|^2 |u_{i,m}|^2\right) + \mathrm{E}\left(\sum_{m\neq l} u_{k,m} u_{k,l}^{*} u_{i,m}^{*} u_{i,l}\right)$$

$$= \mathrm{E}\left(\sum_{m=1}^{LM} |u_{k,m}|^2\, \mathrm{E}\left(|u_{i,m}|^2\right)\right) + \mathrm{E}\left(\sum_{m\neq l} u_{k,m} u_{k,l}^{*} \mathrm{E}\left(u_{i,m}^{*} u_{i,l}\right)\right). \quad (27)$$

Since the amplitudes of $\{h_{i,m}\}$ are identically distributed, so are those of $\{u_{i,m}\}$. Thus we have $\mathrm{E}(|u_{i,m}|^2) = \mathrm{E}(|u_{i,l}|^2)$, $\forall\, m \neq l$. Further recall that $\boldsymbol{u}_i$ is a unit vector, i.e., $\sum_{m=1}^{LM} |u_{i,m}|^2 = 1$, so

$$\mathrm{E}(|u_{i,1}|^2) = \mathrm{E}(|u_{i,2}|^2) = \cdots = \mathrm{E}(|u_{i,LM}|^2) = 1/LM. \quad (28)$$

Hence the first term in (27) can be rewritten as

$$\mathrm{E}\left(\sum_{m=1}^{LM} |u_{k,m}|^2\, \mathrm{E}\left(|u_{i,m}|^2\right)\right) = \mathrm{E}\left(\sum_{m=1}^{LM} \frac{|u_{k,m}|^2}{LM}\right) = 1/LM. \quad (29)$$

Since the phases of $\{h_{k,m}\}$ are i.u.d., the second term in (27) is zero after averaging. Hence we have (6).

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