ROBUST DECENTRALIZED STRUCTURE - CONSTRAINED CONTROLLER DESIGN FOR POWER SYSTEMS: AN LMI APPROACH

G. K. Befekadu and I. Erlich
Department of Power Engineering, University of Duisburg-Essen, 47057 Duisburg, Germany
gkbefekadu@uni-duisburg.de and erlich@uni-duisburg.de

Abstract: This paper presents a linear matrix inequality (LMI) approach for designing a robust decentralized structure - constrained controller for power systems. The problem of designing a fixed-structure H_2/H_∞ dynamic output feedback controller is first reformulated as an extension of a static output feedback controller design problem for the extended system. The resulting optimization problem has bilinear matrix inequalities (BMIs) form which is solved using an iterative LMIs programming method. The approach is demonstrated by designing fixed-structure power system stabilizer (PSS) controllers on a four machine test power system so as to determine the optimal parameters. The paper also presents algorithms for solving the iterative LMI programming problem to determine (sub)-optimally the PSSs parameters.

Keywords: Convex optimization, Lyapunov stability, Power system stabilizers, Robust control

1 INRODUCTION

In the last two decades, the electric power industry has experienced significant changes. The deregulation of the electricity markets has led to increasing uncertainties concerning the power flow within the network. This is further compounded by the physical expansion of interconnected networks such as those in Europe, which makes the prediction of system response to disturbances and severe loading condition more difficult. Besides, the ever-increasing utilization of wind energy may impact the load flow and dynamic behaviour of the system considerably. These and other similar developments prompted both power and control engineers to use new controller design techniques and more accurate model descriptions for the power system with the objective of providing reliable electricity services. To meet modern power systems requirements, controllers have to guarantee robustness over a wide range of system operating conditions and this further highlights the fact that robustness is one of the major issues in power system controllers design.

Recently, a number of efforts have been made to extend the application of robust control techniques to power systems, such as L_∞ optimization [1], [2], H_∞-optimization [3], [4], structured singular value (SSV or μ) technique [5] and linear matrix inequalities (LMIs) technique [6], [7]. Interesting robust decentralized controller schemes that are based on the concept of connectively stabilizing a large-scale nonlinear interconnected system for governor/turbine control and exciter control using the LMIs optimization [8] have been presented in [6] and [7], respectively. However, the designed local state feedback controllers need the corresponding state information of the subsystems, which may be either impossible or simply impractical to obtain measurements of the full state for all individual subsystems. The results of [5] also present a robust centralized controller for power systems based on structured singular value (SSV or μ) technique. The disadvantage of such approach, besides the order of the controller which is at least equal to the order of the system, is that the designed controllers require global information exchange about the measured signals. Furthermore, to some extent the result from this approach is turned out to be conservative.

This paper focuses on the extension of linear matrix inequalities (LMIs) based mixed H_2/H_∞ optimization approach to problems of practical interest in power systems. The design problem considered is the natural extension of the reduced order decentralized dynamic output H_2/H_∞ controllers synthesis for power systems. In the design, the fixed - structure H_2/H_∞ dynamic decentralized output feedback controller problem is first reformulated as an extension of static output feedback controller design problem for the extended system. The resulting optimization problem has a general bilinear matrix inequalities (BMIs) form which can be solved using an iterative LMIs programming method based on linearizing the objective functional with respect to its variables.

The approach has a number of practical relevance among which the following are singled out: i) the stability of the controller can be explicitly stated a priori in the fixed structure of the controller, ii) controller gains can be limited in order to avoid designing high gains that are often undesirable for practical implementation, and, iii) multi-objective optimization technique can easily be incorporated in the design by minimizing the H_2/H_∞ norms of the multiple transfer functions between different input/output channels. Moreover, the paper also presents a general approach that can be used for designing any order robust PSS structure controllers in power system. The application of this approach to a multi-machine power system allows a coordinated tuning of controllers that incorporate robustness to changes in the operating conditions as well as model uncertainties in the system.

This paper is organized as follows. In Section 2, the robust PSS controller design problem is formulated in the framework of linear matrix inequalities (LMIs) based mixed H_2/H_∞ optimization. The application of the approach to design robust PSS structured controllers and simulation results together with performance indices are given in Section 3. Finally, in Section 4, a brief conclusion about the paper is given.

2 OUTLINE OF THE PROBLEM

2.1 System model

Consider the general structure of the i^{th} - generator together with the PSS block in a multi-machine power system shown in Figure 1. The input of the i^{th} - controller is connected to the output of the washout stage filter, which prevents the controller from acting on the system during steady state. Let the structure of this i^{th} - washout stage be given by:

$$\Delta y_i(s) = \frac{sT_{wi}}{1+sT_{wi}}\Delta\omega_i(s) \tag{1}$$

To illustrate the design procedure, consider the following first-order PSS controller with *a-priori* assumption made on the value of T_{i2} :

$$K_i\left[\frac{1+sT_{i1}}{1+sT_{i2}}\right] \tag{2}$$

The PSS structure in (2) can be further rewritten in the following form

$$K_i\left[\frac{1+sT_{i1}}{1+sT_{i2}}\right] = \left[K_{i1} + K_{i2}\frac{1}{1+sT_{i2}}\right] \equiv \left[K_{i1} \quad K_{i2}\right]\begin{bmatrix}1\\ 1/(1+sT_{i2})\end{bmatrix} \tag{3}$$

where K_{i1} and K_{i2} are easily identified as gain parameters that are to be determined during the design. Moreover, the gain parameters K_{i1} and K_{i2} together with T_{i2} determine the original parameters K_i and T_{i1}.

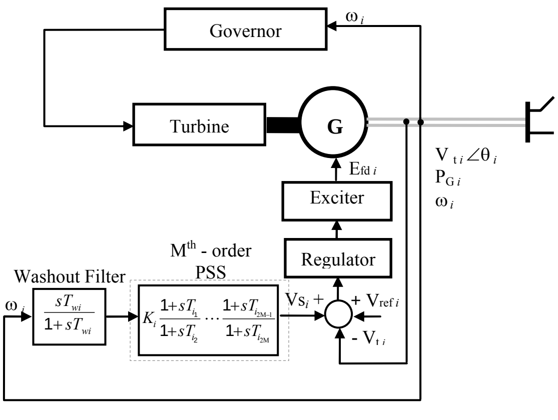

Figure 1: General structure of the i^{th} - generator together with the PSS structure and washout stage in a multimachine power system.

After augmenting the washout stage in the system, the i^{th} - subsystem, within the framework of H_2/H_∞ design, is described by the following state space equation:

$$\begin{aligned}\dot{\mathbf{x}}_i &= \mathbf{A}_{ii}\mathbf{x}_i + \sum_{j\neq i}\mathbf{A}_{ij}\mathbf{x}_j + \mathbf{B}_{i0}\mathbf{w}_{i0} + \mathbf{B}_{i1}\mathbf{w}_{i1} + \mathbf{B}_{i2}\mathbf{u}_i\\ \mathbf{z}_i &= \mathbf{C}_{i1}\mathbf{x}_i + \mathbf{D}_{i10}\mathbf{w}_{i0} + \mathbf{D}_{i11}\mathbf{w}_{i1} + \mathbf{D}_{i12}\mathbf{u}_i\\ \mathbf{y}_i &= \mathbf{C}_{iy}\mathbf{x}_i + \mathbf{D}_{iy0}\mathbf{w}_{i0} + \mathbf{D}_{iy1}\mathbf{w}_{i1}\end{aligned} \tag{4}$$

where $\mathbf{x}_i \in R^{n_{\mathbf{x}_i}}$ is the state variable, $\mathbf{u}_i \in R^{n_{\mathbf{u}_i}}$ is the control input, $\mathbf{y}_i \in R^{n_{\mathbf{y}_i}}$ is the measurement signal, $\mathbf{z}_i \in R^{n_{\mathbf{z}_i}}$ is the regulated variables, $\mathbf{w}_{i0} \in R^{n_{\mathbf{w}_{0i}}}$ and $\mathbf{w}_{i1} \in R^{n_{\mathbf{w}_{1i}}}$ are exogenous signals (assuming that $\mathbf{w}_{i1}$ is either independent of $\mathbf{w}_{i0}$ or dependent causally on $\mathbf{w}_{i0}$) for the i^{th} - subsystem.

Now consider the following approach to design a decentralized robust optimal H_2/H_∞ controllers of the form (3) for the system given in (4), i.e. determining optimally the gains K_{i1} and K_{i2} within the framework of H_2/H_∞ optimization. This implies the incorporation of the dynamic part of the controller first in (4), namely

$$\left[1 \quad 1/(1+sT_{i2})\right]^T \tag{5}$$

and then reformulating the problem as an extension of a static output feedback problem for the extended system. Hence, the state space equation for i^{th} – subsystem becomes:

$$\begin{aligned}\begin{bmatrix}\dot{\mathbf{x}}_i\\ \dot{\mathbf{x}}_{ci}\end{bmatrix} &= \begin{bmatrix}\mathbf{A}_{ii} & \mathbf{0}\\ \mathbf{B}_{ci}\mathbf{C}_{iy} & \mathbf{A}_{ci}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i\\ \mathbf{x}_{ci}\end{bmatrix} + \begin{bmatrix}\sum_{j\neq i}\mathbf{A}_{ij}\mathbf{x}_j\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{B}_{i0}\\ \mathbf{B}_{ci}\mathbf{D}_{iy0}\end{bmatrix}\mathbf{w}_{i0} + \begin{bmatrix}\mathbf{B}_{i1}\\ \mathbf{B}_{ci}\mathbf{D}_{iy1}\end{bmatrix}\mathbf{w}_{i1} + \begin{bmatrix}\mathbf{B}_{i2}\\ \mathbf{0}\end{bmatrix}\mathbf{u}_i\\ \mathbf{z}_i &= \begin{bmatrix}\mathbf{C}_{i1} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i\\ \mathbf{x}_{ci}\end{bmatrix} + \mathbf{D}_{i10}\mathbf{w}_{i0} + \mathbf{D}_{i11}\mathbf{w}_{i1} + \mathbf{D}_{i12}\mathbf{u}_i\\ \tilde{\mathbf{y}}_i &= \begin{bmatrix}\mathbf{D}_{ci}\mathbf{C}_{iy} & \mathbf{C}_{ci}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i\\ \mathbf{x}_{ci}\end{bmatrix} + \mathbf{D}_{ci}\mathbf{D}_{iy0}\mathbf{w}_{i0} + \mathbf{D}_{ci}\mathbf{D}_{iy1}\mathbf{w}_{i1}\end{aligned} \tag{6}$$

where $\mathbf{A}_{ci}$, $\mathbf{B}_{ci}$, $\mathbf{C}_{ci}$ and $\mathbf{D}_{ci}$ are the state space realization of (5) and are given by:

$$\mathbf{A}_{ci} = \left[-1/T_{i2}\right], \qquad \mathbf{B}_{ci} = \left[1/T_{i2}\right], \qquad \mathbf{C}_{ci} = \begin{bmatrix}0\\ 1\end{bmatrix}, \qquad \mathbf{D}_{ci} = \begin{bmatrix}1\\ 0\end{bmatrix} \tag{7}$$

Finally, the overall extended system equation for the system can be rewritten in one state space model as

$$\begin{aligned}\dot{\tilde{\mathbf{x}}} &= \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}_0\mathbf{w}_0 + \tilde{\mathbf{B}}_1\mathbf{w}_1 + \tilde{\mathbf{B}}_2\mathbf{u}\\ \mathbf{z} &= \tilde{\mathbf{C}}_1\mathbf{x} + \tilde{\mathbf{D}}_{10}\mathbf{w}_0 + \tilde{\mathbf{D}}_{11}\mathbf{w}_1 + \tilde{\mathbf{D}}_{12}\mathbf{u}\\ \tilde{\mathbf{y}} &= \tilde{\mathbf{C}}_y\mathbf{x} + \tilde{\mathbf{D}}_{y0}\mathbf{w}_0 + \tilde{\mathbf{D}}_{y1}\mathbf{w}_1 + \tilde{\mathbf{D}}_{y2}\mathbf{u}\end{aligned} \tag{8}$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix}\mathbf{A}_{11} & \mathbf{0} & \mathbf{A}_{12} & \mathbf{0} & \mathbf{A}_{13} & \mathbf{0} & \cdots & \mathbf{A}_{1N} & \mathbf{0}\\ \mathbf{B}_{c1}\mathbf{C}_{12} & \mathbf{A}_{c1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \mathbf{A}_{21} & \mathbf{0} & \mathbf{A}_{22} & \mathbf{0} & \mathbf{A}_{23} & \mathbf{0} & \cdots & \mathbf{A}_{2N} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{c2}\mathbf{C}_{22} & \mathbf{A}_{c2} & \mathbf{0} & \mathbf{0} & \cdots & 0 & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ \mathbf{A}_{N1} & \mathbf{0} & \mathbf{A}_{N2} & \mathbf{0} & \mathbf{A}_{N3} & \mathbf{0} & \cdots & \mathbf{A}_{NN} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_{cN}\mathbf{C}_{N2} & \mathbf{A}_{cN}\end{bmatrix}$$

$$\tilde{\mathbf{B}}_0 = \mathrm{blkdiag}\left\{\begin{bmatrix}\mathbf{B}_{10}\\ \mathbf{B}_{c1}\mathbf{D}_{1y0}\end{bmatrix}, \begin{bmatrix}\mathbf{B}_{20}\\ \mathbf{B}_{c2}\mathbf{D}_{2y0}\end{bmatrix}, \ldots, \begin{bmatrix}\mathbf{B}_{N0}\\ \mathbf{B}_{cN}\mathbf{D}_{Ny0}\end{bmatrix}\right\}$$

$$\tilde{\mathbf{B}}_1 = \mathrm{blkdiag}\left\{\begin{bmatrix}\mathbf{B}_{11}\\ \mathbf{B}_{c1}\mathbf{D}_{1y1}\end{bmatrix}, \begin{bmatrix}\mathbf{B}_{21}\\ \mathbf{B}_{c2}\mathbf{D}_{2y1}\end{bmatrix}, \ldots, \begin{bmatrix}\mathbf{B}_{N1}\\ \mathbf{B}_{cN}\mathbf{D}_{Ny1}\end{bmatrix}\right\}$$

$$\widetilde{\mathbf{B}}_2 = \text{blkdiag}\left\{\begin{bmatrix}\mathbf{B}_{12}\\ \mathbf{0}\end{bmatrix},\begin{bmatrix}\mathbf{B}_{22}\\ \mathbf{0}\end{bmatrix},\ldots,\begin{bmatrix}\mathbf{B}_{N2}\\ \mathbf{0}\end{bmatrix}\right\}$$

$$\widetilde{\mathbf{C}}_1 = \text{blkdiag}\{[\mathbf{C}_{11} \quad \mathbf{0}],[\mathbf{C}_{21} \quad \mathbf{0}],\ldots[\mathbf{C}_{N1} \quad \mathbf{0}]\}$$

$$\widetilde{\mathbf{C}}_2 = \text{blkdiag}\{[\mathbf{D}_{c1}\mathbf{C}_{1y} \quad \mathbf{C}_{c1}],[\mathbf{D}_{c2}\mathbf{C}_{2y} \quad \mathbf{C}_{c2}],\ldots,[\mathbf{D}_{cN}\mathbf{C}_{Ny} \quad \mathbf{C}_{cN}]\}$$

$$\widetilde{\mathbf{D}}_{10} = \text{blkdiag}\{\mathbf{D}_{110},\mathbf{D}_{210},\ldots,\mathbf{D}_{N10}\}$$

$$\widetilde{\mathbf{D}}_{11} = \text{blkdiag}\{\mathbf{D}_{111},\mathbf{D}_{211},\ldots,\mathbf{D}_{N11}\}$$

$$\widetilde{\mathbf{D}}_{12} = \text{blkdiag}\{\mathbf{D}_{112},\mathbf{D}_{212},\ldots,\mathbf{D}_{N12}\}$$

$$\widetilde{\mathbf{D}}_{y0} = \text{blkdiag}\{\mathbf{D}_{c1}\mathbf{D}_{1y0},\mathbf{D}_{c2}\mathbf{D}_{2y0},\ldots,\mathbf{D}_{cN}\mathbf{D}_{Ny0}\}$$

$$\widetilde{\mathbf{D}}_{y1} = \text{blkdiag}\{\mathbf{D}_{c1}\mathbf{D}_{1y1},\mathbf{D}_{c2}\mathbf{D}_{2y1},\ldots,\mathbf{D}_{cN}\mathbf{D}_{Ny1}\}$$

Hence, the static output feedback controller for i^{th}-subsystem is given as:

$$\mathbf{u}_i = \widetilde{\mathbf{K}}_i\,\widetilde{\mathbf{y}}_i \tag{9}$$

where $\widetilde{\mathbf{K}}_i = [K_{i1} \quad K_{i2}]$. Moreover, the decentralized static output feedback controller for the whole system will then have the familiar block structure of the form

$$\mathbf{u} = \widetilde{\mathbf{K}}_D\,\widetilde{\mathbf{y}} \tag{10}$$

where $\widetilde{\mathbf{K}}_D = \text{blkdiag}(\widetilde{\mathbf{K}}_1,\widetilde{\mathbf{K}}_2,\ldots,\widetilde{\mathbf{K}}_N)$.

Substituting the static output feedback strategy (10) into the system equation of (8), the closed loop system will become

$$\begin{aligned}\dot{\widetilde{\mathbf{x}}} &= \mathbf{A}_{cl}\,\widetilde{\mathbf{x}} + \mathbf{B}_{cl0}\mathbf{w}_0 + \mathbf{B}_{cl1}\mathbf{w}_1\\ \mathbf{z} &= \mathbf{C}_{cl1}\mathbf{x} + \mathbf{D}_{cl0}\mathbf{w}_0 + \mathbf{D}_{cl1}\mathbf{w}_1\end{aligned} \tag{11}$$

where

$$\mathbf{A}_{cl} = \widetilde{\mathbf{A}} + \widetilde{\mathbf{B}}_2\widetilde{\mathbf{K}}_D\widetilde{\mathbf{C}}_y,\quad \mathbf{B}_{cl0} = \widetilde{\mathbf{B}}_0 + \widetilde{\mathbf{B}}_2\widetilde{\mathbf{K}}_D\widetilde{\mathbf{D}}_{y0},\quad \mathbf{B}_{cl1} = \widetilde{\mathbf{B}}_1 + \widetilde{\mathbf{B}}_2\widetilde{\mathbf{K}}_D\widetilde{\mathbf{D}}_{y1},$$
$$\mathbf{C}_{cl1} = \widetilde{\mathbf{C}}_1 + \widetilde{\mathbf{D}}_{12}\widetilde{\mathbf{K}}_D\widetilde{\mathbf{C}}_y,\ \mathbf{D}_{cl0} = \widetilde{\mathbf{D}}_{10} + \widetilde{\mathbf{D}}_{12}\widetilde{\mathbf{K}}_D\widetilde{\mathbf{D}}_{y0},\ \mathbf{D}_{cl1} = \widetilde{\mathbf{D}}_{11} + \widetilde{\mathbf{D}}_{12}\widetilde{\mathbf{K}}_D\widetilde{\mathbf{D}}$$

2.2 Suboptimal static H_2/H_∞ output feedback design using iterative LMIs programming method.

Designing a suboptimal static H_2/H_∞ output feedback controller for the extended plant is equivalent to that of finding the gain matrix $\widetilde{\mathbf{K}}_D$ by minimizing the upper bound of the H_2 norm of the transfer function $T_{\mathbf{zw}_0}(s) = \mathbf{C}_{cl0}(s\mathbf{I}-\mathbf{A}_{cl})\mathbf{B}_{cl0} + \mathbf{D}_{cl0}$ from $\mathbf{w}_0$ to measured output $\mathbf{z}$ and which at the same time satisfies an H_∞ norm bound condition on the closed loop transfer function $T_{\mathbf{zw}_1}(s) = \mathbf{C}_{cl1}(s\mathbf{I}-\mathbf{A}_{cl})\mathbf{B}_{cl1} + \mathbf{D}_{cl1}$ from disturbance $\mathbf{w}_1$ to measured output $\mathbf{z}$, i.e. $\|T_{\mathbf{zw}_1}\|_\infty < \gamma$ (for a given scalar constant $\gamma > 0$). Moreover, the transfer functions $T_{\mathbf{zw}_1}(s)$ and $T_{\mathbf{zw}_0}(s)$ must be stable [9].

Designing a static H_∞ output feedback controller for the system given in (8) is reduced to finding of a controller $\widetilde{\mathbf{K}}_D$ and a positive definite matrix $\mathbf{P} > \mathbf{0}$ that satisfy the following equation

$$\begin{bmatrix}\mathbf{P}\mathbf{A}_{cl} + \mathbf{A}_{cl}^T\mathbf{P} & \mathbf{P}\mathbf{B}_{cl1} & \mathbf{C}_{cl1}^T\\ \mathbf{B}_{cl1}^T\mathbf{P} & -\gamma\mathbf{I}_{n_{w1}} & \mathbf{D}_{cl1}^T\\ \mathbf{C}_{cl1} & \mathbf{D}_{cl1} & -\gamma\mathbf{I}_{n_z}\end{bmatrix} < \mathbf{0} \tag{12}$$

Alternatively, using the projection lemma of [10] equation (12) of the BMI form could be transformed into two LMIs equations coupled through a bilinear matrix equation.

$$\mathbf{N}_U^T\begin{bmatrix}\widetilde{\mathbf{A}}\mathbf{Q} + \mathbf{Q}\widetilde{\mathbf{A}}^T + \gamma^{-1}\widetilde{\mathbf{B}}_1\widetilde{\mathbf{B}}_1^T & (\widetilde{\mathbf{C}}_1\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{B}}_1^T)^T\\ \widetilde{\mathbf{C}}_{11}\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{B}}_1^T & \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{D}}_{11}^T - \gamma\mathbf{I}_{n_z}\end{bmatrix}\mathbf{N}_U < \mathbf{0} \tag{13}$$

$$\mathbf{N}_V^T\begin{bmatrix}\mathbf{P}\widetilde{\mathbf{A}} + \widetilde{\mathbf{A}}^T\mathbf{P} + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{C}}_1 & \mathbf{P}\widetilde{\mathbf{B}}_1 + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{D}}_{11}\\ (\mathbf{P}\widetilde{\mathbf{B}}_1 + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{D}}_{11})^T & \gamma^{-1}\widetilde{\mathbf{D}}_{11}^T\widetilde{\mathbf{D}}_{11} - \gamma\mathbf{I}_{n_{w1}}\end{bmatrix}\mathbf{N}_V < \mathbf{0} \tag{14}$$

and

$$\mathbf{P}\mathbf{Q} = \mathbf{I} \tag{15}$$

where $\mathbf{N}_U$ and $\mathbf{N}_V$ denotes arbitrary bases of the nullspaces of $[\widetilde{\mathbf{C}} \quad \widetilde{\mathbf{D}}_{y1}]$ and $[\widetilde{\mathbf{B}}_2^T \quad \widetilde{\mathbf{D}}_{12}^T]$, respectively.

Remark 1: It is possible to find stabilizing controllers which at the same time ensure an α - degree of stability. This additional requirement will introduce terms $2\alpha\mathbf{Q}$ and $2\alpha\mathbf{P}$ for some positive value of α in (13) and (14), respectively.

The coupling nonlinear equality constraint in (15) can be rearranged as $\mathbf{P} - \mathbf{Q}^{-1} = \mathbf{0}$ and which further can be relaxed as an LMI expression

$$\begin{bmatrix}\mathbf{P} & \mathbf{I}\\ \mathbf{I} & \mathbf{Q}\end{bmatrix} \geq \mathbf{0} \tag{16}$$

Moreover, using the cone complementarity approach [11] there exists an H_∞ static output feedback controller $\widetilde{\mathbf{K}}_D$ if and only if the global minimum of the following optimization problem

$$\begin{aligned}&\text{Min} \quad \text{Trace}\ (\mathbf{PQ})\\ &\text{Subject to eq(13), eq(14) and eq(16)}\end{aligned} \tag{17}$$

is $N + \sum_{i=1}^{N} n_{\mathbf{x}_i}$.

The H_2 norm of $T_{\mathbf{zw}_0}(s)$ is finite if and only if $\mathbf{D}_{cl0} \equiv \mathbf{0}$ and which can be computed by the following equation

$$\|T_{\mathbf{zw}_0}\|_{H_2}^2 = \text{Trace}\,(\mathbf{B}_{cl0}^T\mathbf{L}\mathbf{B}_{cl0}) \tag{18}$$

where $\mathbf{L} > \mathbf{0}$ is the *observability Gramian* and satisfies the following Lyapunov equation

$$\mathbf{A}_{cl}^T\mathbf{L} + \mathbf{L}\mathbf{A}_{cl} + \mathbf{C}_{cl0}^T\mathbf{C}_{cl0} = \mathbf{0} \tag{19}$$

Whenever $\mathbf{P}$ satisfies the condition given (12), the H_2 norm of $T_{\mathbf{zw}_0}(s)$ satisfies the following upper bound condition [12]

$$\|T_{\mathbf{zw}_0}\|_{H_2}^2 = \text{Trace}\,(\mathbf{B}_{cl0}^T\mathbf{L}\mathbf{B}_{cl0}) \leq \text{Trace}\,(\mathbf{B}_{cl0}^T\mathbf{P}\mathbf{B}_{cl0}) \tag{20}$$

The above relation suggests minimizing of the upper bound given in (20) for suboptimal static H_2/H_∞ output feedback problem instead of directly minimizing the H_2 norm of $T_{\mathbf{zw}_0}(s)$. Hence, minimizing the upper bound is equivalent to the following optimization problem

$$\begin{aligned}&\text{Min}\quad \mathrm{Trace}(\mathbf{Y}) \\ &\text{Subject to} \\ &\mathbf{P}>\mathbf{0},\ \mathbf{Y}>\mathbf{0} \\ &\text{and} \\ &\begin{bmatrix}\mathbf{Y} & \mathbf{B}_{cl0}^{\mathrm{T}}\mathbf{P}\\ \mathbf{P}\mathbf{B}_{cl0} & \mathbf{P}\end{bmatrix}\geq \mathbf{0}\end{aligned} \tag{21}$$

The LMI expression in (21), i.e. $\mathbf{Y} \geq \mathbf{B}_{cl0}^{\mathrm{T}}\mathbf{P}\mathbf{B}_{cl0} \geq \mathbf{0}$, can be further expanded as follows:

$$\begin{bmatrix}\mathbf{Y} & \tilde{\mathbf{B}}_0^{\mathrm{T}}\mathbf{P}\\ \tilde{\mathbf{B}}_0 & \mathbf{P}\end{bmatrix}+\begin{bmatrix}\mathbf{0}\\ \mathbf{P}\tilde{\mathbf{B}}_2\end{bmatrix}\tilde{\mathbf{K}}_D\begin{bmatrix}\mathbf{D}_{y0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{D}_{y0}^{\mathrm{T}}\\ \mathbf{0}\end{bmatrix}\tilde{\mathbf{K}}_D^{\mathrm{T}}\begin{bmatrix}\mathbf{0} & \tilde{\mathbf{B}}_2^{\mathrm{T}}\mathbf{P}\end{bmatrix}\geq\mathbf{0} \tag{22}$$

Thus, the problem for designing the suboptimal H_2/H_∞ stabilizing static output feedback will be reduced to solving simultaneously the optimization problems in (17) and (21) for the positive definite matrices $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{Y}$:

$$\begin{aligned}&\text{Min}\quad \mathrm{Trace}\,(\mathbf{PQ})+\mathrm{Trace}\,(\mathbf{Y}) \\ &\text{Subject to} \\ &\mathbf{P}>\mathbf{0},\ \mathbf{Q}>\mathbf{0}\ \text{and}\ \mathbf{Y}>\mathbf{0} \\ &\mathbf{PQ}=\mathbf{I} \\ &\mathbf{N}_U^{\mathrm{T}}\begin{bmatrix}\tilde{\mathbf{A}}\mathbf{Q}+\mathbf{Q}\tilde{\mathbf{A}}^{\mathrm{T}}+2\alpha\mathbf{Q}+\gamma^{-1}\tilde{\mathbf{B}}_1\tilde{\mathbf{B}}_1^{\mathrm{T}} & (\tilde{\mathbf{C}}_1\mathbf{Q}+\gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{B}}_1^{\mathrm{T}})^{\mathrm{T}}\\ \tilde{\mathbf{C}}_1\mathbf{Q}+\gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{B}}_1^{\mathrm{T}} & \gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{D}}_{11}^{\mathrm{T}}-\gamma\mathbf{I}\end{bmatrix}\mathbf{N}_U<\mathbf{0} \\ &\mathbf{N}_V^{\mathrm{T}}\begin{bmatrix}\tilde{\mathbf{A}}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\tilde{\mathbf{A}}+2\alpha\mathbf{P}+\gamma^{-1}\tilde{\mathbf{C}}_1^{\mathrm{T}}\tilde{\mathbf{C}}_1 & \mathbf{P}\tilde{\mathbf{B}}_1+\gamma^{-1}\tilde{\mathbf{C}}_1^{\mathrm{T}}\tilde{\mathbf{D}}_{11}\\ (\mathbf{P}\tilde{\mathbf{B}}_1+\gamma^{-1}\tilde{\mathbf{C}}_1^{\mathrm{T}}\tilde{\mathbf{D}}_{11})^{\mathrm{T}} & \gamma^{-1}\tilde{\mathbf{D}}_{11}^{\mathrm{T}}\tilde{\mathbf{D}}_{11}-\gamma\mathbf{I}\end{bmatrix}\mathbf{N}_V<\mathbf{0} \\ &\mathbf{N}_S^{\mathrm{T}}\begin{bmatrix}\mathbf{Y} & \tilde{\mathbf{B}}_0^{\mathrm{T}}\\ \tilde{\mathbf{B}}_0 & \mathbf{Q}\end{bmatrix}\mathbf{N}_S\geq\mathbf{0},\ \mathbf{N}_R^{\mathrm{T}}\begin{bmatrix}\mathbf{Y} & \tilde{\mathbf{B}}_0^{\mathrm{T}}\mathbf{P}\\ \mathbf{P}\tilde{\mathbf{B}}_0 & \mathbf{P}\end{bmatrix}\mathbf{N}_R\geq\mathbf{0}\end{aligned} \tag{23}$$

where $\mathbf{N}_R$ and $\mathbf{N}_S$ denotes arbitrary bases of the nullspaces of $\mathbf{R}=[\tilde{\mathbf{D}}_{y1}\ \ \mathbf{0}]$ and $\mathbf{S}=[\mathbf{0}\ \ \tilde{\mathbf{B}}_2^T]$, respectively.

The optimization in (23) is an NP-hard non-convex optimization problem due to the coupling constraints of the bilinear matrix equation in (15). To compute the (sub-) optimal solution of this problem, an algorithm based on an iterative LMIs programming method is proposed. The idea behind this algorithm is to linearize the cost functional in (23) with respect to its variables and then to solve the resulting convex optimization problem involving only LMI optimization at each iteration. Moreover, the algorithm will set the direction of the feasible solution appropriately by solving a subclass problem for a Newton-type updating coefficient. Furthermore, the solution of the optimization problem is monotonically non-increasing, i.e. the solution decreases in each iteration with the lower bound being $N+\sum_{i=1}^{N} n_{\mathbf{x}_i}$. The convergence behaviour of the whole optimization problem is ensured by checking the norm distances between the current and the previous solutions. Thus, the iterative LMI programming method for finding the stabilizing robust static output gain matrix has the following two-step optimization algorithms.

Algorithm I: Iterative LMI Programming Method

For a given $\alpha>0$, solve the LMI feasibility problems of (13), (14), (16) and (22) simultaneously for $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{Y}$.

Set the solutions $(\mathbf{P}^0,\mathbf{Q}^0,\mathbf{Y}^0):=(\mathbf{P},\mathbf{Q},\mathbf{Y})$

$\mu^0=(2\,\mathrm{Trace}(\mathbf{P}^0\mathbf{Q}^0)+\mathrm{Trace}(\mathbf{Y}^0))/n$

Repeat until $\mu^k\leq\varepsilon$ (for small number $\varepsilon>0$), **do**

(1) Solve the following optimization problem for $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{Y}$

$$\begin{aligned}&\text{Min}\quad \mathrm{Trace}\,(\mathbf{P}\mathbf{Q}^k+\mathbf{P}^k\mathbf{Q})+\mathrm{Trace}(\mathbf{Y}) \\ &\text{Subject to} \\ &\mathbf{P}>\mathbf{0},\ \mathbf{Q}>\mathbf{0},\ \mathbf{Y}>\mathbf{0} \\ &\text{and} \\ &\begin{bmatrix}\mathbf{P} & \mathbf{I}\\ \mathbf{I} & \mathbf{Q}\end{bmatrix}\geq\mathbf{0} \\ &\mathbf{N}_U^{\mathrm{T}}\begin{bmatrix}\tilde{\mathbf{A}}\mathbf{Q}+\mathbf{Q}\tilde{\mathbf{A}}^{\mathrm{T}}+2\alpha\mathbf{Q}+\gamma^{-1}\tilde{\mathbf{B}}_1\tilde{\mathbf{B}}_1^{\mathrm{T}} & (\tilde{\mathbf{C}}_1\mathbf{Q}+\gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{B}}_1^{\mathrm{T}})^{\mathrm{T}}\\ \tilde{\mathbf{C}}_1\mathbf{Q}+\gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{B}}_1^{\mathrm{T}} & \gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{D}}_{11}^{\mathrm{T}}-\gamma\mathbf{I}\end{bmatrix}\mathbf{N}_U<\mathbf{0} \\ &\mathbf{N}_V^{\mathrm{T}}\begin{bmatrix}\tilde{\mathbf{A}}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\tilde{\mathbf{A}}+2\alpha\mathbf{P}+\gamma^{-1}\tilde{\mathbf{C}}_1^{\mathrm{T}}\tilde{\mathbf{C}}_1 & \mathbf{P}\tilde{\mathbf{B}}_1+\gamma^{-1}\tilde{\mathbf{C}}_1^{\mathrm{T}}\tilde{\mathbf{D}}_{11}\\ (\mathbf{P}\tilde{\mathbf{B}}_1+\gamma^{-1}\tilde{\mathbf{C}}_1^{\mathrm{T}}\tilde{\mathbf{D}}_{11})^{\mathrm{T}} & \gamma^{-1}\tilde{\mathbf{D}}_{11}^{\mathrm{T}}\tilde{\mathbf{D}}_{11}-\gamma\mathbf{I}\end{bmatrix}\mathbf{N}_V<\mathbf{0} \\ &\mathbf{N}_S^{\mathrm{T}}\begin{bmatrix}\mathbf{Y} & \tilde{\mathbf{B}}_0^{\mathrm{T}}\\ \tilde{\mathbf{B}}_0 & \mathbf{Q}\end{bmatrix}\mathbf{N}_S\geq\mathbf{0},\ \mathbf{N}_R^{\mathrm{T}}\begin{bmatrix}\mathbf{Y} & \tilde{\mathbf{B}}_0^{\mathrm{T}}\mathbf{P}\\ \mathbf{P}\tilde{\mathbf{B}}_0 & \mathbf{P}\end{bmatrix}\mathbf{N}_R\geq\mathbf{0}\end{aligned} \tag{24}$$

(2) Set the direction

$(\Delta\mathbf{P}^k,\Delta\mathbf{Q}^k,\Delta\mathbf{Y}^k):=(\mathbf{P}-\mathbf{P}^k,\ \mathbf{Q}-\mathbf{Q}^k,\ \mathbf{Y}-\mathbf{Y}^k)$

(3) Set $(\mathbf{P}^{k+1},\mathbf{Q}^{k+1},\mathbf{Y}^{k+1}):=(\mathbf{P}^k,\mathbf{Q}^k,\mathbf{Y}^k)+t_k(\Delta\mathbf{P}^k,\Delta\mathbf{Q}^k,\Delta\mathbf{Y}^k)$ for $t_k\in[0,1]$

(4) Set $\mu^{k+1}=\mathrm{Trace}(\mathbf{P}\mathbf{Q}^k+\mathbf{P}^k\mathbf{Q})+\mathrm{Trace}(\mathbf{Y})$
$-(2\mathrm{Trace}(\mathbf{P}^k\mathbf{Q}^k)+\mathrm{Trace}(\mathbf{Y}^k))$

and increment k by 1.

End do.

Remark 2: The following sub-problem can be used to choose appropriately the value for t_k in Step (3):

$$\begin{aligned}&\text{Min}\ \ \mathrm{Trace}\,(\mathbf{P}^k+t_k(\mathbf{P}-\mathbf{P}^k))(\mathbf{Q}^k+t_k(\mathbf{Q}-\mathbf{Q}^k)) \\ &\qquad +\mathrm{Trace}(\mathbf{Y}^k+t_k(\mathbf{Y}-\mathbf{Y}^k))\end{aligned} \tag{25}$$

Subject to $t_k\in[0,\,1]$

Algorithm II: The LMI Problem to Determine $\tilde{\mathbf{K}}_D$

Solve the following LMI optimization problem for $\tilde{\mathbf{K}}_D$ that gives the *least norm* on the gains of the controller.

Min τ

subject to

$\tau > 0$, and

$$\boldsymbol{\Phi}_B \tilde{\mathbf{K}}_D \boldsymbol{\Theta}_C + (\boldsymbol{\Phi}_B \tilde{\mathbf{K}}_D \boldsymbol{\Theta}_C)^T + \bar{\boldsymbol{\Omega}} \le \mathbf{0},$$

$$\begin{bmatrix} \mathbf{Y}^* & \tilde{\mathbf{B}}_0^T \mathbf{P}^* \\ \tilde{\mathbf{B}}_0 & \mathbf{P}^* \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{P}^* \tilde{\mathbf{B}}_2 \end{bmatrix} \tilde{\mathbf{K}}_D \begin{bmatrix} \mathbf{D}_{y0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{y0}^T \\ 0 \end{bmatrix} \tilde{\mathbf{K}}_D^T \begin{bmatrix} \mathbf{0} & \tilde{\mathbf{B}}_2^T \mathbf{P}^* \end{bmatrix} \ge \mathbf{0}$$

$$\begin{bmatrix} -\tau \mathbf{I} & \tilde{\mathbf{K}}_D^T \\ \tilde{\mathbf{K}}_D & -\mathbf{I} \end{bmatrix} \ge \mathbf{0} \qquad (26)$$

where $\mathbf{P}^*$ and $\mathbf{Y}^*$ are the solutions of Algorithm I, and

$$\bar{\boldsymbol{\Omega}} = \begin{bmatrix} \mathbf{P}^*\tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T\mathbf{P}^* + 2\alpha\mathbf{P}^* & \mathbf{P}^*\tilde{\mathbf{B}}_1 & \tilde{\mathbf{C}}_1^T \\ \tilde{\mathbf{B}}_1^T\mathbf{P}^* & -\gamma\mathbf{I} & \tilde{\mathbf{D}}_{11}^T \\ \tilde{\mathbf{C}}_1 & \tilde{\mathbf{D}}_{11} & -\gamma\mathbf{I} \end{bmatrix},$$

$$\boldsymbol{\Phi}_B = \begin{bmatrix} \mathbf{P}^*\mathbf{B}_2 \\ \mathbf{0} \\ \mathbf{D}_{12} \end{bmatrix}, \quad \boldsymbol{\Theta}_C = \begin{bmatrix} \mathbf{C}_y & \mathbf{D}_{y1} & \mathbf{0} \end{bmatrix}$$

Remark 3: It is possible to include a different set of LMIs constraint in Algorithm II that could be used to limit the upper gain of the controller matrix $\tilde{\mathbf{K}}_D$.

The above two algorithms, i.e. Algorithm I and Algorithm II, involve the following: i) minimization problem of a convex cost functional subject to LMI constraints, i.e. the iterative LMI optimization problem in Algorithm I, will give the optimal values for $\mathbf{P}^*$, $\mathbf{Q}^*$ and $\mathbf{Y}^*$, and ii) minimization problem of a least norm objective functional subject to LMIs constraints, i.e. the optimization problem in Algorithm II, which gives the suboptimal H_2/H_∞ stabilizing static output feedback controller $\tilde{\mathbf{K}}_D$. These two algorithms can be conveniently implemented with the available Semidefinite Optimization (SDO) solvers such as MATLAB *LMI Toolbox* [13].

3 SIMULATION RESULTS

The robust decentralized PSS controllers design approach presented in the previous section is now applied on a four machine test system. This system, which is shown in Figure 2, has been specifically designed to study the fundamental behaviour of large interconnected power systems including inter-area oscillations in power systems [14]. The system has four generators and each generator is equipped with IEEE standard exciter and governor controllers. The parameters for the standard exciter and governor controllers used in the simulation were taken from Kundur [15]. Moreover, the generators used in all simulations were fifth-order generator models. To demonstrate the applicability of the proposed approach a first order PSS controller is used, although it is possible to extend the method to any order and/or combinations of PSS blocks in the design procedure without any difficulty. Figure 3 shows a first - order PSS structure for the i^th^ – machine including the values for T_{wi} and T_{i2} that are used. After including the washout filter for in the system, the design problem is reformulated as a nonconvex optimization problem involving bilinear matrix inequalities (BMIs) and linear matrix inequalities (LMIs) which is solved using the iterative LMI programming method presented in the previous section. Moreover, the robust stability degree of the system was incorporated in the formulation while designing the decentralized robust PSS controllers.

The design procedure has been carried out for loading condition $[P_{L1} = 1600\text{ MW}, Q_{L1} = 150\text{ Mvar}]$ and $[P_{L2} = 2400\text{ MW}, Q_{L2} = 120\text{ Mvar}]$. The speed of each generator and the voltage error signal which is the input to the regulator of the exciter are used as regulated signals within the framework of the design. Moreover, the output of the washout block, i.e., measured output signal, is used as an input signal for the PSS controller in the system.

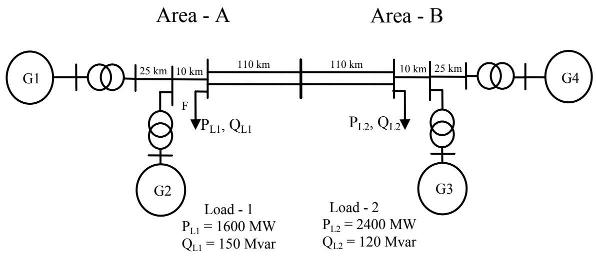

Figure 2: One-line diagram of the test system.

A three-phase fault with different fault durations was applied at different fault locations and operating conditions to verify the performance of the proposed robust PSSs controllers. For a short circuit of 100ms duration at node F in Area-A, the transient responses of generator G2 with and without the PSS controllers in the system are shown in Figure 4. This generator which is the most disturbed generator in the system due to its relative nearness to the fault location shows a good damping behaviour after the PSS included in the system. The calculated PSS gains and parameters for each generator are given in Table 1.

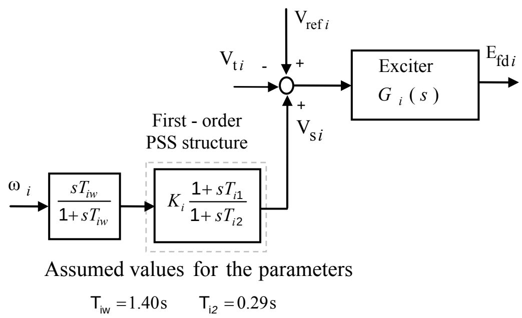

Figure 3: The PSS structure used in the design.

Gains for the PSS	Parameter T_{i1}
$K_1 = 1.2465$	$T_{11} = 0.5347$
$K_2 = 1.0005$	$T_{21} = 0.5342$
$K_3 = 0.8778$	$T_{31} = 0.5338$
$K_4 = 1.1064$	$T_{41} = 0.5338$

Table 1: The computed robust PSS controllers gains and parameters corresponding to each generator.

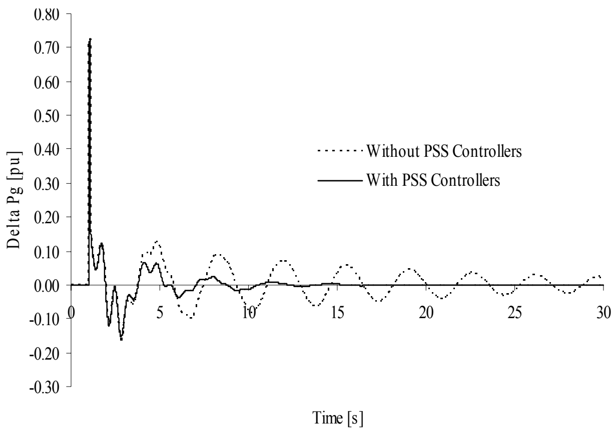

Figure 4: The transient responses of Generator G2 to a short circuit of 100 ms duration at node F in Area A.

To further assess the effectiveness of the proposed approach regarding robustness, the transient performance indices were computed for different loading conditions at node 1 $[P_{L1}, Q_{L1}]$ and node 2 $[P_{L2}, Q_{L2}]$ while keeping constant total load in the system. These transient performance indices, which are used to investigate the behaviour of the system during any fault and/or sudden load changes, are then normalized to the transient performance indices of the base operating condition for which the designed procedure has been carried out. Specifically, these normalizations are computed according to the following formula:

$$I_N = \frac{I_{DLC}}{I_{BLC}} \tag{27}$$

where I_{DLC} is the transient performance index for different loading condition, I_{BLC} is the transient performance index for base loading condition for which the design has been carried out. Moreover, the transient performance indices for generator powers P_{Gi}, generator terminal voltages V_{ti} and excitation voltages E_{fdi} are computed using the following equations.

$$I^{P_G} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| P_{gi}(t) - P_{gi}^{0} \right| dt \tag{28a}$$

$$I^{V_t} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| V_{ti}(t) - V_{ti}^{0} \right| dt \tag{28b}$$

$$I^{E_{fd}} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| E_{fdi}(t) - E_{fdi}^{0} \right| dt \tag{28c}$$

The normalized transient performance indices (NTPIs) for different loading conditions are shown in Figure 5. It can be seen from Figure 5 that the NTPIs for $I_N(E_{fd})$, $I_N(P_G)$ and $I_N(V_t)$ are either near to unity or less than unity for a wide operating conditions. This clearly indicates that the transient responses of the generators for different operating conditions are well damped and the system behaviour exhibits robustness for all loading conditions.

Similarly, the NTPIs after disconnecting one of the tie-lines from the system are shown in Figure 6 for different operating conditions. From these figures the NTPIs are also either near to unity or less than unity for wide operating conditions except for the condition when the total load distribution apparently concentrated to Area-B. This evidently shows the robustness of the proposed approach to structural change in the system.

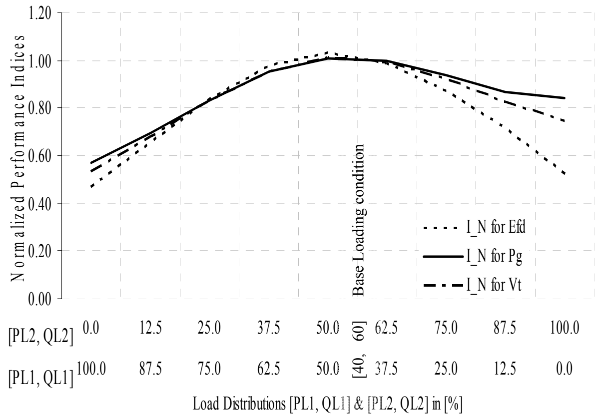

Figure 5: Plot of the normalized transient performance indices.

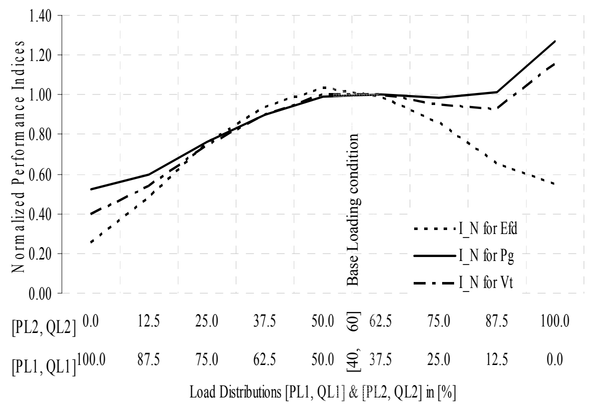

Figure 6: Plot of the normalized transient performance indices after disconnecting one of the tie-lines.

4 CONCLUSION

A framework for robust PSS controller design that takes into account model uncertainties and changes in the operating conditions has been presented. The applicability of the approach has been demonstrated through design example in a four-machine test system. This design problem is formulated as an optimization problem involving bilinear matrix inequalities (BMIs) and solved using an iterative LMIs programming method. Though the proposed approach is demonstrated on a first order PSS block, it is possible to extend the method to any higher order PSS blocks and even a combination of any orders in the design without any difficulty. Moreover the approach is flexible enough to allow the inclusion of additional design parameters such as the size and structure of the gain matrices, the degree of exponential stability and different performance measures for each input/output channel in the system. An additional benefit of this approach is that all the controllers are linear and use minimum local feedback information. The approach could also be used for retuning the existing PSSs in the system.

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