Dynamic models of golf clubs

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Abstract

This paper aims to improve a finite element model of a golf club by using measured experimental natural frequencies and mode shapes. Only the low frequency dynamics of the club are considered, making the improved model suitable for studies of the dynamics of the swing. The shaft was modelled using beam elements and the head was represented as a rigid body. The natural frequencies of the club were measured and compared to those predicted from the analytical model. The flexibility properties of the shaft were modelled using the generic parameter approach. These parameters and the inertia properties of the head were estimated from the measured data to produce an improved analytical model. Subset selection was employed to determine those characteristics of the club that were poorly modelled. The inertia about the shaft axis and the shaft flexibility, particularly the torsional stiffness, have been identified as the most likely to be in error.

Keywords: dynamics, finite element analysis, golf, model updating

Introduction

The quality of the dynamic response of a golf club can have a considerable impact on the 'feel' of the club and the quality of the contact with the ball. Matching the shaft to a player's swing may increase the impact velocity, although in practice this effect is probably small. The size and shape of the 'sweet spot' is related to the natural modes of the club: hitting the ball within the 'sweet-spot' will excite few modes of the club and leave the golfer more satisfied with his shot. Horwood (1994) discussed the dynamics of the shaft in a qualitative manner and described the influence of shaft flexibility and the mass distribution of the head during the swing. The dynamics of the club relate directly to the shaft and the mass and inertia properties of the club head. The low frequency range is of most interest during the swing, and within this range the head may be assumed rigid. Higher frequencies will be of interest during the impact between the club and the ball, but these are not considered further here.

The modelling of the 'static' club, that is when the grip is stationary, is considered as a first step to modelling its dynamics through the swing. One difficulty in the analysis and testing golf clubs is the boundary condition at the grip; if the analysis is required to simulate the club during the swing then the interface between the grip and the golfer must be considered. Mather (1996) and Swider *et al.* (1994) tried to replicate this boundary condition in static tests. The approach here is somewhat different since the purpose of the current work is to validate the model of the club. Thus, it is preferable to fix the grip end of the club so that the boundary condition is less uncertain. The grip is removed so that the shaft is clamped directly. Once the finite element model of the clamped club has been validated, this model may be used in further studies

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by modelling the golfer's grip over a range of stiffnesses. The effect of different mass distributions of the club head or shaft flexibility on the torsion and bending of the shaft during the swing may also be simulated. These simulations can also include the effects of gyroscopic stiffening that may be included more accurately based on the updated finite element model.

Finite element modelling and modal testing were performed on two clubs, a wood and an iron, although only the wood is considered in this paper. The club has a TI Apollo Acculite steel shaft and a Bioedge series no. 1 wood head. The authors previously updated the finite element model of both clubs using only the inertia properties of the head as unknown parameters (Friswell *et al.*, 1996, 1997b); this paper considers updating the flexibility properties of the shaft in addition to the inertia properties.

Finite element modelling

A simple beam model

One of the first questions to be answered when modelling golf clubs is the type of element that should be used. Swider *et al.* (1994) used shell elements to model a composite shaft and head, but while this makes the incorporation of geometry changes relatively easy, it should not be thought that this method will necessarily produce an accurate model. First, the frequency range of interest for the clubs is very low, and is determined by the frequencies that have some effect on the club dynamics during the swing and on impact with the ball. Within this frequency range the modes will involve the bending of the shaft and plate representations of the shaft are therefore not really required. Similarly, within the frequency range of interest, the head will act as a rigid body: the flexible modes of the head will be at very high frequencies. The accuracy of the shell and brick models rely on the accurate measurement or estimation of the shaft thickness and the geometry of the head. Iwata *et al.* (1990) used three-dimensional brick elements to model the impact between the ball and club. This approach may be necessary during impact, although the deformation of the ball is far larger than the deformation of the head.

The approach taken in this paper is to produce a simple model of the shaft using beam elements for the shaft and a rigid body (i.e. just the mass and inertia properties used) for the head. Brylawski (1994) used a similar approach, but modelled the shaft as a continuous beam using partial differential equations, rather than the finite element modelling approach adopted here. The uncertain parameters in this model will be identified from measured data.

The element matrices involve two nodes and six degrees of freedom per node. The element matrices are assembled from the standard bending elements in two planes, the shaft torsion element and the axial extension element (see, for example, Dawe, 1984). Within the element it is assumed that there is no interaction between these four vibration mechanisms. Shear effects could be included in the bending elements, but are likely to be small and are therefore neglected.

Estimating the inertia matrix

The main difficulty with modelling the head as a rigid body is the estimation of its inertia matrix with respect to axes fixed at the end of the shaft. The approach adopted by Johnson (1994) was to measure the inertia matrix directly. Measuring the inertia should produce reasonable estimates, but is time consuming; it requires an isolated club head and will inevitably still contain errors. Since the inertia matrix is available for updating, it will suffice to produce a reasonable estimate of the inertia. This is readily available from a finite element model of the club head (Iwata *et al.*, 1990; Swider *et al.*, 1994) or from a CAD model (Mitchell *et al.*, 1994). In this paper we use an initial estimate of the inertia matrix based on all the mass of the head being located at a single point. This point will be slightly further away from the end of the shaft than the centre of gravity. If the position of the head mass from the end of the shaft is (x_m, y_m, z_m) where the x-axis direction is along the shaft, then the inertia matrix is

$$m\begin{bmatrix} y_m^2+z_m^2 & x_m y_m & x_m z_m \\ x_m y_m & x_m^2+z_m^2 & y_m z_m \\ x_m z_m & y_m z_m & x_m^2+y_m^2 \end{bmatrix} \tag{1}$$

where m is the mass of the head. The choice of the y and z axis directions is arbitrary, save that they are perpendicular to the shaft (the x-axis). The results from the model will not depend on the choice of axis directions, although symmetry in the club response can be retained if the z direction is chosen to be along one of the principal axes of the club head. In the model this implies that $y_m = 0$ and only one pair of the off-diagonal elements in the inertia matrix is nonzero. By the symmetry of the club there will be pure bending modes in the x–z plane, and this will be demonstrated in the example. In the experiment, the principal axes of the head will not be known exactly, but can be roughly estimated from its geometric properties. Slight errors in exciting the club along the principal axis of the head will excite modes in both planes, but the modes in the plane closest to the excitation direction will dominate in the response and may easily be identified.

The finite element model of a golf club

Consider the model of a golf club with a TI Apollo Acculite steel shaft and a Bioedge series no. 1 wood head. The shaft was split into segments of constant diameter and the thickness values were obtained by cutting open a shaft specimen. Note that the first shaft segment does not include the length that is clamped. Similarly the last shaft segment does not include the shaft incorporated into the head. The head mass is taken to be 232 g, located at a position $(x_m, y_m, z_m) = (40, 0, 20)$mm. This gives an inertia matrix of

$$\begin{bmatrix} 0.93 & 0 & 1.86 \\ 0 & 4.65 & 0 \\ 1.86 & 0 & 3.72 \end{bmatrix} \times 10^{-4}\,\mathrm{kg\,m^2}\ . \tag{2}$$

This estimate of the head inertia is likely to be quite inaccurate; a more accurate model could be obtained from a detailed analysis of the head geometry. However, the estimate is good enough to initialise the identification process to be described later. The shaft was modelled using beam elements based on a tube of constant diameter and thickness for each element, using a total of 26 elements and 156 degrees of freedom. The last shaft segment was covered by a plastic sleeve that was modelled as a metal shaft element whose bending stiffness only was multiplied by 3, based on the approximate thickness of the plastic and ratio of Young's modulus of plastic and steel. Typical material properties for steel were used, namely a Young's Modulus of 210 GN m^{-2}, a density of 7800 kg m^{-3} and a shear modulus of 80 GN m^{-2}. The natural frequencies are given in Table 1 and mode shapes are shown in Figs 1 and 2 for this model.

Table 1 Initial, experimental and updated natural frequencies (Hz) of the wood. Note the axial mode was not used in the updating

			Updated model for best parameter subsets of different sizes						
Plane	Experi-mental	Initial Model	1	2	3	4	5	6	7
z	4.45	4.53	4.53	4.53	4.52	4.52	4.52	4.48	4.46
y/ϕ_x	4.50	4.53	4.53	4.53	4.53	4.53	4.53	4.49	4.46
y/ϕ_x	49.0	49.0	44.3	48.5	48.1	47.7	49.0	49.2	49.1
z	49.5	52.6	52.6	52.6	52.4	50.0	49.4	49.9	49.4
y/ϕ_x	66.0	90.8	67.7	66.8	67.2	67.1	66.0	66.1	66.0
z	132	131	131	131	130	130	130	131	132
y/ϕ_x	156	230	162	152	153	152	157	158	157
z	267	286	286	286	274	275	273	273	273
y/ϕ_x	299	452	311	304	290	291	293	293	293
axial	467	471	471	471	471	471	471	466	464

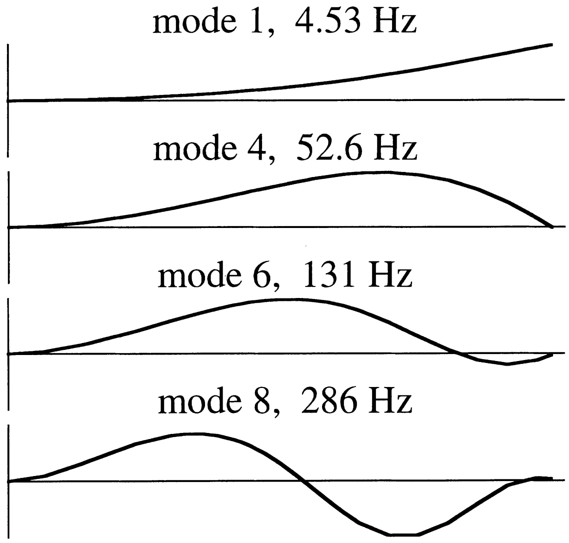

Fig. 1 Vertical modes from the initial finite element model of the club (only the x-z plane is shown)

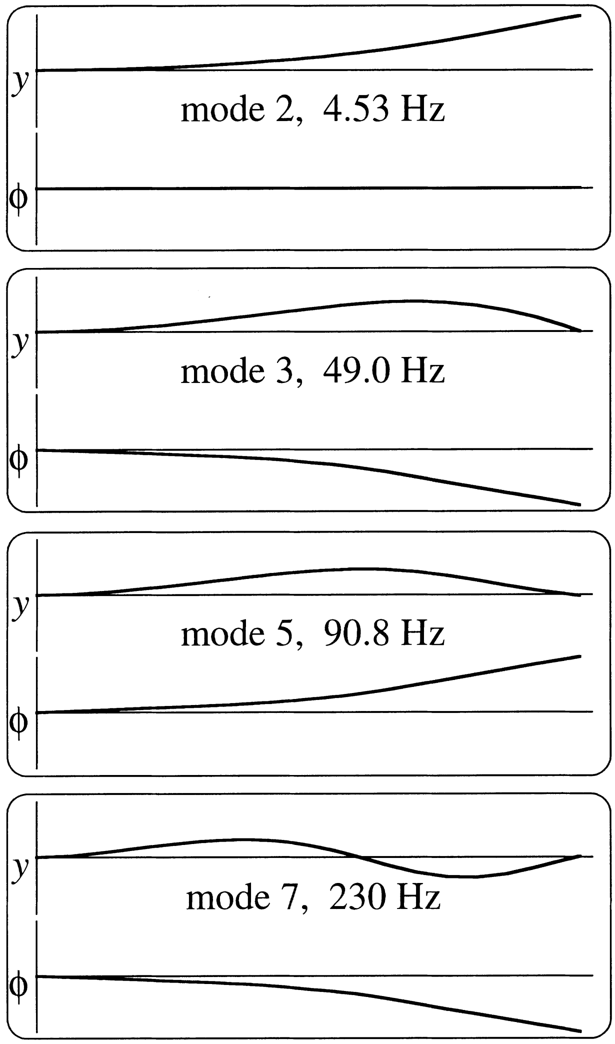

Fig. 2 Combined horizontal and torsional modes from the initial finite element model of the club – y is the horizontal deflection and f is the torsion angle (the modes are scaled so that the x and y directions have the same scaling and ϕ is between ± 30°)

As expected, since the z direction has been assumed to lie along a principal inertia axis of the head, there are modes that just involve bending in the x–z plane. The first bending mode in the x–y plane does not involve much coupling with the torsional modes, but the higher modes show great coupling between bending in the x–y plane and torsion.

Errors in the finite element model

The finite element model was used to calculate the measured outputs and also the sensitivity of those outputs to changes in the unknown parameters. The errors in the model of the club can arise from three main areas which will now be described briefly.

Model structure errors

These errors occur when the governing physical equations or principles are uncertain or complex. For example, the model may be assumed to be linear although the actual structure behaves in a nonlinear way. For the golf club, it may be that the beam and rigid mass/inertia model is not sufficiently accurate and that shell and brick elements would be better. The end of the shaft, where the grip would be, is assumed to be fixed, but some flexibility may be present in the experiment. Damping is very difficult to model, and in many numerical models is ignored completely.

Model parameter errors

Even if the underlying structure of the model was correct, some of the parameters may be uncertain. For example, if the boundary is assumed to be flexible it is often difficult to theoretically estimate the stiffness of the connection. In the case of the golf club, the inertia of the club head is difficult to estimate accurately. These errors are most amenable to correction by model updating.

Discretization errors

Most numerical models of structures, including finite element analysis, approximate the motion of the continuous structure by a discrete system. If the level of discretization, that is the number of degrees of freedom, is insufficient then the model order will be too small to accurately model the dynamics of the structure. The requirements of model updating are considerably more stringent than straightforward design analysis. The natural frequencies should be fully converged, that is the difference between the predicted frequencies, and those for a model with a very large number of degrees of freedom should be much smaller than the difference between the predicted and measured frequencies. In this paper we have ensured that enough elements are used so that the discretization errors are small.

Experimental modal analysis

Experimental modal analysis (EMA) is now an established technique for many industries, for example automobile and aerospace applications. The idea is to apply a force to a structure, and from measurements of the force applied and the response, to estimate the natural frequencies, damping ratios and mode shapes of the structure. The purpose here is not to review EMA in detail but to highlight the special features in testing and updating golf clubs. Ewins (1984) gave more detail on vibration testing and modal analysis.

Errors in experimental data

The quality of measured time series data has improved considerably with the arrival of computerised data acquisition systems. In the hands of an experienced operator, the algorithms available to estimate the frequency response functions and then the natural frequencies, mode shapes and damping ratios are very accurate. However, even with modern, sophisticated systems, errors may still occur.

The data obtained from the structure under test will be used to update the parameters of an analytical model. It is therefore vital to predict, and if possible, eliminate the likely errors in the measurements which may be either random or systematic. Random errors may be reduced by careful experimental technique, the choice of excitation method and by averaging the data. Impact excitation such as hammer excitation puts very little energy into a structure and can produce noisy data. Identification, including modal extraction, work satisfactorily providing the noise is random, with a zero mean and a large quantity of data is used to identify a relatively small number of parameters.

Systematic errors are difficult to remove from the data and are a serious problem in model updating. These errors arise from many sources, including inadequate modelling of the mounting of the club, mass loading due to the accelerometers, a poorly designed stinger and leakage. Two major problems have to be considered in the case of a golf club. The first is that the mass loading of the accelerometer will be significant when the accelerometer is placed on the shaft. Although the accelerometer used only weighs 3.5 g, this compares to a weight per unit length of 0.56 g cm^{-1} at the lightest part of the shaft. This mass loading due to the accelerometer makes the accurate measurement of mode shapes on the club very difficult. A laser system could be used to measure the response of the club, without any mass loading, although the large deflections of the club may cause problems. Strain gauges could also be used, although the gauges will slightly increase the mass and stiffness of the club. Using a shaker to apply the excitation force, via a stinger, can add stiffness to the structure. Only the axial force from the shaker is measured and the assumption is that no other force is transmitted to the club. Since the shaft is very flexible, and many modes involve significant bending, the requirement that the stinger should be flexible in bending is difficult to satisfy. An alternative is to excite the club using an instrumented hammer. This works well at the head, although the flexibility within the shaft makes triggering the analyser very difficult on the shaft.

Errors in the measured data can never be eliminated. The estimates of the natural frequencies are usually very good, whereas mode shape and damping estimates usually contain significant noise. Although the individual elements of the mode shape vector contain relatively high levels of noise, the general shape of the mode will be quite accurate. The object is to reduce the effect of these errors by good experimental technique. There is no substitute for high quality measured data.

Measurements from a golf club

The grip was removed from the clubs which were then clamped using a purpose-made block. This block consisted of a hole only slightly larger than the shaft diameter and a slot to allow the club to be rigidly clamped. The club was excited using hammer excitation at the head and by measuring the response at the head. The impact and response measurements were taken in the same direction during each test and the tests were repeated in two orthogonal directions. The club was arranged so that the vertical, z, direction was approximately parallel to one of the principal axes of inertia of the head. This would decouple the vertical modes from the horizontal/torsional modes. Of course, the principal axis could only be approximated and the alignment error caused excitation of the vertical modes when the club was excited horizontally, and vice-versa. In practice the out of plane modes were visible as low level peaks in the frequency response functions and were easily identified. Frequency bandwidths of 10, 100 and 500 Hz were used; the lower frequency range was required to accurately estimate the first two modes at around 5 Hz. The natural frequencies were easily estimated directly from the frequency response functions since the club was very lightly damped. The modes were well separated in each direction so the experimental and analytical modes may be paired. More formal methods of pairing modes, such as the modal assurance criterion (Allemang & Brown 1982) could not be employed because of the difficulty in obtaining mode shapes due to the mass loading of the accelerometer. Table 1 shows the first 10 natural frequencies of the wood and a comparison with the finite element model estimates.

Finite element model updating

As demonstrated in the previous sections, the dynamic response of the club is not identical to the predicted response. Ideally, if the measurements are accurate, the uncertain parameters of the numerical model should be changed to more closely reflect the properties of the physical structure; this is model updating. Mottershead & Friswell (1993) gave an extensive survey of the field and Friswell & Mottershead (1995) outlined most of the popular methods in detail.

Parameters for updating

A critical decision in model updating is the choice of parameters to update. The primary unknowns will be changes in the inertia properties of the head, and the flexibility properties of the shaft. The change in the inertia properties of the head are given by

$$\begin{bmatrix} \delta I_{xx} & \delta I_{xy} & \delta I_{xz} \\ \delta I_{xy} & \delta I_{yy} & \delta I_{yz} \\ \delta I_{xz} & \delta I_{yz} & \delta I_{zz} \end{bmatrix} \tag{3}$$

A global term for the shaft, namely the change in stiffness of the shaft, is used. Thus the shaft stiffness matrix, $\mathbf{K}$, which is the same as the club stiffness matrix, is:

$$\mathbf{K} = (1 + \theta_k)\mathbf{K}_0 \tag{4}$$

where $\mathbf{K}_0$ is the stiffness matrix of the initial finite element model, and θ_k is the parameter to be updated.

The remaining shaft parameters are based on the generic approach. The generic model updating approach is based on the idea of adjusting the mode shapes and natural frequencies of elements or substructures (Gladwell & Ahmadian, 1995). For example, a joint model that requires updating may be represented as a substructure. The mode shapes of the initial finite element model could be assumed correct, and the natural frequency of the first mode, typically a bending mode, could be updated. Thus the bending flexibility of the joint is changed to produce a model that better represents the measurements. The element mass matrices are assumed to be correct.

In the case of the golf club, the substructure is taken as the whole shaft. Thus the bending stiffness of the shaft bending and torsion (and possibly axial) modes are updated. Since the shaft is axisymmetric the bending modes will occur in pairs, that is bending modes with the same natural frequency in orthogonal planes perpendicular to the shaft cross-section. Thus reference to the first bending mode of the shaft will implicitly mean the two associated bending modes. Of course, attaching the club head will break the axisymmetry of the shaft, and cause the natural frequencies of the bending modes in each plane to be different. Furthermore, the bending and torsion vibration will be coupled.

The stiffness matrix for the shaft may be decomposed as

$$\mathbf{K} = \mathbf{V}\Lambda\mathbf{V}^T \tag{5}$$

where $\mathbf{V}$ is the matrix of eigenvectors of the stiffness matrix, normalized to unit length, and Λ is a diagonal matrix of the corresponding eigenvalues. The elements of Λ are available for updating, and the sensitivity of the stiffness parameters to these parameters is easily computed. The corresponding parameters for updating are the relative changes in the eigenvalues, and are defined in a way analogous to eqn (4).

A sensitivity analysis

If measured eigenvalues (natural frequency squared) only are used for updating, then the sensitivity matrix is easily calculated using the eigenvalue derivatives (Fox & Kapoor, 1968). The sensitivity of the natural frequencies to the uncertain parameters is easily obtained from the eigenvalue derivative. Table 2 shows the sensitivity matrix for the first nine natural frequencies to the uncertain parameters, based on the initial finite element model of the wood. Of particular note are the zero elements in this matrix. The natural frequencies are insensitive to the inertia terms I_{xy} and I_{yz} because of the symmetry imposed by the z axis being coincident with a principal axis of inertia of the head. Furthermore, we will assume that we have the z axis aligned correctly so that these parameters are forced to remain zero. The first bending modes in both directions are relatively insensitive to the inertia terms since these modes do not involve much rotation at the club head. The natural frequencies of the vertical (z direction) modes are only sensitive to I_{yy} and the horizontal/torsional modes are only sensitive to I_{xx}, I_{zz} and I_{xz}. Thus the dynamics of the two planes decouple, although it is important to remember that this occurs because of the alignment of the z axis.

The sensitivity of the generic shaft parameters shows the expected trend, for example the first modes of the club are most sensitive to the first bending mode of the shaft. For the second shaft mode, the horizontal bending and torsion mode couple, and both coupled modes are sensitive to the generic parameter corresponding to this shaft parameter. It should be noted that the vertical bending modes are not coupled to the torsional modes, and so are insensitive to the shaft torsional mode generic parameters. Furthermore, the modes considered are relatively insensitive to the second and higher torsional shaft modes, and only the first torsional mode of the shaft is used for updating.

Updating methods and subset selection

Friswell & Mottershead (1995) described a large number of updating techniques but the weighted least squares method based on the natural frequencies alone is used in this work. The method allows a wide choice of parameters to update and both the measured data and the initial analytical parameter estimates may be weighted. This ability to weight

Table 2 Sensitivities of the natural frequencies to the uncertain parameters for the wood

Sensitivity with respect to	Mode number, vibration plane								
	1, z	2, y/ϕ_x	3, y/ϕ_x	4, z	5, y/ϕ_x	6, z	7, y/ϕ	8, z	9, y/ϕ_x
θ_k	2.3	2.3	25	26	45	65	115	143	226
I_{xx}	0	-1.9×10^{-3}	-2.7×10^{4}	0	-2.8×10^{5}	0	-1.2×10^{6}	0	-4.9×10^{6}
I_{yy}	-4.2×10^{1}	0	0	-2.6×10^{4}	0	-5.6×10^{4}	0	-2.7×10^{4}	0
I_{zz}	0	-4.2×10^{1}	-1.6×10^{4}	0	-4.9×10^{3}	0	-2.4×10^{5}	0	-1.2×10^{6}
I_{xy}	0	0	0	0	0	0	0	0	0
I_{xz}	0	-5.7×10^{-1}	-4.2×10^{4}	0	7.5×10^{4}	0	1.1×10^{6}	0	4.7×10^{6}
I_{yz}	0	0	0	0	0	0	0	0	0
Head									
Mass (m)	-9.2	-9.2	-0.11	-0.44	-4.9	-3.0	-5.6	-6.8	-5.5
Generic									
Parameters									
1st Torsion	0	4.1×10^{-5}	5.0	0	15	0	10	0	11
1st Bending	2.2	2.2	0.18	0.28	0.15	0.037	0.11	0.40	0.43
2nd Torsion	0	7.7×10^{-6}	0.93	0	2.8	0	1.8	0	1.6
2nd Bending	0.016	0.017	16	24	9.2	0.10	12	10	1.2
3rd Torsion	0	1.8×10^{-6}	0.22	0	0.66	0	0.42	0	0.39
3rd Bending	2.3×10^{-3}	2.2×10^{-3}	0.030	5.1×10^{-4}	12	49	57	8.6	44
4th Torsion	0	1.1×10^{-6}	0.13	0	0.39	0	0.24	0	0.20
4th Bending	0.010	0.010	1.2	1.4	0.41	3.5	14	81	89
5th Torsion	0	4.0×10^{-7}	0.048	0	0.15	0	0.092	0	0.084
5th Bending	1.3×10^{-3}	1.3×10^{-3}	7.6×10^{-3}	0.049	2.9	8.5	2.2	8.7×10^{-5}	39

the different data sets gives the method its power and versatility, but requires engineering insight to provide the correct weights.

In general, mode shape data contains far more errors than the natural frequencies, and so the information lost by not measuring mode shapes is relatively small. In addition the mode shapes are not very sensitive to changes in the updating parameters. For example, the first mode of a golf club will approximate the standard first bending mode of a beam, for a large range of shaft cross sections and head properties. This prior knowledge may be incorporated into the updating algorithm through the weighting matrices.

The weighting of changes from the initial parameters is often used to regularise the solution to the ill-conditioned updating problem. This is often necessary because there are only a few measured natural frequencies available for updating, but there are many more potential parameters. Thus the problem is under-determined and extra constraints have to be added. The alternative, used here, is to assume only a subset of parameters is in error (Friswell *et al.*, 1997a). For a given size of parameter subset, the parameters are chosen that produce the smallest residual. Normally this subset has to be chosen using suboptimal methods, but the number of parameters in this example is small enough to find the best subset by an exhaustive search of all the subsets of a given size. The subset is also often chosen based on the sensitivity matrix of the initial model, but in this paper, the difference between the predictions of the initial model and the experimental results are too great to do this. Thus the parameters for each potential subset are updated, and the converged residual used to choose the best subset.

Updating the model of the wood

Only the measured natural frequencies were used to update the golf club model. The sensitivity matrix is easily calculated using the eigenvalue derivatives with a model reduced using the first 25 modes of the initial model. An over-determined problem is generated by choosing the size of the parameter subsets as smaller than the number of measurements. The measurement weighting matrix is a diagonal matrix whose elements are the inverse of the natural frequencies squared. This minimizes the size of the relative natural frequency errors, and weights all natural frequencies equally. The plane of vibration of the analytical modes is determined at each iteration, to ensure that the analytical and experimental natural frequencies are paired correctly. Lower bounds are placed on the parameters, namely that the inertias cannot decrease below half of the initial values, the shaft stiffness cannot reduce by more than 10%, the head mass cannot reduce by more than 30 g and the eigenvalues of the generic shaft parameters cannot reduce by more than 20%. These bounds are really quite generous, in that it should be possible to estimate the parameters with more accuracy than the bounds imply. The initial values of the head inertias are likely to be very low, because the inertia matrix was estimated from the position of the centre of gravity of the head. The inertia parameters should therefore increase substantially.

Table 3 shows the best parameter subsets and the updated parameters. The updated natural frequencies for these parameter subsets are given in Table 1. The best parameter subset does tend to retain parameters chosen in previous subsets, and this indicates the parameters that are likely to be in error. The values of the updated parameters remain reasonably consistent as more parameters are added. As expected, the updated natural frequencies become closer, on average, to the measured natural frequencies as the number of parameters increases.

Conclusions

The golf club is a fascinating structure. The shaft is symmetrical, which would produce repeated natural frequencies if tested in isolation. The addition of the asymmetrical head causes the natural frequencies to separate, and to couple vibration in bending and torsion. By choosing a frame of reference so that one axis is aligned with the principal axis of inertia of the head, the bending vibration in one plane is decoupled from the torsion. This decoupling is a vital aid to inferring the measured mode shapes. The usual modal analysis techniques using a roving accelerometer or roving hammer excitation, are impractical on the golf club. Strain gauges may be suitable, however, and this is the subject of further work.

The natural frequencies from the initial finite element model showed considerable errors when compared to measured frequencies. By updating generic parameters relating to the shaft stiffness and the inertia properties of the head, the agreement between the measurements and the analytical

Table 3 The best subset of parameters for the wood for different subset sizes

	Parameter subset size						
Parameters	1	2	3	4	5	6	7
θ_k						-0.1	-0.0294
δI_{xx} (kg m^{-2})	1.27×10^{-4}	2.86×10^{-4}	2.60×10^{-4}	2.79×10^{-4}	4.15×10^{-4}	4.10×10^{-4}	4.66×10^{-4}
δI_{yy} (kg m^{-2})							-5.22×10^{-5}
δI_{zz} (kg m^{-2})					-6.63×10^{-5}	-6.01×10^{-5}	-8.75×10^{-5}
δI_{xz} (kg m^{-2})							
Head mass, m (kg)						-0.0203	
Generic parameters							
1st Torsion		1.98	1.64	2.32	5.44	6.70	9.42
1st Bending							
2nd Bending				-0.0975	-0.119		-0.137
3rd Bending						0.142	
4th Bending			-0.134	-0.115	-0.134		-0.105
Norm(Residual)	0.149	0.103	0.0810	0.0610	0.0389	0.0354	0.0312

model may be improved. The inertia about the shaft axis and the shaft flexibility, particularly the torsional stiffness, have been identified as those most likely to be in error. The updated model may be used to produce more accurate estimates of the club response during the swing. Parametric studies of the effect of changes in geometry and mass distribution may be undertaken with confidence by using a validated model.

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