Optimizing Polyphase Sequences for Orthogonal Netted Radar

Hammad A. Khan, Yangyang Zhang, Chunlin Ji, Christopher J. Stevens, David J. Edwards, and Dominic O'Brien

***Abstract*—Orthogonal netted radar transmitter signals require a very low aperiodic autocorrelation peak sidelobe level (PSL), low aperiodic cross-correlation, and a good resilience to small Doppler shifts. A new set of polyphase sequences is presented with good correlation properties as well as resilience to Doppler shifts. These sequences are built using numerical optimization based on correlation properties. A structural constraint is imposed on the optimized polyphase sequences, which maintains Doppler tolerance. Cross entropy (CE) technique is used to optimize the sequences. Correlation and Doppler results are compared with best-known sequences on various merit factors and shown to be superior.**

***Index Terms*—Cross entropy (CE), Doppler resilience, netted radar, orthogonal functions, pulse compression radar, radar signal analysis, radar theory, sequences.**

I. Introduction

A NETWORK of radars operating together in the same environment can significantly improve the performance of detection and tracking systems. Recently, Fishler *et al.* have proposed a model of spatial diversity in radars using the name multiple-input multiple-output (MIMO) radars [1], showing detection improvements with such radars. We have earlier provided an experimental demonstration of signal-to-noise ratio (SNR) improvement with MIMO radars [2]. Deng has proposed a set of polyphase sequences for use with orthogonal netted radars, but these sequences suffer from severe degradation, even at small Doppler shifts [3]. Multifrequency radar networks have been in use for long under the title of multisite radars [4], but these radars use different frequencies to cope with interference rejection. However, multifrequency radar receivers are unable to process information from all transmitters. In orthogonal netted radars, a number of transmitters and receivers are spatially distributed with each receiver being capable of processing signals scattered by the target from every transmitter. This paradigm requires the use of transmit signals with very low aperiodic cross-correlation between them, apart from a low peak sidelobe level (PSL) for every signal. These requirements are similar to the multiuser wireless communication, but there is a difference in the requirement of *aperiodic* correlation properties as opposed to *periodic* correlation. Another stringent condition in a highly dynamic sensor environment is the requirement for transmit signals to be tolerant to small Doppler shifts. Deng has presented numerically optimized polyphase sequences, yielding good correlation results [3], but Doppler tolerance levels can be further improved. In this letter, we present new polyphase sequences to include Doppler resilience as well, by incorporating a structural constraint in optimization algorithm. We have also improved the correlation properties by using a cross entropy (CE) optimization technique and compare our new sequences with Deng's method.

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II. Optimization Problem

The aperiodic correlation function between two discrete sequences $s_1(l)$ and $s_2(l)$, $\{l=1,2,\ldots,L\}$ can be defined as

$$C(s_1,s_2,k) = \begin{cases} \frac{1}{L}\sum\limits_{l=1}^{L-k} \exp j\left(s_1(l)-s_2(l+k)\right), & 0\le k<L \\ \frac{1}{L}\sum\limits_{l=-k+1}^{L} \exp j\left(s_1(l)-s_2(l+k)\right), & -L<k<0. \end{cases} \quad (1)$$

When $s_1(l)$ and $s_2(l)$ are the same sequences, (1) gives the autocorrelation function. The total cross-correlation and off-peak autocorrelation energy must be minimized to achieve desired correlation properties. Let $\mathbf{S}$ be an $M \times L$ matrix of all sequences for M number of transmitters and each sequence with L elements;, then the total cross-correlation and off-peak autocorrelation energy of all sequences in $\mathbf{S}$ can be written as

$$E(\mathbf{S}) = \sum_{m=1}^{M}\sum_{\substack{k=-L+1\\k\neq 0}}^{L-1} |C(s_m,s_m,k)|^2 + \mu\sum_{p=1}^{M}\sum_{\substack{q=1\\p\neq q}}^{M}\sum_{k=-L+1}^{L-1} |C(s_p,s_q,k)|^2 \quad (2)$$

where μ is the relative weight assigned to the minimization of each autocorrelation and cross-correlation functions. A similar approach has been used by Deng [3]. This optimization problem does not involve any parameter for consideration of Doppler degradation. Deng has suggested improving Doppler resilience by using an optimization cost function to include reciprocals of the main lobe peaks, sidelobe peaks, and the cross-correlation peaks for all possible Doppler frequencies. This ambiguity function-based cost function, however, becomes very costly, even for short length codes. Another method is therefore desirable to cater for Doppler loss. A detailed analysis of Doppler properties of polyphase sequences was presented by Kretschmer and Lewis [5]. According to the results presented in their paper (although it is not explicitly mentioned in their work), the harmonic relationship of phases from one sequence element to the next appears to be a property of the polyphase sequences, which resists the Doppler loss to a large extent. Sequences such as Frank sequences [6], $P1$, $P2$, $P3$, and $P4$ sequences [5], and PX sequences [7] share these properties. If this constraint on a harmonically related structure (or some partial structure constraint) is added to a correlation optimization algorithm, then Doppler resilience can also be improved.

Each of the M polyphase signals can be represented as $e^{js_m(l)}$, $m = \{1, 2, \ldots, M\}$. Let $\mathbf{S}(M, L)$ be the set of all such sequences expressed in a matrix form with M rows such that each row is a sequence of phases with length L and each row is divided into $(2y+1)$ subsequences (or y subsequences on either side of the central subsequences of every row) for Doppler design purposes—see (3) at the bottom of the page.

This means that there are $(2y+1)$ subsequences (from $-y$ to y) in a row (or y sections on either side of the central subsequence), and each subsequence contains K elements, i.e., $L = (2y+1)K$. Larger y means there is greater structure in the sequences, leading to lesser degrees of freedom in the optimization algorithm. Phases in each subsequence can be chosen from a set of P phases

$$s_{m(i-y)}(l) \in X_i = \left\{ (|i-y|+1) \frac{2\pi}{yP} (0, 1, 2, \ldots\ldots, (P-2)) \right\} \tag{4}$$

such that a $P4$- like structure is maintained from one subsequence to the next and $i = 0, 1, \ldots, 2y$. For example, if $P = 10$, and $y = 4$, then there are nine subsequences in every row. The central subsequence ($i = 4$) values are selected from the set $\{2\pi/40(0, 1, 2 \ldots 8)\}$, and the outer subsequences (from the center) are selected from the set $\{2\pi/40(0, 4, 8 \ldots 40)\}$, which is four times the central set. This ensures that the subsequences have their phase values from higher valued sets center-outwards.

III. Cross Entropy Optimization of Polyphase Sequences

The CE method is generally used for evaluation of rare event probabilities and also for combinatorial optimizations [8], [9]. Denote the required polyphase sequence set as the $M \times L$ matrix $\mathbf{S}(M, L)$, where M is the number transmitters, and L is the length of each polyphase sequence. Denote all possible polyphase sequence sets as $\mathbf{\Omega}$. Thus, $\mathbf{S} \subset \mathbf{\Omega}$. We can obtain the optimum polyphase sequences by optimizing the cost function $E(\mathbf{S})$ given in (2). Thus, the discrete optimization problem becomes

$$e^* = \arg \min_{\mathbf{S} \in \mathbf{\Omega}} (E[\mathbf{S}]) \tag{5}$$

where e^* denotes the global minimum of the cost function. The phase values for $\mathbf{S}$ are chosen from the set X. From (2), E is a function defined on $\mathbf{S}$. Assume that $\mathbf{S}$ is from a family of probability density functions (pdf) $f(\mathbf{S}, \mathbf{P})$, where $\mathbf{P}$ is an $M \times L \times P$ a matrix of probabilities such that the element $\mathbf{P}(m, l, p)$ denotes the probability that the element $\mathbf{S}(m, l)$ has a phase p. We can define a collection of indicator functions on $\mathbf{S}$ for various threshold levels e as $I_{\{E(\mathbf{S}) \leq e\}}$, whose value is 1 if $E(\mathbf{S}) \leq e$ and 0 otherwise. For a given $\widehat{\mathbf{P}}$, we can associate an estimation problem to the optimization in (5)

$$l = P\left(E(\mathbf{S}) \leq e\right) = \sum_{\mathbf{S} \in \mathbf{\Omega}} I_{\{E(\mathbf{S}) \leq m\}} f(\mathbf{S}, \widehat{\mathbf{P}}) = \mathrm{E}_{\widehat{\mathbf{P}}} \left\{ I_{\{E(\mathbf{S}) \leq e\}} \right\} \tag{6}$$

which is the probability that the performance function $E(\mathbf{S})$ is less than or equal to the threshold e. A simple way to estimate l is via importance sampling (IS). Let us take a set of random samples $\boldsymbol{S}_1, \ldots \boldsymbol{S}_N$ from an importance distribution $g(\mathbf{S})$. Then, the unbiased estimator of l is

$$\hat{l} = \frac{1}{N} \sum_{i=1}^{N} I_{\{E(\mathbf{S}_i) \leq e)\}} \frac{f(\mathbf{S}_i, \widehat{\mathbf{P}})}{g(\mathbf{S}_i)}. \tag{7}$$

It is convenient to choose $g(\mathbf{S})$ from the original parameterized family of densities $f(\mathbf{S}, \mathbf{P})$. Choosing an optimal $g(\mathbf{S})$ is crucial to the estimation of (7). The optimal change of measure can be proved to be [9]

$$g^*(\mathbf{S}_i) = \frac{I_{\{E(\mathbf{S}_i) \leq e)\}} f(\mathbf{S}_i, \widehat{\mathbf{P}})}{l}. \tag{8}$$

This optimality condition can be translated into the choice of an optimum parameter $\mathbf{P}$, which can be found by minimizing the CE between the two distributions $f(\mathbf{S}, \widehat{\mathbf{P}})$ and $g^*(\mathbf{S})$ [where $g^*(\mathbf{S})$ is given in (8)]

$$\min_{\mathbf{P}} \left\{ E_{\mathbf{P}} \left\{ I_{\{E(\mathbf{S}) \leq e)\}} \ln \frac{I_{\{E(\mathbf{S}) \leq m)\}} f(\mathbf{S}, \widehat{\mathbf{P}})}{f(\mathbf{S}, \mathbf{P})} \right\} \right\}. \tag{9}$$

Since $\widehat{\mathbf{P}}$ is determinate, the optimum $\mathbf{P}$ for a jth iteration that minimizes can be shown to be

$$\mathbf{P}_i^{(j)} = \frac{\sum_{i=1}^{N} I_{\left\{E\left(\mathbf{S}_i^{(j)} \leq e\right)\right\}} \mathbf{S}_i^{(j)}}{\sum_{i=1}^{N} I_{\left\{E\left(\mathbf{S}_i^{(j)} \leq e\right)\right\}}}. \tag{10}$$

This means that the CE procedure can be divided into a two-phase iteration: 1) generating a random sample data according to some specification and 2) updating the parameters.

We now present the CE algorithm for polyphase sequence design. In order to utilize the Doppler constraint in for the CE optimization, a probability matrix $\mathbf{P}(m, l, k)$ is constructed where $(m = 1, \ldots, M; l = 1, \ldots, L; k = 1, \ldots, P)$. $\mathbf{P}_{m,l,k}$ denotes

$$\mathbf{S}(M, L) = \begin{bmatrix} s_{1(-y)}(1), \ldots, s_{1(-y)}(K), & & s_{1y}(1), \ldots, s_{1y}(K) \\ s_{2(-y)}(1), \ldots, s_{2(-y)}(K), & & s_{2y}(1), \ldots, s_{2y}(K) \\ s_{3(-y)}(1), \ldots, s_{3(-y)}(K), & |\ldots| & s_{3y}(1), \ldots, s_{3y}(K) \\ \vdots & & \vdots \\ s_{M(-y)}(1), \ldots, s_{M(-y)}(K), & & s_{My}(1), \ldots, s_{My}(K) \end{bmatrix} \tag{3}$$

the probability of the matrix element (m, l) being equal to phase k.

Algorithm 1

1) Begin with the initial probability matrix $\mathbf{P}^{(0)} = \mathbf{P}_0$. Set the iteration counter $j := 1$. In this letter, the initial probability matrix $\mathbf{P}^{(0)}_{m,l,k}$ was set to $1/P$, where P is the number of phases in (4).
2) Generate N samples $\mathbf{S}_1, \ldots, \mathbf{S}_N$ according to the probability matrix $\mathbf{P}^{(j-1)}$. Each sample $\mathbf{S}_i$ represents an available solution in the set $\mathbf{\Omega}$, and $\mathbf{S}^{(j)}_{(i,m,l)}$ denotes the (m, l)th element of the sample $\mathbf{S}_i$ at iteration j.
3) Calculate the cost functions for each sample $\{\mathbf{E}(S^{(j)}_i)\}^N_{i=1}$ according to (3) and order these from smallest to biggest, $E_{(1)} \leq \ldots \leq E_{(N)}$. Let $e^{(j)}$ be $(1-\rho)$th sample quantile of the performances: $e^{(j)} := E_{(\lceil(1-\rho)N\rceil)}$.
4) Use the same samples to calculate the updated parameters

$$\mathbf{p}^{(j)}_{m,l,k} = \frac{\sum_i^N I_{\left\{E\left(\mathbf{S}^{(j)}_i\right) \leq e^{(j)}\right\}} I_{\left\{\mathbf{S}^{(j)}_{i,m,n} = k\right\}}}{\sum_i^N I_{\left\{E\left(\mathbf{S}^{(j)}_i\right) \leq e^{(j)}\right\}}}. \tag{11}$$

5) If for some $j \geq d$, say, $d = 10$

$$e^{(j-d)} = \ldots = e^{(j-1)} = e^{(j)} \tag{12}$$

then stop; otherwise go to step 2).

In order to prevent the occurrences of 0 s and 1 s in the parameter vectors in $\mathbf{P}$, parameter $\mathbf{P}^{(j-1)}$ is not updated to $\mathbf{P}^{(j)}$ directly, but a smoothed updating procedure is exploited in which

$$\mathbf{P}^{(j)} = \lambda * \mathbf{P}^{(j)} + (1-\lambda) * \mathbf{P}^{(j-1)} \tag{13}$$

where λ is the smoothing factor. When $\lambda = 1$, we have the original updating formulation.

For the results in this letter, the sample block used was initially set at $N = 4000$, and then, it was decreased to 1000 when iteration counter j increased from 1 to 100. Other parameters were set as follows: $\rho = 0.9$ and the smoothing factor $\lambda = 0.8$ and $\mu = 1$.

IV. Results

Optimization described in Section III can be used to construct arbitrary length sequences for arbitrary number of transmitters. We have constructed many different lengths of sequences using CE method but present only the correlation and Doppler properties of sequences of length 40 and 128 for comparison with Deng's work [3]. Table I lists three polyphase sequences of length 40, and Table II shows autocorrelation PSLs and cross-correlation peaks for these sequences. Mean PSL for these sequences is −17.3, and mean cross-correlation is −14.3 dB, which improve on Deng sequences of the same length [3]. Fig. 1 shows correlation functions for CE sequences of length 128. When code length is increased, correlation properties are further improved with mean PSL being −22.4 dB and mean cross-correlation −25.8 dB because the degrees of freedom for optimization are increased, leading to a better sequence design. Fig. 2 shows a comparison of Doppler properties for a CE sequence and a Deng sequence of same length of 128. The top plot in the figure shows that Deng sequence deteriorates heavily with increased Doppler, and its peak remains below −12 dB after a relative Doppler shift of 0.7. The CE sequence does not deteriorate at the same level and maintains a plateau of around −11 dB, with a small dip at about $|f_dT| = 1.4$ and then rises again. It should be noted that "Doppler resilient" sequences such as Frank codes are also prone to a 4-dB cyclic loss at every $|f_dT| = 1.4$ intervals and have a decaying Doppler curve. This implies that there is a limit to which Doppler loss can be reduced after a big dip has occurred. The challenge, however, would be to avoid the initial dip while maintaining the correlation properties. The bottom plot in Fig. 2 shows the variation of cross-correlation function as Doppler is increased. Mean cross-correlation of both Deng and CE sequence of length 128 are shown. Fig. 3 shows the effects of varying y, the number of subsequences, on correlation and Doppler properties. As hypothesized earlier, correlation properties are worsened, whereas Doppler tolerance becomes better as y is increased. Table III lists the correlation values and Doppler loss levels (up to $|f_dT = 2.0|$) for various length sequence constructions and

TABLE I
Optimized CE Sequences With $L = 40, M = 3, Y = 4$

Sequence 1	Sequence 2	Sequence 2
8π/5	3π/5	4π/5
π	π/5	2π/5
6π/5	π	0
4π/5	6π/5	0
9π/5	9π/5	π
2π/5	9π/5	9π/5
27π/20	3π/20	21π/20
21π/20	3π/5	9π/20
3π/10	27π/20	6π/5
3π/20	3π/10	9π/20
27π/20	27π/20	21π/20
27π/20	6π/5	27π/20
π/5	π/2	4π/5
0	4π/5	9π/10
π/10	π/2	π/10
4π/5	0	π/5
0	7π/10	9π/10
3π/5	4π/5	9π/10
π/20	0	0
π/5	π/20	π/20
π/20	π/10	π/20
π/4	π/20	π/4
9π/10	4π/5	0
9π/10	π/10	9π/10
π/5	4π/5	0
7π/10	π/10	4π/5
0	9π/10	4π/5
π/5	3π/5	π/5
6π/5	6π/5	27π/20
27π/20	9π/20	3π/20
0	9π/20	27π/20
0	0	27π/20
6π/5	27π/20	3π/10
3π/10	6π/5	3π/4
7π/5	2π/5	6π/5
3π/5	6π/5	3π/5
π/5	7π/5	3π/5
6π/5	3π/5	6π/5
6π/5	0	7π/5
8π/5	9π/5	7π/5

TABLE II
AUTOCORRELATION PSL AND CROSS-CORRELATION PEAKS FOR THE CE SEQUENCE SET OF length $= 40$ USING FOUR DESIGN SUBSEQUENCES DURING OPTIMIZATION ($L = 40, M = 3, Y = 4$)

	Sequence 1	Sequence 2	Sequence 3
Sequence 1	-17.3	-14.8	-14.9
Sequence 2	-14.8	-17.7	-14.7
Sequence 3	-14.9	-14.7	-17.0

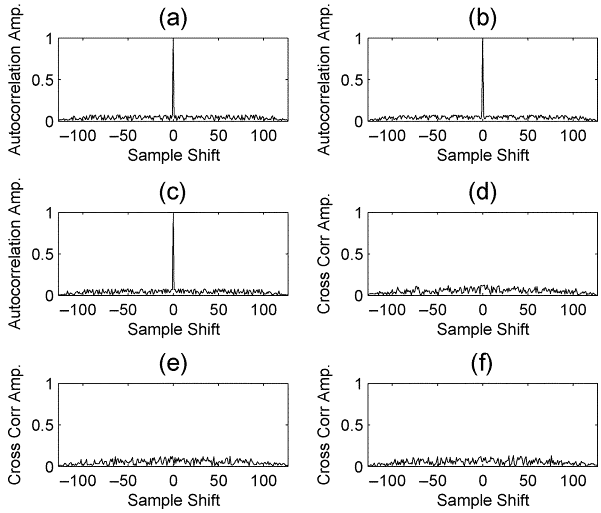

Fig. 1. Autocorrelation and cross-correlation functions of three CE sequences, each of length 128.

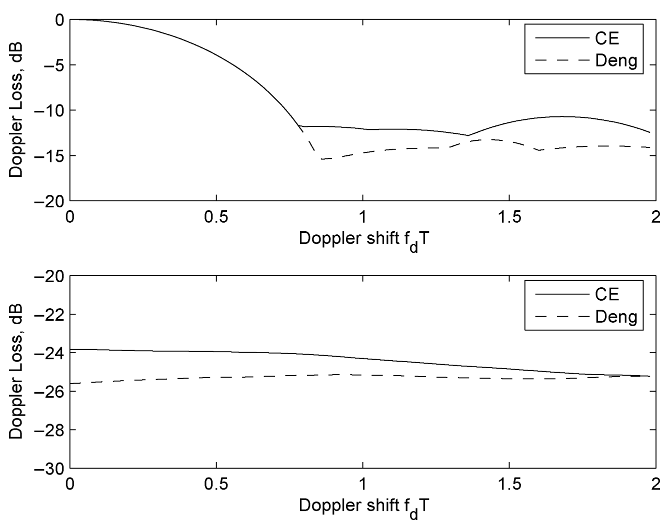

Fig. 2. Doppler variation of autocorrelation function of one CE sequence and one Deng sequence each of length 128 (top) and variation of cross-correlation function with Doppler shift for Deng and CE sequences of length 128 (bottom).

using four subsequences. It can be noted that increasing the length, L, of a sequence only improves correlation properties, and as y is kept constant, Doppler tolerance decreases for higher lengths. This happens because there is more "randomness" in a larger sequence with the same number of subsequences, and hence, it is more prone to Doppler loss. Doppler loss only seems to be affected by the number of subsequences, as shown in Fig. 3.

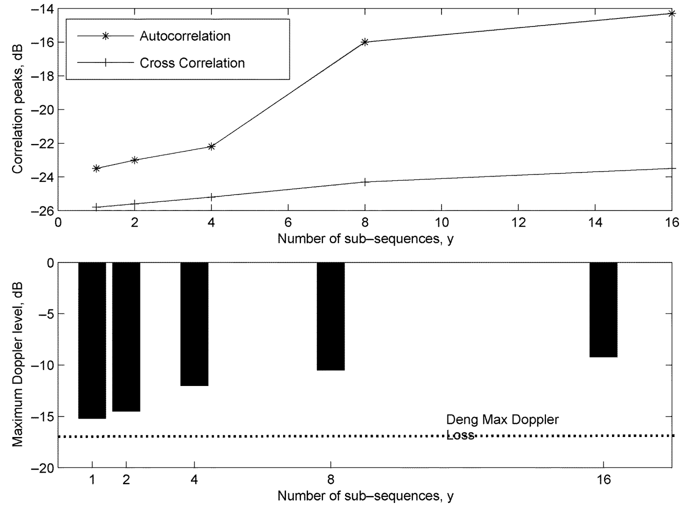

Fig. 3. Variation of PSL and cross-correlation (top) and Doppler loss (bottom) with the number of subsequences used in the CE optimization of length 128.

TABLE III
CORRELATION AND DOPPLER PROPERTIES FOR VARIOUS LENGTH CE SEQUENCES FOR $Y = 4$

L	Autocorrelation, dB	Cross Correlation, dB	Doppler Loss, dB
256	-22.6	-28.3	-14.2
512	-22.7	-31.1	-14.0
1024	-23.6	-33.1	-15.1

V. CONCLUSION

We have presented a new numerically optimized polyphase sequence design method for orthogonal netted radar applications with superior aperiodic correlation and Doppler properties to any existing sequences in literature. Better correlation properties were obtained using the CE optimization technique. Doppler loss decreases as more subsequences are used in design. These results are promising toward improving the aperiodic correlation and Doppler properties simultaneously for multiple sequences.

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