Randomized Space-Time Coding for Distributed Cooperative Communication

Birsen Sirkeci Mergen and Anna Scaglione

Abstract

We study the design of distributed space-time codes for cooperative communication. We assume that each node is equipped with a single antenna; however, to obtain diversity and coding gains, the cooperating nodes act as elements of a multi-antenna system. With few exceptions, most of the literature on the subject proposes coding rules such that each node emulates a predetermined antenna of a multi-antenna system. Since the nodes need to know their specific antenna index, either inter-node communication or a central control unit is required. Our design objective is to obtain diversity and coding gains while eliminating the need for code or antenna allocation. We achieve our objective by introducing novel randomized strategies that decentralize the transmission of a space time code from a set of distributed relays. Our simple idea is to let each node transmit an independent random linear combination of the codewords that would have been transmitted by all the elements of a multi-antenna system. In addition to introducing this new class of designs, we fully characterize the diversity order of the corresponding symbol error probability and also analyze how the performance is linked to different choices of the statistics of the random coefficients. We show that the proposed scheme achieves full diversity (N) if $N < L$, where N is the number of transmitters and L is the number of antennas assumed in the underlying space-time code structure. The diversity order L is achieved if $N > L$. Interestingly, in certain cases (e.g. $N = L = 2$), we show that the achieved diversity order is fractional ($d^* = 1.5$)!

Index Terms

Cooperative transmission, distributed networks, diversity, space-time coding.

B. Sirkeci Mergen {bs233@cornell.edu} is with University of California, Berkeley, CA 94720. This work was done during her PhD at Cornell University, Ithaca, NY, 14853. A. Scaglione {anna@ece.cornell.edu} is with Cornell University, Ithaca NY 14853. Phone:607-254-4959, Fax: 607-255-9072. This work is supported in part by National Science Foundation under grant ITR CCR - 0428427 and in part by Office of Naval Research (ONR) under the contract N00014-00-1-0564. The material in this paper was presented in part at the SPAWC-2005, ICC-2006 and ICASSP-2006.

I. INTRODUCTION

When multiple-antennas are available, the use of space-time codes provides diversity and coding gains that reduce the average error probability over fading channels [1], [2]. In ad-hoc network applications or in distributed large scale wireless networks, the nodes are often constrained in the complexity of their hardware and also in their size. This makes multiple-antenna systems impractical for certain networks.

Recently, several methods have been proposed for cooperation among relay nodes to provide spatial diversity gains without utilizing multiple transmit antennas [3]–[5]. The decode-and-forward strategy is one such method that has been shown to provide various benefits in addition to being information-theoretically optimal in certain scenarios [6]. Common to all decode-and-forward strategies is the fact that the relays first decode the source message reliably and then relay it after re-encoding. Several methods have been proposed for forwarding the common message by the relays, from the simple repetition, to space-time coding [7], to more idealistic approaches derived from the information-theoretic framework established by Cover & El Gamal [8]. In general, space-time coding is superior to repetition, since it provides diversity without a significant loss in spectral efficiency [9].

A major challenge in distributed cooperative transmissions is to find a way to coordinate the relay transmissions without requiring extra control information overhead, which would reduce part of the gain. The coding rule applied by each of the cooperating nodes should, therefore, be identical and independent from node to node. However, most of the distributed space-time codes in the literature do not focus on this issue, see e.g. [7], [10]–[16]. In these schemes, each node emulates a specific array element of a multiple-antenna system; in practice, the implementation requires a centralized code allocation procedure. In addition, in large-scale distributed wireless networks, the set of cooperating nodes is unknown or random in most scenarios. For example, in networks with a single source-destination pair and multiple cooperating relays, the set of nodes that is responsible for retransmission is random due to the error-free decoding constraint. The randomness in the cooperating set may be due to fading, mobility, node failure, expired battery life, or the occurrence of a possible sleep state. In this context, designing codes that provide diversity gains even when the number of cooperating nodes is unknown or random is another issue to address in cooperative networks.

The contribution of this paper is a novel design of a methodology to decentralize the relay transmissions and yet obtain diversity and coding gains analogous to those that can be attained by multi-antenna systems. Our idea is to let each relay transmit an independent, random linear combination of the columns of a space-time code matrix which has a fixed size L, irrespective of the number of cooperative nodes N. Special cases of the proposed scheme include: i) each node emulates one randomly selected antenna; ii) each node transmits the superposition of all antennas with random phases; iii) each node transmits the superposition of all antennas with random gains and phases. We refer to our scheme as *randomized space-time coding* (RSTC). The RSTC entails the specification of a space-time code of size L, and an $L \times N$ random matrix $\boldsymbol{\mathcal{R}}$, whose columns are independent. Random linear mapping are also considered in the context of network coding [19], [20]. Randomization of the transmission signal has also been used in [17], [18] in order to improve the capacity by creating a fast varying channel. In this work, the purpose of randomization, as mentioned before, is to eliminate the need for a centralized code (or *antenna*) allocation procedure.

In order to analyze the performance of the proposed scheme, we express the diversity of the randomized space-time codes as the order of the probability of deep fade event [2] (see Section III-A). The analysis in Section III-A provides the diversity order of any given arbitrary randomization procedure. However, the results are expressed as non-trivial functions of the statistics of $\boldsymbol{\mathcal{R}}$ and, thus, do not lead directly to constructive designs. To provide design guidelines, we resort to a Chernoff bound on the decoding error probability that allows us to derive sufficient conditions under which full diversity is achieved. In our study, we consider random coefficients drawn from both continuous and discrete distributions. For the case of continuous complex coefficients, we provide designs that achieve full diversity under the condition $N \neq L$, where N is the number of active transmitters and L is the number of antennas in the underlying space-time code. We show that, despite the code randomization, the proposed scheme achieves full diversity (N) if $N < L$, and the diversity order L is achieved for $N > L$. Interestingly, for $N = L$ we show that the proposed scheme exhibits a fractional diversity (for example, for $N = L = 2$, the diversity order of the scheme with randomly selected phases is 1.5). For the case of discrete valued random matrices, we observe a multi-slope behavior in the average probability of error for sufficiently large number of nodes ($N > 10$) (see also [21]).

A. Related Work

Other cooperative transmission approaches that apply to a decentralized scenario are in [22] and [23]. In [22], the authors propose a protocol where the relay nodes transmit with randomly chosen delays. Hence, further diversity is obtained by intentionally creating a frequency selective channel. Note that this scheme may not provide diversity gains due to the possibility that each node may choose to use the same delay. In fact, our analysis in Section V provides the performance of a class of forwarding strategies which includes the random delay scheme in [22] as a special case (see also Example 2). In [23], the nodes regenerate the signal at time instants that depend on the energy accumulated per symbol. The decentralized policy produces diversity only if the delays can be resolved at the receiver, which in general requires a large bandwidth.

Other works that address the need for distributed implementation at cooperating nodes are [7], [24]–[27]. In [7], the authors propose orthogonal space-time codes, which may become impractical for large number of nodes. In [24], the authors propose a filtering approach that does not require the knowledge of the number of cooperating nodes in order to achieve maximum diversity. The scheme proposed in [25], has the closest formulation to ours, since each node transmits the product of a space-time code matrix with a pre-assigned vector-code. As a result, this scheme does not require the knowledge of the number of cooperating nodes that are active, but it still requires a preliminary code allocation phase. In one way or another, most of these schemes become impractical in a self-organized networks with a large and/or random number of nodes. The novelty of our work lies in the proposition that the linear coefficients can be chosen randomly and independently at each node, which eliminates the need for antenna/code allocation. Our most interesting finding is that this simple scheme can still provide full diversity as long as $N < L$.

Another linear relaying technique is amplify-and-forward. The schemes in [26], [28] are alternatives to the amplify-and-forward strategy. The authors propose diversity achieving methods that are based on linear mapping of the received message at each relay. Our focus in this paper is, however, on decode and forward strategies.

It is also worth mentioning that, in general, the complexity of the receiver processing (channel estimation, decoding, etc.) increases with the number of cooperative nodes. In order to deal with this, in [29], the authors proposes to utilize space-time codes over group transmissions. The nodes in a specific group transmits a predetermined code with random phases and space-time codes are utilized among the groups. This scheme is a special case of RSTC where the randomization matrix $\boldsymbol{\mathcal{R}}$ takes a block-diagonal form. Note that by changing the fixed size parameter L in RSTC, we can decrease the receiver complexity.

The paper is organized as follows: In Section II, we describe the system model and the proposed scheme. In Section III, we characterize the diversity order of the randomized space-time codes and provide design criteria that leads to full diversity order. In Sections IV, we present specific examples for the randomization matrix $\boldsymbol{\mathcal{R}}$. In Section V, we provide the extended version of antenna selection scheme [21]. In Section VI, we present the simulations. Finally, we conclude in VII.

II. System Model and the Proposed Protocol

We consider a system where a random number of nodes N collaborate in order to transmit a common message to a destination distributively. This problem arises in decode-and-forward communication schemes, where a source node transmits to a group relays (Phase I); N of the relays successfully decode the source message, and transmit the same message simultaneously after re-encoding (Phase II). Fig. 1 describes an analogous scenario, where the end receiver is remotely located relative to the network.

In this paper, we will assume that: 1) the Phase I of the communication has taken place; 2) each relay node can determine whether or not it has reliably decoded the message; 3) only the nodes that has decoded reliably transmit the message; 4) the end receiver uses only the data received from Phase II to decode the message. We will deal exclusively with the Phase II of the communication, and assume that the number of transmitting nodes N (*i.e.*, the *active nodes*) is random due to the error-free decoding constraint.

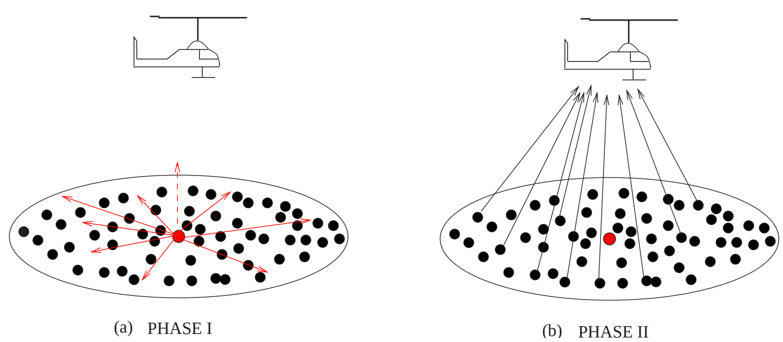

Fig. 1. Two phase cooperative communication.

The output signal for a block space-time coded transmission over a point-to-point $N \times 1$ MISO (multiple-input-single-output) link is generally expressed as follows [9]:

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{w}, \tag{1}$$

where $\mathbf{X} = [X_{ij}] \in \mathbb{C}^{P \times N}$ denotes the transmitted signal (i is the time index, j is the transmitter antenna index), $\mathbf{h} = [h_j] \in \mathbb{C}^{N \times 1}$ denotes the channel gains from different antennas, and $\mathbf{w}$ is the channel noise.

In a block space-time coded cooperative network, the same system model (1) can be used under certain assumptions. For the cooperative system, the j in X_{ij} denotes the user index and h_j is the channel gain from user j to the destination. Furthermore, we assume that the following are satisfied:

a1) The relative receiver and transmitter motion is negligible so that the channels do not change during the course of the transmission of several blocks of data.

a2) Frequency drifts among transmissions from different nodes are negligible. Frequency errors at different nodes are time-invariant over the transmission of several space time codes and the slow phase fluctuations can be incorporated into the channel coefficients $\mathbf{h}$.

a3) There is negligible time-offset among transmissions compared to the symbol interval, *i.e.*, there is no inter-symbol interference (ISI).

We assume a1), a2), a3) to be able to describe the system concisely using equation (1), and also for the analysis of the proposed protocol. Nevertheless, it should be emphasized that for the application of the proposed protocol assumption a3) can be relaxed. The proposed protocol is also applicable to time-asynchronous relays, as discussed in [30]. Note that ISI, which is traditionally viewed as an impairment, can actually improve the system performance by providing frequency diversity. Wei et al. [22] actually proposed introducing random delays to relay transmissions to increase diversity, and showed significant improvements in system performance.

The path-loss and shadowing effects are modelled as a block Rayleigh fading with $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{\Sigma}_h)$, where $\mathbf{\Sigma}_h$ is a positive definite matrix. The receiver noise is modelled by $\mathbf{w} \sim \mathcal{N}_c(0, N_0\mathbf{I})$, where $\mathbf{w}$ is independent of $\mathbf{h}$.

Notation: In the following, $\det(\mathbf{A})$, rank$(\mathbf{A})$, Tr$(\mathbf{A})$ denote the determinant, rank and trace of a matrix $\mathbf{A}$ respectively. In addition, diag$(a_1, a_2, \ldots, a_n)$ denotes $n \times n$ diagonal matrix such that (i, i)'th element is equal to a_i. The identity matrix is denoted by $\mathbf{I}$. All the matrices and vectors will be denoted by bold symbols. A $L \times N$ matrix $\mathbf{A}$ is said to be *full-rank* if rank$(\mathbf{A}) = \min\{L, N\}$.

A. Proposed Diversity Scheme

Let $\mathbf{s} = [s_0 \ s_1 \ldots s_{n-1}]$ be the block of source symbols to be transmitted to the destination. We assume that the message is known perfectly at the active nodes in Phase II. We will consider the transmission of one block of data for simplicity, although the source message will, in general, consist of several blocks. In the following, we describe the processing at each node and analyze the decoding performance at the destination.

At each node, the $\mathbf{s}$ is mapped onto a matrix $\boldsymbol{\mathcal{G}}(\mathbf{s})$ as is done in standard space-time coding:

$$\mathbf{s} \rightarrow \boldsymbol{\mathcal{G}}(\mathbf{s}),$$

where $\boldsymbol{\mathcal{G}}(\mathbf{s})$ is a $P \times L$ space-time code matrix. Here, L denotes the number of antennas in the underlying space-time code. In our scheme each node transmits a block of P symbols, which is a random linear combination of columns of $\boldsymbol{\mathcal{G}}(\mathbf{s})$. Let $\mathbf{r}_i$ be the $L \times 1$ random vector that contains the linear combination coefficients for the i'th node. Define $\mathbf{X} = [\mathbf{x}_1 \ \mathbf{x}_2 \ldots \mathbf{x}_N]$ as the $P \times N$ random code matrix whose rows represent the time and columns represent the space, where $\mathbf{x}_i = \boldsymbol{\mathcal{G}}(\mathbf{s})\mathbf{r}_i$ is the code transmitted by the i'th node. The randomized space time coding can be expressed as the double mapping:

$$\mathbf{s} \rightarrow \boldsymbol{\mathcal{G}}(\mathbf{s}) \rightarrow \boldsymbol{\mathcal{G}}(\mathbf{s})\boldsymbol{\mathcal{R}}, \tag{2}$$

where $\boldsymbol{\mathcal{R}} = [\mathbf{r}_1 \ \mathbf{r}_2 \ldots \mathbf{r}_N]$. In the following, the $L \times N$ matrix $\boldsymbol{\mathcal{R}}$ will be referred to as the *randomization matrix*. Since each node's processing is intended to be local, $\mathbf{r}_i$'s should be independent for each $i = 1 \ldots N$, and we will also assume that they are identically distributed. This property allows the randomized space-time coding to be implemented in a decentralized fashion. In other words, each node chooses a random set of linear combination coefficients from a given distribution, which does not depend on the node index.

Let $\mathbf{y}$ be the received signal at the destination. Using (1), we can rewrite the received signal as

$$\mathbf{y} = \boldsymbol{\mathcal{G}}(\mathbf{s})\boldsymbol{\mathcal{R}}\mathbf{h} + \mathbf{w}, \tag{3}$$

where $\mathbf{w} \sim \mathcal{N}_c(\mathbf{0}, N_0\mathbf{I})$ and $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{\Sigma_h})$.

Definition Define $\mathbf{X} \triangleq \boldsymbol{\mathcal{G}}(\mathbf{s})\boldsymbol{\mathcal{R}}$ as the *randomized space-time code* and $\tilde{\mathbf{h}} \triangleq \boldsymbol{\mathcal{R}}\mathbf{h}$ as the *effective channel*.

These two definitions express two critical interpretations of the proposed scheme. If $\boldsymbol{\mathcal{G}}(\mathbf{s})\boldsymbol{\mathcal{R}}$ is considered as a whole, then the scheme can be viewed as a randomized space-time code $\mathbf{X}$ transmitted over channel $\mathbf{h}$. On the other hand, if $\boldsymbol{\mathcal{R}}\mathbf{h}$ is considered as a whole, then the scheme can be viewed as a deterministic space-time code $\boldsymbol{\mathcal{G}}(\mathbf{s})$ transmitted over a randomized channel $\tilde{\mathbf{h}}$.

The second interpretation is especially important for decoding purposes at the receiver. We assume that the receiver utilizes a coherent detector and in order to perform coherent decoding, the receiver needs to estimate the channel coefficients. Instead of estimating the channel vector $\mathbf{h}$ and the randomization matrix $\boldsymbol{\mathcal{R}}$ separately, the receiver can estimate the effective channel coefficients $\tilde{\mathbf{h}}$. For this, the training data at the transmitters should use the same randomization procedure. Estimating the effective channel provides two main advantages: i) decoders already designed for multiple-antenna space-time codes can be directly used for randomized space-time coding; ii) the number of coefficients that are estimated is less when $L \leq N$, since in this case the effective channel vector $\tilde{\mathbf{h}}$ is shorter that the actual channel vector $\mathbf{h}$.

Yiu et al. [25] proposed a deterministic version of the randomized space-time code scheme (3), where each column of matrix $\boldsymbol{\mathcal{R}}$ is a pre-determined deterministic code allocated to a specific user. The main advantage of this scheme is that it provides robustness to the uncertainty as to which group of relays will transmit in Phase II. That is, the diversity order N is achieved as long as $N \leq L$ irrespective of which N relay nodes transmit. This is different from the orthogonal space-time code approach in [7], because there, if two nodes happen to be allocated the same transmit antenna, then the diversity order is no longer N. Both in [25] and [7], the nodes have to be allocated antennas or codes. The main advantage of randomized space-time coding is that it achieves the full diversity order N for $N < L$ without code or antenna allocation.

In the following, N denotes the number of active relays in Phase II; L and P denote the number of columns and rows of the underlying space time code matrix $\boldsymbol{\mathcal{G}}(\mathbf{s})$ respectively (L is also the maximum diversity order of the underlying space-time code while P is its time duration, in terms of number of symbol intervals).

The signal-to-noise ratio is denoted by SNR; $P_e(\text{SNR})$ is the average error probability; d^* is the diversity order of the randomized space-time code. Often, the notation $\boldsymbol{\mathcal{G}}(\mathbf{s})$ will be replaced simply by $\boldsymbol{\mathcal{G}}$.

B. Performance Metrics

Traditional space-time codes are designed using the probability error as a performance criterion [9]. We will adopt a similar approach for the design of randomized space-time codes. Our main focus is the maximum diversity that can be achieved by the scheme.

Let $\mathcal{M} = \{\mathbf{s}_1, \mathbf{s}_2, \dots \mathbf{s}_{|\mathcal{M}|}\}$ be the message set, where each message is chosen equally likely. Define $\text{SNR} = 1/N_0$ (Eqn. 3). Assume that the effective channel $\tilde{\mathbf{h}}$ is known at the destination (*i.e.*, the receiver has channel state information). Let $P_e(\text{SNR})$ denote the symbol error probability at the destination under the maximum likelihood detection rule, *i.e.*, the probability that a message $\mathbf{s}_i$ is transmitted, but the decoder produces another message $\mathbf{s}_j,\ j \neq i$ (averaged over i and $\tilde{\mathbf{h}}$).

Definition The diversity order d^* of a scheme with probability of error $P_e(\text{SNR})$ is defined as

$$d^* = \lim_{\text{SNR}\to\infty} \frac{-\log P_e(\text{SNR})}{\log \text{SNR}}. \tag{4}$$

We say that the randomized space-time code achieves *diversity order* d if $d \leq d^*$. The randomized space-time code is said to achieve a *coding gain* G if $P_e(\text{SNR}) \leq G\ \text{SNR}^{-d^*}$.

In this paper, we will consider two different types of performance metrics: i) symbol error rate $P_e(\text{SNR})$ (by an upper bound and simulations); ii) diversity order d^* (analytically and by simulations). These metrics do not take channel coding into account. Instead of P_e and d^*, we could analyze outage probability that also takes into account the effect of channel coding. We do not treat this case, however we wish to remark that, in the case of orthogonal space-time codes the outage probability analysis can be easily derived from the error probability analysis carried out here.

III. Design and Analysis of Randomized Space-time Codes

In this section, we analyze the performance of randomized space-time codes and come up with some principles that facilitate the design of the randomization matrix $\boldsymbol{\mathcal{R}}$. Without loss of generality, we assume that $P \geq L$ for the $P \times L$ deterministic space-time code matrix $\boldsymbol{\mathcal{G}}$. Define $\boldsymbol{\mathcal{G}}_i \triangleq \boldsymbol{\mathcal{G}}(\mathbf{s}_i)$.

There is a vast literature on the design of deterministic space-time codes $\{\boldsymbol{\mathcal{G}}_i\}$, and the design of $\{\boldsymbol{\mathcal{G}}_i\}$ problem has been thoroughly investigated by many authors. Our objective in this section is the design of the randomization matrix $\boldsymbol{\mathcal{R}}$ and the analysis of its effect on the diversity order. We will assume that the underlying space-time code satisfies the *rank criterion* [9], which is expected to be satisfied by any optimal design.

C1) *The Rank Criterion for $\boldsymbol{\mathcal{G}}$:* For any pair of space-time code matrices $\{\boldsymbol{\mathcal{G}}_k, \boldsymbol{\mathcal{G}}_i\}$, the matrix $(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$ is full-rank, *i.e.,* of rank L.

A. Exact Characterization of the Diversity Order

The performance degradation in fading channels results from the *deep fade event* as discussed in [2, Ch. 3]. In this section, we first define what the deep fade event means for our communication system and characterize its diversity order. The following lemma asserts that we can equivalently consider the deep fade event instead of P_e for diversity calculations.

Lemma 1: Let $\{||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 \leq \mathrm{SNR}^{-1}\}$ be the deep fade event, and

$$P_{deep}(\mathrm{SNR}) \triangleq \Pr\{||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 \leq \mathrm{SNR}^{-1}\} \tag{5}$$

its probability. If the assumption C1) is satisfied, then the diversity order of P_e is the same as that of the deep fade event, *i.e.,*

$$d^* = \lim_{\mathrm{SNR}\to\infty} \frac{-\log P_{deep}(\mathrm{SNR})}{\mathrm{SNR}}.$$

Remark 1: An interesting corollary from the lemma is that the diversity order d^* is completely independent of the underlying code $\{\boldsymbol{\mathcal{G}}_i\}$ as long as the underlying code is full rank. The main utility of Lemma 1 is that the diversity order of P_{deep} is much easier to analyze than that of P_e.

Proof: The proof is given in Appendix A. ■

In the following, we will equivalently consider $||\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h^{1/2}\hat{\mathbf{h}}||^2$, $\hat{\mathbf{h}} = [\hat{h}_1, \ldots, \hat{h}_N] \sim \mathcal{N}_c(\mathbf{0}, \mathbf{I})$ instead $||\boldsymbol{\mathcal{R}}\mathbf{h}||^2$, $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \boldsymbol{\Sigma}_h)$. Let $\mathbf{U}^H\mathbf{S}\mathbf{U}$ be the eigenvalue decomposition of $\boldsymbol{\Sigma}_h^{1/2}\boldsymbol{\mathcal{R}}^H\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h^{1/2}$, where $\mathbf{U}$ is a random Hermitian matrix and $\mathbf{S} = \mathrm{diag}(\sigma_1^2, \cdots, \sigma_\eta^2)$ are the ordered eigenvalues (squared singular values of $\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h^{1/2}$). Using the properties of the circularly symmetric Gaussian distribution, we obtain

$$P_{deep} = \Pr\left\{\sum_{i=1}^{\eta} \sigma_i^2 |\hat{h}_i|^2 \leq \mathrm{SNR}^{-1}\right\}. \tag{6}$$

The following theorem provides a very general and clean characterization of the diversity order in terms of the distribution of the singular values of $\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h^{1/2}$. Let notation 0^- denote a negative real number that is close to zero and $\Gamma(\alpha_1, \cdots, \alpha_\eta)$ represent the following function:

$$\Gamma(\alpha_1, \ldots, \alpha_\eta) = \lim_{\mathrm{SNR}\to\infty} \frac{-\log \Pr(\sigma_1^2 \leq \mathrm{SNR}^{-\alpha_1}, \cdots, \sigma_\eta^2 \leq \mathrm{SNR}^{-\alpha_\eta})}{\log \mathrm{SNR}}. \tag{7}$$

We call the parameters $\alpha_1, \cdots, \alpha_\eta$ *the deep fade exponents of the singular values.*

Theorem 1: If the assumption C1) is satisfied, then the diversity order of the randomized space-time code is

$$d^* = \inf_{(\alpha_1,\cdots,\alpha_\eta)} \left(\Gamma(\alpha_1,\ldots,\alpha_\eta) + \sum_{i=1}^{\eta}(1-\alpha_i) \right), \tag{8}$$

where the infimum is over $\alpha_i \in [0^-, 1],\ i = 1, \cdots, \eta$.

Intuition and proof: Consider the following events:

i) The singular values are such that $\sigma_i^2 \leq 1/\mathrm{SNR}^{\alpha_i},\ i = 1, \cdots, \eta$ (i.e. σ_i^2 is in deep fade with exponent α_i).

ii) The channel coefficients are such that $|\hat{h}_i|^2 \leq 1/(\eta \mathrm{SNR}^{1-\alpha_i}),\ i = 1, \cdots, \eta$ (i.e. $\hat{h}_i$ is in deep fade with exponent $1 - \alpha_i$).

To calculate the diversity we note that any sufficient condition for the deep fade event provides an upper bound on d^*. If events i) and ii) occur simultaneously, we have a deep fade event $\sum_{i=1}^{\eta} \sigma_i^2 |\hat{h}_i|^2 \leq \mathrm{SNR}^{-1}$, as defined in (6). Because the events i) and ii) are independent, the probability is going to be a product of probabilities and diversity orders are, therefore, additive. With this in mind, the second term ($\sum_{i=1}^{\eta}(1-\alpha_i)$) in equation (8) follows from the Rayleigh distribution; in fact, the diversity order of each event ii) is $(1-\alpha_i)$. The first term follows from the definition in (7). Therefore, $d^* \leq \Gamma(\alpha_1, \ldots, \alpha_\eta) + \sum_{i=1}^{\eta}(1-\alpha_i)$, which implies that

$$d^* \leq \inf_{(\alpha_1,\cdots,\alpha_\eta)} \left(\Gamma(\alpha_1,\ldots,\alpha_\eta) + \sum_{i=1}^{\eta}(1-\alpha_i) \right). \tag{9}$$

For the opposite inequality, see Appendix B for a rigorous proof. ■

The theorem is easiest to understand when $\mathbf{\Sigma}_h = \mathbf{I}$. In this case, σ_i's are the singular values of the randomization matrix $\boldsymbol{\mathcal{R}}$. In simpler terms, the theorem states that the deep fade event happens because of the simultaneous fades of the randomization matrix and the channel coefficients with exponents α_i's and $1 - \alpha_i$'s, respectively. Hence, in our scheme, the randomization of the space-time code matrix may be ill-conditioned.

In order to distinguish between "good" and "bad" design choices for $\boldsymbol{\mathcal{R}}$, we need to understand the conditions under which the σ_i^2's are more likely to be small. Since the singular values $\sigma_\eta^2 \leq \cdots \leq \sigma_1^2$ are ordered, it is easiest for the σ_η^2 to fade. The σ_η^2 fades if and only if the columns of the matrix turn out to be completely or partially confined into a $\eta - 1$ dimensional subspace. This may happen, for example, if two column vectors turn out to be almost parallel to each other, or a column vector approximately lies within the plane spanned by two other column vectors, etc.

In Section IV, for $\mathbf{\Sigma}_h = \mathbf{I}$, we analyze a number of specific designs for $\boldsymbol{\mathcal{R}}$ and conclude that the best designs have random column vectors in $\boldsymbol{\mathcal{R}}$ which have the least probability of being aligned. In fact, the design that performs best among the ones we examine in Section IV has $\boldsymbol{\mathcal{R}}$ with i.i.d. columns uniformly distributed in the complex unit sphere.

A few remarks follow from Theorem 1:

Remark 2: i) In general, finding the distribution of the singular values for a given random matrix distribution is not an easy task. Fortunately, Theorem 1 only requires knowledge of the distribution of the singular values of $\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h^{1/2}$ around zero. We will utilize this observation in Section IV.

ii) Theorem 1 completely characterizes the diversity order of a randomized space-time code for a given $\boldsymbol{\mathcal{R}}$; however, it is non obvious how to use Theorem 1 constructively. In fact, it is unclear how one can choose the singular vector and singular value distributions such that, the singular value distribution has the local properties that are required to maximize d^* in (8) and, at the same time, the columns of $\boldsymbol{\mathcal{R}}$ are statistically independent.

iii) Theorem 1 gives the upper bound

$$d^* \leq \eta = \min(L, N) \tag{10}$$

(choose $\alpha_i = 0^-, \ \forall i$), which says that the diversity order is always bounded by the minimum of the number of relays and the underlying code dimension.

iv) A necessary condition for the randomized code to have maximum diversity order η is that the exponent of the smallest singular value σ_η^2 should be at least 1, *i.e.*,

$$\lim_{\text{SNR}\to\infty} \frac{-\log \Pr(\sigma_\eta^2 \leq \text{SNR}^{-1})}{\log \text{SNR}} \geq 1. \tag{11}$$

This can be seen by substituting $\alpha_i = 0^-, \ i = 1, \cdots, \eta - 1$ in (8) except $\alpha_\eta = 1$. The distribution of the smallest singular value is generally easier to obtain than the joint distribution of all singular values. Consequently, (11) is a simpler condition to check than the condition in Theorem 1.

v) Theorem 1 presents an interesting result. The diversity orders can be *fractional* depending on $\Gamma(\cdot)$. We will see concrete examples of this in Section IV.

B. Upper Bound to the Probability of Error

A brief word about our notation. Let $\mathbf{A}$ be a $n \times n$ Hermitian matrix with eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \lambda_m > 0 \geq \lambda_{m+1} \ldots \geq \lambda_n$. We use the notation $|\mathbf{A}|_{k+}$ to denote the product of k smallest positive eigenvalues of the matrix $\mathbf{A}$, *i.e.*, $|\mathbf{A}|_{k+} = \prod_{i=1}^{k} \lambda_{m-i+1}$. If all eigenvalues are positive, then $|\mathbf{A}|_{n+} = \det(\mathbf{A})$.

We know that the diversity order of the randomized space-time code is always upper bounded by the minimum of the number of relay nodes and the size of the underlying space-time code, *i.e.*, $d^* \leq$

$\min\{N, L\} \triangleq \eta$. The following theorem provides an upper bound to the average error probability and a sufficient condition for the randomized code to have diversity order η.

Theorem 2: Suppose that $\{\boldsymbol{\mathcal{G}}_i\}$ satisfies C1), and the randomization matrix $\boldsymbol{\mathcal{R}}$ satisfies

C2) *Rank criterion for $\boldsymbol{\mathcal{R}}$:* The matrix $\boldsymbol{\mathcal{R}}$ is full-rank with probability 1.

C3) *Finiteness of $\mathbb{E}\{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{\eta+}^{-1}\}$:* The expectation $\mathbb{E}\{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{\eta+}^{-1}\}$ is finite.

Then, the P_e is bounded as

$$P_e \leq \frac{4^{-\eta}(|\mathcal{M}|-1)\text{SNR}^{-\eta}}{\min_{(i,j)}\{\ |(\boldsymbol{\mathcal{G}}_i - \boldsymbol{\mathcal{G}}_j)^H(\boldsymbol{\mathcal{G}}_i - \boldsymbol{\mathcal{G}}_j)|_{\eta+}\}\ |\boldsymbol{\Sigma}_h|_{\eta+}} \mathbb{E}\left\{\frac{1}{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{\eta+}}\right\}. \tag{12}$$

Proof: See Appendix C. ∎

Remark 3: Note that here, it is assumed that the channel $\mathbf{h}$ and the randomization matrix $\boldsymbol{\mathcal{R}}$ changes over the transmission so that the packet experiences multiple realizations.

Remark 4: Notice that the diversity order of the upper bound in (12) is η. Since the diversity order d^* cannot exceed η, we observe from Theorem 2 that the randomized space-time code has maximum diversity order η, as long as C1)-C3) are satisfied.

What kind of random matrices satisfy the rank criterion for $\boldsymbol{\mathcal{R}}$? We know that almost all square matrices over the field of real or complex numbers are invertible, *i.e.,* the set of singular square matrices have Lebesgue measure zero. In general, any random matrix with independent columns drawn from a continuous distribution satisfies the rank criterion. However, this alone does not guarantee the diversity order η. The upper bound in (12) is useful only if $\mathbb{E}\left\{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{\eta+}^{-1}\right\} < \infty$. This is a rather stringent condition, and not all almost-surely full rank matrices satisfy it. In the next section, we will present some sufficient conditions for this to be true.

The bound in Eqn. 12 can be tightened by improving the coding gain. The following conditions are needed: i) $\min_{(i,j)} |(\boldsymbol{\mathcal{G}}_i - \boldsymbol{\mathcal{G}}_j)^H(\boldsymbol{\mathcal{G}}_i - \boldsymbol{\mathcal{G}}_j)|_{\eta+}$ should be maximized with respect to $\{\boldsymbol{\mathcal{G}}_i\}$; ii) $\mathbb{E}\left\{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{\eta+}^{-1}\right\}$ should be minimized with respect the distribution of $\boldsymbol{\mathcal{R}}$. Note that condition i) is a slightly modified version of the determinant criterion in [9].

C. Diversity Order for Randomized Space-time Codes with Power Constraint

In this section we will employ a transmit power constraint on the relay nodes to facilitate the analysis of randomized space-time codes. Let $P_T < \infty$ be the total relay power available to the network such that[1]

$$\text{Tr}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H) \leq P_T \quad \text{with probability } 1. \tag{13}$$

Under the conditions of the following theorem, we show that C3) holds, and therefore the diversity order of the randomized scheme is η.

Theorem 3: Let $\boldsymbol{\mathcal{R}}$ be an $L \times N$ random *complex* matrix and $p(\boldsymbol{\mathcal{R}})$ its probability density function. Assume that the function $p(\boldsymbol{\mathcal{R}})$ is bounded and it satisfies the total power constraint (13). For $N \neq L$, if C1) and C2) are satisfied, then $\mathbb{E}\{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{\eta+}^{-1}\} < \infty$. Therefore, the diversity order of the randomized space-time code is given by

$$d^* = \begin{cases} N & \text{if } N \leq L-1 \\ L & \text{if } N \geq L+1 \end{cases} \tag{14}$$

For $N = L$, the diversity order is such that $N - 1 \leq d^* \leq N$.

Proof: See Appendix D. ■

Remark 5: The above result shows that the randomized space-time codes achieve the maximum diversity order N achievable by any scheme if $N < L$. It also indicates the diversity order saturates at L if the number of relay nodes is greater than or equal to $L + 1$. This problem can be solved by using space-time codes with large enough dimensions. However, N may be random and may take large values in practical networks. In such cases, using smaller L may be preferred for decoding simplicity. For fixed L, randomized space-time codes still give the highest order L for $N \geq L + 1$.

Corollary 1: Let $\boldsymbol{\mathcal{R}}$ be an $L \times N$ random *real* matrix and $p(\boldsymbol{\mathcal{R}})$ its probability density function, which is assumed to be bounded. Suppose that C1) and C2) are satisfied, and the total power constraint (13) holds. Then, the diversity order of the randomized space-time code is given by

$$d^* = \begin{cases} N & \text{if } N \leq L-2 \\ L & \text{if } N \geq L+2 \end{cases} \tag{15}$$

For $N \in \{L-1, L, L+1\}$, the diversity order is such that $N - 2 \leq d^* \leq \min(N, L)$.

Proof: The proof follows from modifying the proof of Theorem 3 for the real valued $\boldsymbol{\mathcal{R}}$. We avoid it for brevity. ■

[1]Notice that there is no expectation in the power condition. We want it to be satisfied almost surely. Condition (13) implies that the pdf of $\boldsymbol{\mathcal{R}}$ has bounded support.

Remark 6: The diversity order of a randomized space-time code is closely related to how ill-conditioned the matrix $\boldsymbol{\mathcal{R}}$ is. This relates to the behavior of the joint distribution of the singular values around origin (Theorem 1). Theorem 3 indicates that, for $N \neq L$ it is quite hard for a complex valued matrix $\boldsymbol{\mathcal{R}}$ to be ill-conditioned. On the other hand, for real valued matrices, ill-conditioned matrices are more likely and, hence, we need at least $|N-L| \geq 2$.

IV. Specific Designs and Their Performance

In this section, we propose different randomized space-time codes and derive the diversity order of these designs using Theorem 1 and Theorem 3. Furthermore, in Section VI, the average error probabilities of these designs are obtained via Monte-Carlo simulations. In the following, we assume that $\mathbf{h} \sim \mathcal{N}(0, \mathbf{I})$.

A. Complex Gaussian distribution

Let us assume elements of the $L \times N$ dimensional randomization matrix $\boldsymbol{\mathcal{R}}$ are zero-mean independent and complex Gaussian. In the random matrix literature, the Gaussian random matrix is one of the most studied [31], [32]. The joint probability density function of the non-zero eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \lambda_\eta$ of the matrix $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ (known as Wishart) is given as

$$f(\lambda_1, \ldots, \lambda_N) = C_{N,L} \exp(-\sum_{i=1}^{\eta} \lambda_i) \prod_{i=1}^{\eta} \lambda_i^{|N-L|} \prod_{i<j} (\lambda_i - \lambda_j)^2, \tag{16}$$

where $C_{N,L}$ is a constant. In the following, we provide the diversity order of this scheme.

1) Case $N \neq L$: Using the results in [31], we obtain

$$\mathbb{E}\{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}|_{\eta+}^{-1}\} = \begin{cases} \frac{(N-L-1)!}{(N-1)!} & \text{if } N \geq L+1 \\ \frac{(L-N-1)!}{(L-1)!} & \text{if } L \geq N+1, \end{cases}$$

where $\eta = \min(L, N)$. Since $\mathbb{E}\{|\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}|_{\eta+}^{-1}\} < \infty$ when $N \neq L$, the upper bound on the average error probability is given as follows (using Theorem 2):

$$P_e \leq \frac{4^{-\eta}(|\mathcal{M}|-1)\text{SNR}^{-\eta}}{\min_{(i,j)}\{|(\boldsymbol{\mathcal{G}}_i - \boldsymbol{\mathcal{G}}_j)^H(\boldsymbol{\mathcal{G}}_i - \boldsymbol{\mathcal{G}}_j)|_{\eta+}\} \; |\boldsymbol{\Sigma}_h|_{\eta+}} \frac{(|N-L|-1)!}{(\max(N,L)-1)!}. \tag{17}$$

Eqn. 17 shows that (14) also holds for $\boldsymbol{\mathcal{R}}$ with i.i.d. complex Gaussian elements. Note that the total power constraint (13) is not satisfied in this scenario. However, we arrive at the same conclusion on the diversity order d^* which we derived previously through Theorem 3.

2) Case $N = L$: We can approximate the probability density of non-zero eigenvalues of the Wishart matrix $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ (Eqn. 16) around zero as

$$f(\lambda_1, \ldots, \lambda_N) \approx c\lambda_1^{2(N-1)}\lambda_2^{2(N-2)} \ldots \lambda_{(N-1)}^2. \tag{18}$$

Using Theorem 1 and (18), the diversity order is

$$d^* = \inf_{\alpha_1,\ldots,\alpha_N} \underbrace{(2N-1)\alpha_1 + (2N-3)\alpha_2 + \ldots + \alpha_N}_{\Gamma(\alpha_1,\ldots,\alpha_N)} + \sum_{i=1}^{N}(1-\alpha_i) = N,$$

where the infimum is obtained when $\alpha_i = 0, \forall i$. Hence, if the elements of the randomization matrix $\boldsymbol{\mathcal{R}}$ are drawn independently and identically from a zero mean complex Gaussian distribution, the full diversity is also achieved for the $N = L$ case.

B. Real Gaussian distribution

Let us assume that the elements of the randomization matrix $\boldsymbol{\mathcal{R}}$ are zero-mean independent and real Gaussian. The joint probability density function of the non-zero eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \lambda_\eta$ of the Wishart matrix $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^T$ is given as

$$f(\lambda_1, \ldots, \lambda_\eta) = \tilde{C}_{N,L} \exp(-\sum_{i=1}^{\eta} \lambda_i) \prod_{i=1}^{\eta} \lambda_i^{\frac{|N-L|-1}{2}} \prod_{i<j}(\lambda_i - \lambda_j), \tag{19}$$

where $\tilde{C}_{N,L}$ is a constant. We can approximate the probability density of the eigenvalues (19) around zero as

$$f(\lambda_1, \ldots, \lambda_\eta) \approx c \prod_{i=1}^{\eta} \lambda_i^{\frac{|N-L|-1}{2}+\eta-i}. \tag{20}$$

We then find $\Gamma(\cdot)$ as $\Gamma(\alpha_1, \ldots, \alpha_\eta) = \sum_{i=1}^{\eta} \left(\frac{|N-L|-1}{2} + \eta\right) \alpha_i$. Using Theorem 1 and (20), the diversity order is obtained as follows:

1) Case $N \neq L$: For this case, $d^* = \eta$ where the infimum is obtained when $\alpha_i = 0, \forall i$.

2) Case $N = L$: For this case, $d^* = \inf_{\alpha_1,\ldots,\alpha_\eta}(\sum_{i=1}^{\eta}(\eta - i/2) + \sum_{i=1}^{\eta}(1-\alpha_i)) = \eta - 0.5$. The infimum is obtained when $\{\alpha_i = 0, i = 1 \ldots \eta, \alpha_\eta = 1\}$.

Therefore, in this case the diversity order d^* is given by

$$d^* = \begin{cases} \eta & \text{if} \quad N \neq L, \\ \eta - 0.5 & \text{if} \quad N = L, \end{cases}$$

where $\eta = \min(N, L)$. Note that the scheme provides a fractional diversity order when $N = L$.

C. Uniform phase distribution

Let us assume that the k'th column of the $L \times N$ randomization matrix is $\mathbf{r}_k = a_k[e^{j\theta_i[0]}, \ldots, e^{j\theta_i[L]}]^t$ where each $\theta_i[N] \sim U(0, 2\pi)$ and $a_k \sim U(1-\epsilon, 1+\epsilon)$ for some small $\epsilon > 0$, where $U(a,b)$ denotes the uniform distribution in the interval (a,b) and all $\theta_i[N], a_k$ are independent of each other. The main advantage of this scheme lies in its ability to control the transmission power at each node. The total power is bounded as

$$\mathrm{Tr}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H) = L\sum_{i=1}^{N} |a_i|^2 \leq NL(1+\epsilon)^2.$$

1) Case $N \neq L$: Using Theorem 3, we conclude that the diversity order d^* satisfies (14). For $\epsilon = 0$, that is $\mathbf{r}_k = [e^{j\theta_i[0]}, \ldots, e^{j\theta_i[L]}]^t$, the randomization matrix $\boldsymbol{\mathcal{R}}$ can be interpreted as a random phase matrix. In this case, unfortunately the probability density function of $\boldsymbol{\mathcal{R}}$ does not exist[2], hence we can not directly use Theorem 3. However, we believe the result (14) is also valid in this scenario and we will see that this is true by numerical examples.

2) Case $N = L = 2$: Consider the random phase matrix $\boldsymbol{\mathcal{R}}$ for $\epsilon = 0$. The eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ can be found as $\lambda_1 = 2 + \sqrt{2 + 2\cos(\theta)}$ and $\lambda_2 = 2 - \sqrt{2 + 2\cos(\theta)}$, where θ is a uniform random variable in the interval $[0, 2\pi)$. Note that $\lambda_1 \in [1, 4]$ with probability 1. Using Theorem 1 and the fact that $\lambda_1 \geq 1$, we can easily see that the optimal $\alpha_1 = 0^-$. Hence, the problem simplifies to determining

$$d^* = \min_{\alpha_2}\ \Gamma(0^-, \alpha_2) + 2 - \alpha_2. \tag{21}$$

One can derive the distribution of λ_2 as

$$F_{\lambda_2}(\lambda) = \Pr\{\lambda_2 \leq \lambda\} = \frac{2}{\pi}\cos^{-1}(1 - \frac{\lambda}{2}),\ 0 \leq \lambda \leq 2.$$

Then, the behavior of the $F_{\lambda_2}(\lambda)$ around zero is given as $F_{\lambda_2}(\lambda) \approx \frac{2}{\pi}\sqrt{\lambda}$, as $\lambda \to 0$. The infimum in (21) is obtained when $\alpha_2 = 1$, which gives us a fractional value $d^* = 1.5$.

D. Uniform distribution on a hypersphere

Let us assume that the k'th column of the $L \times N$ randomization matrix, $\mathbf{r}_k$, is uniformly selected on the surface of a complex/real hypersphere of radius ρ, *i.e.,* $||\mathbf{r}_k|| = \rho$. Note that, in this case, the total power

[2]To see why the pdf of $\boldsymbol{\mathcal{R}}$ does not exist, let's look at the special case where $\boldsymbol{\mathcal{R}}$ is 1×1. Here all the probability mass is concentrated on the unit circle. Hence the "pdf" is what is sometimes referred to as an impulse sheet. Viewed in an engineering sense, this pdf is not bounded (hence Theorem 3 does not apply). From the measure theoretic point of view, the measure induced by $\boldsymbol{\mathcal{R}}$ is not absolutely continuous with respect to the Lebesgue measure on the complex plane [33]. Therefore, its Radon-Nikodym derivative (hence its pdf) with respect to Lebesgue measure does not exist.

TABLE I

DIVERSITY ORDER FOR DIFFERENT SCHEMES.

Distribution of $\mathbf{R}$	Condition	Diversity Order
Complex Gaussian	$N = L$	N
Complex Gaussian	$N \neq L$	$\min(N, L)$
Real Gaussian	$N = L$	$N - 0.5$
Real Gaussian	$N \neq L$	$\min(N, L)$
Uniform Phase	$N \neq L$	$\min(N, L)$
Uniform Phase	$N = L = 2$	1.5
Real Spherical Distribution	$N = L = 2$	2
Complex Spherical Distribution	$N = L = 2$	2
Random antenna selection	any N and L	1

constraint (13) is satisfied, *i.e.,*

$$\mathrm{Tr}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H) = \rho^2 N < \infty.$$

Similar to uniform phase randomization with $\epsilon = 0$ (Section IV-C), the probability density function of $\boldsymbol{\mathcal{R}}$ does not exist in this case. However, we will show through numerical examples that (14) is still valid.

1) Real hypersphere with $N = L = 2$: Let us assume that the columns of the randomization matrix $\boldsymbol{\mathcal{R}}$ are drawn uniformly on a sphere. We can obtain the eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^T$ as $\lambda_1 = 1+\cos^2(\theta), \lambda_2 = 1-\cos^2(\theta)$, where $\theta \sim U(0, 2\pi)$. Note that $\lambda_1 \geq 1$ and $\Pr\{\lambda_2 \leq \lambda\} \approx \lambda/(2\pi)$ as $\lambda \rightarrow 0$. Using Theorem 1, the diversity order is

$$d^* = \min_{\alpha_2} \Gamma(0^-, \alpha_2) + 2 - \alpha_2 = 2,$$

where the infimum is obtained when $\alpha_1 = 0^-$ and α_2 is any value.

2) Complex hypersphere with $N = L = 2$: Let us assume that the columns of the randomization matrix $\boldsymbol{\mathcal{R}}$ are drawn uniformly on complex hypersphere. We obtain the eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ as $\lambda_1 = 1 + \sqrt{\zeta}/2, \lambda_2 = 1 - \sqrt{\zeta}/2$, where $\zeta \sim F_{24}$ and F_{nm} is the F-distribution. Note that $\lambda_1 \geq 1$. Using Theorem 1, the diversity order obtained is $d^* = 2$, where the infimum obtained when $(\alpha_1, \alpha_2) = (0, 0^-)$. Table I summarizes the diversity order of the proposed schemes.

V. ANTENNA SELECTION AND DISCRETE RANDOMIZATION MATRIX

The case considered in this section is that where the randomization matrices $\boldsymbol{\mathcal{R}}$ are drawn from discrete distributions. In the next example, we present the random selection matrices.

Example 1: Let $\boldsymbol{\mathcal{R}} = [\mathbf{r}_1 \ldots \mathbf{r}_N]$ be a random matrix such that $\mathbf{r}_i \in Q \triangleq \{\mathbf{e}_i, i = 1 \ldots L\}$ where $\mathbf{e}_i$ is the vector of all zeros except the i'th position, which is 1. Note that the randomized space-time coding, with the selection matrix $\boldsymbol{\mathcal{R}}$, corresponds to assigning the columns of a given space-time code matrix at random to each of the nodes. This scheme will be referred as *random antenna selection*. In [21], we analyzed the performance of random antenna selection with an underlying orthogonal space-time code. We showed that this simple method almost meets the ideal performance for SNR below a threshold SNR_t, which increases with node density. In the following, we extend the results in [21] to more general scenarios.

When the randomization matrix $\boldsymbol{\mathcal{R}}$ is drawn from a discrete distribution, the probability that the rank of $\boldsymbol{\mathcal{R}}$ is unity, *i.e.*, $\Pr\{\text{rank}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H) = 1\}$ is nonzero. In the light of this observation, the following lemma presents the diversity order of this scheme with finite L, N.

Lemma 2: The randomized space-time coding, with $\boldsymbol{\mathcal{R}}$ drawn from a discrete distribution, has diversity order $d^* = 1$ for $N < \infty$.

Proof: The proof follows from Theorem 1. The diversity order is obtained when $\{\alpha_1 = 0^-, \alpha_i = 1, \forall i = 1\}$. ■

Lemma 2 states that the maximum diversity that can be achieved with schemes based on randomization matrices drawn from discrete distribution is 1, which is quite discouraging. This is somewhat misleading as can be shown by studying the diversity order as the number of nodes increases. We now define the *asymptotic diversity order*.

Definition Let $P_e^{(N)}(\text{SNR})$ denote the probability of error of a randomized space-time code utilizing an $L \times N$ randomization matrix $\frac{\boldsymbol{\mathcal{R}}}{\sqrt{N}}$. Then, the *asymptotic probability of error* $P_e^{\infty}(\text{SNR})$ is defined as

$$P_e^{\infty}(\text{SNR}) \triangleq \lim_{N\to\infty} P_e^{(N)}(\text{SNR}).$$

Also, the *asymptotic diversity order* D of this randomized space-time code is defined as

$$D \triangleq \lim_{\text{SNR}\to\infty} \frac{-\log P_e^{\infty}(\text{SNR})}{\log \text{SNR}}. \tag{22}$$

In the asymptotic case, full diversity conditions are more relaxed. The sufficient conditions in order to achieve the asymptotic diversity order $D = L$ are provided in the following theorem. In order to derive the asymptotic probability of error $P_e^{\infty}(\text{SNR})$, we utilize the behavior of effective channel for large N in the proof of next theorem.

Theorem 4: Let $\boldsymbol{\mathcal{R}} = [\mathbf{r}_1 \ldots \mathbf{r}_N]$ be an $L \times N$ random matrix such that the columns $\mathbf{r}_i$ are i.i.d. with zero-mean and covariance $\boldsymbol{\Sigma}$. Assume that $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \boldsymbol{\Sigma}_h)$, where $\boldsymbol{\Sigma}_h = \text{diag}(\sigma_{h1}^2, \sigma_{h2}^2, \ldots, \sigma_{hN}^2)$. If $L < \infty$, then the asymptotic diversity order $D = L$ is achieved if the following conditions are satisfied:

1) $(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$ is full-rank,
2) $\mathbf{\Sigma}$ is full-rank, *i.e.*, $\det(\mathbf{\Sigma}) > 0$.

Proof: See Appendix E. ∎

The behavior of the schemes utilizing discrete randomization matrices changes dramatically in the high node asymptote due to Theorem 4. From Lemma 2, we know that as SNR→∞, the diversity order of this system is 1 for $N < \infty$. On the other hand, from Theorem 4, in the asymptote that number of nodes goes to infinity, the asymptotic diversity order is $\eta = \min(L, N)$. In addition, an interesting observation on the behavior of networks with finite but sufficiently large number of nodes is made. The average error probability curve (in the typical logarithmic scale, versus SNR in dB) exhibits multiple slopes in different SNR ranges. The justification for this behavior is as follows.

Assume that $(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$ is of rank L for any pair of space-time code matrices $\{\boldsymbol{\mathcal{G}}_k, \boldsymbol{\mathcal{G}}_i\}$. Let $\eta = \min(L, N)$. Let $S = \{\sigma_1^2, \sigma_2^2, \ldots, \sigma_\eta^2\}$ be the set of non-negative eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ ordered such that σ_1^2 is the largest. Let us rewrite the average probability error as a polynomial in $1/\text{SNR}$ (using (43) and (48)): $P_e \leq \bar{P}_e$, where

$$\bar{P}_e = \sum_{m=1}^{\eta} \underbrace{B_m \, \mathbb{E}\left\{ \prod_{i=\eta-m+1}^{\eta} \sigma_i^{-2} | \text{rank}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H) = m \right\}}_{\triangleq C_m} \frac{1}{\text{SNR}^m}, \tag{23}$$

where $B_m \triangleq \frac{4^m(|\mathcal{M}|-1)\Pr\{\text{rank}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H)=m\}}{|(\boldsymbol{\mathcal{G}}_k-\boldsymbol{\mathcal{G}}_i)^H(\boldsymbol{\mathcal{G}}_k-\boldsymbol{\mathcal{G}}_i)|_{m+}|\mathbf{\Sigma}_h|_{m+}}$. The expression (23) helps explain the fact that when the number of nodes is finite but *sufficiently* large, the probability of error curve changes its slope, but above a certain SNR threshold, the expected $O(1/\text{SNR})$ behavior is obtained. The breaking points of the curve change and move towards higher SNRs as the number of nodes increases. In fact, depending on the values of $\{C_m\}$, the range of SNR where the term C_m/SNR^m is dominant in the summation (23) can be derived as follows:

$$\max_{k>m} \left(\frac{C_m}{C_k}\right)^{\frac{1}{m-k}} << \text{SNR} << \min_{k<m} \left(\frac{C_m}{C_k}\right)^{\frac{1}{m-k}}, \tag{24}$$

if $\min_{k<m} (C_m/C_k)^{\frac{1}{m-k}} >> \max_{k>m} (C_m/C_k)^{\frac{1}{m-k}}$ (for $m = 1$, the upper bound is ∞ and for $m = L$, the lower bound is 0). In Section VI, we will show this behavior in a numerical example.

The main advantage of choosing columns of $\boldsymbol{\mathcal{R}}$ from a discrete distribution is the simplification in the encoder, since the random selection can be enforced at the data link layer and hence, the scheme can be implemented in logic without any modification of the existing physical layer modem.

Example 2: (*Frequency Diversity*)

In [22], the authors propose a protocol where the cooperating nodes introduce intentional delays in order to obtain diversity through an artificially created frequency selective channel. In one version of their protocol, they allow the nodes to randomly select the delays from a pool. This scheme can be reexpressed as a randomized space-time code: Let $\boldsymbol{\mathcal{G}}(\mathbf{s})$ be a Toeplitz matrix having all the possible shifted replicas of the transmitted signal and the randomization matrix $\boldsymbol{\mathcal{R}}$ be the selection matrix. The performance of this scheme under coherent detector is analyzed in Section V. Furthermore, the diversity and coding gains can be attained if the strategy is combined with a coded- OFDM modem that includes a cyclic prefix on the order of the maximum allowed delay dispersion among the cooperative relays (see also [11]).

VI. Simulations & Numerical Evaluations

In this section, we present the performance of the proposed *randomized* distributed space-time codes. We obtain the average probability of error through Monte-Carlo methods and validate the conclusions we draw in the analytical sections. We compare the performance of randomized schemes with the centralized space-time codes for different values of N and L. In the following, we assume the nodes channel gains to the destination are i.i.d., *i.e.,* $h_k \sim \mathcal{N}_c(0, 1)$.

In Fig. 2, we look at the performance of Alamouti scheme under different randomization methods and compare it with a centralized space-time coding. Here $L = 2$, and

$$\boldsymbol{\mathcal{G}}(\mathbf{s}) = \begin{bmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{bmatrix},$$

where $\mathbf{s} = [s_1 \ s_2]$ is the transmitted symbol vector and $s_i = \pm 1$ (BPSK symbols). The randomization is done in four different ways: (i) Complex Gaussian randomization (see Section IV-A) (ii) Uniform phase randomization (see Section IV-C) (iii) Uniform spherical randomization (see Section IV-D) and (iv) Random antenna selection (see Section V - Example 1). Let $\mathbf{r}_i$ be the i'th column of the randomization matrix $\boldsymbol{\mathcal{R}}$. In uniform phase randomization, each element of $\mathbf{r}_i$ is equal to $e^{j\theta}$ where θ is a random variable uniformly distributed in $[0, 2\pi)$. In the case of Gaussian randomization, $\mathbf{r}_i$'s are zero-mean independent complex Gaussian vectors with covariance $\mathbf{I}$. In the uniform spherical randomization, $\mathbf{r}_i$'s are chosen as zero-mean independent complex Gaussian vectors with covariance $\mathbf{I}$, and then normalized to have the norm $\rho = ||\mathbf{r}_i|| = 1$ [34], [35].

In the centralized Alamouti, half of the nodes choose to serve as the first antenna, and the other half choose to serve as the second antenna (if N is odd, at one of the nodes the power is equally distributed between two antennas). The transmission power of each node is $P_t = \frac{1}{N}$ for the centralized Alamouti[3], antenna selection, and spherical randomization schemes. For the Gaussian and uniform phase randomization

[3]Note that the centralized scheme using two relays with $P_t = 0.5$ would have the same performance.

schemes, $P_t = \frac{1}{NL}$.[4] This way the average transmission power of each antenna is approximately the same for all schemes, hence the comparison is more fair.

In Fig. 2, we plot the average probability of error with respect to SNR $= 1/N_0$ for $N = 2, 3, 4, 10$. From theoretical analysis, for $N = 2$, we know that the Gaussian and spherical randomization schemes have diversity order $d^* = 2$; on the other hand, uniform phase randomization has diversity order $d^* = 1.5$ and the diversity order of the random antenna selection is 1. This is supported by the simulation results. However, for $N = 2$, the performance of the centralized scheme is much better than the decentralized schemes. We also plot the upper bounds to the average probability of error ($\bar{P}_e$, Eqn. 43), which are very close to the actual P_e curves. For $N = 3, 4$, the Gaussian, uniform phase, and spherical randomization schemes achieve diversity order 2 similar to the centralized scheme. However, the centralized scheme has a better coding gain. Nevertheless, one can observe that as N increases the performance of the distributed schemes approaches the centralized scheme not only in the diversity order but also in the coding gain.

In Fig. 3, we look at the performance of an orthogonal space-time code of order $L = 3$:

$$\boldsymbol{\mathcal{G}}(\mathbf{s}) = \begin{bmatrix} s_1 & 0 & s_2 & -s_3 \\ 0 & s_1 & s_3^* & s_2^* \\ -s_2^* & -s_3 & s_1^* & 0 \end{bmatrix}^t,$$

where $\mathbf{s} = [s_1 \; s_2 \; s_3]$ is the transmitted symbol vector. Note that the rate of this code is $3/4$. In the centralized scheme, for $N \geq L$, the nodes are divided into L equal number groups, and if N is not a multiple of L, then at the remaining nodes, the power is distributed equally among the L antennas. If $N < L$, the nodes imitate N of the preselected antennas. Similar to the Alamouti coding, the transmission power of each node is $P_t = \frac{1}{N}$ for the centralized scheme, antenna selection, and spherical randomization schemes, and $P_t = \frac{1}{NL}$ for the Gaussian and uniform phase randomization schemes. In Fig. 3, for $N = 2$, the diversity order $d^* = 2$ is achieved by centralized, Gaussian randomization and uniform phase, on the other hand, the antenna selection scheme has the worst performance. For $N = 3$, the centralized scheme has diversity order $d^* = 3$ and the performance is much better than the decentralized schemes. In addition, the performance of the randomization via continuous distributions (Gaussian, uniform phase and spherical) is considerably superior to the antennas selection scheme. For $N = 4$, the Gaussian, uniform phase, and spherical randomization schemes achieve diversity order 3. Similar to the Alamouti scheme, the performance of all the randomized

[4]The aim of normalization by $1/L$ is to make the comparison fair among different randomization schemes; the normalization by $1/N$ is just to cancel the effect of power enhancement due to transmission of N nodes; hence we are able to distinguish the diversity order easily. Note that in general, normalization by $1/L$ should depend on the selected code $\boldsymbol{\mathcal{G}}$.

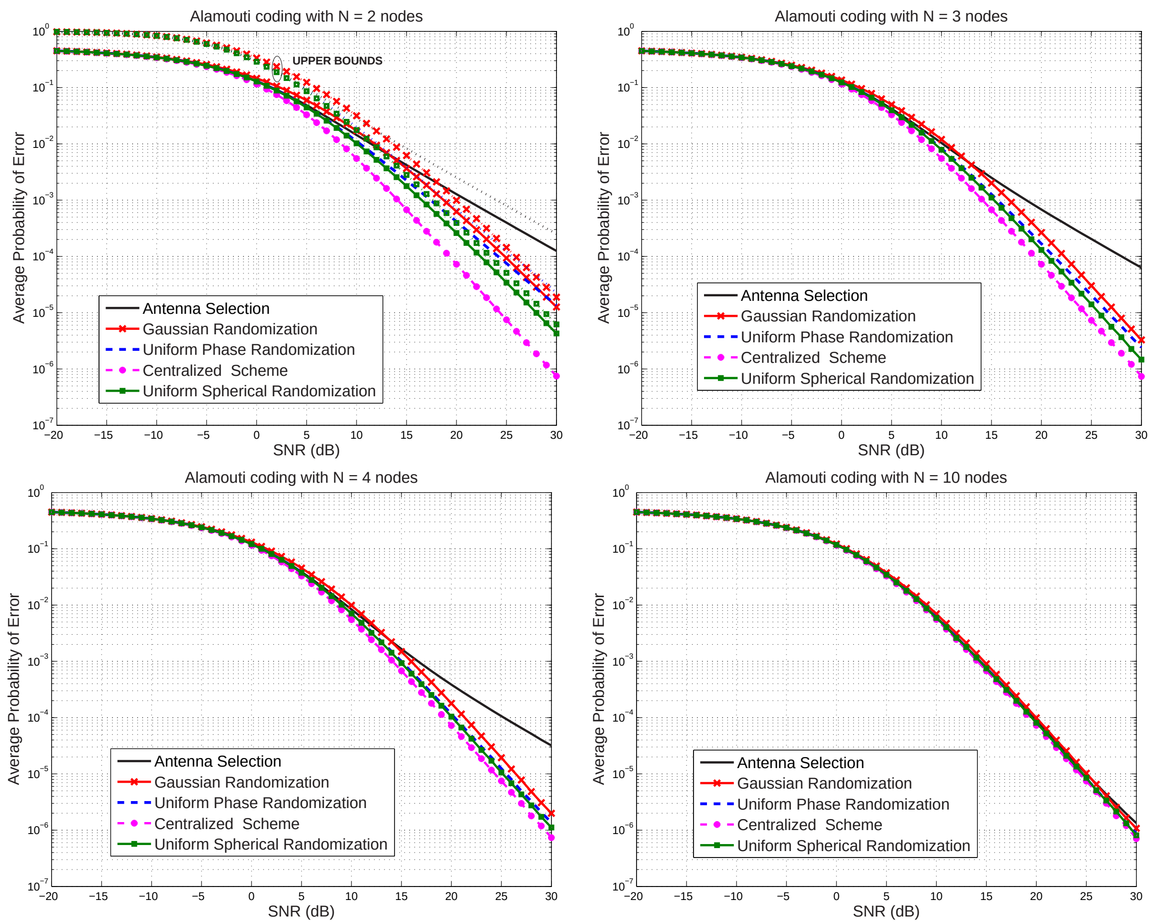

Fig. 2. Average Probability of Error versus SNR (dB): L = 2 (a) N=2 (b) N=3 (c) N=4 (d) N=10 . For $N = 2$, the upper bounds to the average probability of error are drawn for each of the schemes with dotted curves.

schemes converges to the performance of centralized space-time coding as N increases.

In the next experiment, we present the multi-slope behavior of antenna selection scheme with the underlying deterministic Alamouti space-time code. When N is odd and $L = 2$, the analytical expression of the average error probability simplifies to [21],

$$P_e = \frac{1}{2^N} \sum_{k=0}^{N} \binom{N}{k} \frac{g(k) - g(N-k)}{2k - N}, \tag{25}$$

where $g(x) = \frac{x}{2}\left(1 - \sqrt{\frac{x\ \text{SNR}}{x\ \text{SNR}+1}}\right)$. The numerical evaluation of (25) for $N = 3, 5, 7, 9, 11$ is displayed in addition to the asymptotic result (centralized scheme performance) in Fig. 4.

As expected, the P_e curves have a breaking point, which becomes more pronounced as N increases; beyond a certain SNR, they all have the same slope which corresponds to diversity order 1. For SNR values

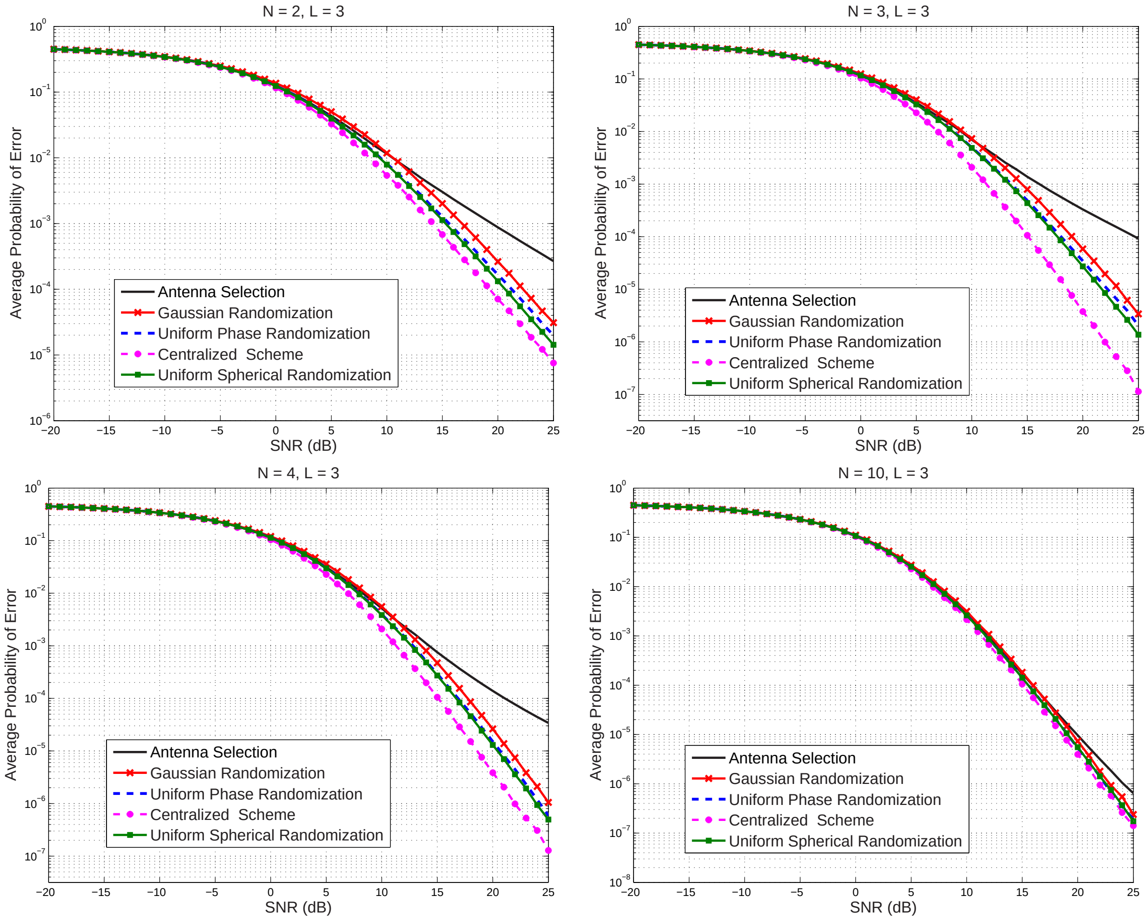

Fig. 3. Average Probability of Error versus SNR (dB): L = 3 (a) N=2 (b) N=3 (c) N=4 (d) N=10

less than a threshold, the diversity order 2 is achieved. This can be clearly seen for $N = 11$ which has a breakpoint around SNR $= 35$ dB.

VII. Conclusion

In this paper, we proposed a decentralized space-time coding for distributed networks. Our scheme is based on independent randomization done at each node. We analyzed its performance and proposed different designs that achieve the diversity order ($\min(N, L)$) when the number of nodes N is different than the number of antennas L in the underlying space-time code. For $N = L$, we presented examples where the diversity order is fractional. In addition, we showed that the randomized schemes achieve the performance of a centralized space-time code in terms of coding gain as the number of nodes increases.

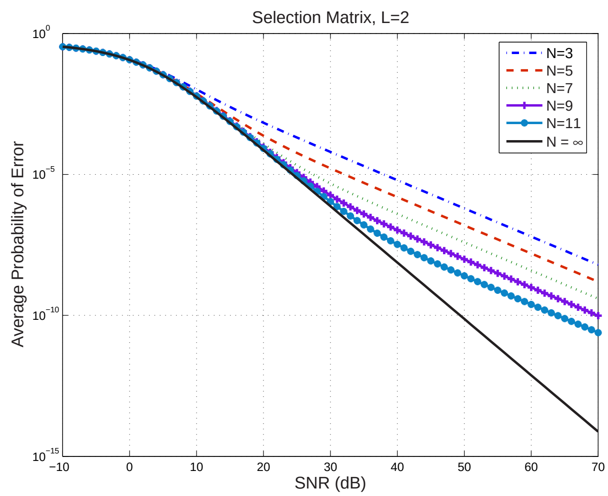

Fig. 4. Average error probability behavior w.r.t. N

APPENDIX

A. Proof of Lemma 1

The average probability of error (41) can be lower bounded as follows ($\forall \mathbf{s}_k, \mathbf{s}_i \in \mathcal{M}, i \neq k$):

$$P_e \geq \frac{1}{|\mathcal{M}|}\Pr(\mathbf{s}_k \to \mathbf{s}_i) \geq \frac{1}{|\mathcal{M}|}\Pr(\mathbf{s}_k \to \mathbf{s}_i \mid ||\mathbf{R}\mathbf{h}||^2 \leq \frac{1}{\mathrm{SNR}})\Pr\{||\mathbf{R}\mathbf{h}||^2 \leq \frac{1}{\mathrm{SNR}}\}. \quad (26)$$

In the following, we will assume that $(\mathcal{G}_k - \mathcal{G}_i)$ is of rank L. Let $\lambda_{ik,max}$ be the maximum eigenvalue of the matrix $(\mathcal{G}_k - \mathcal{G}_i)^H(\mathcal{G}_k - \mathcal{G}_i)$. Define $\lambda_{max} \triangleq \max_{i,k} \lambda_{ik,max}$. Let $Q(\cdot) = \int_x^\infty \frac{1}{\sqrt{2}} e^{-u^2/2} du$. Since $Q(\cdot)$ is a decreasing function, the first part of (26) is lower bounded as

$$\Pr(\mathbf{s}_k \to \mathbf{s}_i \mid ||\mathbf{R}\mathbf{h}||^2 \leq \frac{1}{\mathrm{SNR}}) = \mathbb{E}\left\{Q\left(\sqrt{\mathrm{SNR}/2}||(\mathcal{G}_k - \mathcal{G}_i)\boldsymbol{\mathcal{R}}\mathbf{h}||\right) \mid ||\mathbf{R}\mathbf{h}||^2 \leq \frac{1}{\mathrm{SNR}}\right\} \quad (27)$$

$$\geq \mathbb{E}\left\{Q\left(\sqrt{\mathrm{SNR}}\sqrt{\lambda_{max}/2}\ ||\boldsymbol{\mathcal{R}}\mathbf{h}||\right) \mid ||\mathbf{R}\mathbf{h}||^2 \leq \frac{1}{\mathrm{SNR}}\right\} \geq Q\left(\sqrt{\lambda_{max}/2}\right). \quad (28)$$

Then the average probability of error can be lower bounded as

$$P_e \geq \frac{1}{M} Q\left(\sqrt{\lambda_{max}/2}\right)\Pr\{||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 \leq \mathrm{SNR}^{-1}\}. \quad (29)$$

The pairwise error probability can be upper bounded as follows for some $\alpha \in (0,1)$:

$$\begin{aligned}\Pr(\mathbf{s}_k \to \mathbf{s}_i) &= \Pr(\mathbf{s}_k \to \mathbf{s}_i, ||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 \leq \mathrm{SNR}^{-\alpha}) + \Pr(\mathbf{s}_k \to \mathbf{s}_i, ||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 > \mathrm{SNR}^{-\alpha}) \\ &\leq \Pr\{||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 \leq \mathrm{SNR}^{-\alpha}\} + \underbrace{\Pr\{\mathbf{s}_k \to \mathbf{s}_i \mid ||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 > \mathrm{SNR}^{-\alpha}\}}_{\triangleq P_{ki}}. \quad (30)\end{aligned}$$

Next, we will upper bound the probability P_{ki}. For the system given by Eqn. 3, the conditional pairwise error probability is upper bounded as $\Pr\{\mathbf{s}_k \rightarrow \mathbf{s}_i|\boldsymbol{\mathcal{R}},\mathbf{h}\} \leq \exp(-(\text{SNR}/4)||(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)\boldsymbol{\mathcal{R}}\mathbf{h}||^2)$. Then,

$$P_{ki} \leq \mathbb{E}\{\exp(-\frac{\text{SNR}\lambda_{min,ik}||\boldsymbol{\mathcal{R}}\mathbf{h}||^2}{4}) \mid ||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 > \text{SNR}^{-\alpha}\}, \tag{31}$$

where $\lambda_{min,ik}$ is the minimum eigenvalue of $(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)^H(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$. The right-hand side of (31) converges to zero as SNR$\rightarrow\infty$ since $\alpha \in (0,1)$. Using the union bound, the average probability of error can be upper bounded by the pairwise error probabilities assuming that all source messages $\mathbf{s}_i \in \mathcal{M}$ are equally likely:

$$P_e \leq (|\mathcal{M}|-1)\max_{i,k}\Pr\{\mathbf{s}_k \rightarrow \mathbf{s}_i\}. \tag{32}$$

Using (30),(31) and (32)

$$d^* = \lim_{\text{SNR}\rightarrow\infty}\frac{-\log(P_e)}{\log\text{SNR}} \geq \lim_{\text{SNR}\rightarrow\infty}\frac{-\log\Pr\{||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 \leq \text{SNR}^{-\alpha}\}}{\log\text{SNR}}. \tag{33}$$

Since (33) is valid $\forall\alpha \in (0,1)$, if we take the limit as $\alpha\rightarrow 1$, it is still valid. The lemma follows from this fact and (29).

B. Proof of Theorem 1

The argument after the theorem proves Eqn. 7. In this appendix we will prove the opposite inequality. Using (6) we obtain that

$$\Pr\{||\boldsymbol{\mathcal{R}}\mathbf{h}||^2 \leq \frac{1}{\text{SNR}}\} = \Pr\{\sum_{i=1}^{\eta}\sigma_i^2|\hat{h}_i|^2 \leq \frac{1}{\text{SNR}}\} \leq \Pr\{\underbrace{\sigma_1^2|\hat{h}_1|^2 \leq \text{SNR}^{-1},\ldots,\sigma_\eta^2|\hat{h}_\eta|^2 \leq \text{SNR}^{-1}}_{\triangleq S_\eta}\}.$$

In the following, we will prove the theorem for $\eta = 2$. Generalization to $\eta > 2$ will be obvious. Let $\gamma \triangleq$ SNR. Let $S_2 = \{\sigma_1^2|\hat{h}_1|^2 \leq \gamma^{-1}, \sigma_2^2|\hat{h}_2|^2 \leq \gamma^{-1}\}$. Let $P_2 = \Pr\{S_2\}$ and n be a fixed parameter. We can write P_2 as:

$$P_2 = \sum_{i=1}^{n}\underbrace{\Pr\{\gamma^{(i-1)/n-1} \leq |\hat{h}_1|^2 \leq \gamma^{i/n-1}, S_2\}}_{P_{21i}} + \underbrace{\Pr\{|\hat{h}_1|^2 \leq \gamma^{-1}, S_2\}}_{P_{22}} + \underbrace{\Pr\{|\hat{h}_1|^2 \geq 1, S_2\}}_{P_{23}}. \tag{34}$$

Using the definitions of P_{21i} and the event S_2, we obtain

$$\begin{aligned} P_{21i} &= \Pr\{\gamma^{(i-1)/n-1} \leq |\hat{h}_1|^2 \leq \gamma^{i/n-1}, \sigma_2^2|\hat{h}_2|^2 \leq \gamma^{-1}, \sigma_1^2 \leq \gamma^{(1-i)/n}\}. \\ &\leq \Pr\{|\hat{h}_1|^2 \leq \gamma^{i/n-1}, \sigma_2^2|\hat{h}_2|^2 \leq \gamma^{-1}, \sigma_1^2 \leq \gamma^{(1-i)/n}\}. \end{aligned} \tag{35}$$

Similarly, using the definitions of P_{22}, P_{23} and the event S_2, we obtain

$$\begin{aligned} P_{22} &\leq \Pr\{|\hat{h}_1|^2 \leq \gamma^{-1}, \sigma_2^2|\hat{h}_2|^2 \leq \gamma^{-1}\} = \Pr\{|\hat{h}_1|^2 \leq \gamma^{-1}\}\Pr\{\sigma_2^2|\hat{h}_2|^2 \leq \gamma^{-1}\} \\ P_{23} &= \Pr\{|\hat{h}_1|^2 \geq 1, \sigma_1^2 \leq \gamma^{-1}, \sigma_2^2|\hat{h}_2|^2 \leq \gamma^{-1}\} \leq \Pr\{\sigma_1^2 \leq \gamma^{-1}\}. \end{aligned} \tag{36}$$

Let $P_3 \triangleq \Pr\{\sigma_2^2|\hat{h}_2|^2 \leq \gamma^{-1}\}$. Using similar tricks, we can also upper bound P_3 as

$$P_3 \leq \sum_{j=1}^{n} \Pr\{|\hat{h}_2|^2 \leq \gamma^{j/n-1}, \sigma_2^2 \leq \gamma^{(1-j)/n}\} + \Pr\{|\hat{h}_2|^2 \leq \gamma^{-1}\} + \Pr\{\sigma_2^2 \leq \gamma^{-1}\} \tag{37}$$

Let A, B, C, D be four events. Then we know that,

$$\Pr\{A\} \leq \Pr\{B\} + \Pr\{C\} \quad \Rightarrow \quad \Pr\{A, D\} \leq \Pr\{B, D\} + \Pr\{C, D\}. \tag{38}$$

Using (38), we obtain

$$\begin{aligned} P_{21i} \leq & \sum_{j=1}^{n} \Pr\{|\hat{h}_1|^2 \leq \gamma^{i/n-1}, \sigma_1^2 \leq \gamma^{(1-i)/n}, |\hat{h}_2|^2 \leq \gamma^{j/n-1}, \sigma_2^2 \leq \gamma^{(1-j)/n}\} \\ & + \Pr\{|\hat{h}_1|^2 \leq \gamma^{i/n-1}, \sigma_1^2 \leq \gamma^{(1-i)/n}, |\hat{h}_2|^2 \leq \gamma^{-1}\} \\ & + \Pr\{|\hat{h}_1|^2 \leq \gamma^{i/n-1}, \sigma_1^2 \leq \gamma^{(1-i)/n}, \sigma_2^2 \leq \gamma^{-1}\} \\ = & \sum_{j=1}^{n} \Pr\{|\hat{h}_1|^2 \leq \gamma^{i/n-1}\} \Pr\{|\hat{h}_2|^2 \leq \gamma^{j/n-1}\} \Pr\{\sigma_1^2 \leq \gamma^{(1-i)/n}, \sigma_2^2 \leq \gamma^{(1-j)/n}\} \\ & + \Pr\{|\hat{h}_1|^2 \leq \gamma^{i/n-1}\} \Pr\{|\hat{h}_2|^2 \leq \gamma^{-1}\} \Pr\{\sigma_1^2 \leq \gamma^{(1-i)/n}\} \\ & + \Pr\{|\hat{h}_1|^2 \leq \gamma^{i/n-1}\} \Pr\{\sigma_1^2 \leq \gamma^{(1-i)/n}, \sigma_2^2 \leq \gamma^{-1}\}. \end{aligned} \tag{39}$$

Also,

$$\begin{aligned} P_{22} \leq & \sum_{j=1}^{n} \Pr\{|\hat{h}_1|^2 \leq \gamma^{-1}\} \Pr\{|\hat{h}_2|^2 \leq \gamma^{j/n-1}, \sigma_2^2 \leq \gamma^{(1-j)/n}\} \\ & + \Pr\{|\hat{h}_1|^2 \leq \gamma^{-1}\} \Pr\{|\hat{h}_2|^2 \leq \gamma^{-1}\} + \Pr\{|\hat{h}_1|^2 \leq \gamma^{-1}\} \Pr\{\sigma_2^2 \leq \gamma^{-1}\}. \end{aligned} \tag{40}$$

Compute the diversity order of P_2 from Eqn. (34) (for both sides). Let $\alpha_1 = (i-1)/n, \alpha_2 = (j-1)/n$. It is true that

$$\lim_{\gamma\to\infty} \frac{\log(\Pr\{A\} + \Pr\{B\})}{\log \text{SNR}} = \min\left(\lim_{\gamma\to\infty} \frac{\log(\Pr\{A\})}{\log\gamma}, \lim_{\gamma\to\infty} \frac{\log(\Pr\{B\})}{\log\gamma}\right).$$

Using this fact, we can explicitly find the diversity order of P_{21i} (Eqn. 39), P_{22} (Eqn. 40),P_{23} (Eqn. 36); and hence the diversity order of P_2. Then, consider the asymptote as $n\to\infty$. By further analysis, we can see that

$$\lim_{\gamma\to\infty} \frac{-\log(P_2)}{\log\gamma} \geq \min_{\alpha_1,\alpha_2}\{2 - \alpha_1 - \alpha_2 + \Gamma(\alpha_1, \alpha_2).\}$$

By using similar tricks, we can find the diversity order of P_η as

$$d^* \geq \min_{\boldsymbol{\alpha}}\{\eta - \sum_{i=1}^{\eta} \alpha_i + \Gamma(\alpha_1, \ldots, \alpha_\eta), \}$$

where $0^- \leq \alpha_i \leq 1$.

C. *Proof of Theorem 2*

Using the union bound, the average probability of error can be upper bounded by the pairwise error probabilities assuming that all source messages $\mathbf{s}_i \in \mathcal{M}$ are equally likely:

$$P_e \leq \frac{1}{|\mathcal{M}|} \sum_{\mathbf{s}_k \in \mathcal{M}} \sum_{\mathbf{s}_i \in \mathcal{M}, i \neq k} \Pr(\mathbf{s}_k \to \mathbf{s}_i), \tag{41}$$

where $\Pr(\mathbf{s}_k \to \mathbf{s}_k)$ denotes the probability that a transmitted message $\mathbf{s}_k$ is mistaken for another message $\mathbf{s}_i$. Let $\mathbf{s}_k \in \mathcal{M}$ denote the transmitted symbol. For the system given by Eqn. 3, the conditional pairwise error probability is upper bounded as

$$\Pr\{\mathbf{s}_k \to \mathbf{s}_i | \boldsymbol{\mathcal{R}}, \mathbf{h}\} \leq \exp\left(-\frac{\text{SNR}||(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)\boldsymbol{\mathcal{R}}\mathbf{h}||^2}{4}\right). \tag{42}$$

Assuming $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{\Sigma}_h)$ (for a given positive definite hermitian $\mathbf{\Sigma}_h$); using (41) and (42), the average error probability of coherent detection (averaged over $\{\boldsymbol{\mathcal{R}}, \mathbf{h}\}$) is bounded as,

$$\bar{P}_e \triangleq \mathbb{E}_{\boldsymbol{\mathcal{R}}} \left\{ \frac{|\mathcal{M}| - 1}{\min_{(i,k)} \det(\mathbf{I} + \text{SNR}/4\ (\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)^H (\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i) \boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H)} \right\}. \tag{43}$$

Define $\mathbf{A}_{ik} \triangleq (\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)^H (\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$. Assume conditions C1 is satisfied, and $\boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H$ is of rank d with probability 1. We will upper bound $\bar{P}_e$ (43) for the proof. In the following, we assume that we are given a realization $\boldsymbol{\mathcal{R}}$ of rank $d \leq \min(L, N)$, then the final result follows by averaging over all such realizations.

Under the given conditions, we know that $\boldsymbol{\mathcal{R}}^H \mathbf{A}_{ik} \boldsymbol{\mathcal{R}}$ has non-negative real eigenvalues. Then,

$$\det(\mathbf{I} + \text{SNR}/4\ \mathbf{A}_{ik} \boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H) \quad \geq \quad |\text{SNR}/4\ \mathbf{A}_{ik} \boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H|_{d+}. \tag{44}$$

Let $\boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H = \mathbf{Q}\mathbf{S}\mathbf{Q}^H$ be the eigenvalue decomposition of $\boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H$ where $\mathbf{Q}\mathbf{Q}^H = \mathbf{Q}^H\mathbf{Q} = \mathbf{I}$ and $\mathbf{S} = \text{diag}(\lambda_1, \ldots, \lambda_d, 0, \ldots, 0)$ such that $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_d > 0$. Define $L \times d$ semi-unitary matrix $\mathbf{U}$ as $\mathbf{U} = (q_{ik})_{i=1 \ldots L, k=1 \ldots d}$ where $\mathbf{Q} = (q_{ik})_{i=1 \ldots L, k=1 \ldots L}$. Notice that $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Let $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_d)$, then

$$|\text{SNR}/4\ \mathbf{A}_{ik} \boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H|_{d+} = |\text{SNR}/4\ \mathbf{A}_{ik} \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H|_{d+} = |\text{SNR}/4\ \mathbf{U}^H \mathbf{A}_{ik} \mathbf{U} \mathbf{\Lambda}|_{d+}. \tag{45}$$

We know that $d \times d$ matrix $\mathbf{U}^H \mathbf{A}_{ik} \mathbf{U}$ is of rank d [36, Section 0.4.5(c), page 13]. Furthermore, the eigenvalues of $\mathbf{U}^H \mathbf{A}_{ik} \mathbf{U}$ and the eigenvalues of $\mathbf{A}_{ik}$ have an inequality relation [36, Section 4.3.16, page 190]. That is, let $\lambda_n(\mathbf{C})$ denote the n'th smallest eigenvalue of a matrix $\mathbf{C}$, then $\lambda_n(\mathbf{A}_{ik}) \leq \lambda_n(\mathbf{U}^H \mathbf{A}_{ik} \mathbf{U}),\ n = 1 \ldots d$. Using this result,

$$\begin{aligned}|\text{SNR}/4\ \mathbf{U}^H \mathbf{A}_{ik} \mathbf{U} \mathbf{\Lambda}|_{d+} &= (\text{SNR}/4)^d |\mathbf{U}^H \mathbf{A}_{ik} \mathbf{U}|_{d+}\ |\mathbf{\Lambda}|_{d+} \\ &\geq (\text{SNR}/4)^d |\mathbf{A}_{ik}|_{d+}\ |\mathbf{\Lambda}|_{d+} = (\text{SNR}/4)^d |\mathbf{A}_{ik}|_{d+}\ |\boldsymbol{\mathcal{R}} \mathbf{\Sigma}_h \boldsymbol{\mathcal{R}}^H|_{d+}\end{aligned} \tag{46}$$

We know that the positive eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h\boldsymbol{\mathcal{R}}^H$ are the same as the positive eigenvalues of $\boldsymbol{\mathcal{R}}^H\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h$. Using similar techniques to the derivation of (46), we obtain

$$|\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h\boldsymbol{\mathcal{R}}^H|_{d+} \geq |\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{d+}|\boldsymbol{\Sigma}_h|_{d+}. \tag{47}$$

Using (44), (45), (46) and (47), we obtain

$$\det(\mathbf{I} + \text{SNR}/4\ \mathbf{A}_{ik}\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h\boldsymbol{\mathcal{R}}^H) \geq (\text{SNR}/4)^d|\mathbf{A}_{ik}|_{d+}\ |\boldsymbol{\Sigma}_h|_{d+}\ |\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H|_{d+}. \tag{48}$$

Then the proof follows by letting $d = \eta = \min(N, L)$.

D. Proof of Theorem 3

In the following we will first assume that $L \neq N$. Consider a complex random matrix $\boldsymbol{\mathcal{R}}(L \times N, L < N)$ with the probability density function $p(\boldsymbol{\mathcal{R}})$, then the density of the matrix $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ is called the generalized Wishart density [37]. The following formula allows us to compute the probability density function of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ from the probability density function of $\boldsymbol{\mathcal{R}}$. The formula is the generalization of Theorem 1.3.1 in [37] to the complex random matrices.

Let Γ be the set of unitary $N \times N$ matrices and μ the normalized Haar measure on it [5], $\boldsymbol{\mathcal{R}}$ a rectangular random $L \times N$ matrix ($L \leq N$) with the probability density $p(\boldsymbol{\mathcal{R}})$. Let Λ be the set of Hermitian positive definite matrices. Then the probability density function (pdf) of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$ is equal to

$$f_{\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H}(\mathbf{Z}) = \frac{1}{c_{L,N}}\int_\Gamma p(\sqrt{\mathbf{Z}}\tilde{\mathbf{U}})\det(\mathbf{Z})^{N-L}\mu(d\mathbf{U}), \tag{49}$$

where $L \times L$ matrix $\mathbf{Z} \in \Lambda$, $\mathbf{U} = (u_{ik}) \in \Gamma$, $\tilde{\mathbf{U}} = (u_{ik})_{i=1\ldots L,\ k=1\ldots N}$, $c_{L,N} = \pi^{L(L-1)/2-LN}\prod_{k=1}^{L}(N-k)!$.

By using the formula for the pdf of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$, we conclude that $\mathbb{E}\{\det(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H)^{-1}\} < \infty$ if and only if the integral

$$I \triangleq \int_\Lambda\int_\Gamma p(\sqrt{\mathbf{Z}}\tilde{\mathbf{U}})\det(\mathbf{Z})^{N-L-1}\mu(d\mathbf{U})d\mathbf{Z} \tag{50}$$

is finite. The notation $d\mathbf{Z}$ refers to the Lebesgue measure on the set of $L \times L$ dimensional matrices. The proof of the theorem follows from bounding the integral (50). Remember that the density $p(\cdot)$ is bounded by a constant, say c_1. Therefore, the righthand side of (50) is bounded as

$$\int_\Lambda\int_\Gamma p(\sqrt{\mathbf{Z}}\tilde{\mathbf{U}})\det(\mathbf{Z})^{N-L-1}\mu(d\mathbf{U})d\mathbf{Z} \leq c_1\int_\Lambda\int_\Gamma \det(\mathbf{Z})^{N-L-1}\mu(d\mathbf{U})d\mathbf{Z}. \tag{51}$$

[5] The μ can be viewed as the uniform distribution on Γ. More formally, a measure μ on Γ is called a Haar measure if $\mu(UA) = \mu(A)$ holds $\forall U \in \Gamma$ and measurable set $A \subset \Gamma$, where UA is the set of all matrices of the form Ua where $a \in A$. Haar measure μ on Γ is called normalized if $\mu(\Gamma) = 1$.

Since $\mu(\cdot)$ is the normalized Haar measure,

$$\int_{\Lambda}\int_{\Gamma} \det(\mathbf{Z})^{N-L-1}\mu(d\mathbf{U})d\mathbf{Z} = \int_{\Lambda} \det(\mathbf{Z})^{N-L-1}d\mathbf{Z}. \tag{52}$$

For a Hermitian positive definite matrix $\mathbf{Z}$, Tr$(\mathbf{Z}) \leq P_T \Rightarrow \det(\mathbf{Z}) \leq P_T^L < \infty$. Furthermore, if $N \geq L+1$, the exponent of $\det(\mathbf{Z})$ in (52) is non-negative. Then using the constraint Tr$(\mathbf{Z}) \leq P_T$ on the integration domain Λ,

$$\int_{\Lambda} \det(\mathbf{Z})^{N-L-1}d\mathbf{Z} \;\; = \;\; \int_{\text{Tr}(\mathbf{Z})\leq P_T} \det(\mathbf{Z})^{N-L-1}d\mathbf{Z} \leq P_T^{L(N-L-1)} \int_{\text{Tr}(\mathbf{Z})\leq P_T} d\mathbf{Z}. \tag{53}$$

Note that for a Hermitian positive-definite matrix $\mathbf{Z}$, Tr$(\mathbf{Z}) \leq P_T \Rightarrow ||\mathbf{Z}||_F \leq P_T$, where $||\mathbf{Z}||_F$ is the Frobenious norm of $\mathbf{Z}$. Then,

$$\int_{\text{Tr}(\mathbf{Z})\leq P_T} d\mathbf{Z} \leq \int_{||\mathbf{Z}||_F\leq P_T} d\mathbf{Z} = \frac{\pi^{N^2/2}P_T^{N^2}}{\Gamma(N^2/2+1)}, \tag{54}$$

where $\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt$. The final integration amounts to finding a volume of N^2 dimensional sphere. By combining (51), (52), (53) and (54), we obtain $I \leq c_1 P_T^{L(N-L-1)} \frac{\pi^{N^2/2}P_T^{N^2}}{\Gamma(N^2/2+1)} < \infty$. Hence, $\mathbb{E}\{\det(\boldsymbol{\mathcal{R}}^H\boldsymbol{\mathcal{R}})^{-1}\} < \infty$. Similar to the above derivation, we can easily show that If $L \geq N+1$, then $\mathbb{E}\{\det(\boldsymbol{\mathcal{R}}^H\boldsymbol{\mathcal{R}})^{-1}\} < \infty$. Hence, the result (14) follows.

For $N = L$, we expect that $N-1 \leq d^* \leq N$. We know that $d^* \leq N$ (a conclusion that can be drawn from Theorem 1). The fact that $d^* > N-1$ for $N = L$ can be proved as follows: Consider the randomized code obtained using an $L' \times N'$ dimensional randomization matrix $\boldsymbol{\mathcal{R}}$ such that $L' = N$ and $N' = N-1$. We know that such a system has diversity order $N-1$ (using Eqn. 14). Adding 1 more node to a system would never decrease the diversity order; hence the diversity order of the randomized space-time code with $N \times N$ dimensional randomization matrix is at least $N-1$.

E. Proof of Theorem 4

The effective channel vector $\tilde{\mathbf{h}} = \boldsymbol{\mathcal{R}}\mathbf{h}$ is conditionally Gaussian with zero mean and covariance $\boldsymbol{\mathcal{R}}\boldsymbol{\Sigma}_h\boldsymbol{\mathcal{R}}^H$. In the following, we provide the statistics of $\tilde{\mathbf{h}}$ as $N\rightarrow\infty$. Denote (i,j)'th element of $\boldsymbol{\mathcal{R}}$ by r_{ij}. Define $\mathbf{Z}_k = [h_k r_{1k}, h_k r_{2k}, \ldots, h_k r_{Lk}]^t$ for $k = 1 \ldots N$. We can rewrite $\tilde{\mathbf{h}}$ in terms of the random vectors $\mathbf{Z}_k$, i.e. $\tilde{\mathbf{h}} = [\tilde{h}_1\tilde{h}_2\ldots\tilde{h}_L]^t = \sum_{k=1}^N \mathbf{Z}_k$.

In the following, first we derive the mean and variance of $\mathbf{Z}_k$ and then by using the multivariate central limit theorem [38, pp. 20] as $N\rightarrow\infty$, we show that $\frac{\mathbf{h}}{\sqrt{N}}$ converges in distribution to a Gaussian random variable. We know that $\boldsymbol{\mathcal{R}}$ is a random matrix independent of $\mathbf{h}$, then the mean of $\mathbf{Z}_k$ is $\mathbb{E}\{\mathbf{Z}_k\} = \mathbf{0}$, and the covariance matrix is $\mathbf{\Sigma}_k = \mathbb{E}\{\mathbf{Z}_k\mathbf{Z}_k^H\} = \sigma_k^2\mathbf{\Sigma}$. Since $\mathbf{Z}_k$'s are independent, using the multivariate central limit theorem, we can conclude that

$$\frac{\tilde{\mathbf{h}}}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}\mathbf{Z}_k \to^d \mathcal{N}(\mathbf{0}, \tilde{\mathbf{\Sigma}}) \text{ as } N\to\infty, \tag{55}$$

where $\tilde{\mathbf{\Sigma}} = \mathbf{\Sigma} \ \lim_{N\to}(\sum_{i=1}^{N}\sigma_{hi}^2/N)$. Now we can prove the theorem using (55). We know that

$$\frac{1}{|\mathcal{M}|}\mathbb{E}\left\{Q\left(\frac{||(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)\boldsymbol{\mathcal{R}}\mathbf{h}||}{\sqrt{N}\sqrt{2N_0}}\right)\right\} \leq P_e^N(\text{SNR}) \leq (|\mathcal{M}|-1)\mathbb{E}\left\{Q\left(\frac{||(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)\boldsymbol{\mathcal{R}}\mathbf{h}||}{\sqrt{N}\sqrt{2N_0}}\right)\right\}. \tag{56}$$

Eqn. 55 tells that as $N\to\infty$, $\frac{\boldsymbol{\mathcal{R}}\mathbf{h}}{\sqrt{N}} \xrightarrow{d} \mathcal{N}(\mathbf{0}, \tilde{\mathbf{\Sigma}})$. The *continuous mapping theorem* [33] states that for any continuous and bounded function $h(\cdot)$ and random variables X_n, X, if $X_n \xrightarrow{d} X$, then $\mathbb{E}\{h(X_n)\}\to\mathbb{E}\{h(X)\}$. By taking the limit in (56), we can deduce that as $N\to\infty$, the randomized space-time code $\boldsymbol{\mathcal{G}}(\mathbf{s})\boldsymbol{\mathcal{R}}$ is equivalent to a deterministic space-time code $\boldsymbol{\mathcal{G}}(\mathbf{s})\tilde{\mathbf{\Sigma}}^{1/2}$ and hence, it provides diversity order L when both $\mathbf{\Sigma}$ and $\boldsymbol{\mathcal{G}}$ are full-rank L.

REFERENCES

[1] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. Cambridge Universtiy Press, 2003.

[2] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Proc. of IEEE Inter. Symp. on Inform. Theory (ISIT)*, 16-21 Aug. 1998, p. 156.

[4] ——, "User cooperation diversity. part 1: System description; part 2: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927 – 1948, Nov. 2003.

[5] J. N. Laneman, D. N. C. Tse, , and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[6] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037 – 3063, Sep. 2005.

[7] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2525, Oct. 2003.

[8] T. M. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. on Information Theory*, vol. 25, no. 5, Sep. 1979.

[9] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, March 1998.

[10] P. A., G. Leus, and M. Kaveh, "On the performance of distributed space-time coding systems with one and two non-regenerative relays," *to appear in IEEE Transactions on Wireless Communications*, 2005.

[11] S. Barbarossa and G. Scutari, "Distributed space-time coding for multihop networks," in *Proc. of IEEE Proc. of IEEE International Conference on Communications*, vol. 2, 20-24 June 2004, pp. 916 – 920.

[12] ——, "Distributed space-time coding strategies for wideband multihop networks: regenerative vs. non-regenerative relays," in *Proc. of IEEE Inter. Conf. on Acoustics, Speech, and Signal Process. (ICASSP)*, vol. 4, 17-21 May 2004, pp. 501–504.

[13] J. Mietzner, R. Thobaben, and P. A. Hoeher, "Analysis of the expected error performance of cooperative wireless networks employing distributed space-time codes," in *Proc. IEEE Global Telecomm. Conf. (Globecom 2004)*, vol. 5, 29 Nov. - 3 Dec. 2004, pp. 2854 – 2858.

[14] Y. Hua, Y. Mei, and Y. Chang, "Parallel wireless mobile relays with space-time modulations," in *Statistical Signal Processing, 2003 IEEE Workshop*, 28 Sept.- 1 Oct. 2003, pp. 375 – 378.

[15] Y. Chang and Y. Hua, "Application of space-time linear block codes to parallel wireless relays in mobile ad hoc networks," in *Signals, Systems and Computers, 2003 The Thrity-Seventh Asilomar Conference*, vol. 1, Nov. 2003, pp. 1002 – 1006.

[16] T. Wang, Y.Yao, and G. B. Giannakis, "Non-coherent distributed space-time processing for multiuser cooperative transmissions," *to appear in IEEE Transactions on Wireless Communications*, 2006.

[17] A. Hottinen and T. Heikkinen, "Scheduling in a randomized relay network," in *Proc. of 2006 Conference on Information Sciences and Systems (CISS)*, March 2006.

[18] A. Hottinen and O. Tirkkonen, "A randomization technique for non-orthogonal space-time block codes," in *Proc. of Vehicular Technology Conference*, vol. 2, 6-9 May 2001, pp. 1479 – 1482.

[19] T. Ho, R. Koetter, M. Médard, D. R. Karger, and M. Effros, "The benefits of coding over routing in a randomized setting," in *Proc. of IEEE Inter. Symp. on Inform. Theory (ISIT)*, 29 June-4 July 2003, p. 442.

[20] P. A. Chou, Y. Wu, and K. Jain, "Practical network coding," in *41st Allerton Conf. Communication, Control and Computing*, Oct. 2003.

[21] B. Sirkeci-Mergen and A. Scaglione, "Randomized distributed space-time coding for cooperative communication in self-organized networks," in *Proc. of IEEE Workshop on Signal Process. Advances in Wireless Commun. (SPAWC)*, June 2005, pp. 500–504.

[22] S. Wei, D. Goeckel, and M. Valenti, "Asynchronous cooperative diversity," in *Proc. of 2004 Conference on Information Sciences and Systems (CISS)*, March 2004.

[23] A. Scaglione and Y.-W.Hong, "Opportunistic large arrays:cooperative transmission in wireless multihop adhoc networks to reach far distances," *IEEE Trans. Signal Processing*, vol. 51, no. 8, pp. 2082 – 2092, Aug. 2003.

[24] H. E. Gamal and D. Aktas, "Distributed space-time filtering for cooperative wireless networks," in *Proc. IEEE Global Telecomm. Conf. (Globecom 2003)*, vol. 4, 1-5 Dec. 2003, pp. 1826 – 1830.

[25] S. Yiu, R. Schober, and L. Lampe, "Distributed space-time block coding," *accepted for publication in IEEE Trans. on Communication*, 2006.

[26] Y. Hua, Y. Mei, and Y. Chang, "Wireless-antennas making wireless communications perform like wireline communications," in *IEEE Topical Conference on Wireless Communication Technology*, 15-17 Oct. 2003, pp. 47–73.

[27] Y. Jing and B. Hassibi, "Wireless networks, diversity and space-time codes," in *Proc. of IEEE Information Theory Workshop*, 24-29 Oct. 2004, pp. 463 – 468.

[28] ——, "Distributed space-time coding in wireless relay networks-part 1: basic diversity results, -part 2:tighter upper bounds and a more general case," *submitted to IEEE Trans. On Wireless Communications*, July 2004.

[29] J. Luo, R. Blum, L. Greenstein, and A. Haimovich, "Link-failure probabilities for practical cooperative relay networks," in *Proc. of Vehicular Technology Conference*, vol. 3, 30 May - 1 June 2005, pp. 1489 – 1493.

[30] A. Scaglione, B. Sirkeci, S. Geirhofer, and L. Tong, "Randomized distributed multi-antenna systems in multi-path channels," in *submitted to 14th European Signal Processing Conference - EUSIPCO 2006*, Sep. 2006.

[31] A. M. Tulino and S. Verdú, *Random Matrix Theory and Wireless Communications*. now Publishers Inc., 2004.

[32] A. Edelman, “Eigenvalues and condition numbers of random matrices,” Ph.D. dissertation, Massachusetts Institute of Technology (MIT), May 1989.

[33] P. Billingsley, *Probability and Measure*, 3rd ed. John Wiley & Sons, Inc., 1995.

[34] G. Marsaglia, “Choosing a point from the surface of a sphere,” *Ann. Math. Stat.*, vol. 43, 1972.

[35] M. E. Muller, “A note on a method for generating points uniformly on n-dimensional spheres,” *Comm. Assoc. Comp. Mach.*, vol. 2, 1959.

[36] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.

[37] V. Girko, *Theory of Random Determinants*. Kluwer Academic Publishers, 1990.

[38] A. W. van der Vaart, *Asymptotic statistics*. Cambridge University Press, 1998.