Joint Routing, Scheduling and Power Control for Multihop MIMO Networks

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***Abstract*—We consider the problem of joint routing, scheduling and power control in a multihop wireless network when the nodes have multiple antennas. We present an iterative scheme to compute achievable rates in the interference environment of the network using SINR model. The solution methods have a high computational complexity. Thus we also provide an efficient low complexity sub-optimal solution. We show that the multiple antennas provide linear gain in system throughput.**

***Index Terms*—Multihop wireless networks, MIMO, joint routing scheduling and power control.**

I. Introduction

Multihop wireless networks (MHWN) are essential for ubiquitous computation and communication. Currently there are many experimental setups of multihop wireless networks around the world. Ad-hoc wireless networks and sensor networks are also examples of multihop wireless networks where it is necessary to employ multiple wireless hops for even the connectivity of the nodes deployed in a particular area. However, multiple wireless hops pose many new challenges in a network design. But recent studies have shown that some of these challenges can be converted into opportunities by careful network design. Thus new communication paradigms, e.g., opportunistic scheduling, cooperative communication, network coding and multiple antennas have been developed in recent years. Exploiting these techniques together in a multihop setup to optimize the system performance is very challenging. In this paper we will concentrate on designing multihop networks with multiple antennas at each node.

In multihop wireless networks, unlike in the wire-line networks, even the concept of a link between two nodes is not properly defined. Shadowing can, irrespective of the distance between two nodes, cause the channel to be very bad. Even when there is no shadowing, we may or may not be able to communicate directly between two nodes depending on the transmit power used and also if there are other users transmitting in the neighborhood. Thus, topology of the network, transmit power, link scheduling and routing are all interrelated and for optimal performance one may need to jointly optimize power, scheduling and routing. This problem is computationally intractable and not scalable even for a centralized algorithm [4], [9], [12].

Employing multiple antennas at a transmitter and/or at a receiver can provide transmit diversity, receive diversity, increase the capacity of the link and reduce BER. Thus, in wireless networks where bandwidth is scarce, it is important to employ multiple antennas wherever feasible. This increases the degrees of freedom one can exploit to improve the system performance. Thus even for multihop network it is desirable to have multiple antennas at different nodes. However, as mentioned above, even with single antennas, jointly optimizing routing, scheduling and power (JRSP) in a multihop wireless network is very computationally intensive. Thus with multiple antennas, the problem becomes extremely challenging [7], [13]. At the moment there are very few studies available for this system. We address this problem in this paper.

II. Related Work

In wireless networks, it is well known that the traditional layers of the communication network cannot be considered in isolation. Many authors have studied power control, scheduling, and routing [4], [9], [12], [6]. But there has been little effort in joint optimization of power, link scheduling and routing.

There have been extensive studies on multiple input, multiple output (MIMO) systems for P2P and cellular communications [2]. However, there are limited studies of MIMO in multihop wireless networks.

In [13], a simple phy-layer and a link layer model is developed for MIMO links. The MIMO wireless multihop backhaul network design, with joint routing and scheduling and *overall* power constraint is studied in [10]. They use multi-antenna beamforming in their design and formulate it as a *non-convex* optimization problem. In [1], the system of MIMO-MHWN without power constraints is modelled as a linear program and is provided with a *feasible* solution. A cross-layer scheduling algorithm using minimum mean square error - successive interference cancellation is presented in [3]. In most of these works, the problem formulations are seen to be intractable and authors had to seek heuristic algorithms.

In this paper, we address the joint routing, scheduling and power control problem for MIMO-MHWNs. We seek a fair solution. We pose it as a linear programming problem (LPP). Complexity of this problem is also very large. Then, we provide an efficient suboptimal algorithm based on [4] and [9], which has much lower computational complexity. We show that the throughput due to multiple antennas increases

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linearly with the number of antennas even in the multihop networks. The basic difference from [9] is that, in the current paper each node has multiple antennas and beamforming is used at the time of transmission from a node.

The paper is organised as follows. In Section III, we describe the system model and provide the mathematical model. Section IV provides a computationally simpler heuristic to solve the problem. Section V presents a few examples to show the efficacy of the heuristic algorithm and also show the throughput gains obtained by multiple antennas.

III. System Model And Problem Formulation

We consider a network of half duplex nodes communicating with each other using a common wireless medium. Due to the broadcast nature of the wireless channel, the transmissions from one node interfere with transmissions from the other nodes. In the network, a subset of nodes called *sources* would like to transmit to another subset of nodes called *destinations*. Each source node has one destination. Data from one source in general needs multiple wireless hops to reach its destination. Our objective is to provide a fair (to be defined later) data transmission rate to all sources.

The network is represented by a fully connected directed graph $\mathcal{G}(\mathcal{N}, \mathcal{L})$ where, $\mathcal{N} = \{1, 2, \ldots, N\}$ is the set of nodes and $\mathcal{L} = \{1, 2, \ldots, L\}$ is the set of directed links. The transmission from a source to its destination is a *flow*. The set of flows $\{1, 2, \ldots, F\}$ is denoted by $\mathcal{F}$. The source node and the destination node of flow f are denoted by $src(f)$ and $dest(f)$. Flow $f \in \mathcal{F}$ requires rate d_f. The network may decide to allow rate r_f to flow f, depending on the availability of network resources. The rate $r_f \leq d_f$. But we also allow $r_f > d_f$. We define *fairness* index by $\lambda \triangleq \min_{f \in \mathcal{F}} \left\{ \frac{r_f}{d_f} \right\}$.

Each node has $a \geq 1$ antennas for transmission or reception. The total power used by a node for transmission is from a set of K power levels (including zero). We use point to point transmission model and use beam forming. Hence, a node involves in at most one active link, at any point of time.

Due to interference and half duplex assumption (i.e., a node can only transmit or receive at a time), not all links $\mathcal{L}$ can be activated simultaneously. We define a *mode*, as a valid subset of links being activated simultaneously along with the amounts of powers used on the links. We represent it by an $L \times 1$ vector of powers used on each link. A set of *modes* is given a schedule, which provides the link scheduling. This needs to be done along with routing and power control to provide a solution to JRSP. We will look for a *fair* solution.

We assume slow, flat fading. H_{ij} will denote the $a \times a$ channel gain matrix between node i and node j. We consider a centralized setup. Thus, all channel gains, network topology and rate requirements are known to the node, computing the solution. We also use the notation H'_{lm} to denote the channel between links l, m meaning, the channel between source node of link l and destination node of link m. We will assume H_{ij} to be constant for a certain period for which the solution of the JRSP will be valid. After that, the H_{ij} will change and the algorithm will be run again.

We formulate the problem as in [4] and [9]. This is the *multicommodity flow problem* from network theory, which is modified and exploited to suit the JRSP problem. The problem's objective is to find the solution to the JRSP problem, that gives maximum *fairness* among the users.

We will use the following notation. A is an $N \times L$ matrix with elements a_{ij} defined as:

$$a_{ij} = \begin{cases} 0, & \text{if node } i \text{ is not a part of link } j \\ +1, & \text{if node } i \text{ is the source to link } j, \\ -1, & \text{if node } i \text{ is the dest to link } j. \end{cases}$$

X refers to an $L \times F$ matrix with values X_{lf} representing the average effective rate of transmission on link l corresponding to flow f. $\mathbf{r}$ is an $N \times F$ matrix, which contains the net outwards data rate at each node corresponding to each flow:

$$r_{if} = \begin{cases} 0, & \text{if node } i \text{ is not a source or dest. of flow } j \\ +r_f, & \text{if node } i \text{ is the source to flow } f, \\ -r_f, & \text{if node } i \text{ is the destination to flow } f. \end{cases}$$

$\mathcal{M} = \{1, 2, ..., M\}$ is the set of all possible *modes* in the network. Each mode $m \in \mathcal{M}$ is an L dimensional column vector. The set $\mathcal{M}$ includes an idle state ($\underline{0}$) as well.

$\underline{\alpha}$ represents the vector of time fractions (mode scheduling) given to each of the modes. Its an $M \times 1$ probability vector.

C is an $L \times M$ matrix, where an element C_{lm} represents the rate of a link $l \in \mathcal{L}$ in mode $m \in \mathcal{M}$.

P is an $N \times M$ matrix called *power profile matrix* with values P_{nm} representing the amount of power spent by node n in mode $m \in \mathcal{M}$.

The following is the mathematical statement of JRSP problem with the well-known flow conservation constraints, link capacity constraints and average power constraints. This forms an LPP:

$$\max\ \lambda \triangleq \left(\min_i \left\{ \frac{r_i}{d_i} \right\} \right) \tag{1a}$$

(fairness objective)

subject to:

$$AX = \mathbf{r}\ , \tag{1b}$$

(Flow Conservation)

$$X \cdot \underline{1} \leq C \cdot \underline{\alpha}\ , \tag{1c}$$

(Avg. Link Capacities)

$$P \cdot \underline{\alpha} \leq \underline{P}^{avg}\ , \tag{1d}$$

(Avg. Power Control)

$$\underline{\alpha} \cdot \underline{1} = 1\ , \tag{1e}$$

(Consistency of scheduling)

$$X_l^f \geq 0,\ \forall f \in \mathcal{F},\ \forall l \in \mathcal{L}$$

$$P_n^m \geq 0,\ \alpha_m \geq 0,\ \ \forall n \in \mathcal{N},\ \forall m \in \mathcal{M} \tag{1f}$$

The above problem is a linear programming problem (LPP). It provides the mode (link) scheduling, routing as well as power control. If it is possible to satisfy the requirements of each flow, $\lambda \geq 1$. If not, it gives the solution, which simultaneously satisfies the largest fraction λ of each flows demands. Thus it is a *fair* solution. This concept of fairness has been used before in [9].

But with multiple antennas, we see following issues.
1) The link capacity calculations are interdependent (a joint optimization problem) due to interference.
2) The capacity region of such a vector interference channel is still an open problem.
3) We need a computationally simple method to decide link capacities, at least a good achievable rate point in a mode (columns of C). This is because, we have to calculate the same for a large number of modes.

A. Link Capacity computation

Let $\underline{m} = (m_1, m_2, \ldots, m_L)^T$ be the given vector of L powers spent on the links in mode m. We assume each link to be an additive white Gaussian (AWG) link. Also, the interference from other nodes will be taken as Gaussian noise. Thus to compute the link capacities, one should solve the problem of waterfilling [5] on each of the links. But, the effective noise that each user sees is a sum of AWG noise and the interference from other users. Assuming each source emits independent Gaussian symbols, we need the covariance matrix of the effective noise, for which we need covariance matrices of transmitted symbols from each transmitter in the network. This is clearly not known at the time of computation of link capacities. Thus, we propose the following iterative scheme (Algorithm 1)to greedily calculate one at a time, the capacities of links in the network which are active simultaneously, given the covariances of other nodes. We continue the iterations till we see the convergence in sum rate of all links in the network. The sum rate increases monotonically after each iteration and is observed to converge quickly in less than *ten* iterations. The convergence proof of such an algorithm is an open problem.

We use the following notation. C_i is capacity of the i^{th} link. K_i is the transmit covariance matrix used on link i while m_i is the power spent on link i. The $sumrate$ corresponds to the sum of all link capacities active in that mode. The function "waterfill(.)" performs the elementary waterfilling on any specified link and outputs two values namely the link capacity and the optimum transmit covariance matrix to achieve that capacity under the interference value at that instant. It requires the set of all transmit covariance matrices and channels(H'_{ij}) between link i and every other link j in the network.

B. Complexity Issues

The number of constraints in the LPP is $(NF+L+F+1)$ while the number of variables is $(F+LF+M)$. Also, since the graph is assumed to be directed and fully connected, $N \leq L \leq N(N-1)$. The Number of flows $F \leq N(N-1)$.

Algorithm 1 Iterative Waterfilling algorithm for finding link-capacities in a point-to-point mode

initialize $sumrate = -\infty$, $C_i = 0$, $K_i = (m_i/a) * I$ $\forall i = \{1, 2, \ldots, L\}$
while $|\sum_i C_i - sumrate| \geq \epsilon$ **do**
  $sumrate = \sum_i C_i$
  for $i = 1$ **to** L **do**
    if link i is active **then**
      $[K_i, C_i] = \text{waterfill}(i; \sigma^2; \{K_j, H'_{ji} \forall j \in \mathcal{L}\})$
    end if
  end for
end while

M can be upto K^L. It exceeds a million for a network with $N = 11$ nodes and $K = 4$ power levels. Thus we consider sub-optimal solutions, for networks with more nodes and power levels.

IV. The Heuristic Solution

We follow the algorithm in [4], which is a heuristic column generation method to solve this problem and is claimed to solve networks upto approximately 30 nodes.

The solution method proceeds as follows. First it can be seen [4] that, no more than $N + L + 1$ number of modes are necessary to obtain the optimal basic feasible solution. Then as per the *column generation* procedure, the solution starts by considering a smaller problem called *Master Problem* with just $N + L + 1$ mode variables ($\underline{\alpha}'$) randomly chosen from $\underline{\alpha}$. Rest all is the same as the original optimization problem in (1).

The algorithm also considers the following *Sub-Problem* that chooses a new mode called a *good mode* from the remaining modes, which replaces a bad mode from the master problem.
Sub-Problem:

$$\max_{m \in \mathcal{M} \setminus \mathcal{M}'} \theta(m) \triangleq \underline{u}^T \underline{C}_m - \underline{v}^T \underline{P}^m - \beta$$

subject to:

$$\theta(m) \geq 0. \tag{2}$$

The mode obtained from the sub-problem will replace an already chosen mode variable which is given zero scheduling time in the optimal solution of the *Master* problem. This starts the next iteration, in which the master problem attains an improved solution.

In this procedure, the sub-problem requires an optimum choice of variables by searching exhaustively among the auxiliary function values evaluated at $M - (N + L + 1)$ modes, which is again computationally infeasible. This can be avoided by using a heuristic algorithm for the sub-problem, from [4] and [9]. This comes at the cost of sub-optimality to the solution. But it will be shown via simulations that, the solution that the solution obtained via this heuristic, is close to the optimal solution.

The heuristic algorithm is presented below as Algorithm 2. In this, $nextlevel(x)$ is used to denote the power value which is least among the power values that are $\geq x$ from power level set. $interferers(l)$ denotes the *set* of all the links that *share* a node with link l.

Algorithm 2 The Heuristic Algorithm to obtain an efficient *sub-optimal* solution to the *sub-problem*

Initialize mode vector $\mathbf{goodmode} = \underline{0}$,
$\mathbf{lasttheta} = \theta(\mathbf{goodmode})$, $\mathbf{AllowedSet} = \mathcal{L}$

while $\mathbf{AllowedSet} \neq \phi$ **do**
  for $i = 1$ **to** L **do**
    if $i \in \mathbf{AllowedSet}$ **then**
      $\underline{m} = \mathbf{goodmode}$
      $m_i = nextlevel(m_i)$
      $\theta_i = \theta(\underline{m})$
    end if
  end for
  Let $\mathbf{besttheta} \triangleq \max_i \theta_i$ &
    $\mathbf{bestlink}\ l \triangleq arg\ \max_i \theta_i$
  if $\mathbf{besttheta} \leq \mathbf{lasttheta}$ **then**
    break the loop.
  else
    $\mathbf{lasttheta} = \mathbf{besttheta}$
    $\underline{m} = \mathbf{goodmode}$
    if $m_l = 0$ **then**
      $\mathbf{AllowedSet} = \mathbf{AllowedSet} \backslash interferers(l)$
    end if
    $m_l = nextlevel(m_l)$
    $\mathbf{goodmode} = \underline{m}$
  end if
end while
Declare $\mathbf{goodmode}$ as the solution to sub problem

Since we solve the problem (1) with a heuristic solution at the sub problem level of column generation, we call this method as *Heuristic Column Generation* (HCG).

Further, the final solution (maximum fairness) obtained by HCG depends on the initially selected set of modes. Hence we choose the best value among solutions achieved by solving the problem for a multiple number of initial points, as is done for non-convex optimization problems.

In the next sections, we test the algorithm for several networks with multiple antennas. We first show that the HCG solution is very close to the optimal solution. Via HCG, we can solve the problem for up to 30 nodes, almost similar to the single antennas case, as claimed in earlier papers. Thus, via HCG even for multiple antenna case, which for the optimal solution has a much higher complexity than the single antenna case, one can solve the problems of the same order, as for single antenna systems. Finally, we will show that the MIMO improves the performance of the network and that the gain in rate is linear with the number of antennas a. This is in conformity with the capacity results well known for single link case.

V. SIMULATION RESULTS

Consider a network of 15 nodes in Fig-1. We use the following network parameters: $N = 15$, $L = 60$, $a = 5$. Power levels used by each node = $\{0, 4\}$. Channel gains H_{ij} are generated from unit variance, circularly symmetric Gaussian complex random numbers. Source, destination pairs (flows) are (7,13), (10,5) and (11,8). The rate demands from the flows = $10, 15, 20$ units, respectively. The average power constraint on each node is 3 units. (Note: In this scenario, we have approximately 0.5 million modes in the network).

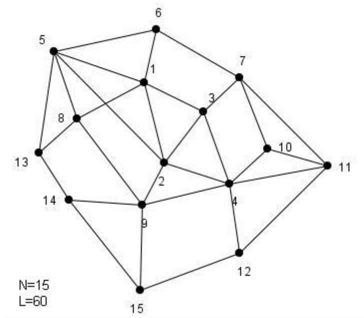

Fig. 1. A network of 15 nodes

By formulating the problem (1) and solving it directly (optimally), we get the following solutions. Allowed user rates after solving the problem: $r_1 = 5.971$ units, $r_2 = 8.957$ units and $r_3 = 11.942$ units.
Hence, the optimum fairness achieved is: $\lambda = 0.5971$

A dominant mode is shown in Fig-2 via bold arrows.

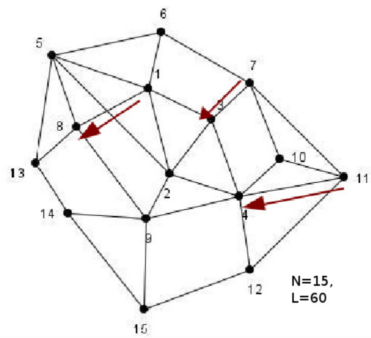

Fig. 2. A dominant mode in the network to achieve optimal throughput

Now we use HCG to solve the problem and see the relative performance. This is shown in Fig-3. We use multiple initial points, for the best initial point. We get allowed rates: $r_1 = 5.8$ units, $r_2 = 8.7$ units and $r_3 = 11.6$ units. Hence the optimum fairness that can be achieved is: $\lambda = 0.58$.

Next, we consider a network of 30 nodes with the following parameters: $N = 30$, $L = 110$ $a = 4$. Source,destination pairs (flows) are = (2,30), (5,10), (1,30), (4,6). The rate demands from the flows are $\{10, 50, 20, 60\}$ units respectively. Each node has average power constraint = 3 units.

This problem was not optimally solvable in a reasonable time. Thus we provide the solution only via HCG for different initial conditions in Fig.4.

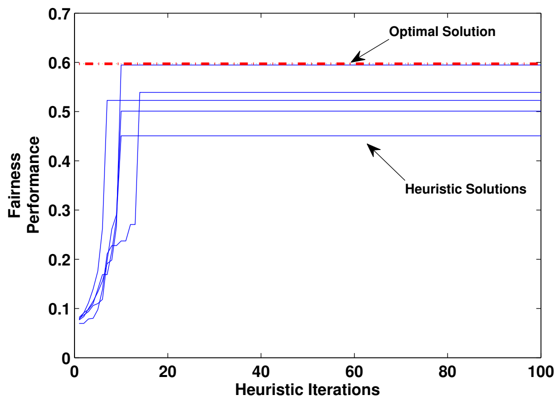

Fig. 3. Fairness Values achieved using HCG, and the Optimal algorithm for the network of 15 nodes.

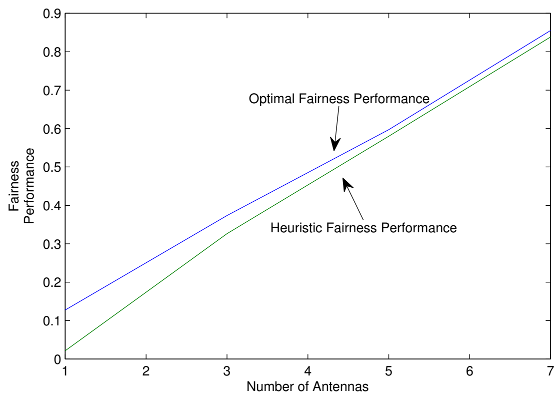

Fig. 5. MIMO gain in optimal and HCG algorithms of network with N=15.

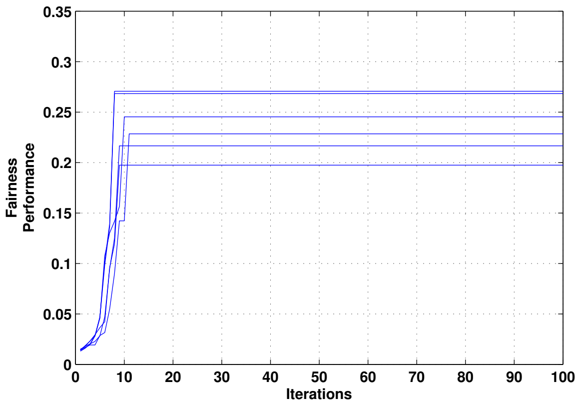

Fig. 4. Heuristic algorithmic performance for network of N=30

Allowed user rates after solving the problem are:

$r_1 = 2.7069$ units, $r_2 = 13.5345$ units,

$r_3 = 5.4138$ units, $r_4 = 16.2414$ units.

Hence the optimum fairness achievable is $\lambda = 0.2707$.

We now show the performance gain achieved using multiple antennas in the system. We consider the 15 node network in Fig-1. All the parameters except a remain same. We vary the number of antennas a and plot the fairness index obtained via the optimal solution as well as via the HCG in Fig.5. The performance improves linearly as we increase the number of antennas on nodes.

VI. Conclusions

We consider a multihop wireless network with multiple antennas. We formulate the joint routing, scheduling and power control problem. A node can use power from a finite set. At each node, we form an optimal beam taking into account the interference from other nodes. The optimal algorithm is a linear program. But its complexity is still very large. Thus, we also consider a heuristic lower complexity solution. Our algorithms provide rates which increase linearly with number of antennas.

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