

# On the Capacity of MIMO Cellular Systems with Base Station Cooperation

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**Abstract**—This paper is concerned with the capacity of multiple-input multiple-output (MIMO) cellular systems. We assume an equal rate constraint for all users and adopt a realistic channel model that incorporates path loss, lognormal fading and Rayleigh fading. Several bounds are derived for the minimum transmission power of such rate-constrained MIMO cellular systems with various base station (BS) cooperation strategies. In particular, the upper power bound is based on the maximum eigenmode beamforming (MEB) scheme. These power bounds are then used to obtain the corresponding cellular capacity bounds when partial BS cooperation strategies are adopted. Our results show that, allowing more users to transmit simultaneously, introducing cooperation among BSs and increasing the number of antennas (especially at each BS) are efficient ways to improve system performance.

**Index Terms**—Cellular capacity, multiple-input multiple-output (MIMO), base station cooperation, rate constraint, simultaneous transmission, maximum eigenmode beamforming (MEB).

## I. INTRODUCTION

THE capacity analysis of cellular systems is an intriguing topic in wireless communication systems. A main challenge in cellular capacity analysis is the treatment of interference among users. In the earlier work [1], the signals of other users (both intra-cell and inter-cell) are treated as additive noise, which simplifies the problem but also underestimates the achievable system performance. Potentially, interference alleviation methods, such as multi-user detection, base station (BS) cooperation and multiple-input multiple-output (MIMO) techniques, can lead to significant performance gain. However, the related capacity analysis problem is difficult due to complicated transmission environments.

The Wyner model [2] provides a simplified approach to this problem. The Wyner model is based on two basic assumptions: (i) inter-cell interference is from adjacent cells only; and (ii) all the same-cell users experience the same path loss to the same BS. Based on the Wyner model, the authors in [3][4] compared the performance of cellular systems with different multiple access techniques. The capacity improvement of cellular systems with BS cooperation and inter-/intra-cell multi-user detection is discussed in [5]-[8]. The impact of the MIMO technique on cellular systems is studied in [8]-[12]. However,

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due to the above two assumptions, the Wyner model cannot always capture the essential aspects of cellular interactions, as pointed out in [13]. Overall, the accurate evaluation of cellular capacity in realistic and complex environments still remains a challenging problem.

This paper addresses the capacity of uplink cellular systems taking into account MIMO configurations and BS cooperation. Most results in this paper can be extended to downlink cellular systems based on the duality principle [14]. We adopt a realistic channel model that incorporates three multiplicative factors, i.e., path loss, lognormal fading that models shadow fading channel process and Rayleigh fading that models fast fading channel process. Various BS cooperation strategies are investigated using bounding techniques. For the full BS cooperation (FBSC) strategy, we derive upper and lower capacity bounds. In particular, the lower bound is based on a simple and realizable maximum eigenmode beamforming (MEB) scheme. These two bounds converge when the number of simultaneous transmission users in each cell is sufficiently large, which indicates that the MEB scheme is asymptotically optimal.

We also consider a more practical partial signal utilization (PSU) strategy. With PSU, the signals of each user are processed at a finite number (denoted by  $B$  below) of BSs. We again use MEB to derive a lower bound of the system capacity. (The asymptotic FBSC capacity serves as an upper bound here.) We show that, when  $B$  is large and the cooperative BSs for every user are independently and identically selected in an adaptive way in each channel realization, PSU can have similar performance as FBSC. However, when BSs are selected based on fixed cluster patterns, performance degradation can be significant. Note that a cellular network involving BS cooperation can be regarded as a distributed antenna system (DAS) [15]. The latter has multiple antennas distributed in a cell, with each antenna (or a group of co-located antennas) equivalent to a BS in the former. Therefore, the results in this paper can also be applied to DASs.

The remainder of this paper is organized as follows. The cellular system model is introduced in Section II. The FBSC and PSU strategies are discussed in detail in Sections III and IV, respectively. Finally, we conclude our paper in Section V.

**Notations:** Boldface lower-case symbols represent vectors. Capital boldface characters denote matrices. The operators  $(\cdot)^T$ ,  $(\cdot)^*$  and  $\|\cdot\|_2$  denote transpose, conjugate-transpose and 2-norm of a matrix or vector, respectively. The operators  $E(\cdot)$  and  $D(\cdot)$  denote the expectation and variance, respectively.

## II. SYSTEM MODEL

Consider the uplink of an MIMO cellular system with  $N_c$  hexagonal cells. The BS at each cell center is equipped with  $M$

antennas and each user is equipped with  $N$  antennas. Denote by  $K$  the density of active users per cell, i.e., on average there are  $K$  users simultaneously transmitting in each cell at any time. Note that the actual user density supported by the system can be higher than  $K$ , e.g., by using a TDMA scheme.

Denote by  $\mathbf{x}_k$  the transmitted length- $N$  signal vector of active user  $k$  ( $k = 1, 2, \dots, N_c K$ ) at any time. The received signal at all BSs can be modeled as

$$\mathbf{y} = \sum_{k=1}^{N_c K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n} \quad (1)$$

where  $\mathbf{n}$  is a length- $N_c M$  vector containing independent and identically distributed (i.i.d.) complex additive white Gaussian noise (AWGN) samples with mean zero and variance  $N_0$ , and  $\mathbf{H}_k = [\mathbf{H}_{k,1}^T, \mathbf{H}_{k,2}^T, \dots, \mathbf{H}_{k,N_c}^T]^T$  is the global channel matrix of user  $k$ , in which  $\mathbf{H}_{k,n}$  ( $n = 1, 2, \dots, N_c$ ) represents the  $M \times N$  channel matrix from user  $k$  to the BS in cell  $n$ . Some detailed assumptions about  $\{\mathbf{H}_k\}$  are listed below.

1. All users experience quasi-static flat fading channels and perfect channel state information (CSI) is assumed at both the transmitter and receiver sides;
2. The number of cells  $N_c$  is assumed to be sufficiently large (i.e.,  $N_c \rightarrow \infty$ ) such that the edge effect can be ignored. The edge-length of every cell is normalized to 1;
3. User locations are independent and uniformly distributed (i.u.d.) over the whole cellular area;
4. Every entry of  $\{\mathbf{H}_k\}$  contains three multiplicative factors, i.e., path loss following a power loss exponent  $\alpha$ , normalized lognormal fading with standard deviation  $\sigma_s$  and Rayleigh fading;
5. The Rayleigh fading factors are i.i.d. for all antenna links. The lognormal fading factors are the same for all antenna links from the same user to the same cell and i.i.d. for the antenna links from different users or to different cells.

From the above assumptions, all  $\{\mathbf{H}_k\}$  are i.i.d.. For simplicity, we assume that each user has the same fixed rate constraint of  $R/K$  in each channel realization, where  $R$  is the system sum rate per cell averaged over time. This assumption is applicable to delay-sensitive services such as voice and real-time video. In this paper, we are interested in verifying whether a transmission rate  $R$  can be supported by the system and, if the rate can be supported, finding the corresponding long-term average transmission power.

### III. FULL BASE STATION COOPERATION

We first study the full BS cooperation (FBSC) strategy by assuming that all BSs in the cellular system are connected to a virtual central processor through error-free links with unlimited link-capacity and no latency. This is an unpractical assumption and hence the FBSC strategy studied here only serves as a reference for the partial BS cooperation strategies to be discussed in the next section. (The work on cellular capacity with finite backhaul can be found in [16][17].)

With FBSC, the system in (1) can be viewed as an overall MIMO multiple access one with  $N$  transmit antennas at each user and  $N_c M$  receive antennas equally distributed at  $N_c$  BSs. For such a system with rate constraints, minimizing the

aggregated transmitted power of all users is an appropriate target due to the battery life concern at mobile units. This requires joint optimization of all users' transmit covariance matrices and the decoding order of successive interference cancellation (SIC) at all BSs. Some algorithms are available for this purpose [18][19], but the related computational costs increase rapidly with the number of concurrently transmitting users. For the system in (1), the total number of concurrent users  $N_c K$  tends to infinity as  $N_c \rightarrow \infty$  and the related optimization complexity is excessively high. In the following, we will take an alternative approach based on bounding techniques.

#### A. Maximum Eigenmode Beamforming (MEB)

We first consider a sub-optimal maximum eigenmode beamforming (MEB) scheme [20] to provide an upper bound for the optimal power efficiency of the system (1). The basic MEB principle is outlined below.

- Each user only transmits in its maximum eigenmode direction;
- SIC is applied at the receiver. The signal of the user with the best channel condition is decoded first;
- A simple correlator receiver is used to collect the received signals of each user from all BS antennas;
- The transmitted power levels of all users are determined via the above transmitting/receiving operations.

More specifically, for each channel realization, let the singular value decomposition (SVD) of  $\mathbf{H}_k$  be

$$\mathbf{H}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^* \quad (2)$$

where  $\mathbf{U}_k$  and  $\mathbf{V}_k$  are unitary matrices.  $\mathbf{D}_k$  an  $N_c M \times N$  diagonal matrix containing all singular values of  $\mathbf{H}_k$ . Denote by  $d_{k,\max}$  the maximum singular value of  $\mathbf{H}_k$ . Let  $\mathbf{u}_{k,\max}$  and  $\mathbf{v}_{k,\max}$  be the corresponding singular vectors in  $\mathbf{U}_k$  and  $\mathbf{V}_k$ , respectively. With MEB, each user only transmits in the direction of  $\mathbf{v}_{k,\max}$ , i.e.,  $\mathbf{x}_k = \mathbf{v}_{k,\max} \sqrt{p_k} x_k$  where  $p_k$  is the transmitted power of user  $k$  and  $x_k$  the coded signal with unit power. Then (1) can be rewritten as

$$\begin{aligned} \mathbf{y} &= \sum_{k=1}^{N_c K} \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^* \mathbf{v}_{k,\max} \sqrt{p_k} x_k + \mathbf{n} \\ &= \sum_{k=1}^{N_c K} d_{k,\max} \sqrt{p_k} \mathbf{u}_{k,\max} x_k + \mathbf{n}. \end{aligned} \quad (3)$$

Without loss of generality, we assume  $d_{1,\max} \leq d_{2,\max} \leq \dots \leq d_{N_c K,\max}$  and apply SIC at all BSs with descending order on  $k$ . When decoding the signal for user  $k$ , we simply use  $\mathbf{u}_{k,\max}$  to correlate the received signal. The corresponding correlation output is written as

$$\begin{aligned} \mathbf{u}_{k,\max}^* \mathbf{y} &= \sum_{i=1}^k d_{i,\max} \sqrt{p_i} \mathbf{u}_{k,\max}^* \mathbf{u}_{i,\max} x_i + \mathbf{u}_{k,\max}^* \mathbf{n} \\ &= d_{k,\max} \sqrt{p_k} x_k + \sum_{i=1}^{k-1} d_{i,\max} \sqrt{p_i} \mathbf{u}_{k,\max}^* \mathbf{u}_{i,\max} x_i \\ &\quad + \mathbf{u}_{k,\max}^* \mathbf{n}. \end{aligned} \quad (4)$$

Note that in (4), we assume that the signals from users  $\{i, i > k\}$  have been successfully decoded and removed by SIC. From

(4), we can calculate the signal-to-noise ratio (SNR) for user  $k$  (denoted by  $SNR_k$ ) as

$$\begin{aligned} SNR_k &= \frac{d_{k,\max}^2 p_k}{\sum_{i=1}^{k-1} d_{i,\max}^2 p_i |\mathbf{u}_{k,\max}^* \mathbf{u}_{i,\max}|^2 + N_0} \\ &= \frac{d_{k,\max}^2 p_k}{\sum_{i=1}^{k-1} d_{i,\max}^2 p_i \phi_{k,i} + N_0} \end{aligned} \quad (5)$$

where  $\phi_{k,i} = |\mathbf{u}_{k,\max}^* \mathbf{u}_{i,\max}|^2$ .

Assume rate- $R/K$  ideal coding (e.g., the random Gaussian coding introduced by Shannon) for each user. From the Shannon capacity formula  $R/K = \log_2(1 + SNR_k)$ ,  $\forall k$ , we can rewrite (5) as

$$p_k = \frac{2^{R/K} - 1}{d_{k,\max}^2} \cdot \left( \sum_{i=1}^{k-1} d_{i,\max}^2 p_i \phi_{k,i} + N_0 \right). \quad (6)$$

We now proceed to derive the average minimum transmitted sum power (MTSP) per cell of MEB in FBSC cellular systems. To avoid an extremely large transmitted power level due to deep fading, we introduce an outage for each user, i.e., a user does not transmit if its channel gain is below a given threshold  $G_0$ . For convenience, we define an indicator function  $I(k, i)$  for any two users  $k$  and  $i$  in each channel realization as follows<sup>1</sup>.

$$I(k, i) = \begin{cases} 0, & \text{if } d_{i,\max}^2 < G_0 \text{ or } d_{i,\max}^2 \leq d_{k,\max}^2; \\ 1, & \text{if } G_0 \leq d_{i,\max}^2 < d_{k,\max}^2. \end{cases} \quad (7)$$

Taking outage into consideration, we can rewrite (6) as

$$p_k = \frac{2^{R/K} - 1}{d_{k,\max}^2} \cdot \left( \sum_{i=1}^{N_c K} d_{i,\max}^2 p_i \phi_{k,i} I(k, i) + N_0 \right). \quad (8)$$

The proof of the lemma below is given in Appendix I.

**Lemma 1:**

$$\mathbb{E}(\phi_{k,i} | d_{k,\max}, d_{i,\max}) = \frac{1}{N_c M}, \forall k \neq i. \quad (9)$$

Denote by  $F(\cdot)$  and  $f(\cdot)$ , respectively, the cumulative distribution function (CDF) and the probability density function (PDF)<sup>2</sup> of  $\{d_{k,\max}^2\}$  (both of which can be obtained by the Monte Carlo method). The proof of Theorem 1 below is given in Appendix II.

*Theorem 1:* The average MTSP per cell of the MEB scheme (denoted by  $P^{MEB}(R, K)$ ) in an FBSC cellular system is given by

$$\begin{aligned} &P^{MEB}(R, K) \\ &= N_0 \int_{G_0}^{\infty} \frac{K(2^{\frac{R}{K}} - 1) e^{K(2^{R/K} - 1)(F(g) - \varepsilon)/M}}{g} f(g) dg \end{aligned} \quad (10)$$

where  $\varepsilon = F(G_0)$  is the outage probability of each user.

Since MEB is realizable, (10) serves as an upper bound for the average MTSP of the FBSC cellular system in (1) with

<sup>1</sup>Note that when  $d_{k,\max}^2 = d_{i,\max}^2, \forall k \neq i$ , we have  $I(k, i) = 0$  from (7), which makes (8) inconsistent with (6). However, this will not affect the derivation below as such an event occurs with zero probability if  $\{d_{k,\max}^2\}$  are continuously distributed.

<sup>2</sup>Since all  $\{\mathbf{H}_k\}$  are i.i.d., so are  $\{d_{k,\max}^2\}$

the optimal scheme discussed in [18][19]. Hence we have the following.

**Corollary 1:** The average MTSP per cell of the optimal scheme (denoted by  $P^{Opt}(R, K)$ ) in an FBSC cellular system is upper-bounded by that of MEB, i.e.,

$$P^{Opt}(R, K) \leq P^{MEB}(R, K). \quad (11)$$

Later we will see this upper bound is tight for a large  $K$ .

## B. Asymptotic Performance of FBSC

The following lemma provides a lower bound for  $P^{Opt}(R, K)$ . Its proof is given in Appendix III.

**Lemma 2:** The average MTSP per cell of the optimal scheme in an FBSC cellular system is lower-bounded by

$$\begin{aligned} P^{Opt}(R, K) &\geq N_0 \int_{G_0}^{\infty} \frac{R \ln 2 \cdot 2^{R(F(g) - \varepsilon)/M}}{g} f(g) dg \\ &\triangleq P^{LB}(R). \end{aligned} \quad (12)$$

Returning to Theorem 1, when  $K \rightarrow \infty$ , we can rewrite (10) as

$$\begin{aligned} &P^{MEB}(R, \infty) \\ &= \lim_{K \rightarrow \infty} N_0 \int_{G_0}^{\infty} \frac{K(2^{\frac{R}{K}} - 1) e^{K(2^{R/K} - 1)(F(g) - \varepsilon)/M}}{g} f(g) dg \\ &= N_0 \int_{G_0}^{\infty} \frac{R \ln 2 \cdot 2^{R(F(g) - \varepsilon)/M}}{g} f(g) dg \\ &= P^{LB}(R, K). \end{aligned} \quad (13)$$

Note that we exchanged the order of limit and integration in (13). This exchange is valid since the term inside the integration in (10) is continuous and uniformly convergent when  $f(\cdot)$  is continuous. Equation (13) indicates that the upper/lower bounds (11) and (12) converge when  $K \rightarrow \infty$ . Hence we have the following.

*Theorem 2:* The asymptotic average MTSP per cell of the optimal scheme in an FBSC cellular system with  $K \rightarrow \infty$  is given by

$$P^{Opt}(R, \infty) = N_0 \int_{G_0}^{\infty} \frac{R \ln 2 \cdot 2^{R(F(g) - \varepsilon)/M}}{g} f(g) dg, \quad (14)$$

which is asymptotically achievable by the MEB scheme.

Intuitively, the asymptotic optimality of MEB in FBSC cellular systems can be explained as follows. With MEB, although every user only transmits in one direction, the signals of all users arrive at the BSs in all possible directions and, when  $K$  is large, statistically span the whole received signal space. Hence MEB can approximately achieve all the available degrees of freedom (DOF) and is therefore asymptotically optimal. The corollary below can be obtained from Theorem 2 directly. The proof is omitted here for brevity.

**Corollary 2:** Given the average sum power  $P$  per cell and equal rate allocation among all users, the asymptotic capacity

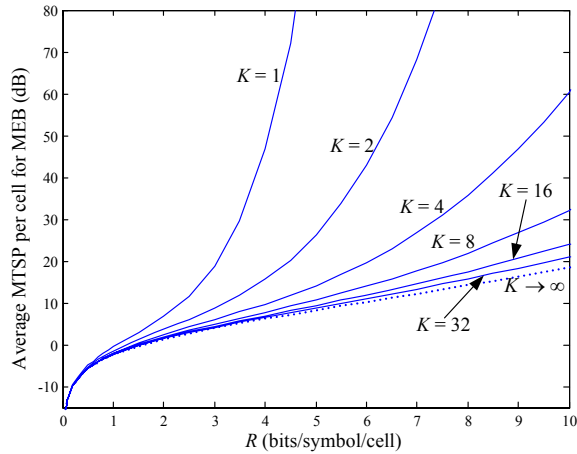


Fig. 1. The average MTSP of MEB in an FBSC cellular system with  $M = N = 1$  and different simultaneous user densities  $K$ .

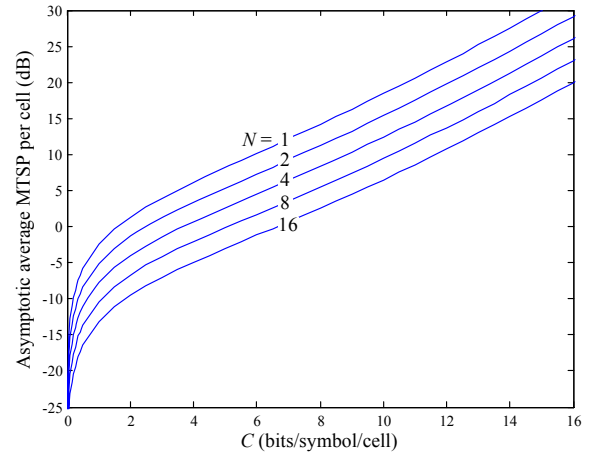
of an FBSC cellular system with  $K \rightarrow \infty$  is the solution of  $C$  (in terms of bits/symbol/cell) in the following equation.

$$P = N_0 \int_{G_0}^{\infty} \frac{C \ln 2 \cdot 2^{C(F(g)-\varepsilon)/M}}{g} f(g) dg. \quad (15)$$

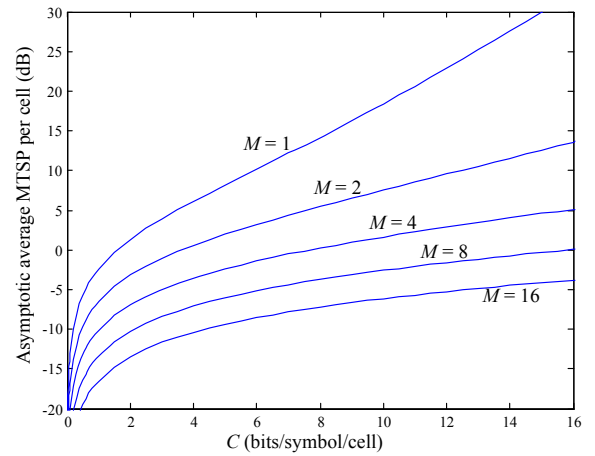
### C. Numerical Examples

We now present some examples based on Theorems 1, 2 and Corollary 2. The noise power  $N_0$  is normalized to 1. The path loss exponent and the standard deviation of lognormal fading are set at  $\alpha = 4$  and  $\sigma_s = 8$ , respectively. The outage probability for each user is  $\varepsilon = 0.01$ . Since it is impossible to simulate a cellular system with infinite size, both  $F(\cdot)$  and  $f(\cdot)$  are obtained by generating users only in a central cell and considering their channel matrices to the BSs within the first 20 cycles around (i.e., the nearest 187 cells). Figure 1 plots the upper and lower power bounds, i.e.,  $P^{MEB}(R, K)$  and  $P^{LB}(R)$ , for an FBSC cellular system with  $M = N = 1$ . We can see from Fig. 1 that, for a fixed sum rate  $R$  per cell, significant power savings can be achieved when  $K$  increases, indicating that allowing more simultaneous users is advantageous. When  $K = 16$ , these two bounds are very close to each other and they provide a reasonably accurate estimation for  $P^{Opt}(R, K)$ .

Figure 2 compares the asymptotic capacities of FBSC cellular systems with different  $M$  and  $N$  when  $K \rightarrow \infty$ . We fix  $M = 1$  and  $N = 1$  in Figs. 2(a) and 2(b), respectively. From these figures, we can see that doubling  $N$  achieves only about 3dB of power gain, regardless of the value of  $C$ . In contrast, increasing  $M$  can increase the available DOF of the system and significant power savings can be achieved, especially when  $C$  is large. This observation is in line with the discussion in [21] that the performance of a multi-antenna system is dominated by the minimum of transmit and receive antenna numbers. It is also interesting to see from Fig. 2(b) that the slope of  $P^{Opt}(R, \infty)$  is very small when both  $C$  and  $M$  are large, indicating that the cellular capacity can be significantly enhanced using minimal additional transmission power.



(a)  $M = 1$



(b)  $N = 1$

Fig. 2. The asymptotic capacities of FBSC cellular systems with different  $M$  and  $N$  when the density of simultaneous users is sufficiently high ( $K \rightarrow \infty$ ).

## IV. PARTIAL SIGNAL UTILIZATION

### A. System Model

The discussion in Section III is based on full utilization of each user's signals at all BSs, which is unrealistic in practice. We now consider a more practical partial signal utilization (PSU) strategy. Specifically, for each user, we only process its signals received at a limited number (denoted by  $B$  below) of BSs, and those received at other BSs are treated as additive noise. We have seen from Fig. 2 that the capacity of an FBSC cellular system increases indefinitely with the average MTSP (i.e., the system is power limited). In the following, we will see that the cellular capacity with PSU is upper-bounded, which is caused by interfering signals that cannot be suppressed by SIC (i.e., the system is interference limited).

For each channel realization, we write the channel matrix  $\mathbf{H}_k$  for each user  $k$  as the sum of two parts, i.e.,

$$\mathbf{H}_k = \mathbf{H}_k^u + \overline{\mathbf{H}}_k^u \quad (16)$$

where  $\mathbf{H}_k^u$  corresponds to the  $B$  BSs at which the signals of user  $k$  are processed.  $\overline{\mathbf{H}}_k^u$  then corresponds to the rest unused BSs. For example, if the signal of user  $k$  is only processed at

the BS in cell 1, we have

$$\mathbf{H}_k^u = [\mathbf{H}_{k,1}^T, \mathbf{0}^T, \dots, \mathbf{0}^T]^T$$

and

$$\overline{\mathbf{H}}_k^u = [\mathbf{0}^T, \mathbf{H}_{k,2}^T, \dots, \mathbf{H}_{K,N_c}^T]^T$$

where  $\mathbf{0}$  represents an  $M \times N$  all-zero matrix. Such an extreme case of PSU with  $B = 1$  is referred to as non-BS cooperation (NBSC).

In this section, we will derive lower bounds for the capacity achieved by PSU using MEB. The optimal FBSC capacity can be used as an upper bound here. The following two PSU strategies will be studied.

**Adaptive PSU (APSU):** In each channel realization, the signals of each user are collected, decoded and cancelled only at  $B$  adaptively selected BSs. The BS selection criterion is independent and identical for all users and can be based on either the user location or the channel condition. For example, we can select the  $B$  BSs with the nearest distances to user  $k$ , or those with the highest channel gains  $\{\|\mathbf{H}_{k,n}\|_2^2\}$  among all BSs  $\{n, n = 1, 2, \dots, N_c\}$ .

**Fixed PSU (FPSU)<sup>3</sup>:** All cells in the system are grouped into fixed and non-overlapped clusters, each containing  $B$  cells. For each channel realization, we find a cluster for each user  $k$  that either contains this user geographically or has the best channel condition to this user among all clusters. The signals of user  $k$  are then processed cooperatively only by the BSs in this cluster. Those signals received at the BSs in other clusters are treated as interference and never canceled.

The term ‘‘cluster’’ in FPSU is the same as that used for frequency reuse. Figure 3 shows two clustering examples with  $B = 3$  and 7. Note that FPSU only involves local cooperation among the BSs in each cluster. In contrast, APSU may need collecting the signals of each user from any  $B$  BSs and requires global cooperation among all BSs in general. However, later we will see that the APSU performance approaches to that of FBSC faster than FPSU when  $B$  increases. The global BS cooperation involved in APSU can be implemented in a distributed way [23]-[25] (e.g., using the message passing technique in [23]).

### B. SNR Derivation for PSU

The discussion in this subsection applies to both APSU and FPSU. Let the SVD of  $\mathbf{H}_k^u$  be

$$\mathbf{H}_k^u = \mathbf{U}_k^u \mathbf{D}_k^u (\mathbf{V}_k^u)^*. \quad (17)$$

Denote by  $d_{k,\max}^u$ ,  $\mathbf{u}_{k,\max}^u$  and  $\mathbf{v}_{k,\max}^u$  the maximum singular value of  $\mathbf{H}_k^u$  and the corresponding singular vectors, respectively. With MEB, we have  $\mathbf{x}_k = \sqrt{p_k} \mathbf{v}_{k,\max}^u x_k$  for each user

<sup>3</sup>A similar clustered joint processing strategy is discussed in [22] based on a linear cellular model.

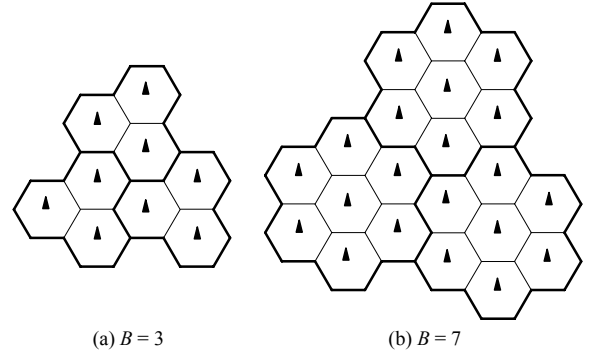


Fig. 3. Illustrations of clusters in FPSU with  $B = 3$  and 7. Note that in each cluster with  $B = 7$ , except for the central cell, the other 6 cells are all outermost cells (see Section IV-E for details).

$k$  and (1) can be rewritten as

$$\begin{aligned} \mathbf{y} &= \sum_{k=1}^{N_c K} \mathbf{U}_k^u \mathbf{D}_k^u (\mathbf{V}_k^u)^* \sqrt{p_k} \mathbf{v}_{k,\max}^u x_k \\ &\quad + \sum_{k=1}^{N_c K} \overline{\mathbf{H}}_k^u \sqrt{p_k} \mathbf{v}_{k,\max}^u x_k + \mathbf{n} \\ &= \sum_{k=1}^{N_c K} d_{k,\max}^u \sqrt{p_k} \mathbf{u}_{k,\max}^u x_k + \boldsymbol{\xi}^u + \mathbf{n} \quad (18a) \end{aligned}$$

where

$$\boldsymbol{\xi}^u = \sum_{k=1}^{N_c K} \overline{\mathbf{H}}_k^u \sqrt{p_k} \mathbf{v}_{k,\max}^u x_k \quad (18b)$$

represents the interference components that will not be suppressed by SIC.

Similar to (4), we assume  $d_{1,\max}^u \leq d_{2,\max}^u \leq \dots \leq d_{N_c K,\max}^u$  and apply SIC as well as correlators  $\{\mathbf{u}_{k,\max}^u\}$  at the receiver with descending order on  $k$ . The signal at user  $k$ 's correlator output is now written as

$$\begin{aligned} &(\mathbf{u}_{k,\max}^u)^* \mathbf{y} \\ &= (\mathbf{u}_{k,\max}^u)^* \left( \sum_{i=1}^k d_{i,\max}^u \sqrt{p_i} \mathbf{u}_{i,\max}^u x_i + \boldsymbol{\xi}^u + \mathbf{n} \right) \\ &= d_{k,\max}^u \sqrt{p_k} x_k + \sum_{i=1}^{k-1} d_{i,\max}^u \sqrt{p_i} (\mathbf{u}_{k,\max}^u)^* \mathbf{u}_{i,\max}^u x_i \\ &\quad + \sum_{i=1}^{N_c K} \sqrt{p_i} (\mathbf{u}_{k,\max}^u)^* \overline{\mathbf{H}}_i^u \mathbf{v}_{i,\max}^u x_i + (\mathbf{u}_{k,\max}^u)^* \mathbf{n}. \quad (19) \end{aligned}$$

The four terms in (19) are, respectively, the signal from user  $k$  intending to be decoded currently, the signals that interfere with user  $k$  but will be decoded and cancelled via SIC later, the interference signals that are not suppressed by SIC, and additive noise. From (19), the SNR value for user  $k$

is calculated as

$$\begin{aligned} SNR_k &= \frac{(d_{k,\max}^u)^2 p_k}{\sum_{i=1}^{k-1} (d_{i,\max}^u)^2 p_i \phi_{k,i}^u + \sum_{i=1}^{N_c K} \overline{(d_{i,\max}^u)^2 p_i \phi_{k,i}^u} + N_0} \\ &= \frac{(d_{k,\max}^u)^2 p_k}{\sum_{i=1}^{k-1} (d_{i,\max}^u)^2 p_i \phi_{k,i}^u + I_k + N_0} \end{aligned} \quad (20a)$$

where

$$\phi_{k,i}^u = |(\mathbf{u}_{k,\max}^u)^* \mathbf{u}_{i,\max}^u|^2, \quad (20b)$$

$$\overline{d_{i,\max}^u} = \|\overline{\mathbf{H}_i^u} \mathbf{v}_{i,\max}^u\|_2, \quad (20c)$$

$$\overline{\phi_{k,i}^u} = |(\mathbf{u}_{k,\max}^u)^* \overline{\mathbf{H}_i^u} \mathbf{v}_{i,\max}^u|^2 / (\overline{d_{i,\max}^u})^2, \quad (20d)$$

and

$$I_k = \mathbb{E}(|(\mathbf{u}_{k,\max}^u)^* \boldsymbol{\xi}|^2) = \sum_{i=1}^{N_c K} \overline{(d_{i,\max}^u)^2 p_i \phi_{k,i}^u}. \quad (20e)$$

### C. PSU with Finite $K$

So far, we are unable to obtain a closed-form characterization for the PSU performance with a finite  $K$ . A brute-force numerical evaluation is also unrealistic due to the excessive computational cost involved when  $N_c$  tends to infinity. In the following, we use an example to roughly illustrate the PSU performance with  $B = 1$  and different finite values of  $K$ . We assume that the channel conditions of users in different cells are symmetric to each other in each channel realization, i.e., the users in one cell experience exactly the same channel conditions to the BSs around as those experienced by the users in any other cells. Based on this assumption, all cells have exactly the same power allocation in each channel realization and the complexity of calculating  $\{p_k\}$  is greatly reduced. Provided that  $\{I_k\}$  are known, the power allocation  $\{p_k\}$  can be obtained recursively from (20a) and Shannon capacity formula in a similar form as given in (8). However,  $\{I_k\}$  are in turn determined by  $\{p_k\}$ . We therefore use the following algorithm to verify the existence of feasible  $\{p_k\}$  for a given user density  $K$  and sum rate  $R$  bits/symbol/cell.

#### Algorithm 1:

- Step 1. Ignore the additive noise  $\mathbf{n}$  and normalize the contribution of all users to  $\boldsymbol{\xi}^u$  by assuming  $I_k = 1, \forall k$ ;
- Step 2. Calculate  $\{p_k\}$  from (20a) and Shannon capacity formula recursively;
- Step 3. Calculate  $\{I_k\}$  from (20e);
- Step 4. Claim a successful transmission if all  $\{I_k\}$  calculated in step 3 are smaller than 1, and claim a system transmission failure otherwise.

Figure 4 shows the system transmission failure probabilities of a cellular system with finite  $K$  based on Algorithm 1. We set  $M = N = 1$  and  $B = 1$ . Other conditions are the same as those used in Figs. 1 and 2. With  $B = 1$ , the system effectively reduces to an NBSC one, and it further reduces to a conventional TDMA system when  $K = 1$ . From Fig.

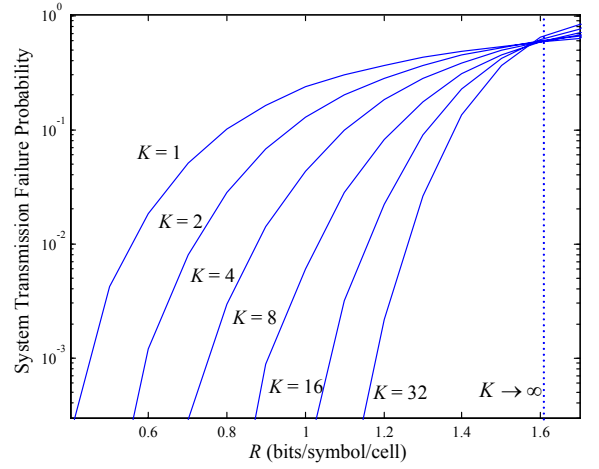


Fig. 4. System transmission failure probability versus average sum rate per cell in NBSC cellular systems with  $M = N = 1$ . The values of simultaneous user density  $K$  are marked on the curves.

4 we can see that, though the single-user rate is decreased, allowing more users to transmit simultaneously can greatly increase the system throughput per cell. For example, when the system transmission failure probability is  $10^{-3}$ , the system throughput achieved by  $K = 32$  is about 1.2 bits/symbol/cell, which is about three times of that achieved by  $K = 1$  (i.e., 0.45 bit/symbol/cell). The dashed line in Fig. 4 is for the limiting case of  $K \rightarrow \infty$ , which will be discussed in detail below. Again, the curves for finite  $K$  in Fig. 4 are only rough illustrations due to the symmetric assumption above. Nevertheless, we can still see the trend of performance improvement when  $K$  increases, indicating that multi-user concurrent transmission is advantageous.

### D. Asymptotic Performance of APSU

From definition, the BS antenna selection criterion for APSU is independent and identical for all users. Recalling the i.u.d. assumption for all users in the system, we can conclude that all  $\{\mathbf{H}_k^u\}$  are i.i.d. and all entries of each  $\mathbf{H}_k^u$  are identically distributed, so are  $\{\overline{\mathbf{H}_k^u}\}$  and their entries. Then we have the following.

#### Lemma 3:

$$\begin{aligned} \mathbb{E}(\phi_{k,i}^u | d_{k,\max}^u, \overline{d_{i,\max}^u}) &= \mathbb{E}(\overline{\phi_{k,i}^u} | d_{k,\max}^u, \overline{d_{i,\max}^u}) \\ &= \frac{1}{N_c M}, \forall k \neq i. \end{aligned} \quad (21)$$

The proof of Lemma 3 is similar to that of Lemma 1 and omitted here for brevity. On the basis of Lemma 3, we can obtain the corollary below. Its proof is given in Appendix IV.

**Corollary 3:** When  $K \rightarrow \infty$ , all  $\{I_k\}$  defined in (20e) take the same value with probability 1, i.e.,

$$\lim_{K \rightarrow \infty} I_k = \lim_{K \rightarrow \infty} \sum_{i=1}^{N_c K} \frac{\overline{(d_{i,\max}^u)^2 p_i}}{N_c M} \triangleq I_0, \forall k. \quad (22)$$

Corollary 3 greatly simplifies the discussion below. In this case, (20a) has a similar form as (5). Then if the term  $I_k + N_0$

(or equivalently  $I_0 + N_0$ ) in (20a) is known, we can obtain

$$p_k = \frac{(I_0 + N_0)(2^{R/K} - 1) \left(1 + (2^{R/K} - 1)/N_c M\right)^{k-1}}{(d_{k,\max}^u)^2}. \quad (23)$$

Note that we have assumed  $K \rightarrow \infty$  and  $N_c \rightarrow \infty$ . Hence (23) holds for almost all  $k$  even without taking average over  $\{\phi_{k,i}^u\}$ .

We now reuse  $F(\cdot)$  and  $f(\cdot)$  to denote the CDF and PDF of  $\{(d_{k,\max}^u)^2\}$ . (Note that all  $\{H_k^u\}$  are i.i.d., so are  $\{(d_{k,\max}^u)^2\}$ .) From Lemma 3 and following a similar process as the one for deriving (10) and (13), we obtain the asymptotic average MTSP per cell of MEB in an APSU cellular system (denoted by  $P^{APSU}(R)$ ) as

$$P^{APSU}(R) = (I_0 + N_0) \int_{G_0}^{\infty} \frac{R \ln 2 \cdot 2^{R(F(g)-\varepsilon)/M}}{g} f(g) dg. \quad (24)$$

Next we consider the calculation of  $I_0$ . Substituting (23) into (22), we have

$$I_0 = \lim_{K \rightarrow \infty} \sum_{k=1}^{N_c K} \frac{(I_0 + N_0)(2^{\frac{R}{K}} - 1) \left(1 + \frac{2^{\frac{R}{K}} - 1}{N_c M}\right)^{k-1}}{(d_{k,\max}^u)^2 N_c M}. \quad (25)$$

For convenience, we define

$$\Psi(x) \triangleq \mathbb{E} \left( \frac{1}{(d_{k,\max}^u)^2} \mid (d_{k,\max}^u)^2 = x \right), \quad (26)$$

which is a deterministic function of  $d_{k,\max}^u$  and can be obtained via the Monte Carlo method. Similar to the treatment in (23), when  $K \rightarrow \infty$ , we replace the term  $(d_{k,\max}^u)^2$  in (25) by its conditional mean given  $d_{k,\max}^u$ , i.e.,  $\Psi((d_{k,\max}^u)^2)$ , and obtain

$$\begin{aligned} I_0 &= \lim_{K \rightarrow \infty} \sum_{k=1}^{N_c K} \Psi((d_{k,\max}^u)^2) \\ &\quad \cdot \frac{(I_0 + N_0)(2^{\frac{R}{K}} - 1) \left(1 + \frac{2^{\frac{R}{K}} - 1}{N_c M}\right)^{k-1}}{(d_{k,\max}^u)^2 N_c M} \\ &= \lim_{K \rightarrow \infty} \frac{I_0 + N_0}{M} \int_{G_0}^{\infty} \Psi(g) K(2^{\frac{R}{K}} - 1) \\ &\quad \cdot \frac{\left(1 + (2^{\frac{R}{K}} - 1)(F(g) - \varepsilon)/N_c M\right)^{N_c K - 1}}{g} f(g) dg \\ &= \frac{I_0 + N_0}{M} \int_{G_0}^{\infty} R \ln 2 \cdot 2^{R(F(g)-\varepsilon)/M} \frac{\Psi(g)}{g} f(g) dg \quad (27) \end{aligned}$$

or equivalently

$$I_0 = \frac{N_0 \int_{G_0}^{\infty} R \ln 2 \cdot 2^{R(F(g)-\varepsilon)/M} \frac{\Psi(g)}{g} f(g) dg}{M - \int_{G_0}^{\infty} R \ln 2 \cdot 2^{R(F(g)-\varepsilon)/M} \frac{\Psi(g)}{g} f(g) dg}. \quad (28)$$

The right-hand side of (28) can be either positive or negative. When it is positive, we can substitute (28) into (24) to obtain the asymptotical average MTSP per cell of MEB, otherwise the related sum rate  $R$  cannot be supported by MEB. The critical event occurs when the denominator of (28) equals zero. The corresponding value of  $R$  (denoted by  $R_{\max}$ ) then serves

as a lower bound for the cellular capacity with APSU. Hence we have the following.

*Theorem 3:* The asymptotic capacity of an APSU cellular system with  $K \rightarrow \infty$  is lower-bounded by the solution of  $R_{\max}$  in the following equation.

$$\int_{G_0}^{\infty} R_{\max} \ln 2 \cdot 2^{R_{\max}(F(g)-\varepsilon)/M} \frac{\Psi(g)}{g} f(g) dg = M. \quad (29)$$

For an achievable throughput below  $R_{\max}$ , the asymptotic average MTSP per cell of MEB is given by (24) and (28).

Similar to the asymptotic optimality of MEB in Section III, we conjecture that the lower bound above achieved by MEB is asymptotically tight when  $K \rightarrow \infty$ . However, we have no rigorous proof so far.

An interesting conclusion from Theorem 3 is that the number of BS antennas  $M$  plays an important role in the performance of APSU cellular systems with MEB. To see this, let us re-write (29) as

$$\Omega(R_{\max}) = 1 \quad (30a)$$

where

$$\Omega(R) = \int_{\varepsilon}^1 \frac{R}{M} \ln 2 \cdot 2^{\frac{R}{M}(t-\varepsilon)} \frac{\Psi(F^{-1}(t))}{F^{-1}(t)} dt \quad (30b)$$

and  $F^{-1}(\cdot)$  is the inverse of  $F(\cdot)$ . Note that both  $\Psi(F^{-1}(t))$  and  $F^{-1}(t)$  are implicit functions of  $M$ . We observed numerically that the ratio  $\Psi(F^{-1}(t))/F^{-1}(t)$  remains almost unchanged with  $M$ . Thus we expect that the ratio between the solution of  $R_{\max}$  in (30a) and  $M$  remains approximately a constant for different  $M$ , indicating that the maximum achievable throughput of MEB in APSU cellular systems increases almost linearly with  $M$ .

### E. Asymptotic Performance of FPSU

With FPSU, all cells in the system are grouped into fixed and non-overlapped clusters. Every cluster can be regarded as a virtual cell with  $BM$  distributed BS antennas. However, these  $BM$  BS antennas are in general asymmetric to each other in terms of received signal power as well as interference power (with the exception of  $B = 3$ ). Hence Lemma 3 does not hold, which makes the problem complicated. Let us consider Fig. 3(b) with  $B = 7$  for example. Compared with the BSs in six outer-side cells of each cluster, the central-cell BS statistically receives higher signal power and lower interference power from intra- and inter-cluster users, respectively. In what follows, we derive a lower bound for the capacity achieved by FPSU with MEB. When decoding the signals of each user in its own cluster, we will deal with intra- and inter-cluster interference separately as follows. For the intra-cluster interference, we define

$$\rho(g) = \max_{\tilde{g} > g} \mathbb{E} \left( \phi_{k,i}^u \mid (d_{k,\max}^u)^2 = \tilde{g}, (d_{i,\max}^u)^2 = g \right) \quad (31)$$

where users  $k$  and  $i$  are assumed to belong to the same cluster. The deterministic function  $\rho(\cdot)$  can be obtained via the Monte

$$\begin{aligned}
E(p_k) &= \frac{(2^{R/K} - 1)}{(d_{k,\max}^u)^2} \left( \sum_{i=(c-1)BK+1}^{(c-1)BK+k-1} (d_{i,\max}^u)^2 E(p_i) E(\phi_{k,i}^u | d_{k,\max}^u, d_{i,\max}^u) + I_k + N_0 \right) \\
&\leq \frac{(2^{R/K} - 1)}{(d_{k,\max}^u)^2} \left( \sum_{i=(c-1)BK+1}^{(c-1)BK+k-1} (d_{i,\max}^u)^2 E(p_i) \rho((d_{i,\max}^u)^2) + I_k + N_0 \right) \\
&\leq \frac{(2^{R/K} - 1)}{(d_{k,\max}^u)^2} \left( \sum_{i=(c-1)BK+1}^{(c-1)BK+k-1} (d_{i,\max}^u)^2 E(p_i) \rho((d_{i,\max}^u)^2) + \Sigma_{\max} + N_0 \right). \tag{35}
\end{aligned}$$

Carlo method. Clearly, we have

$$\begin{aligned}
E(\phi_{k,i}^u | (d_{k,\max}^u)^2, (d_{i,\max}^u)^2) &\leq \rho((d_{i,\max}^u)^2), \\
&\forall d_{k,\max}^u > d_{i,\max}^u. \tag{32}
\end{aligned}$$

For the inter-cluster interference, we model  $\xi^u$  in (18b) as a random vector with joint Gaussian distribution when  $K \rightarrow \infty$ . Note that for each user, the fading coefficients for all antenna links are complex Gaussian random variables with independent and uniform phase distribution. Hence the phases of all entries in  $\xi^u$  are also i.u.d.. This indicates that the mean and variance of  $\xi^u$  are, respectively, a zero vector and a diagonal matrix when  $K \rightarrow \infty$ . Therefore we have

$$E(\xi^u (\xi^u)^*) = \text{diag}(\Sigma_1, \Sigma_2, \dots, \Sigma_{N_c M}) \tag{33}$$

where  $\text{diag}(\cdot)$  denotes a diagonal matrix and  $\Sigma_i$  the inter-cluster interference power received at the  $i$ -th BS antenna. In general,  $\{\Sigma_i\}$  are not identical to each other. When  $K \rightarrow \infty$ , we have (recall (20e))

$$I_k = E(|(\mathbf{u}_{k,\max}^u)^* \xi^u|^2) \leq \max_i \Sigma_i \triangleq \Sigma_{\max}. \tag{34}$$

Without loss of generality, we assume that the users belonging to cluster  $c$  (here  $c = 1, 2, \dots$ ) are indexed as  $\{k, k = (c-1)BK + 1, (c-1)BK + 2, \dots, cBK\}$  with  $d_{(c-1)BK+1,\max}^u \leq d_{(c-1)BK+2,\max}^u \leq \dots \leq d_{cBK,\max}^u$ . When focusing on user  $k$  in cluster  $c$ , we fix  $\{d_{i,\max}^u\}$  and take average over  $\{\phi_{k,i}^u\}$ . Then (20a) can be rewritten as equation (35) shown at the top of this page, where the first and second inequalities are based on (32) and (34), respectively.

After a similar procedure as those for (10) and (24), we can obtain an upper bound from (35) for the asymptotic average MTSP per cell of the FPSU strategy (denoted by  $P^{FPSU}(R)$ ) as

$$\begin{aligned}
P^{FPSU}(R) &\leq (\Sigma_{\max} + N_0) \\
&\cdot \int_{G_0}^{\infty} \frac{R \ln 2 \cdot 2^{R \cdot \int_{G_0}^g \rho(g') f(g') dg'}{g} f(g) dg. \tag{36}
\end{aligned}$$

Note that the value of  $\Sigma_{\max}$  depends on the power allocation  $\{p_k\}$  in other clusters. Following a similar procedure as that for the computation of  $I_0$  in (28), we obtain

$$\Sigma_{\max} = \frac{N_0 \int_{G_0}^{\infty} R \ln 2 \cdot 2^{R \cdot \int_{G_0}^g \rho(g') f(g') dg'} \frac{\Psi'(g)}{g} f(g) dg}{M - \int_{G_0}^{\infty} R \ln 2 \cdot 2^{R \cdot \int_{G_0}^g \rho(g') f(g') dg'} \frac{\Psi'(g)}{g} f(g) dg} \tag{37a}$$

where

$$\Psi'(x) \triangleq B \cdot E \left( \sum_{c>1} \|\mathbf{H}_{k,(c-1)B+l^*} \mathbf{v}_{k,\max}^u\|_2^2 \middle| (d_{k,\max}^u)^2 = x \right). \tag{37b}$$

The deterministic function  $\Psi'(\cdot)$  in (37b) can be obtained via the Monte Carlo method and is explained as follows. We call a cell in one cluster an outermost cell if it is located the farthest away from the geometrical center of this cluster. Statistically, a BS antenna in an outermost cell receives the largest inter-cluster interference power  $\Sigma_{\max}$ . Now consider a user  $k$  in cluster 1 and let  $l^*$  be the index of one outermost cell in this cluster. Then  $\Psi'(\cdot)$  represents the average total channel gain from user  $k$  to the BSs  $\{(c-1)B+l^*\}$  in other clusters  $\{c, c > 1\}$ . When  $K \rightarrow \infty$  and by symmetry, there exists a mirroring user of user  $k$  in any cluster  $c > 1$  with probability one (i.e., these users have exactly the same channel condition to the BSs around as user  $k$ ). Hence (37b) also equals to the average sum channel gain from all these mirroring users to the BS of cell  $l^*$  in cluster 1. From the above discussion, we obtain the theorem below.

*Theorem 4:* The asymptotic capacity of an FPSU cellular system with  $K \rightarrow \infty$  is lower-bounded by the solution of  $R_{\max}$  in the following equation.

$$\int_{G_0}^{\infty} R_{\max} \ln 2 \cdot 2^{R_{\max} \cdot \int_{G_0}^g \rho(g') f(g') dg'} \frac{\Psi'(g)}{g} f(g) dg = M. \tag{38}$$

For a throughput below  $R_{\max}$ , the asymptotic average MTSP per cell of MEB is upper-bounded by (36) and (37).

## F. Numerical Results

Figure 5 shows the asymptotic average MTSP of some MEB-based NBSC systems with different  $M$  and  $N$  when  $K \rightarrow \infty$ . The signal of each user is decoded only in the cell with the best channel condition to this user. The corresponding maximum throughputs  $R_{\max}$  (computed using Theorem 3) are also plotted. We fix  $N = 1$  and  $M = 1$  in Figs. 5(a) and 5(b), respectively. From Fig. 5(a) we can see that the maximum achievable throughput  $R_{\max}$  can be approximately expressed as a linear function of  $M$ , i.e.,

$$R_{\max} \approx 1.62M \text{ bits/symbol/cell}. \tag{39}$$

This is consistent with the observation made from (30). The key limitation on throughput is the term  $2^{R(t-\varepsilon)/M}$  inside the



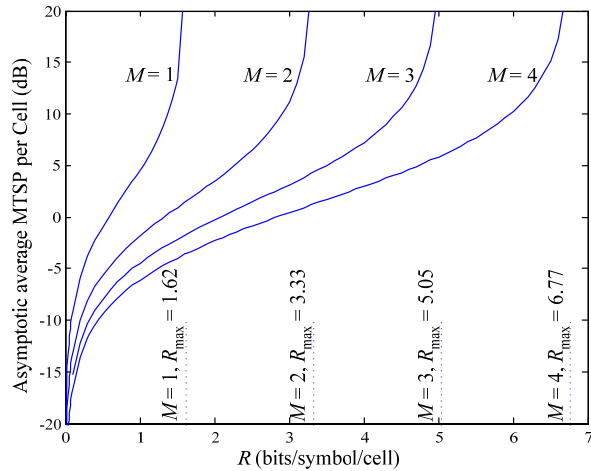
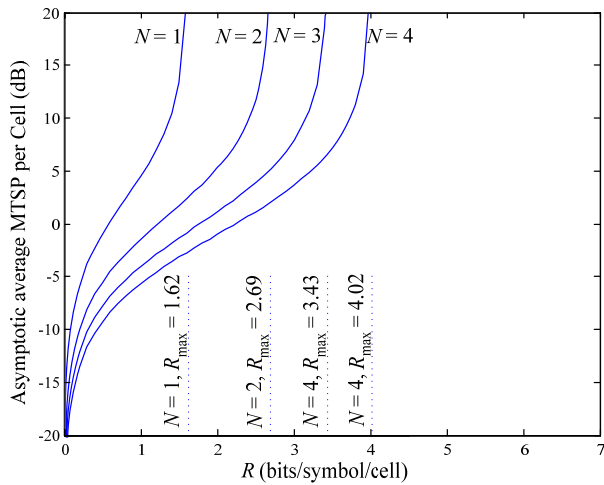
(a)  $N = 1$ (b)  $M = 1$ 

Fig. 5. The asymptotic minimum transmission power (solid curves) and maximum achievable throughput  $R_{\max}$  (dashed lines) of MEB in various NBSC cellular systems with different  $M$  and  $N$  when  $K \rightarrow \infty$ .

function  $\Omega(R)$  in (30b). This term increases rapidly when  $R$  increases, which causes severe interference to other cells and finally imposes a limit on throughput. Also note that the constant 1.62 is determined by fading and path loss, which may take different values under different channel assumptions.

As a comparison, it is seen from Fig. 5(b) that the gain obtained by increasing  $N$  is, although less impressive than that of increasing  $M$ , still very significant. For example, the throughput gain by increasing  $N$  from 1 to 2 is about 1 bit/symbol/cell. The gain achieved is decreasing with further increasing  $N$ . In summary, increasing  $M$  is more efficient to improve cellular performance than increasing  $N$ . However, when both  $M$  and  $N$  are small, significant performance gain is still achievable by increasing  $N$  only.

Figure 6 compares the asymptotic performance of cellular systems with various BS cooperation strategies when  $K \rightarrow \infty$ . The BS (or cluster) selection for each user  $k$  in APSU (or FPSU) is based on channel gains  $\{\|\mathbf{H}_{k,n}\|_2^2, n = 1, 2, \dots, N_c\}$ . For FPSU with  $B = 3$ , all BS antennas in each cluster are symmetrically located in terms of both wanted and

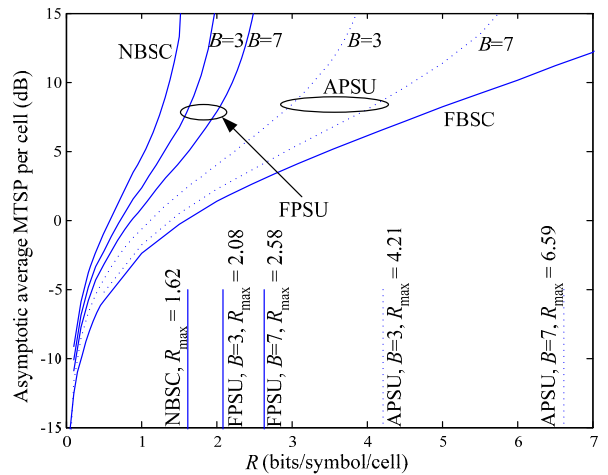


Fig. 6. Performance comparison among NBSC, APSU, FPSU and FBSC in cellular systems with  $M = N = 1$ . The values of  $B$  are marked on the curves. The corresponding capacity lower bounds are also plotted for reference.

TABLE I  
VALUES OF  $E((\overline{d_{k,\max}^u}^2)/(d_{k,\max}^u)^2)$  IN DIFFERENT PSU STRATEGIES.

$E((\overline{d_{k,\max}^u}^2)/(d_{k,\max}^u)^2)$	$B = 3$	$B = 7$
APSU	0.1561	0.0525
FPSU	0.4641	0.3106

unwanted power received, so we can still use Theorem 3 to evaluate the system performance. The curve for FPSU with  $B = 7$  is obtained based on Theorem 4. We have used the algorithm 1 in Section IV-C with  $K = 10^4$  and obtained a result very close to the curve of FPSU with  $B = 7$  in Fig. 6, indicating that the lower bound provided by Theorem 4 is tight in this case. We can see from Fig. 6 that FPSU is considerably inferior to APSU for the same  $B$  in terms of both achievable throughput and minimum transmission power. An intuitive explanation for this is as follows. The PSU-based cellular system performance is mainly determined by the ratio between the received signal and interference power when the noise power is neglectable. With PSU, the received signals of each user are either processed for signal detection or treated as additive noise. The system performance is then dominated by the average ratio between the channel gains of unprocessed and processed signals from each user, i.e.,  $E((\overline{d_{k,\max}^u}^2)/(d_{k,\max}^u)^2)$ . Table I lists the values of this ratio in different PSU strategies. It is seen that, given the same  $B$ , the ratio for FPSU is significantly larger than that for APSU, indicating that the former suffers more from interference than the latter. This is caused by the fact that all cells are grouped into fixed clusters for FPSU and we cannot select BSs as flexible as that for APSU.

A key observation from Fig. 6 is that BS cooperation can provide significant performance gain over the conventional NBSC. In particular, the MEB scheme with APSU and a large  $B$  can achieve performance similar to that of the optimal FBSC strategy. Note that with APSU, global cooperation among all BSs is required in general. However, as demonstrated in [23], it can be implemented in a distributed way.

## V. CONCLUSIONS

In this paper, we have considered a realistic MIMO cellular channel model and derived upper and lower bounds with closed-form expressions for the minimum transmission power and cellular capacity of various BS cooperation strategies. It is shown that allowing more users to transmit simultaneously, introducing cooperation among BSs and increasing the number of antennas (especially at each BS) are efficient ways to improve the cellular performance. In particular, the APSU strategy with MEB is a promising approach to the tradeoff between the performance and implementation complexity. Our focus in this paper is on the joint interference cancelation via BS cooperation. Further performance improvement is possible by adopting other advanced techniques such as relaying and network coding [26]. The related capacity analysis is an interesting topic under investigation.

### APPENDIX A PROOF OF LEMMA 1

Since all entries of each  $\mathbf{H}_k$  are identically distributed, so are those of each unit vector  $\mathbf{u}_{k,\max}$ . Denote by  $u_{k,m}$  the  $m$ -th entry of  $\mathbf{u}_{k,\max}$ . For each user  $k$  with channel gain  $d_{k,\max}^2$  we have

$$\mathbb{E}(|u_{k,i}|^2 | d_{k,\max}^2) = \frac{1}{N_c M}, \forall i = 1, 2, \dots, N_c M. \quad (40)$$

In addition, from the i.i.d. property of all  $\{\mathbf{H}_k\}$ , all  $\{\mathbf{u}_{k,\max}\}$  are i.i.d. as well. By definition, we have

$$\begin{aligned} & \mathbb{E}(\phi_{k,i} | d_{k,\max}^2, d_{i,\max}^2) \\ &= \mathbb{E} \left( \left( \sum_{m=1}^{N_c M} u_{k,m}^* u_{i,m} \right) \left( \sum_{m'=1}^{N_c M} u_{k,m'}^* u_{i,m'} \right)^* \middle| d_{k,\max}^2, d_{i,\max}^2 \right) \\ &= \sum_{m=1}^{N_c M} \mathbb{E}(|u_{k,m}|^2 | d_{k,\max}^2) \mathbb{E}(|u_{i,m}|^2 | d_{i,\max}^2) \\ & \quad + \sum_{m \neq m'} \mathbb{E}(u_{k,m}^* u_{k,m'} | d_{k,\max}^2) \mathbb{E}(u_{i,m}^* u_{i,m'} | d_{i,\max}^2). \quad (41) \end{aligned}$$

In (41), the second term is zero since the phases of  $\{u_{k,m}\}$  are i.i.d. (which follows the i.i.d. property of the phases of all entries in  $\{\mathbf{H}_k\}$ ). Substituting this and (40) into (41), we have

$$\mathbb{E}(\phi_{k,i} | d_{k,\max}^2, d_{i,\max}^2) = \sum_{m=1}^{N_c M} \frac{1}{N_c M} \cdot \frac{1}{N_c M} + 0 = \frac{1}{N_c M}. \quad (42)$$

### APPENDIX B PROOF OF THEOREM 1

Let us focus on a user  $k$  with channel gain  $d_{k,\max}^2 = g$ . Our derivation includes the following steps.

*First*, we fix  $\{d_{i,\max}^2, \forall i \neq k\}$  (and  $\{I(k,i)\}$  as well) and take averages on both sides of (8) over  $\{\phi_{k,i}\}$ . From lemma

1, we have

$$\mathbb{E}_{\{\phi_{k,i}\}}(p_k) = \frac{2^{R/K} - 1}{g} \left( \sum_{i=1}^{N_c K} \frac{\mathbb{E}_{\{\phi_{k,i}\}}(p_i) d_{i,\max}^2 I(k,i)}{N_c M} + N_0 \right) \quad (43)$$

or in a non-recursive form

$$\mathbb{E}_{\{\phi_{k,i}\}}(p_k) = \frac{N_0(2^{R/K} - 1)}{g} \prod_{i=1}^{N_c K} \left( 1 + \frac{(2^{R/K} - 1)I(k,i)}{N_c M} \right). \quad (44)$$

*Second*, we take averages on both sides of (44) over  $\{I(k,i)\}$ , i.e., the decoding order. For user  $k$  with channel gain  $g$ , the probability that user  $i$ 's channel gain is smaller than  $g$  is simply given by  $F(g)$ . This implies  $\mathbb{E}(I(k,i)) = F(g) - \varepsilon$  where the term  $\varepsilon$  is related to the situation when user  $i$  is in outage. Then from (44) we have

$$\begin{aligned} & \mathbb{E}_{\{\phi_{k,i}\}, \{I(k,i)\}}(p_k) \\ &= \frac{N_0(2^{R/K} - 1)}{g} \prod_{i=1, i \neq k}^{N_c K} \left( 1 + \frac{(2^{R/K} - 1)(F(g) - \varepsilon)}{N_c M} \right) \\ &= \frac{N_0(2^{R/K} - 1)}{g} \left( 1 + \frac{(2^{R/K} - 1)(F(g) - \varepsilon)}{N_c M} \right)^{N_c K - 1}. \quad (45) \end{aligned}$$

Let  $N_c \rightarrow \infty$  and define a deterministic function of  $g$  as

$$\begin{aligned} p(g) &\triangleq \lim_{N_c \rightarrow \infty} \mathbb{E}_{\{\phi_{k,i}\}, \{I(k,i)\}}(p_k) \\ &= \frac{N_0(2^{R/K} - 1) e^{K(2^{R/K} - 1)(F(g) - \varepsilon)/M}}{g}. \quad (46) \end{aligned}$$

Clearly,  $p(g)$  is the average transmitted power for a user experiencing channel gain  $g$ .

*Finally*, the average of  $p(g)$  with respect to channel gain  $g$  is calculated as

$$\begin{aligned} \mathbb{E}(p(g)) &= \int_{G_0}^{\infty} p(g) f(g) dg \\ &= N_0 \int_{G_0}^{\infty} \frac{(2^{\frac{R}{K}} - 1) e^{K(2^{\frac{R}{K}} - 1)(F(g) - \varepsilon)/M}}{g} f(g) dg. \quad (47) \end{aligned}$$

From the assumptions for  $\{\mathbf{H}_k\}$  in Section II, all users have the same fading distribution  $f(\cdot)$ . Hence the long-term average MTSP per cell is simply  $K$  times of that of a single user, i.e.,

$$\begin{aligned} P^{MEB}(R, K) &= K \mathbb{E}(p(g)) \\ &= N_0 \int_{G_0}^{\infty} \frac{K(2^{\frac{R}{K}} - 1) e^{K(2^{\frac{R}{K}} - 1)(F(g) - \varepsilon)/M}}{g} f(g) dg. \quad (48) \end{aligned}$$

### APPENDIX C PROOF OF LEMMA 2

We define a new system below based on (1).

$$\tilde{\mathbf{y}} = \sum_{k=1}^{N_c K} d_{k,\max} \mathbf{I}_{N_c M} \tilde{\mathbf{x}}_k + \mathbf{n} \quad (49)$$

where  $\mathbf{I}_{N_c M}$  denotes an identity matrix with dimension  $N_c M$ . Suppose that in each channel realization,  $\{\mathbf{x}_k\}$  achieve the

MTSP of the system in (1) that supports the rate constraint  $R/K$  for each user. Then for the system in (49), we can design the transmitted signal of user  $k$  as

$$\tilde{\mathbf{x}}_k = d_{k,\max}^{-1} \mathbf{H}_k \mathbf{x}_k. \quad (50)$$

It is obvious that  $\tilde{\mathbf{x}}_k$  has no larger power than  $\mathbf{x}_k$  because

$$\|\tilde{\mathbf{x}}_k\|_2^2 = \|d_{k,\max}^{-1} \mathbf{H}_k \mathbf{x}_k\|_2^2 \leq d_{k,\max}^{-2} \|\mathbf{H}_k\|_2^2 \|\mathbf{x}_k\|_2^2 = \|\mathbf{x}_k\|_2^2. \quad (51)$$

Substituting (50) into (49), we can obtain

$$\begin{aligned} \tilde{\mathbf{y}} &= \sum_{k=1}^{N_c K} d_{k,\max} \mathbf{I}_{N_c M \times N_c M} \cdot d_{k,\max}^{-1} \mathbf{H}_k \mathbf{x}_k + \mathbf{n} \\ &= \sum_{k=1}^{N_c K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n} = \mathbf{y}. \end{aligned} \quad (52)$$

This indicates that the same rate constraint  $R/K$  can be supported in the system (49) with no larger transmitted sum power than that required in the system (1). Hence the average MTSP per cell of the optimal scheme for the system in (49) (denoted by  $P^{LB}(R)$ ) serves as a lower bound for that of the system in (1). In addition, the system in (49) can be viewed as a parallel of  $N_c M$  identical single-input single-output sub-systems, each with  $N_c K$  users and rate constraint  $R/N_c M K$  (the sum rate of each sub-system is then  $R/M$ ). The optimal average MTSP for each of such sub-systems is given by (see [20] for details)

$$\begin{aligned} P(R/M, N_c K) &= N_0 \int_{G_0}^{\infty} \frac{N_c K (2^{\frac{R}{M N_c K}} - 1)}{g} \\ &\cdot \left(1 + (2^{\frac{R}{M N_c K}} - 1)(F(g) - \varepsilon)\right)^{N_c K - 1} \cdot f(g) dg. \end{aligned} \quad (53)$$

Hence the average MTSP per cell of the system in (49) when  $N_c \rightarrow \infty$  is

$$\begin{aligned} P^{LB}(R) &= \lim_{N_c \rightarrow \infty} \frac{N_c M}{N_c} P(R/M, N_c K) \\ &= N_0 \int_{G_0}^{\infty} \frac{R \ln 2 \cdot 2^{R(F(g) - \varepsilon)/M}}{g} f(g) dg. \end{aligned} \quad (54)$$

#### APPENDIX D PROOF OF COROLLARY 3

For each user  $k$ , from Lemma 3 we have

$$E(I_k) = \sum_{i=1}^{N_c K} (\overline{d_{i,\max}^u})^2 p_i E(\overline{\phi_{k,i}^u}) = \sum_{i=1}^{N_c K} \frac{(\overline{d_{i,\max}^u})^2 p_i}{N_c M} = I_0. \quad (55)$$

On the other hand, the variance of each  $I_k$  can be written as

$$\begin{aligned} D(I_k) &= \sum_{i=1}^{N_c K} ((\overline{d_{i,\max}^u})^2 p_i)^2 \cdot D(\overline{\phi_{k,i}^u}) \\ &\leq I_0^2 \frac{\sum_{i=1}^{N_c K} ((\overline{d_{i,\max}^u})^2 p_i)^2}{\left(\sum_{i=1}^{N_c K} (\overline{d_{i,\max}^u})^2 p_i / N_c M\right)^2} \\ &\leq I_0^2 (N_c M)^2 \cdot \frac{\max_i (\overline{d_{i,\max}^u})^2 p_i \cdot \sum_{i=1}^{N_c K} (\overline{d_{i,\max}^u})^2 p_i}{\left(\sum_{i=1}^{N_c K} (\overline{d_{i,\max}^u})^2 p_i\right)^2} \\ &= I_0^2 (N_c M)^2 \cdot \frac{\max_i (\overline{d_{i,\max}^u})^2 p_i}{\sum_{i=1}^{N_c K} (\overline{d_{i,\max}^u})^2 p_i} \end{aligned} \quad (56)$$

where the first inequality holds because we always have  $D(I_k) \leq 1$  by definition.

For any given finite  $R$ , we have  $R/K \rightarrow 0$  when  $K \rightarrow \infty$ . This in turn leads to  $p_k \rightarrow 0$  for all  $k$ . Hence we have

$$\lim_{K \rightarrow \infty} \frac{\max_i (\overline{d_{i,\max}^u})^2 p_i}{\sum_{i=1}^{N_c K} (\overline{d_{i,\max}^u})^2 p_i} = 0 \quad (57)$$

and (56) can be rewritten as

$$\lim_{K \rightarrow \infty} D(I_k) \leq I_0^2 (N_c M)^2 \cdot 0 = 0. \quad (58)$$

Combining (55) and (58), we can conclude that all  $\{I_k\}$  converge to  $I_0$  with probability 1 when  $K \rightarrow \infty$ . Hence (22) holds, which concludes the proof.

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